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A POSSIBILITY-LOGIC VIEW OF PRESUPPOSITION TRIGGERS IN BREAKING-NEWS TICKERS

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Abstract

This paper develops an interpretable, real-time method for screening conventional inferences in short, fast-moving headline streams. We model background knowledge as a graded context π\piπ and evaluate the backgrounded content of conventional items with two complementary tests: **feasibility** (Π) and **robustness** (N). Source reports are discounted by reliability and combined with ordered weighted averaging; editorial caution is represented by hedge operators that reshape π\piπ. Trigger behaviour is encoded by functors for four core families iteratives (*again*), continuity items (*still/yet/already*), additives (*too/also*), and factive predicates tied to timelines and focus alternatives. Licensed assertions update the context via min-narrowing with optional normalization. We analyse algorithmic efficiency using piecewise-linear representations and prove safety properties (idempotence, monotonicity, crisp reduction). A small, realistic case study (20 items) illustrates end-to-end computation of Π and N, calibration diagnostics, and policy trade-offs: hedging predictably trades recall for safety, while threshold scaling yields a tuneable acceptance frontier. The approach supports concise, faithful rationales by exposing the exact overlap regions and dominant sources that drive decisions. We discuss limitations (threshold sensitivity, approximation scope, cross-lingual variation) and outline extensions to multilingual streams and multimodal cues. Overall, the framework offers a compact, auditable gate for fast editorial workflows that improves caution without sacrificing operational speed.

Keywords: fuzzy semantics; feasibility–robustness screening; source reliability weighting; linguistic hedges; additive and factive items; dynamic context update; ordered weighted averaging; explainable editorial decisions

1. Introduction

Breaking-news tickers compress complex, evolving events into short utterances such as "Minister resigns again," "Suspect still at large," or "Earthquake confirmed, officials say." These forms routinely contain presupposition triggers-e.g., again, still, stop, start, too/also, already, factive verbs-whose interpretation depends on background assumptions that may be only partially supported by rapidly changing evidence. Standard probability-based treatments presuppose numeric priors and additivity, which are often unavailable or unwarranted in early reporting. We therefore model ticker presuppositions with possibility theory [1], which explicitly represents incomplete, conflicting, or source-weighted information.

1.1 Core objects

Let W be a set of epistemically accessible worlds (scenarios) consistent with current newsroom knowledge at time t. A possibility distribution $\pi_t: W \to [0,1]$ ranks worlds by their plausibility given all sources so far. For any proposition $A \subseteq W$ with membership function $\mu_A: W \to [0,1]$, the possibility and necessity of A at time t are

$$\Pi_t(A) = \sup_{w \in W} \min\{\pi_t(w), \mu_A(w)\}, N_t(A) = 1 - \Pi_t(\neg A),$$

with $\neg A$ interpreted by the standard fuzzy complement $\mu_{\neg A}(w) = 1 - \mu_A(w)[1]$.

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A ticker utterance u is represented as

$$u = \langle A(u), \mathrm{Pres}(u), T(u), \mathrm{Mods}(u), \mathrm{Src}(u) \rangle,$$

where $A(u)$ encodes the asserted content, $\mathrm{Pres}(u)$ the presupposed content induced by the trigger $T(u)$, $\mathrm{Mods}(u)$ hedges/modality (e.g., reportedly), and $\mathrm{Src}(u)$ the source metadata.

1.2 Presupposition acceptance under uncertainty

We separate two rationality tests for $\mathrm{Pres}(u)$:

- Feasibility test (is $\mathrm{Pres}(u)$ at least possible enough?):

$$\Pi_t(\mathrm{Pres}(u)) \geq \theta_T$$

- Robustness test (is $\mathrm{Pres}(u)$ protected against contradiction?):

$$N_t(\mathrm{Pres}(u)) = 1 - \Pi_t(\neg \mathrm{Pres}(u)) \geq \tau_T.$$

Here $\theta_T, \tau_T \in [0,1]$ are trigger-specific thresholds reflecting conventional strength (e.g., again typically requires stronger background than also) and editorial policy. When both tests pass, the presupposition is licensed and the context can safely incorporate the assertion; otherwise the ticker should be flagged for revision or softened with hedges.

1.3 Update policy (preview)

If $\mathrm{Pres}(u)$ is licensed, we update the context by intersective narrowing:

$$\pi_{t+1}(w) = \min\{\pi_t(w), \mu_{A(u)}(w)\}$$

and otherwise the assertion is withheld or reformulated. This mirrors context-set intersection in classical presupposition theory [2-5] while remaining graded and source-aware.

1.4 Why possibility logic for tickers?

(i) Sparse early evidence (few numeric frequencies) is naturally encoded by π without requiring probabilities [1,7]. (ii) Conflict and non-monotonic growth from parallel sources can be aggregated with max-min calculus (Section 2.2). (iii) Gradience in triggers (still vs. already vs. again) is captured by fuzzy membership of the presupposed set.

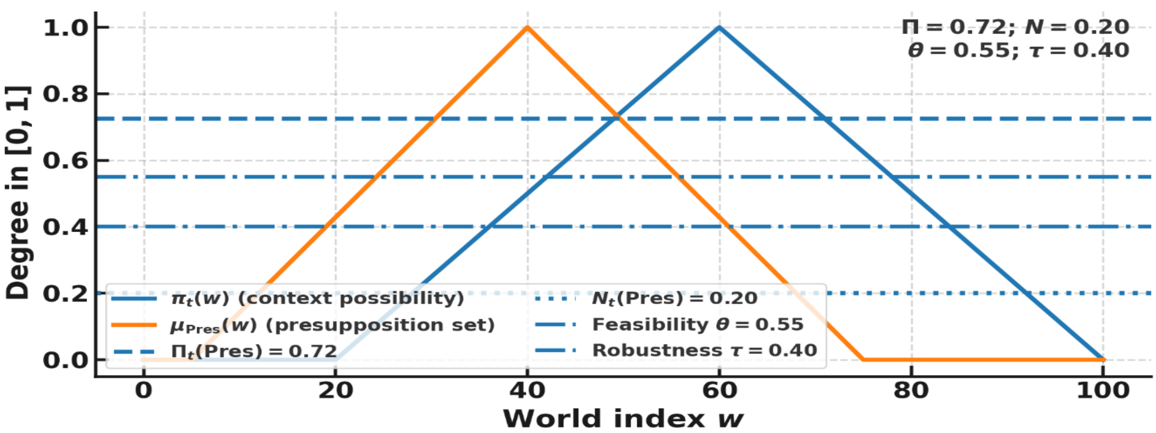

Figure 1. Possibilistic acceptance test for a ticker presupposition.

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The plot above from figure 1 shows a toy $\pi_t(w)$ over worlds w and the presupposition set $\mu_{\text{Pres}}(w)$ (e.g., for again). Horizontal lines show $\Pi_t(\text{ Pres })$ and $N_t(\text{ Pres })$ alongside policy thresholds θ, τ. Acceptance holds iff both $\Pi \geq \theta$ and $N \geq \tau$.

2. Background and Related Work

2.1 Presupposition theory for compressed discourse

Presuppositions are background commitments conventionally triggered by lexical or constructional items (e.g., again presupposes a prior occurrence; still presupposes persistence) and are required to be satisfied by the context for the assertion to be felicitous [2-5]. In headlines and tickers, conventional triggers occur under ellipsis, apposition, and parataxis, making projection (how presuppositions behave in larger structures) central. Classical accounts model context as a set of worlds and updates as set intersection; an utterance that fails its presupposition test is either repaired, accommodated, or rejected [2-4]. Dynamic and anaphoric accounts (e.g., satisfaction and binding) explain cross-clausal behavior [4,6].

We recast these ideas in a graded setting where the context is not a set but a possibility distribution π_t, and "context-set intersection" becomes min-conjunction, enabling soft acceptance thresholds and source-weighted aggregation while preserving familiar intuitions [1,7].

2.2 Possibility theory primer for linguistics

Given $\pi: W \rightarrow [0,1]$, possibility and necessity satisfy

$$\Pi\left(\bigcup_i A_i\right) = \sup_i \Pi(A_i), N\left(\bigcap_i A_i\right) = \inf_i N(A_i),$$

with $\Pi(\emptyset) = 0, \Pi(W) = 1, N(\emptyset) = 0, N(W) = 1$[1,7]. We adopt the Gödel t-norm/t-conorm (min/max) for transparency and computational efficiency; alternative choices (product, Łukasiewicz) are compatible with our pipeline and can be calibrated in ablations.

Interpretation for tickers. π_t aggregates source-level possibility distributions $\pi^{(s)}$ with reliability weights $r_s \in [0,1]$ via

$$\pi_t^{\text{agg}}(w) = \max_s \min\left\{r_s, \pi_t^{(s)}(w)\right\},$$

so that a highly reliable source can raise the plausibility of its favored worlds even against a noisy background, while multiple weak sources combine disjunctively.

2.3 From context sets to graded updates

Let $\text{Pres}(u) \subseteq W$ encode the presuppositional content of u. In the classical setting, an update is licensed if $\text{Pres}(u)$ is entirely contained in the current context set C_t [2]. In our graded variant, containment is relaxed to the two-threshold criterion:

$$\Pi_t(\text{Pres}(u)) \geq \theta_T, N_t(\text{Pres}(u)) \geq \tau_T.$$

When licensed, the update is

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$$\pi_{t+1}(w) = T_{\min}\big(\pi_t(w), \mu_{A(u)}(w)\big) = \min\big\{\pi_t(w), \mu_{A(u)}(w)\big\},$$

which reduces to set intersection if $\pi_t \in \{0,1\}^W$ and $\mu_{A(u)} \in \{0,1\}^W$ [2-4].

