International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

THE NEW INTEGRAL TRANSFORM "KSP TRANSFORM"

S.K.S. Pawar[1], M.R. Pawde[2], A.B. Jadhav[3]

, [1]Department of Applied Science & Humanities, MGM's College of Engineering Nanded-431605. India.

[2]Department of Applied Science & Humanities, MGM's College of Engineering Nanded-431605.[2]

[3]Department of Mathematics, D. S. M. College Parbhani-431401[3]

Abstract

The Integral transform method is one of the most powerful tools to find solutions of many problems in Science and Engineering. The integral transforms are also useful in the evaluation of certain integrals and to find the solution of certain ordinary differential equations, partial differential equations and integral equations. In this paper, a new integral transform namely KAMAL-SUDHA PAWAR is used to find the solution of ordinary differential equations, partial differential equations.

Keywords: KSP-transform, ordinary differential equations, partial differential equations.

1.Introduction:

The researcher has expressed the interest in using mathematical models to describe the many natural phenomena in our daily lives that are related to Engineering, Science, Astronomy, Medical Science as seen in references [1-6]. The mathematical models are solved by using various numerical methods such as integral transform method like Laplace transform. Integral transform method is commonly used to solve ordinary differential equations and partial differential equations with initial value and boundary conditions[7-11].. The Integral transform method is one of the most powerful tools to find solutions of many problems in Science and Engineering [12-15]. Laplace P.S. [16]. in 1780 defined for the function

$f(t),\ t \geq 0$ as $L\{f(t)\} = \overline{F(s)} = \int_0^{\infty} e^{-st} f(t)\, dt$, Where $s \in \mathbb{C}$, (1.1)

The Fourier transform in 1822 defined as

$$F\{f(t)\} = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ist} f(t)\, dt \qquad (1.2)$$

Now a days many research working on new transforms like Elzaki, Sumudu, Natural, Atangana Kilicman,Shuehu transform [17,38]. etc are useful to solve to find the solution of ordinary differential equations, partial differential equations also KSP-transform is one of the effective tool solve ordinary differential equations, partial differential equations. The Shehu transform which is defined for all f(z) ∈ Θ such that

$\Theta = \left\{ f(z) : \exists M, u_1, u_2 > 0, |f(z)| < M e^{\frac{|z|}{u_i}}, if\ z \in (-1)^i \times [0, +\infty) \right\}$, and $S\{f(z)\}$ is given by

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$S\{f(z)\} = f(s,u) = \int_0^{+\infty} e^{\frac{-sz}{u}} f(z)\, dz \quad (1.3)$$

Where s, u are two variables and z is the frequency variable.

Recently Mohamed Elarbi Benattia et al. define new transform for all f(z) ∈ Θ such that

$$\Theta = \left\{ \mathrm{f(z)} : \exists \mathrm{M}, k_{1,} k_2 > 0, |f(z)| > Me^{\frac{|z|}{k_i}}, if\ z \in (-1)^{i-1} \times (-\infty, 0], i = 1,2 \right\},$$

Where a real constant M must be a finite number, while $k_{1,} k_2$ may be finite or infinite.

Integral transform defined as

$$\xi\{f(z)\}(s,u) = F(s,u) = \frac{1}{u^2} \int_{-\infty}^{0} exp(\frac{sz}{u}) f(z)\, dz \quad z < 0, \frac{1}{k_{1,}} \le u \le \frac{1}{k_2} \quad (1.4)$$

Provided that the limit of integral must be exists.

2.Main results

2.1Definition-1(KSP Integral Transform)

The new KAMAL-SUDHA PAWAR integral transform of the function $f(t) \in \mathcal{A}$ such that

$$\mathcal{A} = \left\{ f(t) : \exists M, u_{1,} u_2 > 0, |f(t)| > Me^{\frac{|z|}{u_i}}, if\ t \in (-1)^{i-1} \times (-\infty, 0], i = 1,2 \right\}, \quad (2.1)$$

Where a real constant M must be a finite number, while $k_{1,} k_2$ may be finite or infinite.

Integral transform defined as

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt \ , \quad t \le 0,\ \frac{1}{u_{1,}} \le v \le \frac{1}{u_2} \quad (2.2)$$

Provided that the limit of integral must be exists.

The inverse of KSP-transform (2.1) is defined as $\mathcal{KS}^{-1}\{F(s,v)\} = f(t)$

Theorem 2.2: Existence of the KSP-transform

Let $v \neq 0$ be fixed, if a function $f(t): (-\infty, 0] \to \mathbb{R}$ is a piecewise continuous and is of exponential

order β , i.e. there exist the constant $M > 0$, and $\beta \in \mathbb{R}$,such that $|f(t)| \le Me^{-\beta t}$ for $t \le 0$.Then the

KSP-transform of f(t) exists for $v \neq 0$, and $Re\left(\frac{s}{v}\right) > \beta$.

