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SEMILINEAR SPACES OVER SPECIAL SEMIRING

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Abstract

The main motive of this paper is to develop results for semimodules over special semiring identical to linear spaces over fields. This paper is divided into two parts; in first part we have given some preliminaries. Also we obtain analogue of ring and module theoretic concepts for special semiring. In second part we prove rank-nullity theorem in semilinear spaces over special semiring called strict semidomain. To give examples of special semirings, we have used Lattice theory.

Keywords: strict semidomain, semilinear spaces, kernel, lattice, dimension.

Introduction

Cuninghame evolved a theory over strict semirings similar to how linear algebra works over fields. [1]. A great deal of research on strict semirings is released in [2, 3, 4, 5, 6, 7, 8, 9]. Qian Shu and Wang proved some results about the dimensions of semilinear subspaces and direct sums in 2013[10]. Here we proved Rank-Nullity theorem for semimodules over special semiring called strict semidomain similar to that of linear spaces over fields.
Some preliminaries are given in part 2 and Rank-Nullity theorem in semilinear spaces over strict semiring is proved in part 3.

Preliminaries

Definition 2.1[11]: A non-empty set S with the binary operations "+" and "•" represented by (S, +, •) is called a semiring subject to the following conditions

i) Commutative monoid (S, +, 0) with identity member 0.
ii) A monoid with identity element 1 is (S, •, 1).
iii) $(a + b) \bullet c = a \bullet c + b \bullet c$ and $a \bullet (b + c) = (a \bullet b) + (a \bullet c)$, for all $a, b, c \in S$.

Definition 2.2 [11, 12]: If for every $a, b \in S$, $a \bullet b = b \bullet a$, then the semiring (S, +, •) is commutative.

Note that unless otherwise mentioned S is a commutative semiring throughout this paper.
Example 2.1 [12]: Q_0^+, the set containing all positive rational numbers with zero is a commutative semiring.

Theorem 2.1 [12]: Let $(\mathscr{L}, \wedge, \vee)$ be a distributive lattice. Then $\mathscr{L}$ is a commutative semiring.
Example 2.2 [12]: Let S be a finite semiring, as indicated by the distributive Lattice below.

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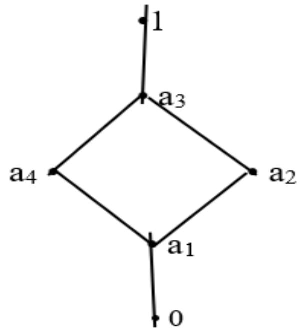
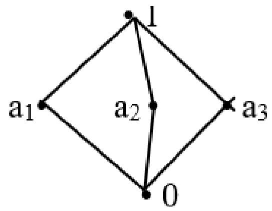
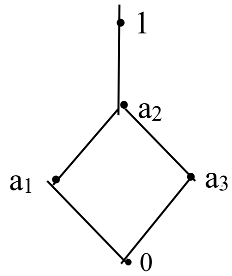


Corollary 2.1 [12]: Every chain lattice is a semiring.

Note 2.1 [12]: Every lattice is not a semiring.
Example 2.3 [12]: Following Lattice $\mathscr{L}$ doesnot hold distributive property. Hence $\mathscr{L}$ is not a semiring.
Hasse diagram:



Definition 2.3 [11]: $\mathcal{T}$, subset of a semiring S. If $\mathcal{T}$ is a semiring, then $\mathcal{T}$ is a subsemiring of S.
Example 2.4 [12]: A subsemiring of the set of non-negative rational numbers is the set of non-negative integers.
Definition 2.4 [12]: Let S be a semiring then an element a ∈ S, a ≠ 0 is called zero divisor of S if a • b = 0 for some b ∈ S, b ≠ 0.
Example 2.5: See the semiring S indicated by below lattice



Here $a_1 \neq 0, a_3 \neq 0$ but $a_1 \bullet a_3 = 0$ for a_1, a_3 ∈ S i.e. a_1 and a_3 are zero divisors of S.
Definition 2.5[11]: Let S be a semiring. S is said to be a Semidomain if in S, $\alpha \bullet \beta = 0$ implies either $\alpha = 0$ or $\beta = 0$ for $\alpha, \beta \in$ S (that is S has no zero divisors).
Example 2. 6 Every Chain lattice is a Semidomain.