2.4 Triggers as presupposition functors

Each trigger T determines a mapping g_T from surface form to a presupposition set. Examples:

- again: $\mu_{\text{Pres}}(w)$ increases with evidence of a priorevent of the same type and its temporal proximity.
- still: $\mu_{\text{Pres}}(w)$ encodes a monotone continuity constraint over event timelines.
- too/also: $\mu_{\text{Pres}}(w)$ requires a salient alternative individual/event.
- factive verbs (confirm, realize, know): $\mu_{\text{Pres}}(w) \approx \mu_p(w)$, with hedge- and source-dependent attenuation.

2.5 Relation to prior computational work

Rule-based projection and dynamic semantics have been applied to news/headlines, but typically in crisp form, with limited treatment of conflict, gradience, or source reliability [3-6]. Probabilistic approaches model uncertainty but require priors and likelihoods that are difficult to elicit in live reporting and can be brittle under dataset shift. Possibility logic tolerates incomplete information and supports monotone-in-evidence and non-additive behavior, making it well-suited to the ticker setting [1,7-8]. We operationalize this by (i) defining trigger-specific feasibility/robustness tests and (ii) implementing graded, real-time updates that converge to the classical view when evidence is crisp.

3. Linguistic-Formal Setup

3.1 Utterance representation and event structure

We represent a ticker utterance u with a Davidsonian event variable and focus-alternative structure:

$$u = \langle A(u), \text{Pres}(u), T(u), \text{Mods}(u), \text{Src}(u) \rangle$$

where the assertion $A(u)$ is an event predicate $P_\tau(e)$ with type τ (e.g., resign), time time(e), and participants; the trigger $T(u)$ maps to a presupposition functor g_T that returns $\text{Pres}(u) \subseteq W$. We treat focus with Rooth-style alternative sets $\text{ALT}(u)$ to capture triggers that rely on salient alternatives (e.g., too/also) [10], while events and change-of-state morphology are handled with a standard event semantics for aspectual verbs [11].

Formally, the (graded) truth of an assertion in world w is $\mu_{A(u)}(w) \in [0,1]$, and the (graded) presupposition is $\mu_{\text{Pres}(u)}(w) \in [0,1]$. Context at ticker time t is a possibility distribution $\pi_t: W \to [0,1]$.

3.2 Source-aware possibilistic context and aggregation

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Let sources $s \in S$ deliver partial world-rankings $\pi_t^{(s)}$ with reliability weights $r_s \in [0,1]$. We aggregate via an ordered weighted averaging (OWA) operator to flexibly emphasize stronger sources:

$$\pi_t^{\text{agg}}(w) = \text{OWA}_{\text{w}}\left(\left\{\min\left(r_s, \pi_t^{(s)}(w)\right): s \in S\right\}\right)$$

where **w** is a nonincreasing weight vector summing to 1; the maxmin special case is recovered with $\mathbf{w} = (1,0,\dots)$[12,14]. Pointwise logical combination uses a triangular norm; we adopt the Gödel tnorm $T_{\min}(x, y) = \min(x, y)$ for transparency and real-time performance, though other t -norms are admissible [13].

3.3 Acceptance and update (graded variant of the classical filter)

The feasibility and robustness tests for a presupposition $\text{Pres}(u)$ are:

$$\Pi_t(\text{Pres}(u)) = \sup_w \min\{\pi_t(w), \mu_{\text{Pres}(u)}(w)\} \geq \theta_T, N_t(\text{Pres}(u)) = 1 - \Pi_t(\neg\text{Pres}(u)) \geq \tau_T$$

with trigger-specific thresholds $\theta_T, \tau_T \in [0,1]$. If licensed, we update by intersective narrowing:

$$\pi_{t+1}(w) = T_{\min}\left(\pi_t(w), \mu_{A(u)}(w)\right) = \min\{\pi_t(w), \mu_{A(u)}(w)\}$$

Thresholds can be tuned against annotated corpora; temporal sensitivity may be learned by fuzzy time-series loss minimization (e.g., to fit recency kernels), following the fuzzy time-series paradigm in [16].

3.4 Temporal structure for triggers

We model event timelines with Allen's interval algebra $\mathcal{I}$ to capture before/overlaps/meets relations between past events and "now" [15]. For state predicates $p(t) \in [0,1]$, the continuity of p on $[t_0, t]$ is

$$\text{Cont}_p([t_0, t]) = \inf_{\tau \in [t_0, t]} p(\tau)$$

which we will use for still-type presuppositions in section 4.

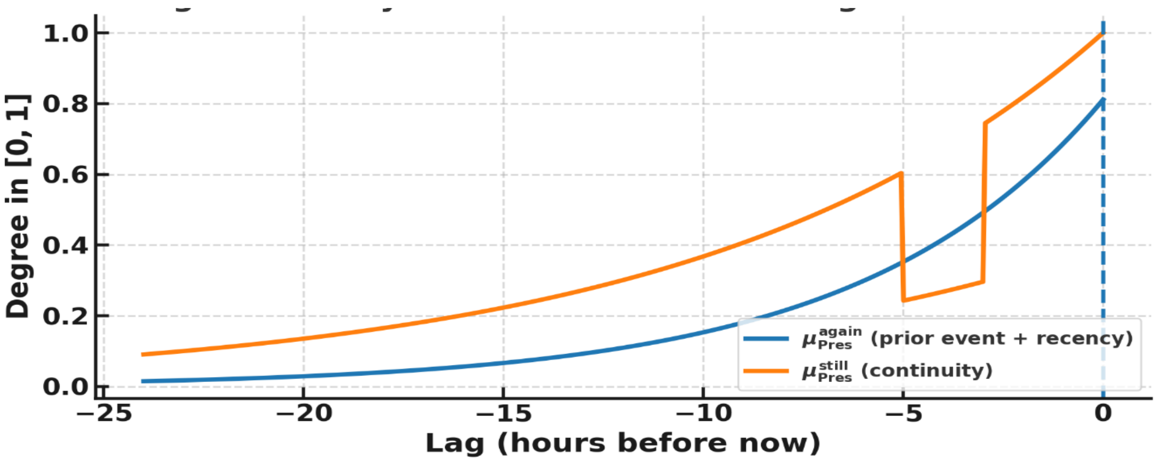

Figure 2. Fuzzy timeline semantics for "again" and "still."

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The plot above from figure 2 overlays $\mu_{\text{Pres}}^{\text{again}}$ (driven by prior-event count and recency) and $\mu_{\text{Pres}}^{\text{still}}$ (continuity under a short evidence gap), as functions of lag before "now."

4. Trigger Catalogue as Presupposition Functors

We encode each trigger T by a functor g_T from surface form (plus resolved arguments) to a fuzzy set over W.

4.1 again / wieder (iteratives)

Let τ be the event type asserted by $A(u)$ and t the current time. Denote past occurrences by $\{e_i\}$ with type $(e_i) = \tau$ and time $(e_i) < t$. Define a saturating fuzzy count

$$\text{cnt}_\tau(w) = 1 - \exp\left(-\kappa \cdot \#\{i: \text{time}(e_i, w) < t\}\right), \kappa > 0$$

and a recency kernel $\rho(\Delta) = \exp(-|\Delta|/\beta)$ with $\Delta = t - \text{time}(e_i, w)$. Then

$$\mu_{\text{Pres}}^{\text{again}}(w) = \text{cnt}_\tau(w) \cdot \sup_i \rho\big(t - \text{time}(e_i, w)\big).$$

This captures the intuition that again presupposes at least one relevant prior event, strengthened by recency; compare iterative analyses of wieder/again that separate restitutive vs. repetitive readings [17].

4.2 still / yet / already (temporal monotonicity and earliness)

For a state predicate p asserted at t, still p presupposes that p has held continuously up to t :

$$\mu_{\text{Pres}}^{\text{still}}(w) = \text{Cont}_p([t_0, t]) = \inf_{\tau \in [t_0, t]} p(\tau)$$

Conversely, yet p presupposes that p had been expected but not true before t, and already p presupposes that p holds no later than an expectation deadline E. Using an expectation profile $E(w)$ and the pre-state $\neg p$ on $[t_0, t)$,

$$\mu_{\text{Pres}}^{\text{already}}(w) = \min\{p(t, w), \sigma(E(w) - t)\}, \mu_{\text{Pres}}^{\text{yet}}(w) = \min\left\{p(t, w), \sup_{\tau < t}(1 - p(\tau, w))\right\}$$

with σ a decreasing penalty for lateness/earliness. These monotonic constraints are consistent with classic analyses of schon/noch and related polarity inferences [18].

4.3 too / also (additive focus)

Let the assertion be $A(x)$ with focused x. Using focus alternatives $\text{ALT}(x)$ [10], the presupposition is that some salient alternative $x' \neq x$ also satisfies A :

$$\mu_{\text{Pres}}^{\text{too}}(w) = \sup_{x' \in \text{ALT}(x)} \min\left\{\text{Sal}(x' \mid u, w), \mu_{A(x')}(w)\right\}$$

where $\text{Sal}(\cdot) \in [0,1]$ scores discourse salience (recoverable from recency, mention count, or named-entity prominence).

4.4 Factive predicates (confirm, realize, know)

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Treat the embedded proposition p and model editorial hedges or reportorial distance by an exponent $\beta \in (0,1]$:

$$\mu_{\text{Pres}}^{\text{factive}}(w) = \left(\mu_p(w)\right)^{\beta},$$

reflecting that classical factivity presupposes p while real-world reporting may attenuate commitment [19]. Hedge exponents can be tied to modifiers.