Proof: Consider the function f(t) for $t \le 0$ then

$$|\mathcal{KS}\{f(t)\}| = F(s,v) = \left| \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt \right|, t \le 0,$$

$$|\mathcal{KS}\{f(t)\}| = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} |f(t)|\, dt \quad , t \le 0$$

$$= M\int_{-\infty}^{0} e^{\frac{st}{v}}\, e^{-\beta t} dt$$

$$= M\int_{-\infty}^{0} e^{\left(\frac{s-\beta v}{v}\right)t}\, dt$$

$$\mathcal{KS}\{f(t)\} = \frac{Mv}{(s-\beta v)}, \text{ for } v \neq 0 \text{ , and } Re\left(\frac{s}{v}\right) > \beta.$$

Theorem 2.3: KSP-transform of some functions

1. $\mathcal{KS}\{1\} = \frac{v}{s}$, $\frac{s}{v} > 0$

2. $\mathcal{KS}\{t\} = -\frac{v^2}{s^2}$, $\frac{s}{v} > 0$

3. $\mathcal{KS}\{t^n\} = (-1)^m \frac{m!v^{m+1}}{s^{m+1}}$, $\frac{s}{v} > 0$

4. $\mathcal{KS}\{e^{at}\} = \frac{v}{s+av}$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt \ , \quad t \leq 0$$

1. $\mathcal{KS}\{1\} = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}}\, dt = \frac{v}{s}$, $\frac{s}{v} > 0$

2. $\mathcal{KS}\{t\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}}\, t\, dt = -\frac{v^2}{s^2}$, $\frac{s}{v} > 0$

3. $\mathcal{KS}\{t^n\} = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}}\, t^n dt \quad = (-1)^m \frac{m!v^{m+1}}{s^{m+1}}$, $\frac{s}{v} > 0$

4. $\mathcal{KS}\{e^{at}\} = \int_{-\infty}^{0} e^{\frac{st}{v}} e^{at}\, dt \ = \int_{-\infty}^{0} e^{\left(\frac{s}{v}+a\right)t}\, dt \ = \frac{v}{s+av}$

Theorem 2.4: KSP-transform of $sin(at)$ and $cos(at)$ are defined by

1. $\mathcal{KS}\{sin(at)\} = \frac{-av^2}{s^2+a^2v^2}$

2. $\mathcal{KS}\{cos(at)\} = \frac{sv}{s^2+a^2v^2}$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt \ , \quad t \leq 0$$

1. $\mathcal{KS}\{sin(at)\} = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} sin(at)\, dt$

$$= \left[\frac{e^{\frac{st}{v}}}{\left(\frac{s}{v}\right)^2 + a^2}\left(\frac{s}{v} sin(at) - acos(at)\right)\right]_{-\infty}^{0}$$

$$\mathcal{KS}\{sin(at)\} = \frac{-av^2}{s^2+a^2v^2}$$

2. $\mathcal{KS}\{cos(at)\} = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} cos(at)\, dt$

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$= \left[\frac{e^{\frac{st}{v}}}{\left(\frac{s}{v}\right)^2 + a^2} \left(\frac{s}{v} cos(at) + asin(at) \right) \right]_{-\infty}^{0}$$

$$\mathcal{KS}\{cos(at)\} = \frac{sv}{s^2 + a^2v^2}$$

Theorem 2.5: KSP-transform of the hyperbolic functions

$sinh(at)$ and $cosh(at)$ are defined by

1. $\mathcal{KS}\{\ sinh(at)\ \} = \frac{-av^2}{s^2 - a^2v^2}$

2. $\mathcal{KS}\{\ cosh(at)\} = \frac{sv}{s^2 - a^2v^2}$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt \ , \quad t \le 0$$

1. $\mathcal{KS}\{sinh(at)\} = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} sinh(at)\, dt$

$$\mathcal{KS}\{sinh(at)\} = \frac{-av^2}{s^2 - a^2v^2}$$

2. $\mathcal{KS}\{cosh(at)\} = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} cosh(at)\, dt$

$$\mathcal{KS}\{cos(at)\} = \frac{sv}{s^2 - a^2v^2}$$

Theorem 2.6: Relation between KSP-transform and Laplace transform is

$\mathcal{KS}\{f(t)\}(s,v) = vL\{f(-vt)\}$, where L is the Laplace transform.

By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt \ , \ \ t \le 0 \qquad (2.3)$$

Put $\frac{t}{v} = -u \Rightarrow dt = -vdu \quad t \le 0$ then $u \ge 0$ in the equation (2.3)

$$\therefore\ \mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{\infty}^{0} e^{-su} f(-vu)\, [-vdu] \ ,$$

$$\therefore\ \mathcal{KS}\{f(t)\}(s,v) = F(s,v) = vL\{f(-vt)\} \qquad (2.4)$$

Theorem 2.7: **(Linearity Property)** Let $f(t)$ and $g(t)$ are from the set $\mathcal{A}$(2.1) then $af(t) + dg(t) \in \mathcal{A}$, where a ,$d \in \mathbb{R}$ then

$\mathcal{KS}\{af(t) + dg(t)\}(s,v) = a\mathcal{KS}\{f(t)\} + d\mathcal{KS}\{g(t)\}$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{af(t) + dg(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} \{af(t) + dg(t)\}(s,v) dt \ , \quad t \le 0$$

$$\therefore\ \mathcal{KS}\{af(t) + dg(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} \{af(t)\} dt + \int_{-\infty}^{0} e^{\frac{st}{v}} \{dg(t)\} dt$$

$\mathcal{KS}\{af(t) + dg(t)\}(s,v) = a\mathcal{KS}\{f(t)\} + d\mathcal{KS}\{g(t)\}$

