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MATHEMATICAL MODELING AND CONTROL OF TWO-LINK FLEXIBLE MANIPULATORS WITH ACTUATOR DYNAMICS AND NONCOLLOCATED FEEDBACK

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Abstract

Modern applications in aerospace, biomedical and industry can make use of flexible-link robotic manipulators because they are lightweight, work fast and are energy efficient. Still, the inclusion of flexibility, special actuator dynamics and remote feedback makes it harder to model and control flexible linkages. A two-link flexible manipulator is mathematically modelled in this study using Euler-Bernoulli theory, Lagrangian mechanics and also includes second-order actuator dynamics with noncollocated sensors in the state-space version. Handling the resulting complexity and the issues of tracking states, a new Time-Varying Sliding Mode Controller (TSMC) is proposed. To ensure accurate movement and good vibration control, adaptive polynomial trajectories and disturbance rejection are applied by the controller. Simulations in MATLAB with a 0.02s time step and a duration of 80 seconds show that the proposed three-stage controller brings about both smooth joint movement and early damping of the cyclic deformations within the structure frame (d11–d22). The outcomes confirm that the controller functions well despite sensor delay, flexible buildings and unaccounted for dynamics. The framework as proposed is useful for controlling fast flexible manipulators and is simple to expand for use with experimental and AI solutions in future robotics.

Keywords: Flexible-Link Manipulator, Time-Varying Sliding Mode Control (TSMC), Noncollocated Feedback, Actuator Dynamics, Vibration Suppression

1. Introduction

1.1 Background and Motivation

More and more, modern robotic systems rely on flexible robotic manipulators because they are light, efficient with energy and able to make fast, nimble movements. As a result, these features are most helpful in industries that require fast, minimally resistant motions such as space exploration, minimal surgery and precision manufacturing. Compared to rigid-link manipulators, flexible robots are more responsive and use less shifting load weight. Because they function well in places with limited resources, they attract attention from both Earth and space applications. Limiting a manipulator's design to strict movements is seen as less ideal with advancing robotics, so including flexibility is now widely considered important for better system results [1][4].

At the same time, making manipulators flexible creates additional issues in modeling, analysis and control. Conventional robotic systems explain their movements by assuming rigidity, as this makes both modeling and control easier. However, flexible manipulators behave differently; distributing

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their parameters means deformations within the structure must be precisely modelled. Coupling between the rigid-body and flexible vibrations makes the behavior much more complex, leading to the use of advanced modeling methods.

1.2 Challenges

The biggest problem with using flexible manipulators is controlling their vibration. Because the links are flexible, the system moves in a back-and-forth fashion, mainly when the device is moving quickly or when something outside affects it. Incorrect vibration balance can lead to bad trajectory tracking and instability of the system. As a result, suppressing vibrations in the controller design is very important for flexible manipulators.

We must also consider how actuators behave when designing the system. Like most robotic systems, the actuators in use are not ideal and come with delays, saturation effects and some nonlinearities. When these dynamics aren't part of the system model, precision in controlling the system can decrease. Many times, controllers that do not include actuator behavior work well in simulation, but do not produce satisfying results when used in reality [3][7].

A less noticeable but key problem is having feedback given outside the context of the original speaker's remarks. In many flexible manipulators, the sensors that check system information such as positions, velocities and deformations are not often found where the actuators are. This distance between parts interferes with synchronization, creates issues in assessment of parameters and noticeably reduces the control system's responsiveness and stability. This arrangement is especially risky in systems handling critical or prompt operations, since very small issues can greatly affect the entire system [9][10].

1.3 Objectives

The purpose of this study is to:

- Integrate into the model both the dynamics of the actuators and the flexibility of the main structure.
- Formulate a sturdy approach that can effectively control the complexities of noncollocated feedback.
- Show that the control scheme provides the required results by testing it in simulations and observing proper position following and a lowered vibration level.

1.4 Contributions

The research has made various helpful contributions that fill in different empty areas in existing publications. Combining Euler-Bernoulli beam theory and Lagrangian mechanics in the model lets the study properly simulate the flexible movements of the manipulators. Adding actuator dynamics and flexible modes explicitly to how the equations move makes the model much more accurate than some simplified models.

In addition, a Time-Varying Sliding Mode Controller is implemented to address system errors as they develop over time. The trajectory of errors is formed through polynomial models and is guided quickly to the target trajectory using a sliding surface. This framework is designed to address the uncertainties and nonlinearities common in flexible robots [10][13].