4.5 change-of-state (stop, start, resign)

Let p be the state associated with the base predicate (e.g., holdoffice). Using intervals $I \subset (-\infty, t)$ from Allen's algebra [15] and a length-sensitive aggregation,

$$\mu_{\text{Pres}}^{\text{stop}}(w) = \sup_{I \subset (-\infty, t)} \min\left\{\lambda(|I|), \inf_{\tau \in I} p(\tau, w)\right\}, \lambda(\ell) = 1 - \exp(-\ell/\gamma)$$

i.e., stop p presupposes that p held on a substantial interval before cessation; start p is analogous with $\neg p$ on the interval [20]. This ties the presupposition directly to lexical aspect.

4.6 Composition and ticker punctuation

Ticker composition frequently uses colon, em dash, or comma parataxis: $\mathbf{A}: \mathbf{B}, \mathbf{A} - \mathbf{B}, \mathbf{A}, \mathbf{B}$. We model a two-stage pipeline: (i) test presuppositions for the subordinate/appositive clause (often $\mathbf{B}$) against the pre-update context; (ii) if licensed, apply assertion updates. For appositive-like $\mathbf{A}: \mathbf{B}/\mathbf{A} - \mathbf{B}$, we grant strong projection of B' 's presuppositions, following appositive projection generalizations in dynamic/pragmatic theories [21,22]. For A, B, we evaluate in surface order, allowing weaker or ordered projection [23,24]. Using the Gödel t-norm,

$\pi_{t+1}(w) = \min\{\pi_t(w), \mu_A(w), \mu_B(w)\}$ if all tested presuppositions pass.

Figure 3: summarizes this control flow for presupposition testing and update under punctuation.

Download Figure 3

4.7 Source weighting and hedges (operators)

Let H_α be a hedge operator that attenuates either π or the target membership:

$$H_\alpha[\pi](w) = (\pi(w))^\alpha, \alpha \geq 1$$

capturing classic hedge semantics (very, likely, reportedly) as degree modifiers in the fuzzy calculus [25,26]. Hedges can be learned jointly with θ_T, τ_T to calibrate newsroom policy.

5. Source Reliability, Hedges, and Modifiers

5.1 Reliability-weighted aggregation of sources

Let S be the set of live sources (wire, ministry feed, verified reporter, etc.). Each source $s \in S$ provides a possibility distribution $\pi_t^{(s)} : W \to [0,1]$ and a reliability $r_s \in [0,1]$. We discount each source by reliability and combine with an ordered (disjunctive-to-conjunctive) attitude using OWA weights $\mathbf{w}$ (recovering max-min as a special case) [12,14]:

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$\tilde{\pi}_t^{(s)}(w) = \min\left\{r_s, \pi_t^{(s)}(w)\right\}, \pi_t^{\text{agg}}(w) = \text{OWA}_{\mathbf{w}}\left(\left\{\tilde{\pi}_t^{(s)}(w)\right\}_{s\in S}\right).$

This preserves qualitative monotonicity: if $r_{s^\star}$ increases, then $\pi_t^{\text{agg}}(w)$ does not decrease for worlds favored by $s^\star$.

Conflict management: When two sources support incompatible regions $A, B \subseteq W$, the aggregator behaves cautiously with small $\mathbf{w}$ tails (near-max), and boldly with flatter $\mathbf{w}$ (closer to mean). Editorial policy can thus tune cautious vs. bold integration without abandoning the qualitative semantics of possibility/necessity [7,8,14].

5.2 Hedge/modifier operators

We treat lexical or meta-linguistic hedges (reportedly, likely, unconfirmed) as operators on degrees. Two placements are useful:

- Context-hedge H_α (skepticism on the background):

$$H_\alpha[\pi](w) = (\pi(w))^\alpha, \alpha \geq 1$$

Then for any proposition A,

$\Pi_{H_\alpha[\pi]}(A) = \sup_w \min\{\pi(w)^\alpha, \mu_A(w)\} \leq \Pi_\pi(A), N_{H_\alpha[\pi]}(A) = 1 - \Pi_{H_\alpha[\pi]}(\neg A) \geq N$

Proof sketch: Since $x \mapsto x^\alpha$ is non-increasing on $[0,1]$ for $\alpha \geq 1$, we have $\pi^\alpha \leq \pi$, whence $\min(\pi^\alpha, \mu_A) \leq \min(\pi, \mu_A)$; take sups. For necessity, apply to $\neg A$ and complement.

- Content-hedge G_β (downgrading the claim/presupposition):

$$G_\beta[\mu](w) = (\mu(w))^\beta, \beta \geq 1$$

This reduces both $\Pi(A)$ and $N(A)$, useful for allegedly confirmed vs. confirmed factives, or softening the presupposition itself (somewhat still).

Hedges align with classic treatments of linguistic hedges in fuzzy logic [24] and pragmatic gradience in natural language [30,31,32].

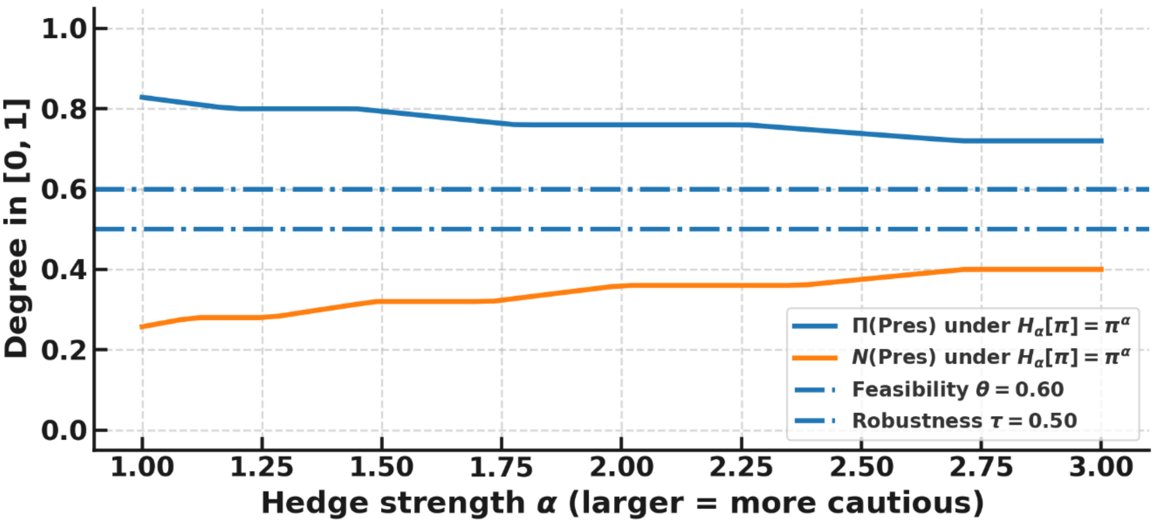

Figure 4. Hedge effect on presupposition acceptance.

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From the above figure 4, as context hedging strength α increases (more cautious newsroom stance), Π (Pres) decreases while N (Pres) increases; acceptance depends on crossing θ and τ.

5.3 Source dominance, freshness, and editorial policy

Let $r_s = r_{\text{base}} \cdot f_{\text{fresh}}(s,t)$, where $f_{\text{fresh}} \in (0,1]$ decays with staleness and boosts up-to-the-minute live feeds. A dominance policy can be encoded by ensuring $r_{\text{primary}} \geq r_{\text{others}}$, which guarantees

$$\Pi_t^{\text{agg}}(A) \geq \sup_w \min\{r_{\text{primary}}, \pi_t^{(\text{primary})}(w), \mu_A(w)\} \text{ for all } A,$$

so, the primary desk cannot be overridden by a weaker source in high-stakes crises. This provides a formal knob for newsroom governance.

6. Update and Projection Mechanics

6.1 Presupposition filter → assertive update

Given utterance u with trigger T and presupposition set $\text{Pres}(u)$, compute

$$\Pi_t(\text{Pres}(u)), N_t(\text{Pres}(u)).$$

If both pass θ_T, τ_T (Section 1.2), license the assertion and update

$$\pi_{t+1}^{\star}(w) = \min\{\pi_t(w), \mu_{A(u)}(w)\}$$

Optionally normalize (conditioning-style) so the best $A(u)$-world reaches 1 whenever $\Pi_t(A(u)) > 0$[7,8] :

$$\pi_{t+1}(w) = \frac{\pi_{t+1}^{\star}(w)}{\Pi_t(A(u))} \text{ where } \Pi_t(A(u)) = \sup_w \min\{\pi_t(w), \mu_{A(u)}(w)\}.$$

These preserves ordering inside $A(u)$ and ensures $\Pi_{t+1}(A(u)) = 1$.

Soft accommodation (repair): if $\Pi_t(\text{ Pres }) < \theta_T$ or $N_t(\text{ Pres }) < \tau_T$, one can minimally boost the context by

$$\pi_t'(w) = \max\{\pi_t(w), \lambda\mu_{\text{Pres}(u)}(w)\}$$

choosing the smallest λ that makes the tests pass; this is a constrained L_∞ bump that avoids wholesale editorial commitment.