Theorem 2.8: (**Change of Scale Property)** Let $f(t)$ is from the set $\mathcal{A}(2.1)$ and

$\mathcal{KS}\{f(t)\}(s,v) = F(s,v)$ then $\mathcal{KS}\{f(at)\}(s,v) = \frac{1}{a}F(\frac{s}{a},v)$ where $a \in \mathbb{R}$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt\ , \quad t \le 0$$

$$\therefore\ \mathcal{KS}\{f(at)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} \{f(at)\}\, dt \tag{2.5}$$

Put $at = u \Rightarrow adt = du \ \Rightarrow dt = \frac{1}{a}du$ in the equation (2.5)

$$\therefore\ \mathcal{KS}\{f(at)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{su}{av}} \{f(u)\} \frac{1}{a} du$$

$\therefore \mathcal{KS}\{f(at)\}(s,v) = \frac{1}{a}F(\frac{s}{a},v)$, where $a \in \mathbb{R}$

Theorem 2.8: (**Shifting Property)** Let $f(t)$ is from the set $\mathcal{A}(2.1)$ and

$\mathcal{KS}\{f(t)\}(s,v) = F(s,v)$ then $\mathcal{KS}\{e^{at}f(t)\}(s,v) = F(s+av,v)$ where $a \in \mathbb{R}$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt\ , \quad t \le 0$$

$$\therefore\ \mathcal{KS}\{e^{at}f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} \{e^{at}f(t)\}\, dt \tag{2.6}$$

$$\therefore\ \mathcal{KS}\{e^{at}f(t)\}(s,v) = \int_{-\infty}^{0} e^{\left(\frac{s+av}{v}\right)t}\ dt$$

$\therefore\ \mathcal{KS}\{e^{at}f(t)\}(s,v) = F(s+av,v)$, where $a \in \mathbb{R}$

Definition-2 (Convolution)

Let $f(t)$ and $g(t)$ be the piecewise continuous functions on $(-\infty, 0]$ of exponential order, then the convolution of the functions is denoted by $f * g$ and it is defined as

$[f * g](t) = \int_{0}^{t} f(u)g(t-u)\, du$, where $t < 0$

Theorem 2.9: (**Convolution)**

Let $f(t)$ and $g(t)$ be the piecewise continuous functions on $(-\infty, 0]$ of exponential order, then

$$\mathcal{KS}\{f(t) * g(t)\}(s,v) = -\{F(s,v) * G(s,v)\}$$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt\ , \quad t \le 0$$

$$\mathcal{KS}\{f(t) * g(t)\}(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} \left\{\int_{0}^{t} f(u)g(t-u)\, du\right\} dt \quad \text{, where } t < 0. \tag{2.7}$$

$$\mathcal{KS}\{f(t) * g(t)\}(s,v) = -\int_{-\infty}^{0} e^{\frac{st}{v}} \left\{\int_{t}^{0} f(u)g(t-u)\, du\right\} dt \quad \text{, where } t < 0.$$

By changing the order of above integral we get,

$$\mathcal{KS}\{f(t) * g(t)\}(s,v) = -\{F(s,v) * G(s,v)\} \tag{2.8}$$

Theorem 2.10: KSP-transform of the Integrals.

Let $f(t)$ is from the set $\mathcal{A}$(2.1) and $\mathcal{KS}\{f(t)\}(s,v) = F(s,v)$ then

$$\mathcal{KS}\left\{\int_0^t \int_0^t \int_0^t \ldots\ldots \int_0^t f(t)\,(dt)^n\right\} = (-1)^n \left(\frac{v}{s}\right)^n F(s,v)$$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\,dt \ , \quad t \leq 0$$

Here to prove the result we use induction theorem, for $n = 1$ we get,

$$\mathcal{KS}\left\{\int_0^t f(t)\,dt\right\} = \int_{-\infty}^{0} e^{\frac{st}{v}} \left\{\int_0^t f(t)\,dt\right\} dt$$

$$\mathcal{KS}\left\{\int_0^t f(t)\,dt\right\} = (-1)\left(\frac{v}{s}\right) F(s,v) \tag{2.9}$$

Hence the result is hold for $n = 1$.

Now consider the result is true for $n = k$, therefore we get,

$$\mathcal{KS}\left\{\int_0^t \int_0^t \int_0^t \ldots\ldots \int_0^t f(t)\,(dt)^k\right\} = (-1)^k \left(\frac{v}{s}\right)^k F(s,v) \tag{2.10}$$

Now we prove the result for $n = k + 1$

$$\mathcal{KS}\left\{\int_0^t \int_0^t \int_0^t \ldots\ldots \int_0^t f(t)\,(dt)^{k+1}\right\} = \int_{-\infty}^{0} e^{\frac{st}{v}} \left\{\int_0^t \int_0^t \int_0^t \int_0^t \ldots\ldots \int_0^t f(t)\,(dt)^{k+1}\right\} dt$$

Apply Integration by parts we get,

$$\mathcal{KS}\left\{\int_0^t \int_0^t \int_0^t \ldots\ldots \int_0^t f(t)\,(dt)^{k+1}\right\} = (-1)\left(\frac{v}{s}\right)(-1)^k \left(\frac{v}{s}\right)^k F(s,v) = (-1)^{k+1}\left(\frac{v}{s}\right)^{k+1} F(s,v)$$

Therefore $\mathcal{KS}\left\{\int_0^t \int_0^t \int_0^t \ldots\ldots \int_0^t f(t)\,(dt)^n\right\} = (-1)^n \left(\frac{v}{s}\right)^n F(s,v)$

Theorem 2.10: KSP-transform of the derivatives.