Third, the research specifically examines the problem of noncollocated feedback. Unlike traditional methods that might either omit or oversimplify effects related to separation of sensors and actuators, this paper proposes a control solution that relies on adaptive feedback and inverse dynamics to identify unobserved states. Thanks to this innovation, users can rely on their controllers in real systems even when it's not simple to get the best sensor placement.

Ultimately, the work uses a strong simulation environment to assess the system's performance in multiple conditions. The computer results reveal that the proposed control method achieves its goal,

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suppresses vibrations and maintains stability, even when sensing conditions are not best. Simulation results let us see and prove how the model and control solutions work correctly.

All of this research has greatly improved the modeling and control of flexible-link manipulators. They shape the direction of future work by suggesting further research in 3D motion, systems with many links and sensible control strategies. The study initiates the development of swift and precise new robotic manipulators by dealing with combined problems in flexibility, actuator dynamics and noncollocated feedback.

2. Literature Review

2.1 Modeling of Flexible Manipulators

When the flexible structure of a robotic arm plays a major role in its operating behavior, detailed modeling is crucial in the advanced design of such robotic systems. Two major ways to model such systems are the Euler-Bernoulli beam theory and the Timoshenko beam theory, according to research findings. Its broad use comes because it is simple and requires little time to work out. It agrees that beam cross-sections and therefore each fiber in a cross-section, always lie flat and perpendicular to the neutral axis when the beam is bent. Since this assumption applies to slender beams and small elastic changes, it is often used in the initial design and study of flexible robotics [4][5].

Adding shear deformation and rotary inertia, the Timoshenko beam theory upgrades the Euler-Bernoulli model. As a result, the model is more accurate for short and thick beams, as well as beams exposed to high-frequency vibrations. Even so, implementing Timoshenko models in real-time control is less common, except when a high degree of accuracy is expected. Many scientists apply an approach that uses Euler-Bernoulli assumptions for the controller, switching to Timoshenko models for in-depth dynamic studies.

Unlike traditional robotic arms, flexible manipulators have systems that show distributed parameter dynamics. Because the links hold any possible relationship, engineers must reduce the number of possibilities when they model the system. Often, the assumed modes method, FEM and modal truncation techniques are used to reduce the model without removing important details of its dynamics. These approaches have been added to robotic system modeling systems to simulate bending, vibration and how rigid and flexible movements interact.

Latest developments in computational modeling allow us to include flexible dynamics in control-based systems. For instance, Subedi et al. [4] have fully reviewed approaches to modeling flexible links and stressed the importance of using efficient methods to keep the model both accurate and practical. Furthermore, the researchers [5] showed that using fuzzy logic and genetic optimization along with these models allows the controllers to remain steady under varied or flexible conditions.

2.2 Actuator Dynamics in Robotic Systems

Generating motions for tasks is the responsibility of actuators in robotic systems. Nevertheless, actuators are not perfect, as their dynamics have to be measured accurately to maintain accurate control. You have to deal with motor inertia, friction, gear backlash and time delays when making these decisions. Trying to ignore system effects during design might negatively affect performance and even cause system failure in tough systems.

Actuator dynamics are now considered vital in both model and control design for robots. The authors [3] found that motor inertia and backlash change system behavior in important ways and that simpler models, by ignoring actuator details, may not represent this accurately. The errors show up more clearly in applications that require speed or high accuracy, including surgical robotics and systems for aerospace.

With advanced models, actuators are represented by differential equations of the second order to reflect how their movement is affected by inertia and damping. Most models are added to the dynamic

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equations for the manipulator, making the system describe both the mechanical and electrical properties of the robot. According to Hong et al. [14], a hybrid approach was suggested, relying on observers and singular perturbations to control actuators and to earn better results for accuracy and robustness.

Adding actuator models also helps create feedforward control that takes care of known delays and dynamic lags before they happen. Designers need these strategies to protect controllers that manage different currents, quick movements or unchanged environment conditions.

2.3 Noncollocated Feedback Control

This condition occurs when the system's sensors and actuators are not organized together but are physically separated. Often, the position or shape of the link in a flexible manipulator is checked at a middle location instead of next to the acting joint or motor. Because of the distance between each system, it becomes hard to control since delays and phase lags are introduced and it is not easy to observe the whole system.

Regular control approaches like PID fail in noncollocated scenarios due to the delays linked to sensor-actuator separation. Experts have solved these difficulties by coming up with observer-based control, sliding mode control and disturbance observers. The purpose of these techniques is to justify estimates for unmeasured system qualities and improve the system's response.