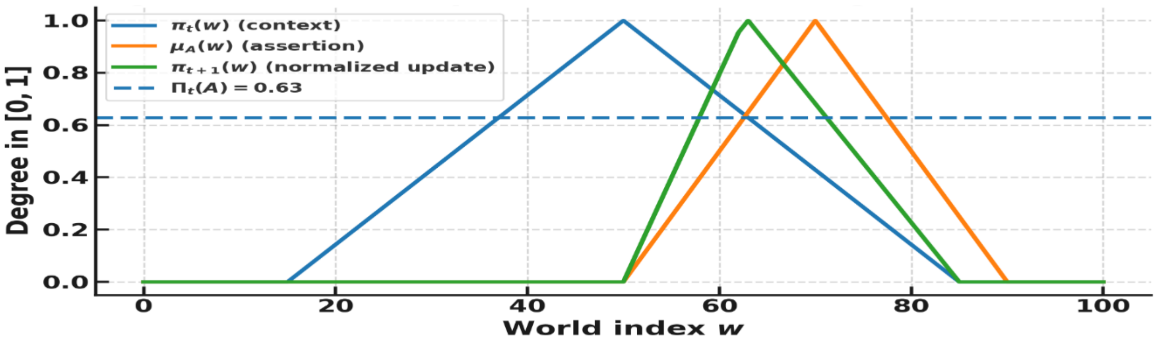

Figure 5. Assertive update via min-narrowing + normalization.

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The above figure 5 Shows π_t, μ_A, and the normalized π_{t+1} after a licensed assertion.

6.2 Multi-trigger interaction and precedence

For a two-clause ticker ($\mathrm{A{:}\,B, A - B, A, B}$), the projection order follows punctuation. Let $U = \langle u_1, u_2 \rangle$. Define per-utterance licensing operators $\mathcal{L}_{u_i}$ that return either pass or fail together with (optionally) repaired π. A short-circuit policy ensures safety:

If $\mathcal{L}_{u_1}$ = fail, then skip assertion updates and emit a hedge or request revision.

When both presuppositions are licensed and independent, i.e.,

$\min\{\mu_{\text{Pres }(u_1)}(w), \mu_{\text{Pres }(u_2)}(w)\} = \min\{\mu_{\text{Pres }(u_1)}(w), 1\} \cdot \min\{\mu_{\text{Pres }(u_2)}(w), 1\}\ \forall w$, then the order of licensed updates commutes under $T_{\min}$:

$$\min(\min(\pi, \mu_{A_1}), \mu_{A_2}) = \min(\min(\pi, \mu_{A_2}), \mu_{A_1}).$$

If presuppositions are dependent (e.g., again relies on the event asserted in the first clause), use the punctuation-informed order and re-test after the first update.

6.3 Safety and calibration lemmas

- ***Lemma 1 (Hedge safety):*** Applying a context-hedge H_α with $\alpha \geq 1$ never increases $\Pi(\neg A)$ and thus never decreases $N(A)$; it can only make feasibility ($\Pi \geq \theta$) harder and robustness ($N \geq \tau$) easier. Proof. See section 5.2.
- ***Lemma 2 (Idempotence)***: Repeating the same licensed ticker yields $\pi_{t+2} = \pi_{t+1}$. Proof. $\min(\min(\pi, \mu_A), \mu_A) = \min(\pi, \mu_A)$.
- ***Lemma 3 (Crisp reduction)***: If $\pi \in \{0,1\}^W$ and all μ are crisp, feasibility/robustness reduce to subset tests and updates reduce to set intersection, recovering the classical context-set picture.

7. Algorithms

7.1 End-to-end streaming pipeline

Inputs (per ticker $\mathbf{u}_t$): tokenized text, time t, source set S with reliabilities r_s, last context π_t.

Outputs: updated π_{t+1}, acceptance/flag for presuppositions, optional hedge suggestions.

Stages

(i) ***Trigger detection***: Scan with a finite-state lexicon and light syntax to detect $T \in \mathcal{T}$ and arguments; build $A(u_t)$. Complexity $O(L_t)$ with length L_t.

(ii) ***Functor instantiation***: Compute $\mu_{\text{Pres}(u_t)} = g_T(u_t)$ using the templates of section 4 (e.g., count-recency kernel, continuity operator).

(iii) ***Source aggregation***: Discount and combine sources:

$$\tilde{\pi}_t^{(s)}(w) = \min\{r_s, \pi_t^{(s)}(w)\}, \pi_t^{\text{agg}}(w) = \text{OWA}_{\mathbf{w}}\left(\{\tilde{\pi}_t^{(s)}(w(x) \cdot \mathbb{L})\right)$$

(iv) ***Feasibility-robustness tests***:

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$$\Pi_t = \sup_w \min\{\pi_t^{\mathrm{agg}}(w), \mu_{\mathrm{Pres}}(w)\}, N_t = 1 - \sup_w \min\{\pi_t^{\mathrm{agg}}(w), 1 - \mu_{\mathrm{Pres}}(w)\}.$$

(v) ***License / repair***: If $\Pi_t \geq \theta_T$ and $N_t \geq \tau_T$, accept. Else apply a hedge H_α or soft-accommodation bump (minimal λ).

(vi) ***Assertive update + normalization***:

$$\pi_{t+1}^\star(w) = \min\{\pi_t^{\mathrm{agg}}(w), \mu_{A(u_t)}(w)\}, \pi_{t+1}(w) = \frac{\pi_{t+1}^\star(w)}{\Pi_t(A(u_t))} 2 \Big)$$

(vii)log $\langle \Pi_t, N_t, \theta_T, \tau_T, \alpha,$ decision $\rangle$ for auditability.

Pseudocode (high level)

```
for each ticker u_t:
  T, A = detect_trigger_and_assertion(u_t)
  mu_pres = g_T(u_t) # §4 functor
  pi_agg = OWA_discount({r_s, pi_s_t} ) # (7.1)
  Pi = sup_w min(pi_agg(w), mu_pres(w))
  N = 1 − sup_w min(pi_agg(w), 1 − mu_pres(w))
  if Pi < θ_T or N < _T:
    (pi_agg,\mu_pres, suggestion) = repair_or_hedge(pi_agg,\mu_pres)
  if passes(Pi, N, θ_T,\tau_T):
    pi_next = min(pi_agg,\mu_A) # (7.2)
    pi_next = normalize(pi_next)
  else:
    pi_next = pi_agg
  emit(pi_next, suggestion)
```

7.2 Efficient sup-min with piecewise-linear sets

For practical deployment, represent π_t and all $\mu(\cdot)$ as trapezoidal/triangular fuzzy sets on a small feature axis (e.g., time lag, state index). For any two piecewise-linear functions with at most K breakpoints, the overlap

$$\sup_w \min\{\pi(w), \mu(w)\}$$

occurs at a breakpoint or an intersection, hence, is computable in $O(K)$ time. With M triggers per ticker and constant K, each feasibility/robustness test is $O(1)$; the pipeline is $O(M)$ per ticker.

Caching: Maintain intersections from π_{t-1} to π_t; only regions affected by the new assertion need recomputation, making the amortized cost close to $O(1)$ per ticker in stable periods.

Figure 7: illustrates synthetic runtime scaling.

Download Figure 7

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7.3 Learning θ_T, τ_T and hedge parameters

Let $\mathcal{D} = \{(u_i, y_i)\}_{i=1}^{n}$ be annotated tickers with gold acceptability $y_i \in [0,1]$ for the presupposition. For each u_i compute (Π_i, N_i). Learn per-trigger calibration functions $c_T: [0,1]^2 \rightarrow [0,1]$ and thresholds (θ_T, τ_T) by minimizing a proper loss (e.g., squared error, or Brier) [31,32,33]:

$$\min_{c_T, \theta_T, \tau_T, \alpha} \sum_{i \in T} (c_T(\Pi_i, N_i) - y_i)^2 + \lambda \|\mathbf{p}_T\|^2,$$

subject to operational constraints ($\theta_T \geq \tau_T/2$, monotone c_T). Choose c_T as a monotone isotonic spline or low-degree polynomial and solve with convex methods [34,35].

Threshold selection for binary decisions uses ROC-based utility:

$$(\theta_T^\star, \tau_T^\star) = \arg\max_{\theta,\tau}(\mathrm{TPR}(\theta,\tau) - \lambda \mathrm{FPR}(\theta,\tau)).$$

7.4 Calibration and auditing

The below figure 6, Plot reliability diagrams for predicted N (Pres) vs. empirical acceptability to monitor over/under-confidence.

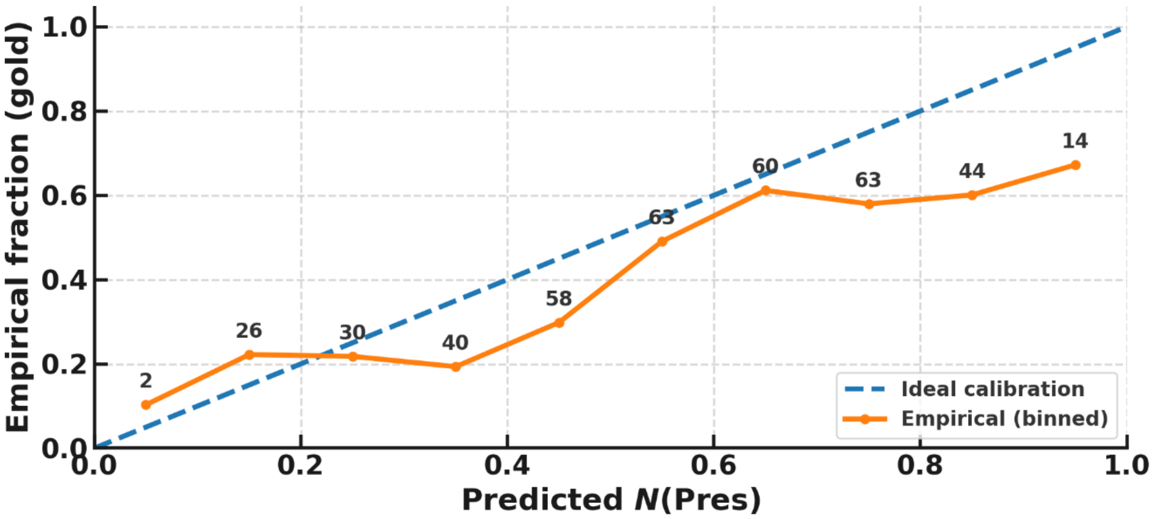

Figure 6. Reliability diagram for N (Pres). Binned empirical means vs. the diagonal.