Definition 2.6 [9, 11, 12]: Let the semiring be S. If there is an element β ∈ S for α ∈ S such that $\alpha \bullet \beta = \beta \bullet \alpha = 1$ then in S, α is referred to as an invertible element while element β, represented by α^{-1}, is the multiplicative inverse of α.
Note: In semiring S, U (S) represents the set of all invertible elements.
Definition 2.7 [11]: A commutative semiring S in which every nonzero element has an inverse with respect to multiplication is called a semifield.
Example 2.7 [12] Q_0^+ (set of non negative rational numbers) is a semifield.
Note that every semifield is a semidomain. However, the opposite is untrue.
Example 2.8 Z_0^+, the set containing all positive integers

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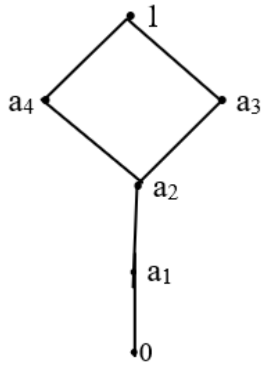
with zero is a Semidomain which is not a semifield.
In case of ring theory, every finite integral domain is a field but in case of semiring, a finite semidomain is not necessarily a semifield.

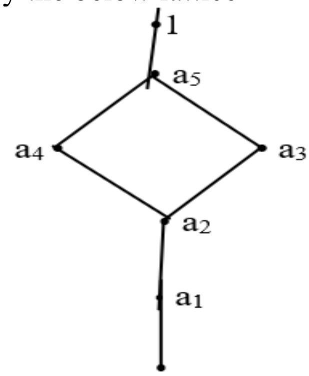
Example 2.9 Every finite chain lattice other than C_2 is a finite semidoamin but it is not a semifield.
Definition 2.8 [11, 12]: Let the semiring be S. If $\alpha + \beta = 0$ indicates that $\alpha = 0$ and $\beta = 0$ for $\alpha, \beta \in$ S, then S is referred to as a Strict Semiring.
Note that strict semiring is also called as zerosumfree semiring.
Example 2.10 [12]: Z_0^+, the set containing all positive integers with zero is a Strict Semiring.
Example 2.11: Consider a semiring S given by the following lattice



Here S is a strict semiring.

Definition 2.9: Let the semiring be S. If
i) $\alpha + \beta = 0$ indicates that $\alpha = 0$ and $\beta = 0$ for $\alpha, \beta \in$ S and
ii) $\alpha \bullet \beta = 0$ indicates that $\alpha = 0$ or $\beta = 0$ for $\alpha, \beta \in$ S
then S is said to be a Strict Semidomain.

Example 2.12 1) Z_0^+, the set containing all positive integers with zero is a strict semidomain.
2) Consider a semiring S indicated by the below lattice



Then S is a strict semidomain.
Note that every Chain lattice is a Strict Semidomain.

Definition 2.10: [12]: Let the two semirings be S and S′. Semiring homomorphism refers to the mapping ψ**:** S → S' if $\psi(\alpha + \beta) = \psi(\alpha) + \psi(\beta)$ and $\psi(\alpha \bullet \beta) = \psi(\alpha) \bullet \psi(\beta)$ for every $\alpha, \beta \in$ S.
ψ is a semiring isomorphism if ψ is one one and onto map.

Definition 2.11 [11, 12]: Let the strict semiring be S. A commutative monoid (M, +) with additive identity 0_M for which the function S × M → M represented by (s, m) → s m, is known as scalar multiplication and which satisfies the following conditions for all elements s, s'∈ S and all elements

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m, m' ∈ M
1) (s s') m = s (s' m)
2) s (m + m') = s m + s m'
3) (s + s') m = s m + s' m
4) 1_S m = m
5) s $0_M = 0_M = 0_s$ m
is called a S-Semimodule.

Example1 2.13: 1) Let S be a semiring. The cartesian product S × S × …× S (n times) = S^n such that ($x_1, x_2, \dots x_n$), ($y_1, y_2, \dots, y_n$) in S^n, $\alpha \in$ S under the addition ($x_1, x_2, \dots, x_n$) + ($y_1, y_2, \dots, y_n$) = ($x_1 + y_1, x_2 + y_2, \dots, x_n + y_n$) and scalar multiplication α ($x_1, x_2, \dots, x_n$) = ($\alpha x_1, \alpha x_2, \dots, \alpha x_n$) is a semimodule over S .
2) Let C_n be a chain Lattice then M = $C_n \times C_n \times \dots \times C_n$ (m times) is a semimodule over C_n and M is denoted by C_n^m.

Definition 2.12[11]: If and only if N is closed under addition and scalar multiplication and N contains additive identity 0 then a non-empty subset N of a S -Semimodule M is a subsemimodule of M.