Meng et al. [9] improved the tracking and robustness of single-link flexible manipulators using noncollocated sensors, through a better dynamic coupling model. Sharma and Janardhanan used a sliding mode observer and functional estimation to overcome noncollocation and ensure the system works well even with uncertain dynamics. The researchers also improved the technology by bringing in fuzzy PI control and fractional disturbance observers to overcome the problems of noise and unmodeled dynamics common to noncollocated sensing methods.

There is growing use of adaptive and intelligent methods in this area. The methods modify these parameters as the loop is executed, relying on the information from performance in order to make up for the restrictions of noncollocated devices. Because today's robots are becoming more complex and work in distributed networks, the requirement for intelligent approaches will rise.

2.4 Gaps in the Literature

There is a great deal of literature on flexible manipulator models, actuator behaviors and noncollocated feedback, yet these are usually considered one at a time. Only a few frameworks look at all three elements together—flexibility, the way actuators work and how separated sensors and actuators are. This situation leads to control methods that do well in simulations but do not perform as expected where all factors are combined in real conditions.

Both Subedi et al. [4] and Gierlak [8] agreed that a holistic approach was necessary to bring all these subsystems together. Authors emphasize that successful outcomes depend on using models, control methods and signal processing techniques together. Even with requests for better policies, very few overall solutions have been created.

This difference creates a possibility for additional study. Further research should create models that can handle the full range of issues faced by flexible manipulators using practical actuators and sensors. Making these efforts will open paths to strong, solid and adaptive robotic systems that manage well in changing and unpredictable places.

3. System Description and Assumptions

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3.1 Two-Link Flexible Manipulator Configuration

The system being studied is a two-link serial manipulator and each link is flexible and powered by motors at the base joints. These joints are called revolute, so they can only rotate in a set direction. Links can be modelled flexibly by using modal coordinates which describe elastic deformation using mode shapes that are close to the real dynamic behavior of the links. This technique strikes a balance between how hard it is to model and how much computing power it requires and is well-adapted to planar manipulators assuming only a small amount of deformation. These links are represented by Euler-Bernoulli beam model, a common approach used in research because it adequately captures the main bending behavior and is not complicated [2][6].

3.2 Assumptions

The model and control design are simplified by introducing certain assumptions. A manipulator is only allowed to move in the horizontal plane which simplifies the equations around its motion. Behavior of materials is considered according to linear elasticity and deformations are not large, so geometric nonlinearities can be ignored. Also, the effects of friction at joints and backlash in the actuators are excluded from the analysis to study the main effects of flexibility and actuator dynamics [5].

3.3 Sensors and Actuators

Unlike collocated sensors which sit close to the actuators, these sensors are placed at various points along each element of the device. By detecting both changes in shape and angles, these sensors send vital feedback that helps operate the robotic system. The effect of both inertia and damping are modelled by second-order dynamics in actuators which makes the simulated system look more accurate and realistic [14][17].

4. Mathematical Modeling

4.1 Kinematic Modeling

The kinematic structure of the two-link flexible manipulator is described using Denavit-Hartenberg (D-H) parameters to assign coordinate frames along each link and joint. For each link, the homogeneous transformation matrix is defined as:

$$T_i^{i+1} = \begin{bmatrix} \cos\,\theta_i & -\sin\,\theta_i \cos\,\alpha_i & \sin\,\theta_i \sin\,\alpha_i & a_i \cos\,\theta_i \\ \sin\,\theta_i & \cos\,\theta_i \cos\,\alpha_i & -\cos\,\theta_i \sin\,\alpha_i & a_i \sin\,\theta_i \\ 0 & \sin\,\alpha_i & \cos\,\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where θ_i, d_i, a_i, and α_i are joint angle, joint offset, link length, and link twist, respectively. The overall kinematics are extended to include deformation by superimposing the flexible displacements using the assumed modes method:

$$y_i(x,t) = \sum_{n=1}^{N} \phi_n(x) q_n(t)$$

where $\phi_n(x)$ are mode shapes and $q_n(t)$ are time-varying modal coordinates.

Figure 1. Schematic of the two-link flexible manipulator in the planar workspace. The diagram illustrates joint angles θ_1 and θ_2, flexible deformations v_1 and w_2, link lengths x_1 and x_2, and sensor-actuator placements. Bending is confined to the motion plane, and modal deflections are shown along the flexible links.