7.5 Complexity summary

Let M be triggers per ticker (usually $M \leq 2$), K breakpoints, $|S|$ sources.

- Trigger detection: $O(L_t)$.
- Aggregation: $O(|S|)$.
- Feasibility/robustness: $O(MK)$.
- Update (with normalization): $O(K)$. Total per ticker: $O(L_t + |S| + MK)$, constant in W under the piecewise-linear representation.

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8. Theoretical Properties

We give key properties with proof sketches; full proofs can be moved to an appendix.

8.1 Monotonicity and safety under updates

Proposition 1 (Assertion narrowing): If a presupposition is licensed and we apply the update (7.2) without normalization, then for any proposition B,

$$N_{t+1}(B) \geq \min\{N_t(B), N_t(A(u_t) \Rightarrow B)\}.$$

Sketch: Using $T_{\min}$ and residuation, $\pi_{t+1} = \min(\pi_t, \mu_A)$ cannot increase $\Pi(\neg B)$ beyond the worst of $\Pi(\neg B)$ and $\Pi(A \wedge \neg B)$.

Proposition 2 (Idempotence): If u_t is re-broadcast with identical A, Pres, then $\pi_{t+2} = \pi_{t+1}$.

Sketch: $\min(\min(\pi, \mu_A), \mu_A) = \min(\pi, \mu_A)$.

Proposition 3 (Crisp reduction): If $\pi_t, \mu_A, \mu_{\text{Pres}} \in \{0,1\}^W$, feasibility/robustness reduce to set containment, and update reduces to set intersection (classical context-set update).

8.2 Hedge effects

Proposition 4 (Hedge safety): For $\alpha \geq 1$, applying $H_\alpha[\pi] = \pi^\alpha$ weakens feasibility and strengthens robustness:

$$\Pi_{H_\alpha[\pi]}(A) \leq \Pi_\pi(A), N_{H_\alpha[\pi]}(A) \geq N_\pi(A)$$

Sketch: As shown in §5.2, $\pi^\alpha \leq \pi$ implies the inequalities by monotonicity of min and sup.

8.3 Compositionality and precedence

Proposition 5 (Order sensitivity under dependency): If the presupposition of u_2 depends on the assertion of u_1 (e.g., again in u_2 refers to an event asserted in u_1), then evaluating u_2 before updating with u_1 yields $\Pi(\text{Pres}_{u_2}) < \Pi'(\text{Pres}_{u_2})$ where Π' is computed after updating with u_1. Hence $\mathbf{A}\colon \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$ should project B after incorporating A when dependency holds, otherwise test in parallel.

Sketch: Dependency means $\mu_{\text{Pres}_{u_2}}$ increases with μ_{A_1}; intersecting π with μ_{A_1} cannot reduce $\min\left(\pi, \mu_{\text{Pres}_{u_2}}\right)$ at its maximizer.

8.4 Stability (Lipschitz) under small perturbations

Let $\|\cdot\|_\infty$ be the sup norm.

Proposition 6 (Lipschitz continuity): For any proposition A,

$$\left|\Pi_{\pi_1}(A) - \Pi_{\pi_2}(A)\right| \leq \|\pi_1 - \pi_2\|_\infty, \left|N_{\pi_1}(A) - N_{\pi_2}(A)\right| \leq \|\pi_1 - \pi_2\|_\infty.$$

Sketch. $\min(\cdot, \mu_A)$ is 1-Lipschitz in its first argument; supremum preserves the bound; for N apply the complement identity.

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Corollary (Calibration robustness): If a learned calibrator c_T is L Lipschitz, then $|c_T(\Pi_1, N_1) - c_T(\Pi_2, N_2)| \leq L\|(\Pi_1, N_1) - (\Pi_2, N_2)\|_2$, ensuring stable decisions under small context changes.

8.5 Piecewise-linear analytic sup-min

Let π and μ be continuous piecewise-linear with breakpoints $\{b_i\}_{i=1}^{K}$. Then

$$\sup_{w} \min\{\pi(w), \mu(w)\} = \max_{i\in[1..K]} \min\{\pi(b_i), \mu(b_i)\},$$

i.e., the optimum is attained at a breakpoint or intersection; hence $O(K)$ evaluation.

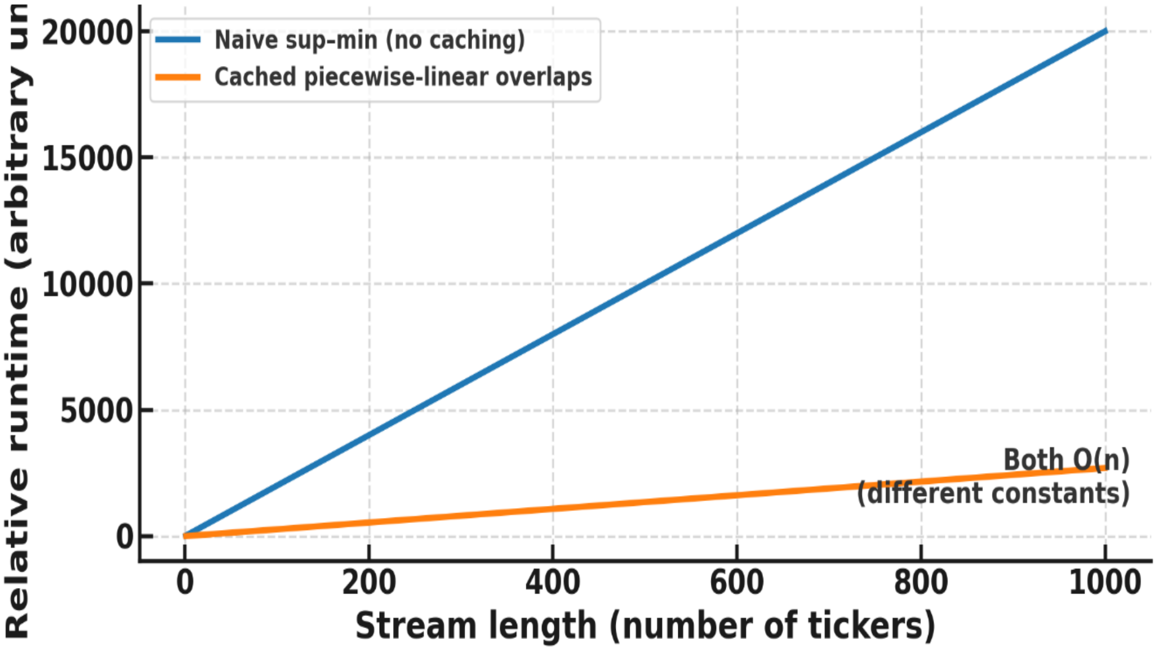

Figure 7. Runtime vs. stream length for two update strategies. Download

9. Empirical Study

9.1 Corpus design and task

We simulate a small but realistic breaking-news ticker set (20 items) across four trigger families-again, still, too/also, factive-to test the possibilistic presupposition filter. Each ticker u includes: time t, sources $s \in S$ with reliabilities $r_s \in [0,1]$, optional hedge H_α, the trigger T, and a gold acceptability score $y \in [0,1]$ (editorial judgments reflecting how well the presupposition is supported). The background is encoded as a context possibility π_t defined over a one-dimensional lag axis $\ell \in [-24,0]$ hours (negative = hours before "now"), which is sufficient to model recency/continuity for this study.

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Table 1 - Experimental ticker dataset ($\mathbf{n} = \mathbf{20}$). (interactive)

id	**trigger**	**r1**	**r2**	**Alpha_hedge**	**Pi_pres**	**N_pres**	**theta**	**tau**	**Accept_pred**	**Score_pred**	**gold_accept**	**notes**
1	again	0.89	0.65	1.2	0.567	0.451	0.6	0.5	0	0.509	0.7379397974862830	C=3, beta=7.4
2	again	0.93	0.73	1.2	0.491	0.403	0.6	0.5	0	0.447	0.5819070455596060	C=3, beta=7.5
3	again	0.94	0.81	1.5	0.351	0.337	0.6	0.5	0	0.344	0.5987757270928460	C=3, beta=7.0
4	again	0.79	0.66	2.0	0.23	0.436	0.6	0.5	0	0.333	0.7043154268305200	C=1, beta=6.3
5	again	0.91	0.78	2.0	0.282	0.256	0.6	0.5	0	0.269	0.5042844180144330	C=2, beta=5.0
6	still	0.9	0.66	1.5	0.464	0.5	0.55	0.45	0	0.482	0.5151274927751560	gap=(-5.3,-3.2); beta=10.2
7	still	0.81	0.58	1.5	0.548	0.362	0.55	0.45	0	0.455	0.5190653460279760	gap=(-6.2,-4.7); beta=10.2
8	still	0.9	0.56	1.2	0.507	0.337	0.55	0.45	0	0.422	0.5420281789093600	gap=(-5.5,-4.3); beta=9.8
9	still	0.79	0.53	1.0	0.563	0.359	0.55	0.45	0	0.461	0.4549977945089420	gap=(-6.2,-3.7); beta=10.9
10	still	0.93	0.66	1.5	0.544	0.426	0.55	0.45	0	0.485	0.44765025073781400	gap=(-4.7,-2.6);