Definition 2.13[9]: The semimodule M over strict semiring S is called an S-semilinear space.
A subspace of an S- semilinear space is the subsemimodule of semimodule M over strict semiring S. Unless otherwise mentioned semiring S is assumed to be semidomain.
Note 2.2: Semilinear space elements are called vectors.

Definition 2.14[9, 12]: A subset A of a semilinear space M over a strict semiring S is said to form spanning set of M, if every V in M is a linear combination of vectors $V_1, V_2, \dots, V_m \in$ A that is there exists scalars $a_1, a_2, \dots, a_m$ in S such that V = $a_1V_1 + a_2V_2 + \dots + a_mV_m$.
Note 2.3: Spanning set of a semilinear space M is also called generating set of M. M is called finitely generated semilinear space if it has finite spanning set.

Definition 2.15[9, 12]: The vectors of a semilinear space M over strict semiring S are called linearly independent, if none of them can be represented as linear combination of the remaining others. Otherwise these vectors are linearly dependent.

Definition 2.16[9, 12]: Let A be a linearly independent subset of a semilinear space M over the strict semiring S. A is referred to as a basis of the semilinear space M if it spans the semilinear space M.
For a semiring S we have defined S^n as follows
S^n = S × S ×….× S (n times) is a semilinear space over strict semiring S, under the following addition and scalar multiplication: For X =($x_1, x_2, \dots, x_n$), Y =($y_1, y_2, \dots, y_n$) ∈ S^n and $\alpha \in$ S, X + Y = ($x_1, x_2, \dots, x_n$) + ($y_1, y_2, \dots, y_n$) = ($x_1 + y_1, x_2 + y_2, \dots, x_n + y_n$).
and α X = α ($x_1, x_2, \dots, x_n$) = ($\alpha x_1, \alpha x_2, \dots, \alpha x_n$).

Note 2.4[9, 12]: In S-semilinear space, the number of elements in each basis may vary.
Definition 2.17[9]: A free S-semilinear space is an S-semilinear space which has a basis over S, where S is a strict semiring.
Definition 2.18[9]: In an S-Semilinear space S^n, two elements $V_1 = \left(v_{11}, v_{12}, \dots, v_{1n}\right)$, $V_2 = \left(v_{21}, v_{22}, \dots, v_{2n}\right) \in S^n$ are said to be orthogonal if V_1

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$$V_2^T = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \\ \vdots \\ v_{2n} \end{bmatrix} = 0 \text{ and } V_1 V_1^T, V_2 V_2^T \in U(S).$$

Theorem 2.2[9]: An orthogonal subset of an S-semilinear space S^n over strict semiring S is linearly independent.

Note 2.5 The set $\{e_1, e_2, \ldots, e_n\}$, where $e_1 = (1, 0, \ldots, 0_n)$, $e_2 = (0, 1, \ldots, 0_n)$, …, $e_n = (0, 0, \ldots, 1_n)$ is an orthogonal subset of S^n. Therefore the set $\{e_1, e_2, \ldots, e_n\}$ is linearly independent (by theorem 2.3). Also $\{e_1, e_2, \ldots, e_n\}$ spans S^n and hence it is a basis of S^n. $\{e_1, e_2, \ldots, e_n\}$ is called as standard orthogonal basis of S^n.

Definition 2.19[9]: If s = $\alpha + \beta$ implies that s = α or s = β, for all $\alpha, \beta \in$S then s ∈ S is called an additive irreducible element of S, where S is a strict semiring.

Theorem 2.3[9]: In S^n, each basis has the same number of elements if and only if 1 is an additive irreducible element, where S^n is a semilinear space over strict semiring S.

Corollary 2.2[9]: 1 being additive irreducible element in strict semiring S, $\{X_1, X_2, \ldots, X_n\}$ is a basis of S-Semilinear space S^n if and only if, $X_1 = (x_{11}, 0, \ldots, 0)$, $X_2 = (0, x_{22}, \ldots, 0)$, …, $X_n = (0, 0, \ldots, x_{nn})$, with $x_{ii} \in U(S)$ for any i, $1 \leq i \leq n$.

Corollary 2.3[9]: If U (S) = {1} then $\{e_1, e_2, \ldots, e_n\}$ is the unique basis of S^n, where S^n be a semilinear space over strict semiring S with 1 as an additive irreducible element.

Note that in corollary 2.2 for all $x_{ii} \in U(S) = \{1\}$ means $x_{ii} = 1$. Therefore each $X_i = e_i$, $1 \leq i \leq n$.