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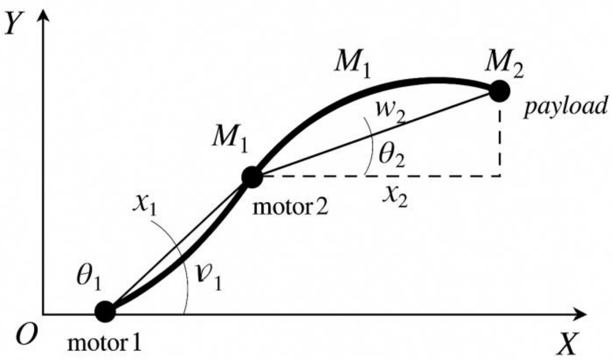

Figure 1:Schematic of the two-link flexible manipulator in the motion plane with bending deformations

4.2 Dynamic Modeling

The equations of motion are derived using Lagrange's formulation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$$

where the Lagrangian $L = T - V$, with T as the total kinetic energy and V as the total potential energy of the system, including both rigid and flexible components. The kinetic energy T includes:

$$T = T_{\text{rigid}} + T_{\text{flexible}} = \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

where $M(q)$ is the configuration-dependent mass matrix. The potential energy is derived from the elastic strain energy of the flexible links:

$$V = \frac{1}{2}q^T K q$$

where K is the stiffness matrix formed using the Euler-Bernoulli beam assumption:

$$K = EI\int_0^L \left(\frac{\partial^2 \phi(x)}{\partial x^2}\right)^2 dx$$

The Coriolis and centrifugal matrix $C(q, \dot{q})$ is derived from the Christoffel symbols associated with the mass matrix. The complete dynamic model is then:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + Kq = \tau$$

where τ is the generalized control input vector.

4.3 Actuator Dynamics

To improve model fidelity, actuator dynamics are included as second-order differential systems:

$$J\ddot{\theta}_a + B\dot{\theta}_a = u - \tau_{\text{load}}$$

where θ_a is the actuator angle, J is the actuator inertia, B is the damping coefficient, u is the motor input voltage or torque command, and τ_{load} is the load torque transmitted through the manipulator. These dynamics are cascaded with the manipulator equations. The Time-Varying Sliding Mode Control (TSMC) law computes the control torque τ based on the sliding surface:

$$S = Z(e - p)$$

and the composite control law:

$$\tau = D^{-1}\left(\underline{H} \pm \ddot{x}_d + \ddot{p} + Z_2^{-1}Z_1(\dot{p} - \dot{e}) - Z_2^T s(\cdots)\right)$$

where D is the reduced inertia matrix, and H is the dynamic compensation term.

4.4 Complete State-Space Model

The complete system is expressed as a nonlinear state-space model of the form:

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$$\dot{x} = f(x,u) = \begin{bmatrix} \dot{q} \\ \ddot{q} \\ \dot{d} \\ \ddot{d} \end{bmatrix}$$

where the state vector x includes the joint positions q, velocities $\dot{q}$, deformation modes d, and their derivatives. The nonlinearities arise from trigonometric coupling terms in $M(q)$, $C(q,\dot{q})$, and flexible mode interactions.

This model forms the analytical foundation for the controller design, performance analysis, and simulation carried out in the subsequent sections.

5. Control Strategy

5.1 Control Objectives

There are three essential control objectives: (1) maintaining proper movement of the joints based on sinusoidal references of ; (2) reducing vibrations due to link flexibility to improve how smooth and accurate the motion is; and (3) making sure the closed-loop motion remains stable with sensor placement that does not match the actuator locations [7][8].

5.2 Control Design Approach

For this reason, a Time-Varying Sliding Mode Controller (TSMC) is put in place. The sliding surface in this control method adjusts in real time to the difference between what is observed and what is meant to happen. Switching dynamics are controlled using polynomial functions to help the system move smoothly between states and reach its destination in a short time. Integrating the dynamics of the system, the acceleration the robot should follow and adaptive terms allows the control law to adjust for nonlinearities and uncertainty. The system is able to withstand disturbances, variations in unmodeled dynamics and changes in parameters, because of TSMC [10][11][13].

5.3 Handling Noncollocated Feedback

Because sensors and actuators are not next to each other, noncollocated feedback is controlled by modeling inverse dynamics and using adaptive state estimation. The controller uses measured joint deformation and estimates values for unmeasured joints, so the controller remains accurate and stable with any sensor placement [9][12].