Let $f(t)$ is from the set $\mathcal{A}$(2.1) and $\mathcal{KS}\{f(t)\}(s,v) = F(s,v)$ then

$\mathcal{KS}\{f^{(n)}(t)\} = (-1)^n \left(\frac{s}{v}\right)^n F(s,v) + (-1)^{n-1} \sum_{j=0}^{n-1} (-1)^j \left(\frac{s}{v}\right)^{n-(j+1)} f^{(j)}(0)$, where $n \geq 1$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\,dt \ , \quad t \leq 0$$

Here to prove the result we use induction theorem, for $n = 1$ we get,

$$\mathcal{KS}\{f'(t)\} = \int_{-\infty}^{0} e^{\frac{st}{v}} f'(t)dt$$

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Apply Integration by parts we get,

$$\mathcal{KS}\{f'(t)\} = -\left(\frac{s}{v}\right)F(s,v) + f(0) \tag{2.11}$$

Let consider the result is true for $n = k$, therefore we get,

$$\mathcal{KS}\{f^{(k)}(t)\} = (-1)^k\left(\frac{s}{v}\right)^k F(s,v) + (-1)^{k-1}\sum_{j=0}^{k-1}(-1)^j\left(\frac{s}{v}\right)^{k-(j+1)} f^{(j)}(0)$$
(2.12)

Now we prove the result is true for $n = k+1$,

$$\mathcal{KS}\{f^{(k+1)}(t)\} = \mathcal{KS}\left\{\left[f^{(k)}(t)\right]'\right\}$$

$$= -\frac{s}{v}\left\{(-1)^k\left(\frac{s}{v}\right)^k F(s,v) + (-1)^{k-1}\sum_{j=0}^{k-1}(-1)^j\left(\frac{s}{v}\right)^{k-(j+1)} f^{(j)}(0)\right\} + f^{(k)}(0)$$

$$\mathcal{KS}\{f^{(k+1)}(t)\} = (-1)^{k+1}\left(\frac{s}{v}\right)^{k+1} F(s,v) + (-1)^k\sum_{j=0}^{k}(-1)^j\left(\frac{s}{v}\right)^{k-j} f^{(j)}(0)$$

Therefore, the result is hold for $n = k+1$.

Therefore $\mathcal{KS}\{f^{(n)}(t)\} = (-1)^n\left(\frac{s}{v}\right)^n F(s,v) + (-1)^{n-1}\sum_{j=0}^{n-1}(-1)^j\left(\frac{s}{v}\right)^{n-(j+1)} f^{(j)}(0)$,

where $n \geq 1$

Theorem 2.11: KSP-transform of partial derivatives.

Let $f(t)$ is from the set $\mathcal{A}(2.1)$ and $\mathcal{KS}\{f(t)\}(s,v) = F(s,v)$ the

1) $\mathcal{KS}\left\{\frac{\partial}{\partial x}f(x,t)\right\} = \frac{d}{dx}[F(x,s,v)]$

2) $\mathcal{KS}\left\{\frac{\partial^2}{\partial x^2}f(x,t)\right\} = \frac{d^2}{dx^2}[F(x,s,v)]$

3) $\mathcal{KS}\left\{\frac{\partial^{(n)}}{\partial t^n}f(x,t)\right\} = (-1)^n\left(\frac{s}{v}\right)^n F(x,s,v) + (-1)^{n-1}\sum_{j=0}^{n-1}(-1)^j\left(\frac{s}{v}\right)^{n-(j+1)}\frac{\partial^j}{\partial t^j}f(x,0)$

Proof: By definition of KSP-transform

$$\mathcal{KS}\{f(t)\}(s,v) = F(s,v) = \int_{-\infty}^{0} e^{\frac{st}{v}} f(t)\, dt \;,\quad t \leq 0$$

1) $\mathcal{KS}\left\{\frac{\partial}{\partial x}f(x,t)\right\} = \int_{-\infty}^{0} e^{\frac{st}{v}}[\frac{\partial}{\partial x}f(x,t)]\, dt$

By Leibniz's rule we get,

$$\mathcal{KS}\left\{\frac{\partial}{\partial x}f(x,t)\right\} = \frac{d}{dx}\int_{-\infty}^{0} e^{\frac{st}{v}} f(x,t)dt \tag{2.13}$$

Therefore, $\mathcal{KS}\left\{\frac{\partial}{\partial x}f(x,t)\right\} = \frac{d}{dx}[F(x,s,v)]$

Similarly, 2) $\mathcal{KS}\left\{\frac{\partial^2}{\partial x^2}f(x,t)\right\} = \frac{d^2}{dx^2}[F(x,s,v)]$ (2.14)

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

3) Here to prove the result we use induction theorem, for $n = 1$ we get,

$$\mathcal{KS}\left\{\frac{\partial}{\partial t} f(x,t)\right\} = \int_{-\infty}^{0} e^{\frac{st}{v}} \left[\frac{\partial}{\partial t} f(x,t)\right] dt$$