Definition 2.20[9]: If any two bases of a free semilinear space M over strict semiring S have the same number of elements then the number elements in a basis is the dimension of M over S and is represented by $dim_S\, M$.

Note that any two bases of S^n have the same number of elements only when S is a strict semiring with 1 as an additive irreducible element. Therefore, $dim_s\,(S^n) = n$.

Theorem 2.4[9]: A set $\{X_1, X_2, \ldots, X_n\}$ is a basis of S^n if and only if it is orthogonal, where S^n be n-dimensional Semilinear space over a strict semiring.

Corollary 2.4[9]: In S^n, the number of elements in every orthogonal set is not more than n where S^n is n-dimensional S- Semilinear space.

Note 2.6: As S is finite strict semidomain, S^n and every semilinear subspace M of S^n is finite. Thus M has finite generating set which implies it has a basis. Therefore every semilinear subspace of S^n is free.

Definition 2.21[14]: T: $S^n \to S^m$ be a linear transformation which fulfills the constraints

i) T $(X_1 + X_2)$ = T (X_1) + T (X_2), $X_1, X_2 \in S^n$ ii) T (aX) = a T (X), a ∈S, X∈ S^n

Constraints (i) and (ii) are equal to just one constraint

T (aX_1+bX_2) = a T (X_1) + b T (X_2), a, b ∈S, $X_1, X_2 \in S^n$.

Definition 2.22[14]: T: $S^n \to S^m$ be a linear transformation for which kernel is the set of all vectors X ∈ S^n such that T(X) = 0.

Definition 2.23[14]: T: $S^n \to S^m$ be a linear transformation for which range is the set of all vectors X' ∈ S^m such that T (X) = X' where X ∈ S^n.

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Theorem 2.5[14] If T: $S^n \to S^m$ be a linear transformation then kernel and range of T are semilinear subspaces of S^n.

Definition 2.24[14]: The dimension of range of a linear transformation T is referred to as a rank and it is represented by Rank (T). Also the dimension of kernel of a linear transformation T is referred to as a nullity and represented by Nullity (T).
Hamming Weight: Let $X \in S^n$, $X = (x_1, x_2, \ldots, x_n)$ then w (X) = Number of non-zero x_i, $1 \le i \le n$.

Example 2.14: Let $X \in S^5$, X = (2, 0, 3, 0, 1) then w(X) = 3.
Rank-Nullity theorem in semilinear spaces over strict semidomain
Theorem 3.1 Let S be a finite strict semidomain, $1 \in S$ is an additive irreducible element, 1 + 1 = 1, U (S) = 1 and T: $S^n \longrightarrow S^n$ be a linear transformation. If $V_1, \ldots\ldots, V_k$ is an orthogonal basis of range of T with $w(V_i) = w_i$, $w_1 + w_2 + \ldots\ldots + w_k = m$ then Rank (T) + Nullity (T) = k + n - m.
Proof: Let $\{e_1, e_2, \ldots\ldots, e_n\}$ be a standard orthogonal basis of S^n.
Now $\{V_1, \ldots, V_k\}$ is an orthogonal basis of range of T and hence without loss of generality we can assume

$V_1 = (1, 1, \ldots, 1w_1, 0, \ldots, 0)$

$V_2 = (0, 0, \ldots, 0w_1, 1, 1, \ldots, 1w_2, 0, \ldots, 0)$

$V_k = (0, 0, \ldots, 0w_1, 0, 0, \ldots, 0w_2, 0, \cdots, 0, 0, 0, \ldots, 0w_{k-1}, 1, 1, \ldots, 1wk, 0, \ldots, 0)$

Let $M = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \end{bmatrix}$ and $X = (x_1, x_2, \ldots, x_n) \in \ker T$.

$Therefore$, T (X) = $MX^T = 0$

It means $T(X) = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = 0$

$Therefore$, T (X) = $MX^T = 0$

$$T(X) = \begin{bmatrix} 1 & 1 & \cdots & 1w_1 & 0 & 0 & \cdots & 0w_2 & \cdots & 0 & 0 & \cdots & 0w_k & \cdots & 0 \\ 0 & 0 & \cdots & 0w_1 & 1 & 1 & \cdots & 1w_2 & \cdots & 0 & 0 & \cdots & 0w_k & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0w_1 & 0 & 0 & \cdots & 0w_2 & \cdots & 1 & 1 & \cdots & 1w_k & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

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Thus, $T(X) = \begin{bmatrix} x_1 + x_2 + \ldots + x_{w_1} \\ x_{w_1+1} + \ldots + x_{w_1+w_2} \\ \vdots \\ x_{w_1+w_2+\ldots+w_{k-1}+1} + \ldots + x_{w_1+w_2+\ldots+w_k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