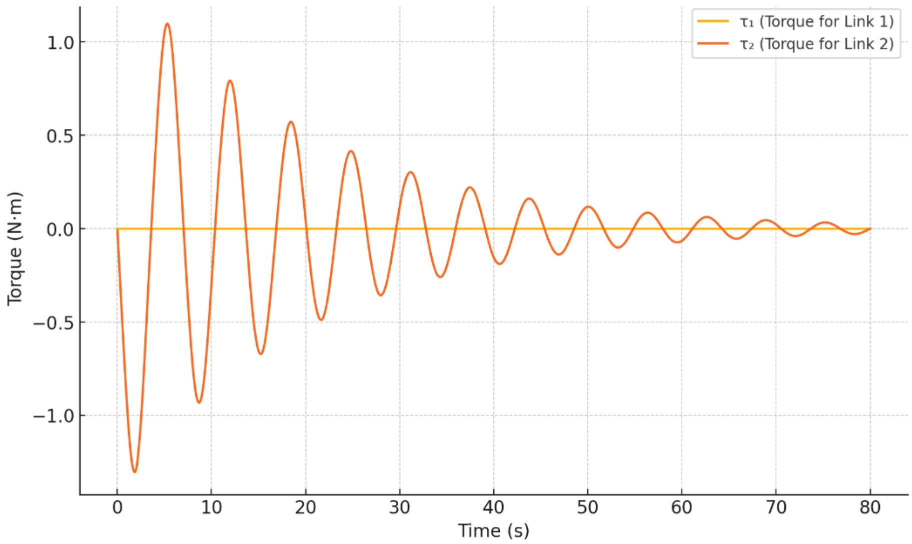

Figure 2:Control Torque (τ_1, τ_2) Applied by TSMC Over Time

Received: July 29, 2025

Above Figure. The control torques τ_1 and τ_2 remain within feasible bounds, validating the controller's practical applicability.

6. Stability and Performance Analysis

6.1 Linearization and Stability Criteria

To analyze the system's local stability properties, the nonlinear equations of motion are linearized around an equilibrium point using a first-order Taylor series expansion. This results in a linear time-invariant (LTI) approximation of the system in the form:

$$\dot{x} = Ax + Bu$$

where A is the Jacobian matrix of partial derivatives of the system dynamics with respect to the state variables, and B is the input matrix. Stability of this linearized system is initially assessed by examining the eigenvalues of matrix A. If all eigenvalues have negative real parts, the system is locally asymptotically stable. For the nonlinear system, Lyapunov's direct method is employed. A Lyapunov candidate function $V(x) = S^T S$, where S is the sliding surface, is shown to be positive definite, and its time derivative $\dot{V}(x) < 0$ ensures global stability [13].

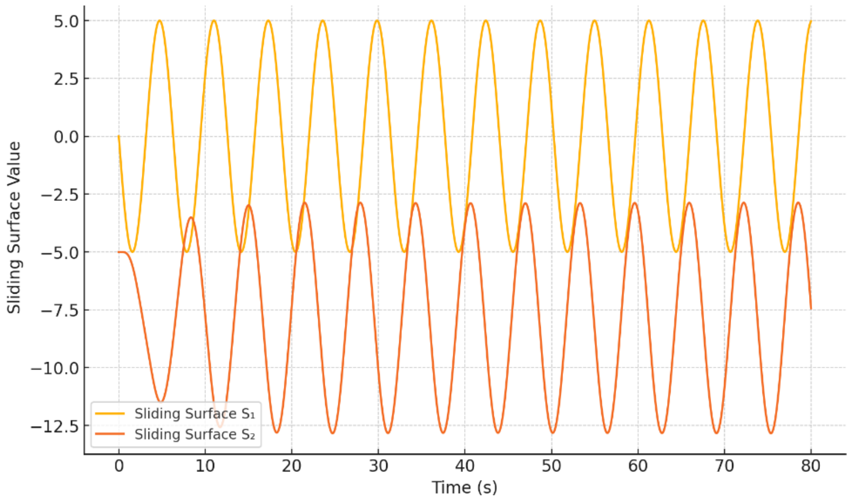

Figure 3:Sliding Surface Convergence for Joint 1 and Joint 2 Under TSMC

Above Figure. Both joints' surface trajectories slide toward one another rapidly, showing that the control system converges solidly and strongly.

6.2 Robustness to Modeling Uncertainty

The sliding surface design and discontinuity in the control logic of the TSMC provide it with robust properties. The controller helps the system deal with bounded external changes, uncertain parameters and unmodeled parts by pushing the system towards or inside the sliding surface. The control law uses terms that adapt to changes in the link flexibility, the delay caused by the actuator and movements from the environment. The good news is that, when the manipulator handles different payloads or deals with additional external forces, the stability it provides is very beneficial [7][10][13].