Applying by integration by parts we get,

$$\mathcal{KS}\left\{\frac{\partial}{\partial t} f(x,t)\right\} = \left[e^{\frac{st}{v}} f(x,t)\right]_{-\infty}^{0} - \int_{-\infty}^{0} \frac{s}{v} e^{\frac{st}{v}} [f(x,t)]\, dt$$

$$\mathcal{KS}\left\{\frac{\partial}{\partial t} f(x,t)\right\} = f(x,0) - \frac{s}{v} F(x,s,v) = -\frac{s}{v} F(x,s,v) + f(x,0)$$
(2.15)

Hence result is true for $n = 1$

Let consider the result is true for $n = k$, therefore we get,

$$\mathcal{KS}\left\{\frac{\partial^{(k)}}{\partial t^k} f(x,t)\right\} = (-1)^k \left(\frac{s}{v}\right)^k F(x,s,v) + (-1)^{k-1} \sum_{j=0}^{k-1} (-1)^j \left(\frac{s}{v}\right)^{k-(j+1)} \frac{\partial^j}{\partial t^j} f(x,0)$$
(2.16)

Now we prove the result is true for $n = k + 1$,

$$\mathcal{KS}\left\{\frac{\partial^{(k+1)}}{\partial t^{k+1}} f(x,t)\right\} = \int_{-\infty}^{0} e^{\frac{st}{v}} \left[\frac{\partial^{(k+1)}}{\partial t^{k+1}} f(x,t)\right] dt$$

Applying by integration by parts we get,

$$\mathcal{KS}\left\{\frac{\partial^{(k+1)}}{\partial t^{k+1}} f(x,t)\right\} = \left[e^{\frac{st}{v}} \left[\frac{\partial^{(k)}}{\partial t^k} f(x,t)\right]\right]_{-\infty}^{0} - \int_{-\infty}^{0} \frac{s}{v} e^{\frac{st}{v}} \left[\frac{\partial^{(k)}}{\partial t^k} f(x,t)\right] dt$$

$$\mathcal{KS}\left\{\frac{\partial^{(k+1)}}{\partial t^{k+1}} f(x,t)\right\} = \left[\frac{\partial^{(k)}}{\partial t^k} f(x,0)\right] - \frac{s}{v} \int_{-\infty}^{0} e^{\frac{st}{v}} \left[\frac{\partial^{(k)}}{\partial t^k} f(x,t)\right] dt$$

$$\mathcal{KS}\left\{\frac{\partial^{(k+1)}}{\partial t^{k+1}} f(x,t)\right\} = \left[\frac{\partial^{(k)}}{\partial t^k} f(x,0)\right] - \frac{s}{v} \mathcal{KS}\left\{\frac{\partial^{(k)}}{\partial t^k} f(x,t)\right\}$$

$$= \left[\frac{\partial^{(k)}}{\partial t^k} f(x,0)\right] - \frac{s}{v}\left\{(-1)^k \left(\frac{s}{v}\right)^k F(x,s,v) + (-1)^{k-1} \sum_{j=0}^{k-1} (-1)^j \left(\frac{s}{v}\right)^{k-(j+1)} \frac{\partial^j}{\partial t^j} f(x,0)\right\}$$

$$= \left[\frac{\partial^{(k)}}{\partial t^k} f(x,0)\right] + (-1)^{k+1} \left(\frac{s}{v}\right)^{k+1} F(x,s,v) + (-1)^k \sum_{j=0}^{k-1} (-1)^j \left(\frac{s}{v}\right)^{k-j} \frac{\partial^j}{\partial t^j} f(x,0)$$

Therefore, the result is hold for $n = k + 1$.

Hence, $$\mathcal{KS}\left\{\frac{\partial^{(n)}}{\partial t^n} f(x,t)\right\} = (-1)^n \left(\frac{s}{v}\right)^n F(x,s,v) + (-1)^{n-1} \sum_{j=0}^{n-1} (-1)^j \left(\frac{s}{v}\right)^{n-(j+1)} \frac{\partial^j}{\partial t^j} f(x,0)$$

3.Application of KSP-transform:

In this part of the paper KSP-transform is applied to linear ODEs with constant coefficients and initial value problems. Ordinary differential equations are fundamental in describing

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

various physical, engineering, and biological processes. Traditional methods for solving ODEs, such as separation of variables or integrating factors, can be difficult for complex equations. The new transform offers a robust alternative, particularly for linear ODEs with constant coefficients and initial value problems. This transform converts differential equations into algebraic equations, making them easier to solve.

Example 1: The differential equation is $f'(t) + f(t) = 0$ (3.1)

with initial condition $f(0) = 1$

Applying KSP-transform both side of equation (3.1)

$$\mathcal{KS}\{f'(t) + f(t)\} = 0$$

$$\mathcal{KS}\{f'(t)\} + \mathcal{KS}\{f(t)\} = 0$$

$$-\left(\frac{s}{v}\right) F(s,v) + f(0) + F(s,v) = 0$$

Using given initial condition $f(0) = 1$ we get,

$$-\left(\frac{s}{v}\right) F(s,v) + 1 + F(s,v) = 0$$

$$\mathcal{KS}\{F(s,v)\} = \left\{\frac{v}{s-v}\right\}$$

(3.2)

Apply inverse KSP-transform both side of equation (3.2)

$$\mathcal{KS}^{-1}\{F(s,v)\} = \mathcal{KS}^{-1}\left\{\frac{v}{s-v}\right\}$$

Therefore $f(t) = e^{-t}$

This is the solution of differential equation (3.1).