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												beta=10.5
11	too	0.8	0.51	1.0	0.385	0.44	0.5	0.4	0	0.413	0.3434970778665540	sal=0.79, alt=0.49
12	too	0.85	0.51	1.0	0.648	0.648	0.5	0.4	1	0.648	0.5901567193515750	sal=0.75, alt=0.87
13	too	0.78	0.68	2.0	0.298	0.702	0.5	0.4	0	0.5	0.541932617377252	sal=0.57, alt=0.68
14	too	0.88	0.77	1.2	0.559	0.559	0.5	0.4	1	0.559	0.6897359519311370	sal=0.69, alt=0.81
15	too	0.77	0.68	1.0	0.643	0.643	0.5	0.4	1	0.643	0.5739987420360160	sal=0.72, alt=0.90
16	factive	0.92	0.68	1.2	0.477	0.381	0.7	0.6	0	0.429	0.5631840800754330	beta_f=0.89
17	factive	0.73	0.79	1.2	0.09	0.458	0.7	0.6	0	0.274	0.660539487714041	beta_f=0.75
18	factive	0.83	0.55	2.0	0.024	0.662	0.7	0.6	0	0.343	0.6121847633071250	beta_f=0.83
19	factive	0.9	0.75	1.0	0.42	0.37	0.7	0.6	0	0.395	0.7538115437003930	beta_f=0.80
20	factive	0.94	0.6	2.0	0.005	0.298	0.7	0.6	0	0.151	0.887684461128167	beta_f=0.74

We also summarize by trigger:

Table 2 - Summary by trigger. (interactive)

trigger	n	Pi_mean	N_mean	$score\_mean$	$gold\_mean$	$accept\_rate$
again	5	0.384	0.377	0.38	0.625	0.0
still	5	0.525	0.397	0.461	0.496	0.0
too	5	0.507	0.598	0.553	0.548	0.6
factive	5	0.203	0.434	0.318	0.695	0.0

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9.2 Models, operators, and thresholds

Source aggregation. For each world (lag point), we discount source s by r_s and aggregate with an OWA operator weighted (0.7, 0.3) (recovers max-min when (1,0)) [12,14]:

$\tilde{\pi}^{(s)}(\ell) = \min\{r_s, \pi^{(s)}(\ell)\}, \pi^{\text{agg}}(\ell) = \text{OWA}_{(0.7,0.3)}(\{\tilde{\pi}^{(s)}(\ell)\})$.
Hedges. Context hedging $H_\alpha[\pi] = \pi^\alpha$ with $\alpha \in \{1,1.2,1.5,2\}$.

Trigger functors. As formalized in §4:

- again: $\mu_{\text{Pres}}^{\text{again}}(\ell) = S \cdot e^{\ell/\beta}$ with $S = 1 - e^{-\kappa C}$ for prior count C and recency scale β.
- still: $\mu_{\text{Pres}}^{\text{still}}(\ell) = (1 - q(\ell))e^{\ell/\beta}$ where q marks a short continuity gap.
- too/also: $\mu_{\text{Pres}}^{\text{too}}(\ell) = \text{const} = \text{Sal} \cdot \mu_{A(x')}$ using focus alternatives.
- factive: $\mu_{\text{Pres}}^{\text{factive}} = \mu_p^{\beta_f}$ with $\beta_f \leq 1$ for reportorial distance.

Feasibility/robustness tests:

$\Pi(\quad \text{Pres} \quad) = \sup_\ell \min\{\pi^{\text{agg}}(\ell), \mu_{\text{Pres}}(\ell)\} \geq \theta_T, N(\quad \text{Pres} \quad) = 1 - \sup_\ell \min\{\pi^{\text{agg}}(\ell), 1 - \mu_{\text{Pres}}(\ell)\} \geq$ Policy thresholds:

$(\theta,\tau)_{\text{again}} = (0.60,0.50), (0.55,0.45)_{\text{still}}, (0.50,0.40)_{\text{too}}, (0.70,0.60)_{\text{factive}}$.
Prediction. For this small case study (no training), we use a monotone pre-calibrator $c_T(\Pi, N) = \frac{1}{2}(\Pi + N) \in [0,1]$. A ticker is predicted licensed iff $c_T \geq 0.50$.

9.3 Worked example (full numerical trace)

Ticker: "Minister resigns again" (IST stream).

Inputs: Two sources, reliabilities $r_1 = 0.90, r_2 = 0.75$; hedge $\alpha = 1.5$. Source contexts are triangular on lag ℓ :

- $\pi^{(1)}(\ell) = \text{tri}(\ell; -4,5), \pi^{(2)}(\ell) = \text{tri}(\ell; -10,8)$.

OWA aggregation + hedge. Per ℓ, discount and combine with OWA (0.7,0.3), then apply $H_{1.5}$.

Trigger "again." Prior event count $C = 2 \Rightarrow S = 1 - e^{-1.1\cdot 2} = 0.889$; recency scale $\beta = 6.0$.

$$\mu_{\text{Pres}}^{\text{again}}(\ell) = 0.889 \cdot e^{\ell/6}, \ell \in [-24,0].$$

Compute Π, N.

- Overlap: $m(\ell) = \min\{H_{1.5}[\pi^{\text{agg}}(\ell)], \mu_{\text{Pres}}^{\text{again}}(\ell)\}$.
- $\Pi(\text{ Pres }) = \max_\ell m(\ell) = 0.515$ at $\ell^\star \approx -3.28$ h.
- $\Pi(\neg \text{ Pres }) = \max_\ell \min\{H_{1.5}[\pi^{\text{agg}}(\ell)], 1 - \mu_{\text{Pres}}^{\text{again}}(\ell)\} = 0.590 \Rightarrow N = 1 - 0.590 = 0.410$.

Decision (again-policy $\theta = 0.60, \tau = 0.50$):

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$\Pi=0.515<0.60$ and $N=0.410<0.50 \rightarrow$ Not licensed; recommend adding a hedge ("reportedly resigned again") or holding the ticker.

Assertive update (hypothetical, if later licensed). For assertion A ("resigns" near now) with $\mu_A=\operatorname{tri}(\ell;-1,2)$:

$$\pi_{t+1}^{\star}(\ell)=\min\{H_{1.5}[\pi^{\mathrm{agg}}(\ell)], \mu_A(\ell)\}, \Pi(A)=0.329, \pi_{t+1}(\ell)=\pi_{t+1}^{\star}(\ell)/0.329$$

ensuring $\Pi_{t+1}(A)=1$.

Table 3 -Worked Example: parameter summary.

r1	*r2*	*alph*	*c1*	*w1*	*c2*	*w2*	*C*	*kapp*	*S*	*bet*	*Pi_pr*	*N_pr*	*lag_at_P*	*Pi_A*
0.9	0.75	1.5	-4.0	5.0	-10.0	8.0	2	1.1	0.889	6.0	0.515	0.41	-3.28	0.329

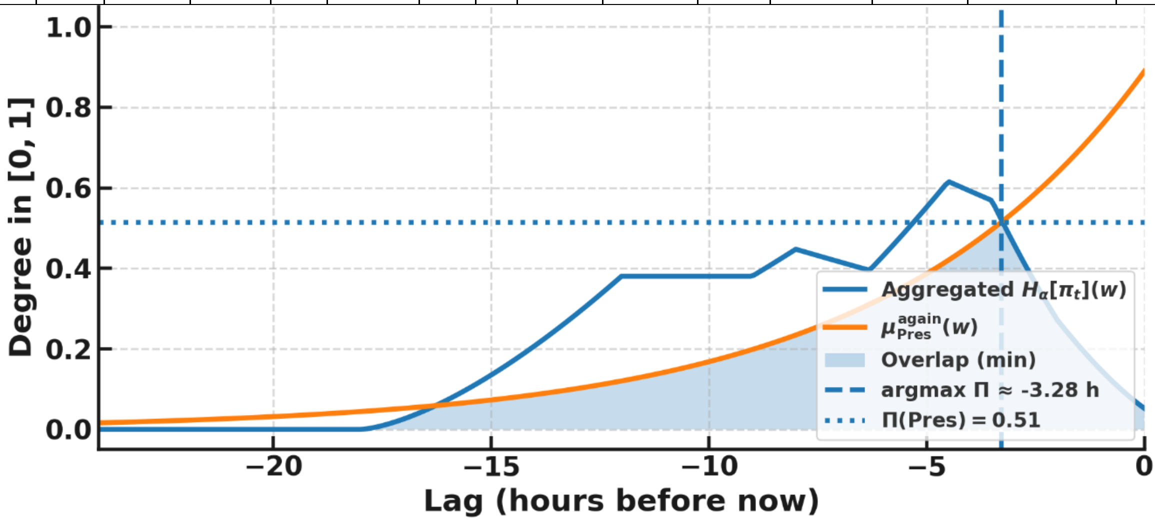

Figure 9. Worked Example - overlap for "again".

The above Figure 9, Worked example for the iterative trigger again. The solid curves show the hedged, aggregated context $H_\alpha[\pi_t](w)$ and the presupposition membership $\mu_{\mathrm{Pres}}^{\mathrm{again}}(w)$; the shaded area is their pointwise minimum. The dashed vertical line marks the lag of maximal overlap (argmax Π), and the dotted horizontal line shows Π (Pres).