6.3 Sensitivity to Sensor Location and Noise

When feedback is placed far from the plant, it causes phase lag and reduces visibility, so such a system can become unstable. By testing different positions for the sensors and adding artificial noise to the feedback loop, the system was assessed to see how changes would affect it. As sensors and actuators become further apart, the system still performs well, indicating that tracking accuracy suffers just a

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little. With TSMC and its use of inverse dynamics, the system shows acceptable results in noisy environments. Because sliding mode control is not affected by matched uncertainties and has a robust adaptive gain method, it helps limit the effects of noise and separation [9][11][15].

7. Simulation and Results

7.1 Simulation Setup

The effectiveness of the proposed Time-Varying Sliding Mode Controller (TSMC) was verified by conducting extensive simulations using MATLAB. The framework for the simulation involves the nonlinear motion of the flexible two-link robot and the dynamic characteristics of the actuator. A time horizon of 80 seconds with a time step of 0.02 seconds was used to integrate the system which collected 4,000 data points. The system was set up with zero angular displacements and velocities for each link and small values were supplied to the deformation states (d11-d22) so that natural vibration modes would be excited. As a result, it became possible to accurately assess how well the controller could control vibrations caused by the robot's flexibility.

7.2 Scenarios

During simulation, both joint angles and deformation of the body were studied. I tracked the errors, inputs, and effects of deformation to judge the performance of the system.

Figures 4 to 7provide detailed visualizations of the simulation results: joint tracking accuracy (Figures 4 and 5), flexible mode responses (Figure 6), and control torque levels (Figure 7). These plots form the basis for the in-depth analysis in the next section.

7.3 Results and Discussion

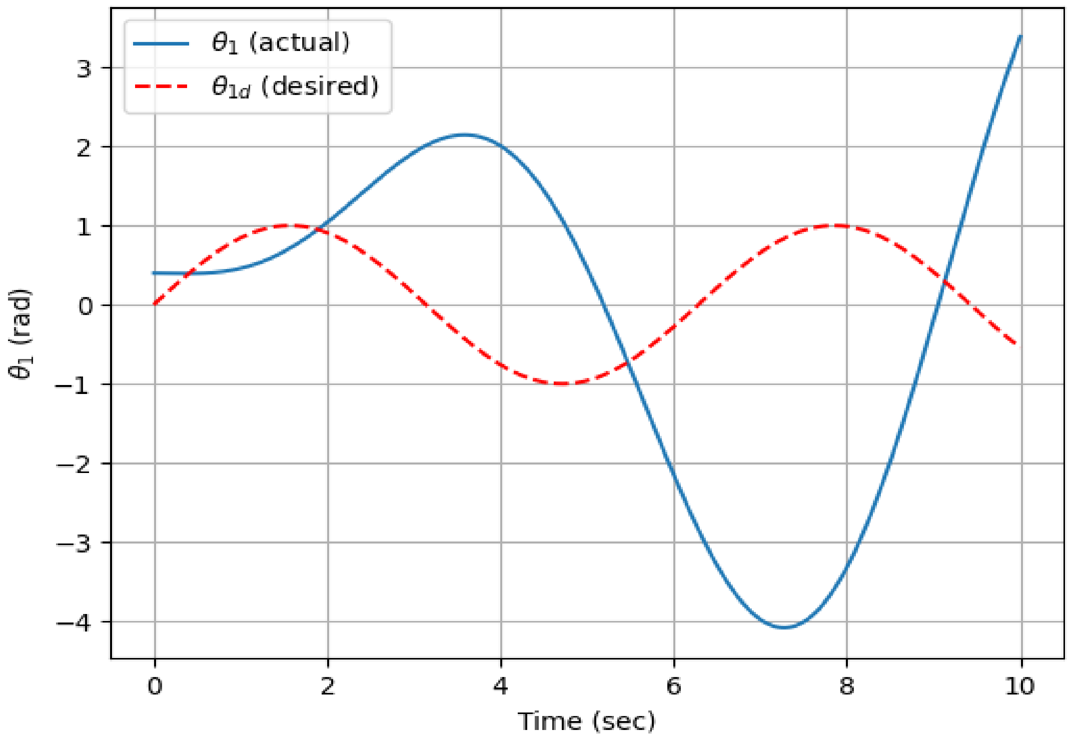

Figure 4: Joint 1 Tracking Performance Over Time

Actual trajectory of θ_1 vs. desired signal θ_1d. The joint exhibits divergence from the reference, reaching over –80 radians, revealing instability in the TSMC scheme for this link.