Which implies that $x_1 + x_2 + \ldots + x_{w_1} = 0$,

$x_{w_1+1} + \ldots + x_{w_1+w_2} = 0$,

$x_{w_1+w_2+\ldots+w_{k-1}+1} + \ldots + x_{w_1+w_2+\ldots+w_k} = 0$

Since S is a strict semiring, hence

$x_1 = x_2 = \ldots = x_{w_1} = x_{w_1} + x_{w_2} = x_{w_1+w_2+\ldots.+w_{k-1}+1} = \ldots = x_{w_1+w_2+\ldots+w_k} = 0$

But $w_1 + w_2 + \ldots + w_k = m$

Therefore, $x_1 = x_2 = \ldots = x_m = 0$.

Thus, $X = (0\ldots, 0_m, x_{m+1}\ldots, x_n)$.

Hence $X \in kerT$ implies that $X = \sum_{i=m+1}^{n} x_i e_i$

Also each $e_i \in kerT$ for $m + 1 \leq i \leq n$

Therefore, $kerT = \langle e_{m+1}, e_{m+2}, \ldots, e_n \rangle$.

Thus, $\{e_{m+1}, e_{m+2}, \ldots, e_n\}$ is a basis of kerT.

Hence dim (kerT) = n - m

Therefore, dim (Range T) + dim (ker T) = k + n – m.

That is Rank (T) + Nulity (T) = k + n – m (by definition 2.24).

Example 3.1 Let T: $S^5 \to S^5$ be a linear transformation on semilinear space S^5 defined as $T(a_1, a_2, a_3, a_4, a_5) = (a_1, a_1, a_1, a_2, 0)$.

Then we can write $T(a_1, a_2, a_3, a_4, a_5) = (a_1, a_1, a_1, a_2, 0) = a_1 (1, 1, 1, 0, 0) + a_2 (0, 0, 0, 1, 0)$.
Denote $V_1 = (1, 1, 1, 0, 0)$ and $V_2 = (0, 0, 0, 1, 0)$. Here $V_1V_2^T = 0$. Therefore, the set $\{V_1, V_2\}$ is orthogonal set. Hence it is linearly independent. Also the set $\{V_1, V_2\}$ spans range of T. Therefore, it is a basis of range T. Thus, *dim* (*RangeT*) = 2.

Here $w(V_1) = 3$ and $w(V_2) = 1$. Therefore, $m = w(V_1) + w(V_2) = 4$.

Let $M = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ and $B = (b_1, b_2, b_3, b_4, b_5) \in kerT$.

Therefore, $T(\mathrm{B}) = MB^T = 0$.

Implies that $\begin{bmatrix} b_1 + b_2 + b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Therefore, $b_1 + b_2 + b_3 = 0$, $b_4 = 0$.

Since S is a strict semiring. It implies that $b_1 = b_2 = b_3 = b_4 = 0$. Thus, $X = (0, 0, 0, 0, x_5)$. Hence $kerT = \langle (0, 0, 0, 0, 1) \rangle$.

Therefore, *dim* (*kerT*) = 1.

Thus, *dim* (*RangeT*) + *dim* (*kerT*) = 2 + 1 = 3 and $k + n - m = 2 + 5 - 4 = 3$. Hence the result.

Corollary 3.1 Let S be a finite strict semidomain, S^n be a finite dimensional S-semilinear space, T: $S^n \longrightarrow S^n$ be a linear transformation. If $\{V_1, V_2, \ldots, V_k\}$ is an orthogonal basis of RangeT with $w(V_i)$

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= 1, for all i then Range T = S^k and Rank (T) + Nullity (T) = n.
Proof: Let S^n be a finite dimensional S-semilinear space and $\{e_1, e_2\ldots, e_n\}$ be a standard orthogonal basis of S^n. Also T: $S^n \longrightarrow S^n$ be a linear transformation and range of T is a semilinear subspace of S^n (by theorem 2.5).
$\{V_1, V_2\ldots, V_k\}$ is an orthogonal basis of RangeT and w (V_i) = 1, for all i. Then $w1 + w2 + \cdots + wk = k$.

Hence Range T = S^k .
Thus, dim (RangeT) + dim (kerT) = k + n – k = n.
That is Rank (T) + Nullity (T) = k + n – k = n (by definition 2.24).

Conclusions:

In this contribution we have proved Rank-Nullity theorem in semilinear spaces over special semiring called strict semidomain.

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Declarations
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