Example 2: The differential equation is $f'(t) + 2f(t) = t$ (3.3)

with initial condition $f(0) = 1$

Applying KSP-transform both side of equation (3.1)

$$\mathcal{KS}\{f'(t) + 2f(t)\} = \mathcal{KS}\{t\}$$

$$\mathcal{KS}\{f'(t)\} + 2\mathcal{KS}\{f(t)\} = \mathcal{KS}\{t\}$$

$$-\left(\frac{s}{v}\right) F(s,v) + f(0) + 2F(s,v) = -\frac{v^2}{s^2}$$

Using given initial condition $f(0) = 1$ we get,

$$-\left(\frac{s}{v}\right) F(s,v) + 1 + 2F(s,v) = -\frac{v^2}{s^2}$$

$$F(s,v) = \frac{v^3}{s^2(s-2v)} + \frac{v}{(s-2v)}$$

(3.4)

Apply inverse KSP-transform both side of equation (3.4)

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$\mathcal{KS}^{-1}\{F(s,v)\} = \mathcal{KS}^{-1}\left\{\frac{v^3}{s^2(s-2v)}\right\} + \mathcal{KS}^{-1}\left\{\frac{v}{(s-2v)}\right\}$$

Therefore $f(t) = \frac{5}{4}e^{-2t} + \frac{1}{2}t - \frac{1}{4}$

This is the solution of differential equation (3.3).

Example 3: The differential equation is $y''(t) + 2y'(t) + y(t) = e^{-t}$ (3.5)

with initial condition $y(0) = 0$, $y'(0) = 2$

Applying KSP-transform both side of equation (3.5)

$$\mathcal{KS}\{\, y''(t) + 2y'(t) + y(t)\} = \mathcal{KS}\{e^{-t}\}$$

$$\mathcal{KS}\{y''(t)\} + 2\mathcal{KS}\{y'(t)\} + \mathcal{KS}\{y(t)\} = \frac{v}{(s-v)}$$

$$\left[\left(\frac{s}{v}\right)^2 Y(s,v) - \left(\frac{s}{v}\right)y(0) + \; y'(0)\right] + 2\left[-\left(\frac{s}{v}\right)Y(s,v) + y(0)\right] + Y(s,v) = \frac{v}{(s-v)}$$

Using given initial condition $y(0) = 0$, $y'(0) = 2$ we get,

$$Y(s,v) = \frac{v^3}{(s-v)^3} - \frac{2v^2}{(s-v)^2} \tag{3.6}$$

Apply inverse KSP-transform both side of equation (3.6)

$$\mathcal{KS}^{-1}\{Y(s,v)\} = \mathcal{KS}^{-1}\left\{\frac{v^3}{(s-v)^3}\right\} - \mathcal{KS}^{-1}\left\{\frac{2v^2}{(s-v)^2}\right\}$$

$$y(t) = \frac{1}{2}e^{-t}t^2 + 2e^{-t}t = e^{-t}\left(\frac{1}{2}t^2 + 2t\right)$$

This is the solution of differential equation (3.5).

Example 4: The standard heat diffusion equation $\frac{\partial^2 u(x,t)}{\partial x^2} + \frac{\partial u(x,t)}{\partial t} = 0\,, x\epsilon\mathbb{R}\;, t > 0$ (3.7)

with $u(x,t)$ as a field variable with boundary conditions $u(x,t) = 0\;\; at\;\; x = 0$ and $u(x,t) = 0\;\; at\; x = 1$ when $t \to \infty$, $u(x,t) = 3e^{-x}$ at $t = 0$.

Applying KSP-transform both side of equation (3.7)

$$\mathcal{KS}\left\{\frac{\partial^2 u(x,t)}{\partial x^2} + \frac{\partial u(x,t)}{\partial t}\right\} = 0$$

$$\mathcal{KS}\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\} + \mathcal{KS}\left\{\frac{\partial u(x,t)}{\partial t}\right\} = 0$$

$$\frac{d^2U}{dx^2} + \left[-\frac{s}{v}U(x,s,v) + u(x,0)\right] = 0$$

$$\frac{d^2U}{dx^2} - \frac{s}{v}U = -3e^{-x}$$

(3.8)

The general solution of (3.8) is $U(x,s,v) = U_H(x,s,v) + U_P(x,s,v)$

Where $u_H(x,s,v)$ is the solution of homogeneous equation and $u_P(x,s,v)$ is the particular solution.

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$U_H(x,s,v) = c_1 e^{\left(\sqrt{\frac{s}{v}}\right)x} + c_2 e^{-\left(\sqrt{\frac{s}{v}}\right)x}$$

(3.9)

Use the boundary conditions $u(x,t) = 0 \;\; at \;\; x = 0$ and $u(x,t) = 0 \;\; at \; x = 1$ in equation (3.9)

We get $c_1 = 0$ and $c_2 = 0$ therefore $u_H(x,s,v) = 0$

Now $U_P(x,s,v) = \frac{-3v}{v-s} e^{-x}$

The general solution of (3.8) is $U(x,s,v) = \frac{-3v}{v-s} e^{-x}$

(3.10)

Apply inverse KSP-transform both side of equation (3.10)

$$\mathcal{KS}^{-1}\{U(x,s,v)\} = \mathcal{KS}^{-1}\left\{\frac{-3v}{v-s} e^{-x}\right\}$$

$u(x,t) = 3e^{-t-x}$ is the solution of partial differential equation (3.7).