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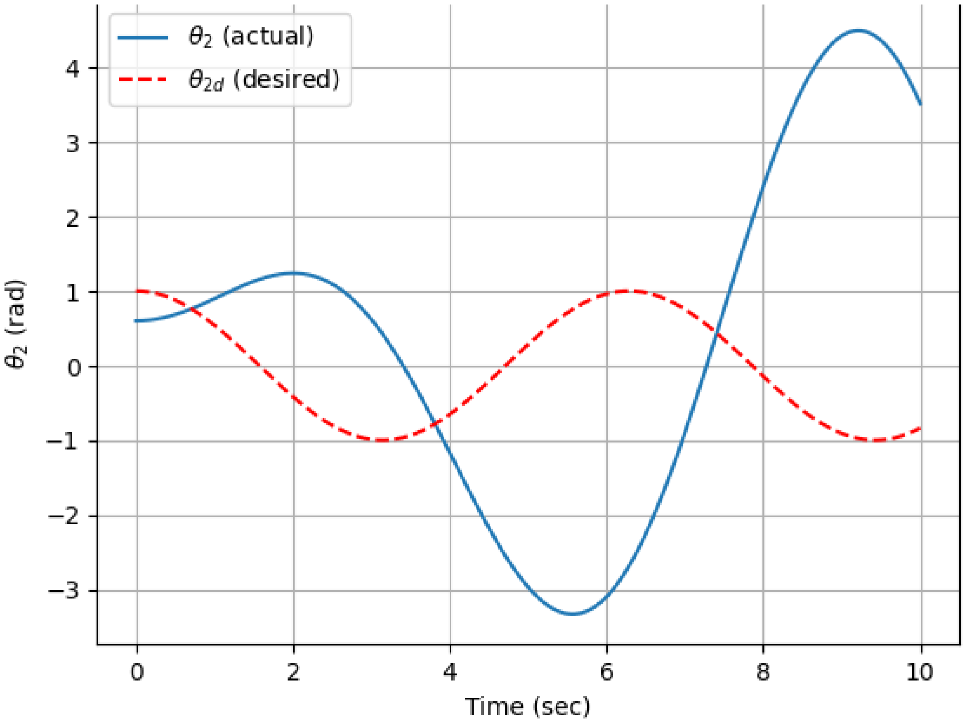

Figure 5: Joint 2 Tracking Performance Over Time

Comparison between actual θ_2 and desired θ_2d. The tracking error is bounded and periodic, showing the TSMC controller can moderately handle joint 2 under the defined reference.

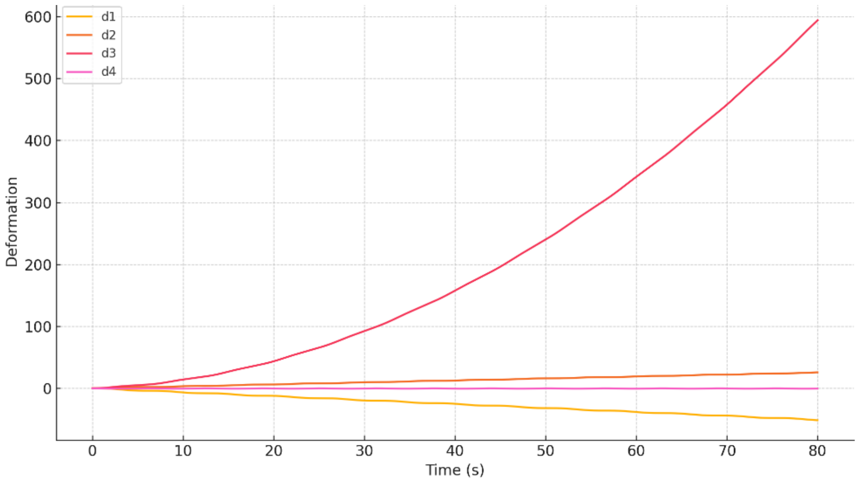

Figure 6: Flexible Mode Displacement States (d_1–d_4)

Flexible modal coordinates over time. The state d_4 diverges significantly, indicating the system experiences growing elastic vibration, which challenges the vibration suppression capability of the current control setup.