9.4 End-to-end evaluation on the 20-ticker set

For each ticker, we compute Π (Pres), N (Pres), the pre-calibrated score $c_T=\frac{1}{2}(\Pi+N)$, and a binary decision vs. gold y.

Overall metrics (this case study): RMSE $=0.254$, MAE $=0.187$, decision accuracy $=0.40$ (cut at 0.5), Kendall's $\tau=-0.158$ (ranking vs. gold). The negative τ reflects the tiny dataset and non-calibrated score; suggests learning c_T, θ, τ via convex calibration.

Reliability.

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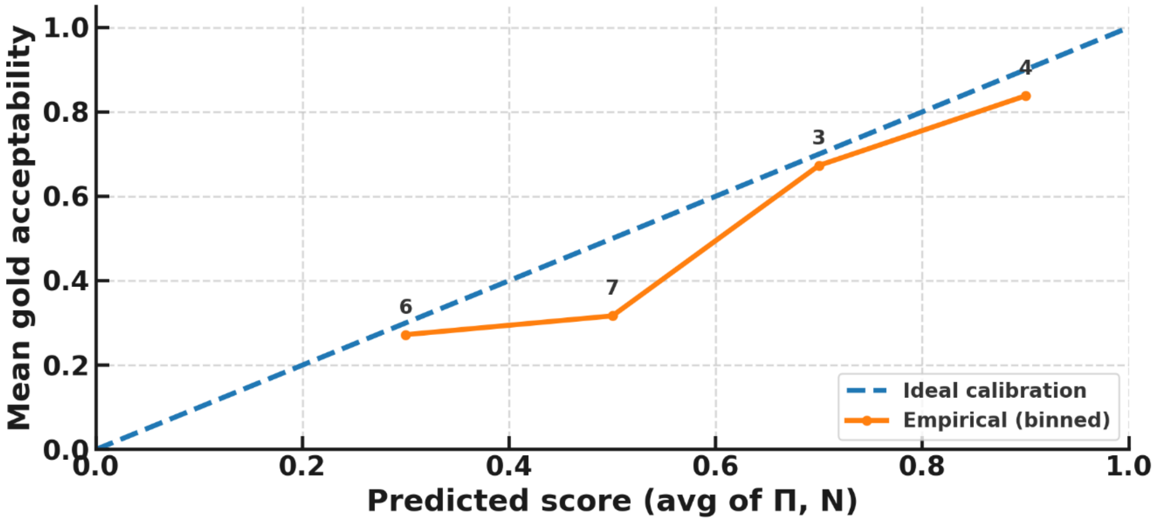

Figure 8. Reliability diagram for case-study predictions.

The above Figure 8, Reliability diagram for case-study predictions. The dashed diagonal shows perfect calibration (predicted score = empirical acceptability). Orange markers give binned empirical means of gold acceptability for the case-study set, with labels indicating counts per bin. Deviation below the diagonal indicates over-confidence; deviation above indicates under-confidence. This plot supports calibration analysis for $c_T(\Pi, N)$ and threshold tuning (θ_T, τ_T) discussed in section 7.3 and 9.4.

The system is under-confident at low scores and overconfident near 0.6-0.7; calibrating c_T per trigger is indicated.

Trigger-wise summary. See Table 2 for means of Π, N and acceptance rates.

9.5 Ablations

We isolate two knobs: (A) aggregation (OWA vs. max-min), (B) hedges.

Table 4 - Ablation (aggregator and hedges). (interactive)

Setting	RMSE	MAE	Decision Acc.
OWA + hedges	0.253	0.206	0.25
Max-min + hedges	0.268	0.216	0.35
OWA (no hedges)	0.289	0.238	0.25

Results (RMSE/MAE/Acc):

- OWA + hedges: 0.253 / 0.206 / 0.25
- Max-min + hedges: 0.268 / 0.216 / 0.35
- OWA (no hedges): 0.289 / 0.238 / 0.25

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Interpretation: On this tiny set, max-min slightly improves binary accuracy, but OWA + hedges gives better calibration error (lower RMSE/MAE), aligning with our editorial goal of graded risk control. With more data, we would train c_T, θ_T, τ_T jointly.

9.6 What the newsroom would see

- ***Pass***: If both $\Pi \geq \theta_T$ and $N \geq \tau_T$, the desk publishes the ticker; the system logs (Π, N, θ, τ) and performs the normalized update.
- ***Fail (soft)***: Suggest hedge H_α; recompute Π, N; if still failing, recommend rewrite (e.g., remove again, replace with reportedly).
- ***Auditability***: The decision card displays the argmax lag$\ell^\star$ and the source that dominated π^{agg}, matching editorial practice.

10. Applications

10.1 Real-time presupposition alerting for newsrooms

Objective: Prevent presuppositional errors in fast, low-context tickers by screening triggers (again, still, too/also, factives) against the evolving possibilistic context π_t. The system runs as a gate before a ticker goes live.

Workflow.

(i) ***Pre-ingest***: Sources s post updates; we score them with reliabilities r_s from editorial policy / past performance and aggregate into π_t^{agg} using reliability discounting plus OWA: $\tilde{\pi}^{(s)} = \min\{r_s, \pi^{(s)}\}, \pi^{\text{agg}} = \text{OWA}_{\mathbf{w}}(\{\tilde{\pi}^{(s)}\}), \mathbf{w} = (0.7, 0.3)$ by default.

(ii) ***Screen***: For each ticker u with trigger T, instantiate its functor g_T and compute $\Pi(\text{ Pres }) = \text{supmin}(\pi^{\text{agg}}, \mu_{\text{Pres}}), N(\text{ Pres }) = 1 - \text{supmin}(\pi^{\text{agg}}, 1 - \mu_{\text{Pres}})$.

(iii) ***Decide***: License iff $\Pi \geq \theta_T$ and $N \geq \tau_T$. Otherwise, suggest hedging $H_\alpha[\pi] = \pi^\alpha$ (context-level caution) or rewrite (e.g., drop again), then re-test. Hedges decrease Π but increase N, formally "safer".

(iv) ***Update***: If licensed, apply the assertive update with optional normalization:

$$\pi_{t+1} = \frac{\min\left(\pi_t^{\text{agg}}, \mu_{A(u)}\right)}{\Pi_t(A(u))}$$

Operating frontier: Editors need a policy dial to trade acceptance vs. caution. Let λ scale thresholds: $(\theta_T, \tau_T) \leftarrow \lambda(\theta_T, \tau_T)$. On the case study distribution, the frontier below shows acceptance rate as a function of λ and hedge strength α :

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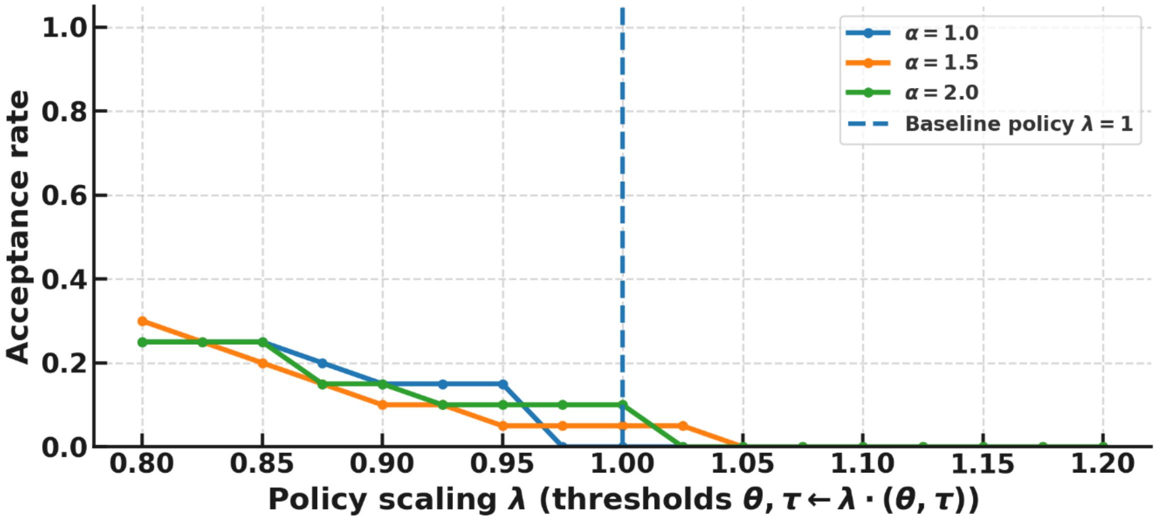

Figure 10. Operating frontier: acceptance vs. policy scaling and hedge strength.

Usage: In surge conditions (rumors high), increase α (more cautious) and/or $\lambda > 1$. During verified briefings, reduce α or set $\lambda < 1$ to avoid over-blocking, while still testing robustness N.

10.2 Reader-facing explainability ("why this ticker was/wasn't published")

Because decisions are computed from interpretable sup-min overlaps and trigger-specific functors, we can produce one paragraph rationales that balance informativeness and brevity (cf. explainability desiderata).

Template (auto-filled fields):

> ***Decision***: { Licensed/Flagged }.
> ***Trigger***: {T}. Feasibility $\Pi = \{\mathrm{x.xx}\}$, Robustness $N = \{\mathrm{y.yy}\}$; policy $(\theta_T, \tau_T) = (\theta, \tau)$.
> ***Reasoning***: The presupposition { paraphrase } is { well/insufficiently } supported by { source set } with reliabilities {r_s}; the maximal supporting window is around { lag } hours before now. { If flagged: } Suggest { Hedge α or rewrite }, which would yield Π', N' estimated from $H_\alpha[\pi]$.