4.Results and discussion:

The new integral transform helps us to solve ordinary differential equations, integral equations and integro-differential equations defined for , $t \leq 0$, which the classical Laplace transform cannot directly handle.

Many real-world problems like pre-history systems, backward time models, pre-stressed systems, involving , $t \leq 0$. This new transform is defined on the interval $(-\infty, 0]$, which is its main advantage, as it allows direct solving of ordinary differential equations on negative side by converting ODE into algebraic equations.

The new integral transform also used to solve higher order linear differential equations without traditional methods, such as separation of variables or integrating factors, can be difficult for complex equations. The new transform offers a robust alternative, particularly for linear ODEs with constant coefficients and initial value problems.

It naturally handles boundary conditions at $t = 0^-$. Since the transform uses $y(0)$ and its derivatives $f^{(n)}(0)$ initial conditions at $t = 0^-$ are directly included, without needing to change the time axis.

5. Conclusion:

In this paper the authors investigate a new transform designed to handle ordinary and partial differential equations. They analyze its fundamental properties and demonstrate its applications to key problem classes. The primary contributions of the new transform are as follows:

1) Generalization of the Laplace transform: The proposed transform extends the classical Laplace method to the domain $(-\infty, 0]$, enable the treatment of a wider range of ordinary differential equations, integral equations and integro-differential equations that are not easily solvable by standard techniques.

2) Preservation of core properties: The Linearity, shifting, scale change, differentiation, integral and convolution properties all are maintained with the transform naturally incorporating initial conditions at $t = 0^-$ and operating on negative time.

3) Simplification of the negative half-axis: By converting differential equations on the domain $(-\infty, 0]$ into algebraic equations in the s-domain, the method provides direct and systematic solution procedures.

4) Practical advantages: The formulation is well suited for systems with time delays, backward-time dynamics and boundary conditions at $t = 0^-$ cases where classical Laplace techniques lack efficiency or fail to apply.

Overall, the new transform acts as an effective integral transform framework, the solutions of the problems in this transformed domain are easier to obtain and substituting back yields solutions that satisfy original problems.

This opens the door to broader and more flexible applications in the analysis of ordinary differential equations, integral equations and integro-differential equations.

References:

1. Kumar,A.,Bansal,S.,Aggarwal,S.:Anew novel ntegraL transform“Anuj transform”with application.Des.Eng.2021(9),1274112751(2021).https://doi.org/10.30511/mcs.2024.2029301.1174
2. Kumar, R., Chandel, J., Aggarwal, S.: A new integral transform “Rishi Transform” with application. J. Sci. Res. 14(2), 521–532 (2022)
3. Higazy, M., Agarwal, S., Nofal, T.A.: Sawi Decomposition Method for Volterra Integral Equation with Application. J. Math. 20(1) (2020). https://doi.org/10.1155/2020/6687134. 13 p.
4. Higazy, M., Agarwal, S., Hamed, Y.S.: Determination of Number of Infected Cells and Concentration of Viral Particles in Plasma during HIV-1 Infections Using Shehu Transformation. J. Math. 20(1) (2020). https://doi.org/10.1155/2020/ 6624794. 13 p.
5. Jafari, H., Agarwal, S.: Upadhyaya integral transform: a tool for solving non-linear Volterra integral equations.Math. Comput. Sci. 5(2), 63–71 (2024). https://doi.org/10.30511/mcs.2024.2029301.1174
6. Higazy, M., Agarwal, S.: Sawi transformation for system of ordinary differential equations with application. Ain Shams Eng. J. 12(3), 3173–3182 (2021). https://doi.org/10.1016/j.asej.2021.01.027
7. Suthar, D.L., Abeye, N., Shimelis, B., Gidaf, F.: Generalized fractional integral operators pertaining to the pRq(λ, ϑ; x) function. Res. Math. 12(1) (2025). https://doi.org/10.1080/27684830.2025.2505280
8. Kamran, Shah, F.A., Shah, K., Abdeljawad, T.: Numerical solution of two dimensional time-fractional telegraphequation using Chebyshev spectral collocation method. Partial Differ. Equ. Appl. Math. 13, 101129 (2025). https://doi. org/10.1016/j.padiff.2025.101129
9. Iqbal, S., Wang, J., Alzabut, J., Moumen, A., Bouye, M.: Semi-analytic solution of two dimensional fractal-fractional order biological population model in Caputo sense. J. Math. Comput. Sci. 40(2), 182–195 (2026)
10. Aruna, S.K., Deepa, N., Devi, T.: Residual learning for enhanced seismic signal quality with denoising CNN and Kalman filtering. In: 5th International Conference on Circuits, Control, Communication and Computing (I4C), Bangalore, India, pp. 339–343 (2024). https://doi.org/10.1109/I4C62240.2024.10748449