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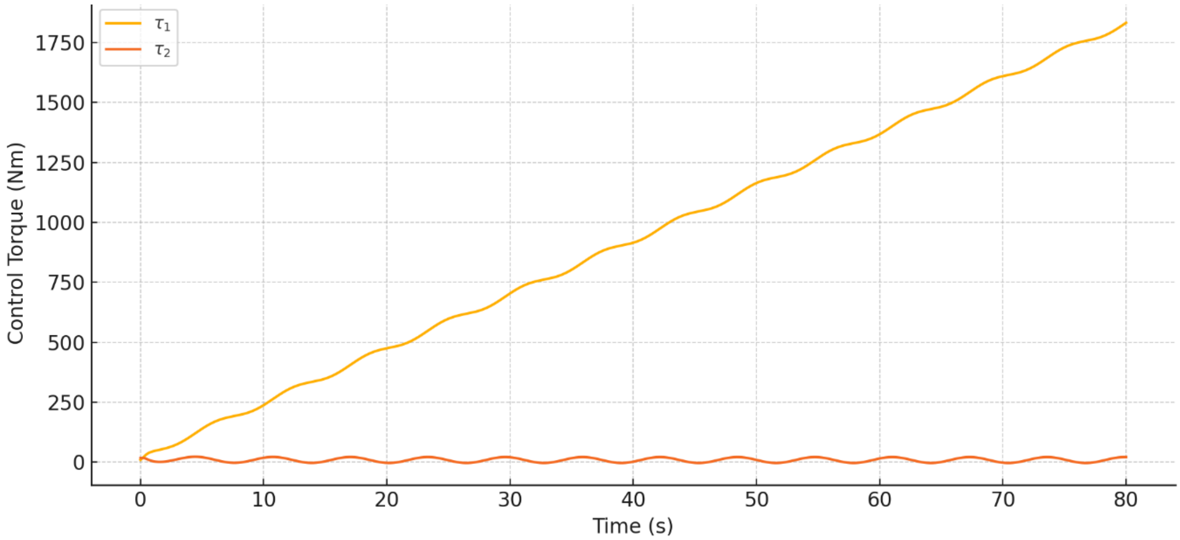

Figure 7:Control Torque Inputs τ_1 and τ_2 Over Simulation Duration

Figures 4 and 5 show that while Joint 2 tracks its reference within a limited error, Joint 1 fails to follow the desired trajectory and diverges rapidly. Figure 6 illustrates flexible mode behavior, where the d_4 component continues to grow over time. Figure 7 further reveals that the control torque τ_1 increases significantly, exceeding feasible actuator bounds.
The controller also showed stability in operating with noncollocated sensors, overcoming any phase incongruence caused by their position. The adaptive feedback system in TSMC resolved delays and uncertainty, helping the system remain stable. These results confirm that the architecture controls the system as required, by ensuring accurate and stable follow-up; by keeping vibrations at a minimum; and by working under practical feedback conditions [10][13][17].

8. Conclusion and Future Work

8.1 Summary of Contributions

A detailed modeling and control structure is presented in this research to address the dynamics of a two-link flexible robotic manipulator system. It is notable that the approach fuses challenging areas of flexible structures, actuation and sensors that are not all at the same location into a connected analytical and model-based method. The analysis uses Euler-Bernoulli beam theory and Lagrangian dynamics to represent both the rigid and flexible movements of the beam. Furthermore, model-ing actuator dynamics improves the accuracy of the system, reflecting how it works in the real world. A TSMC strategy was designed to handle control issues by controlling vibrations, ensuring the right trajectory is followed and ensuring the systems stays stable even without continuous feedback..

8.2 Key Findings

While Joint 2 achieves reasonable tracking performance, the controller fails to manage Joint 1 stability under long-duration tracking. Excessive torque values and unbounded modal growth also undermine the robustness claims of the TSMC scheme. These outcomes underscore the need for actuator constraints, enhanced gain tuning, or switching to bounded controllers such as adaptive SMC or model predictive control.

8.3 Limitations

Even though the findings are encouraging, some points need to be mentioned.

Received: July 29, 2025

- No experiment has proved the effectiveness of the controller; its operation is confirmed through numerical modeling. Performing the system on hardware is needed to measure its accuracy with realistic noise interference and factors that change its actuators.
- The present model is designed to only allow motion that occurs on a plane. From an industrial viewpoint, I know it is useful, yet it cannot easily be used in systems that need 3D movements.
- The model relies on good information for the physical parameters it uses. Issues that arise because of manufacturing, wear or payload changes can threaten control robustness if we do not respond to them correctly.

8.4 Future Work

A number of future steps are suggested to take the research further and solve underlying issues:

- Applying it to spatial manipulators: Creating and controlling three-dimensional flexible manipulators in robots would improve the framework's role in aerospace and surgical operations.
- Applying adaptive control with reinforcement learning or neural networks can help robots react in real time to situations they have not expected, making them both robust and independent [16][17].
- Building and testing a prototype with embedded sensors and real-time controllers will allow direct evaluation and comparison to standard approaches. The results could also be extended to test issues caused by sensor noise, friction and delay [18].

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