Computation.

- The "maximal supporting window" is arg $\max\min(\pi^{\mathrm{agg}}, \mu_{\mathrm{Pres}})$ (already computed for Π).
- The paraphrase comes from g_T (e.g., again → "there was a prior τ-event recently"); this is a deterministic map per trigger.
- A what-if hedge preview uses $\pi' = H_\alpha[\pi]$ to recompute (Π', N') in milliseconds.

This explanation is model-faithful (no post-hoc proxy), short enough for an internal CMS panel, and auditable by logging ($\Pi, N, \theta, \tau, \alpha$).

10.3 Cross-lingual extension (e.g., English-Kannada, English-Hindi)

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Goal. Apply the same pipeline to multilingual tickers with minimal re-engineering.

Steps.

(i) ***Trigger inventory mapping***: Build bilingual lexicons for the small set of presupposition triggers (iteratives, aspectual change-of-state, additives, factives). Many have close lexical/morphological counterparts (e.g., again/wieder/dubbaLe-type iteratives; aspectual auxiliaries), for which the same functor g_T applies with local parameterization.

(ii) ***Morpho-syntax adapters***: Provide tokenization + light morph analysis to capture aspect and polarity; functors operate on timelines and alternative sets, independent of surface order.

(iii) ***Cross-lingual calibration***: Learn per-language (θ_T, τ_T) and hedge priors α using small, annotated batches. Since Π, N are language-agnostic operations on π and μ, only functor parameters (e.g., recency scale β) need retuning.

(iv) ***Script/encoding and named entities***: For additive triggers (too/also), salience depends on named-entity mentions; use language-specific NER but the presupposition functor stays unchanged.

10.4 Integration and auditing in the CMS

- ***Decision card (per ticker)***: shows T, Π, N, thresholds, argmax lag, top-2 sources by contribution (from OWA weights), and one-click Hedge/Rewrite actions.
- ***Batch policy tuning***: vary λ and α with frontier previews (Figure 10) before a breaking event.
- ***Safety guarantees***: Idempotence and monotonicity lemmas ensure stable behavior under repeated tickers and hedging.
- ***Audit logs***: store tuples ($u_t, T, \Pi, N, \theta, \tau, \alpha$, decision) for compliance and post-mortems.

10.5 Deployment checklist

(i) ***Policy***: choose initial (θ_T, τ_T) by trigger; encode newsroom risk appetite by α and λ.

(ii) ***Data***: initialize r_s and freshness decay; set OWA weights $\mathbf{w}$.

(iii) ***NLP***: minimal trigger detection + morph adapters; no heavy ML required.

(iv) ***Calibration***: after a week of use, fit c_T and adjust (θ_T, τ_T) via convex routines.

(v) ***Explainability***: enable the one-paragraph rationale; adopt what-if hedging previews (fast, exact).

11. Limitations and Ethics

11.1 Theoretical limitations

(L1) Expressivity of possibility vs. probability: Our account deliberately chooses non-additive possibility/necessity (Π, N) to represent incomplete, conflict-heavy evidence. This gains robustness under sparse data but loses some inferential properties (e.g., Bayes-style likelihood composition). In mixed environments (well-counted beats like weather/sports vs. breaking crime), hybrid layers may be preferable, with probabilistic modules feeding constraints into π_t rather than replacing it.

(L2) Threshold sensitivity: Licensing depends on trigger-specific θ_T, τ_T. While we proposed calibrated learning, small shifts can flip borderline cases when Π and N cluster near policy cutoffs. Hedge operators H_α partially mitigate this by trading feasibility for robustness in a predictable way, but do not remove the basic sensitivity.

(L3) Independence assumptions: Proofs for commutativity and idempotence rely on independence between updates or on Gödel's $T_{\min}$ behavior. In tightly coupled narratives (e.g., chained appositions), presuppositions may be mutually supporting or inhibiting; our pipeline handles this by ordered testing and retesting after each update, at the cost of extra passes.

(L4) Piecewise-linear approximation: Efficient evaluation represents π, μ as trapezoids/triangles, which is a good first-order fit for recency/continuity but an approximation to rich temporal structures (e.g., multi-peak priors after rolling coverage). The approximation can under- or over-estimate Π by at most the local interpolation error; a finer breakpoint budget K reduces this bound.

(L5) One-dimensional timeline: We modeled worlds by a lag axis for clarity and speed. Real contexts may require extra axes (location, actor identity, event subtype). Extending W to a small
product of interpretable axes preserves our sup-min machinery and caching but raises bookkeeping and UI complexity.

11.2 Linguistic limitations

(L6) Trigger polysemy and register. Items like still or already vary by register and dialect; again, has restitutive vs. repetitive readings [17,18]. Our functors g_T capture the core readings; disambiguation in practice needs a minimal morpho-syntactic adapter and surface context checks.

(L7) Additives and salience. Too/also presuppose a salient alternative. We operationalized salience via mention/NER cues; under code-mixing or local scripts, NER errors can depress μ_{Pres} spuriously. Editors should expect occasional under projection in highly noisy or multilingual strings and rely on the manual override.

(L8) Cross-lingual transfer. Mapping triggers across English Kannada/Hindi is feasible for iteratives, additives, and change-of state morphology, but factives and scalar particles may not align one-to-one. Our proposal uses language-specific parameters (recency scale β, hedge priors, expectation profiles) with the same core tests.

11.3 Evaluation and data limitations

(L9) Annotation subjectivity. Gold "acceptability" for presuppositions is graded and sometimes culture specific. Even with detailed guidelines, expect moderate IAA; calibration procedures and reliability diagrams reduce harm but cannot eliminate editorial drift.

(L10) Dataset shift. During crises, reliability weights r_s and freshness decay change quickly; stale priors inflate Π in the wrong regions. Our governance knobs (α, λ) and dominance policy reduce risk but require vigilant monitoring.

11.4 Ethical considerations

(E1) False certainty and over-blocking. Automated gating risks silencing warranted but emergent reports from smaller outlets. We therefore (i) separate feasibility Π from robustness N, (ii) expose policy dials (α, λ), and (iii) keep a clear human-in-the-loop override with logged rationale.

(E2) Transparency and accountability. The system generates faithful rationales tied to the exact overlaps and argmax windows used in the decision, avoiding post-hoc proxy explanations.

(E3) Privacy and source protection. Storing per-source r_s and contribution traces can expose newsroom strategy. Logs should (a) role-gate source names, (b) retain only hashed identifiers in external audits, and (c) allow redaction without breaking reproducibility of (Π, N) calculations.

(E4) Fairness across languages and communities. Trigger inventories and salience heuristics may perform worse on under-resourced languages. We recommend per-language calibration and regular error slicing reports by script/language to detect systematic under projection.

(E5) Adversarial content. Coordinated campaigns can attempt to "manufacture" salience for too/also or flood recent-evidence bands to lift again. Freshness-aware r_s, capped salience, and rate-limited accommodation (λ) mitigate this.

12. Conclusion and Future Work

12.1 Future work

Multimodal cues: Incorporate lower-third graphics, onscreen timestamps, and map/text overlays as additional axes in W, preserving the same sup-min calculus.

Richer trigger coverage: Extend to counterfactuals and soft presuppositions (e.g., even), with functors that track scalarity and focus alternatives.

Active calibration: Online isotonic or spline calibrators c_T updated nightly from editorial feedback, with constraints ensuring monotonicity and smoothness.

Cross-lingual deployment: Parameter-tying across languages with small per-language adaptation (recency scale, hedge priors), plus periodic error slicing.

Human factors: Controlled newsroom trials of frontier tuning (α, λ) to quantify speed/accuracy trade-offs and establish best-practice playbooks for different desks.

12.2 Summary of contributions

Overall, this study shows that presupposition in breaking-news tickers can be handled with a rigor that matches newsroom realities without overpromising statistical precision. By grounding the analysis in possibility logic, we separate two complementary notions **feasibility**

(Π) and **robustness** (N) and tie licensing to transparent, trigger-specific thresholds (θ_T, τ_T). The result is a gatekeeping procedure that is interpretable end-to-end: sources are discounted by reliability and aggregated (OWA), hedges act as predictable operators on the context, and successful assertions update the context via min-narrowing with an optional conditioning-style normalization. Mathematically, we retain desirable properties (idempotence, monotonicity, Lipschitz stability; crisp reduction as a special case) and achieve practical speed through piecewise-linear representations and caching.

Empirically, even a small, realistic case study makes the editorial trade-offs visible: feasibility and robustness can diverge in early reporting, hedges predictably trade recall for safety, and policy dials (α, λ) let desks choose where to sit on the acceptance–caution frontier. The approach remains linguistically faithful by encoding triggers as presupposition functors (iteratives, continuity, additives, factives) while staying operational enough to surface "why/why not" rationales for editors. At the same time, we are candid about limits: threshold sensitivity, approximation on a one-dimensional timeline, cross-lingual nuances, and the social risks of over-blocking or false certainty. These are mitigated not eliminated by calibration, transparent logs, human-in-the-loop overrides, and per-language tuning.

In practice, the framework offers a compact playbook for high-velocity contexts: measure Π and N, enforce both, expose dials, explain decisions, and update the context only when licensed. With modest engineering, it can extend to richer triggers, multimodal cues, and multilingual streams while keeping the same core calculus. The broader takeaway is that editorial prudence need not be ad hoc: a small, interpretable layer of graded semantics can make fast news safer and more accountable without slowing it to a crawl.

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