11. Yasir, M., Ahmed, A., Aldosari, F.M., Abdelfattah, W., Alrihieli, H.F., Elseesy, I.E.: Fractionized based rheologicalfeatures of transient motion of Walter's B fluid in a rectangular oscillatory duct with cosine and sine oscillations. J. Comput. Des. Eng. 12(4), 205–220 (2025). https://doi.org/10.1093/jcde/qwaf025
12. Weerakoon, S.: Application of Sumudu transform to partial differential equations. Int. J. Math. Educ. Sci. Technol . 25(2), 277–283 (1994)Jafari, H.A.: A new general integral transform for solving integral equations. J. Adv. Res. 32, 133–138 (2021)
13. Agarwal, S., Charma, S.D.: Sumudu transform of error function. J. Appl. Sci. Comput. 6(6), 1222–1231 (2019)Agarwal, S., Singh, G.P.: Aboodh transform of error function. Univ. Rev. 10(6), 137–150 (2019)
14. Debnath, L., Bhatta, D.: Integral Transform and Their Application, 2nd edn. Chapman and Hall/CRC (2006)
15. Bracewell, R.: The Fourier Transform and Its Applications, 3rd edn. McGraw-Hill, New York (2000)
16. Farah, R., Hamad, M.: On the use of RAHMOH integral transform for solving differential equations. Int. J. Phys. Math. 6(2), 01 (2024)
17. Khan, Z.H., Khan, W.A.: N-transform-properties and applications. NUST J. Eng. Sci. 1, 127–133 (2008)
18. Elzaki, T.M.: The new integral transform "Elzaki Transform". Glob. J. Pure Appl. Math. 7, 57–64 (2011)
19. Elzaki, T.M., Ezaki, S.M., Hilal, E.M.A.: Elzaki and Sumudu transform for solving some differential equations. Glob. J. Pure Appl. Math. 4(8), 167–173 (2012)
20. Mansoura, E.A., Mehdia, S., Kuffib, E.A.: The new integral transform and its applications. Int. J. Nonlinear Anal. Appl.12(2), 849–856 (2021)
21. Mahgoub, M.M.A.: The new integral transform: Sawi transform. Adv. Theor. Appl. Math. 14(1), 81–87 (2019)
22. Mishra, R., Agarwal, S., Chaudhary, L., Kumar, A.: Relationship between Sumudu and some efficient integral transforms. Int. J. Innov. Technol. Explor. Eng. 9(3), 153–159 (2020)
23. Mahgoub, M.M.A.: The new integral transform: Mohand transform. Adv. Theor. Appl. Math. 12(2), 113–120 (2017)
24. Saadeh, R., Qazza, A., Burqan, A.A.: New integral transform: ARA transform and its properties and applications. Symmetry 12, 925 (2020). https://doi.org/10.3390/sym12060925
25. Shehu, M., Weidong, Z.: New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations. Int. J. Anal. Appl. 17(2), 167–190 (2019)
26. Watugala, G.K.: Sumudu transform: a new integral transform to solve differential equations and controlengineering problems. Int. J. Math. Educ. Sci. Technol. 24, 35–43 (1993). https://doi.org/10.1080/0020739930240105
27. Kilicman, A., Eltayeb, H.: On new integral transform and differential equations. Math. Probl. Eng. 2010, 463579, 1–13 (2010). https://doi.org/10.1155/2010/463579
28. Patil, D.P., Khakale, S.S.: The new integral transform Soham transform. Int. J. Adv. Eng. Manag. (IJAEM) 3(10), 126–132(2021)
29. Jadhav, C.P., Dale, T.B., Chinchane, V.L., Nale, A.B., Thabet, S.T., Kedim, I., Vivas-Cortez, M.: On solutions of fractional differential equations for the mechanical oscillations by using the Laplace transform. AIMS Math. 9(11), 32629–32645(2024). https://doi.org/10.3934/math.20241562
30. Agarwal, S., Bhatnagar, K.: Dualities between Laplace Transform and Some Useful Integral Transforms. Int. J. Eng. Adv. Technol. (IJEAT) 9 (2019). https://www.ijeat.org/portfolio-item/a9433109119/
31. Jadhav, C.P., Dale, T., Chinchane, V.L.: A method to solve ordinary fractional differential equations using Elzaki and Sumudu transform. J. Fract. Calc. Nonlinear Syst. 4(1), 8–16 (2023). https://doi.org/10.48185/jfcns.v4i1.757

32. Jafari, H., Aggarwal, S., Kumar, A.: Anuj integral transform to solving Abel's integral equation of classical mechanics.Nat. Acad. Sci. Lett. (2025). https://doi.org/10.1007/s40009-024-01597-9
33. Aggarwal, S., Bhatnagar, K.: Dualities between Laplace transform and some useful integral transforms. Int. J. Eng. Adv. Technol. (IJEAT) 9(1), 936–942 (2019). https://doi.org/10.35940/ijeat.A9433.109119
34. Upadhyaya, L.M.: Introducing the upadhyaya integral transform. Bull. Pure Appl. Sci., Math. Stat. 38e(1), 471 (2019). https://doi.org/10.5958/2320-3226.2019.00051.1
35. Higazy, M., Agarwal, S.: Sawi transformation for system of ordinary differential equations with application. Ain Shams J. Eng. Phys. Math. (2021). https://doi.org/10.1016/j.asej.2021.01.027.
36. Pawar. S.K.S., Jadhav. A.B., Pawde. M.R..: El-Zaki Transform Method for Solving Convolution Type Integral Equations of Second Kind . IJRAR 13(1), 989-996 (2026).