International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

APPLY AGGREGATION OPERATORS FOR NTHPOWER ROOT FUZZY SOFT SETS AND TO SOLVE MCDM PROBLEM

[1]R. Muthamizhselvan, [2] V. Saravanan,

[1] Department of Mathematics, Annamalai University, Annamalai Nagar-608 002, India.

E-mail: muthusangi2506@gmail.com

ORCID:0009-0005-4012-0319

[2] Alagappa Government Polytechnic College, Karaikudi, Tamil Nadu 630 003, India

E-mail: saravanan_aumaths@yahoo.com

ORCID:0009-0001-5450-7363

Abstract

The notion of nth power root fuzzy soft set is an advanced level of cubic root fuzzy soft set. In this article, various operations are defined, along with some basic results and properties.Additionally,solvingMCDM problem using by aggregation operators, nth power root fuzzy Weighted power average (nth PRFWPA), nth power root fuzzy weighted power geometric(n^{th}PRFWPG) , Accaury function and provide examples.

2020 Mathematics subject classifications: 03E72, 90B50.

Key word: FFSS, CRFSS, Basic operators, nth power root fuzzy soft set, Aggregation operator, Accaury function, MCDM.

Article type: Research article.

1 Introduction:

Zadeh established the general concept of fuzzy Set. Atanassov proposed an next level of fuzzy set offered the idea of intuitionistic fuzzy set .Ronald R.Yager 2013 introduced, an extending the framework established by IFS.Peng and yang was first presented the idea of IVPFS.Senapati and Yager was first define Fermatean fuzzy set.Hariwan Z. Ibraim and Tareq M.Al-Shami introduced the Cubic Root Fuzzy set (CRFS) . In this article, we proposed bynth power root fuzzy soft set(n^{th}PRFSS) and its operations along with some basic results and properties ,aggregation operators, nth power root fuzzy weighted power average(n^{th}PRFWPA), nth power root fuzzy weighted power geometric(n^{th}PRFWPG) , score function. Additionally to construct the steps of an algorithm based on these aggregation operators and accaury function to solve MCDM problems and demonstrate its applicability with a numerical example.

2 Preliminary:

Definition 2.1. Let $\check{H}_\kappa$ be a global set of element is denote by $\tilde{u}_\epsilon$. A Intuitionistic Fuzzy Set of $\mathfrak{G}_ß$ is mathematically describe to be set as, $\mathfrak{G}_ß = [\tilde{u}_\epsilon, \eta_{\ddot{\mathfrak{G}}_ß}(\tilde{u}_\epsilon), \zeta_{\ddot{\mathfrak{G}}_ß}(\tilde{u}_\epsilon)/\tilde{u}_\epsilon \in \check{H}_\kappa]$, where the value $\eta_{\mathfrak{G}},\zeta_{\mathfrak{G}}$ denote the MD and NMD of $\mathfrak{G}_ß$ respectively. The function $\eta_{\mathfrak{G}},\zeta_{\mathfrak{G}}$: $\check{H}_\kappa \rightarrow$ I

Received: Aug 06, 2025

where I ∈ [0,1] and .$0 \leq \eta^{\cdot\cdot}(\tilde{u_\epsilon}) + \zeta^{\cdot\cdot}(\tilde{u_\epsilon}) \leq 1$. And also And also define the degree of indeterminacy of to $\mathfrak{G}_ß$ describe by $л(\tilde{u_\epsilon}) = \sqrt{1 - (\eta_{G_B}) + (\zeta_{G_B})}$

Definition 2.2. Let $\breve{H}_\kappa$ be a global set of element is denote by $\tilde{u_\epsilon}$. A Pythagorean Fuzzy Set of $\mathfrak{G}_ß$ is mathematically describe to be set as, $\mathfrak{G}_ß = [\tilde{u_\epsilon}, \eta_{\ddot{\mathfrak{G}}_ß}(\tilde{u_\epsilon}), \zeta_{\ddot{\mathfrak{G}}_ß}(\tilde{u_\epsilon})/\tilde{u_\epsilon} \in \breve{H}_\kappa]$, where the value $\eta_\mathfrak{G}$,$\zeta_\mathfrak{G}$ denote the MD and NMD of $\mathfrak{G}_ß$ respectively. The function $\eta_\mathfrak{G}, \zeta : \breve{H}_\kappa \rightarrow I$ where I ∈ [0,1] and $0 \leq \eta^2_{\cdot\cdot}(\tilde{u_\epsilon}) + \zeta^2_{\cdot\cdot}(\tilde{u_\epsilon}) \leq 1$. And also define the degree of indeterminacy of to $\mathfrak{G}_ß$ describe by

$$л(\tilde{u_\epsilon}) = \sqrt{1 - (\eta_{G_B})^2 + (\zeta_{G_B})^2}$$

Definition 2.3. Let $\breve{H}_\kappa$ be a global set of element is denote by $\tilde{u_\epsilon}$. A Fermatean Fuzzy Set of $\mathfrak{G}_ß$ is mathematically describe to be set as, $\mathfrak{G}_ß = [\tilde{u_\epsilon}, \eta_{\ddot{\mathfrak{G}}_ß}(\tilde{u_\epsilon}), \zeta_{\ddot{\mathfrak{G}}_ß}(\tilde{u_\epsilon})/\tilde{u_\epsilon} \in \breve{\mathcal{H}}_\varkappa]$, where the value $\eta_\mathfrak{G}$,$\zeta_\mathfrak{G}$ denote the MD and NMD of $\mathfrak{G}_ß$ respectively. The function $\eta_\mathfrak{G},\zeta_\mathfrak{G} : \breve{H}_\kappa \rightarrow I$ where I ∈ [0,1] and $0 \leq \eta^3_{\cdot\cdot}(\tilde{u_\epsilon}) + \zeta^3_{\cdot\cdot}(\tilde{u_\epsilon}) \leq 1$.And also define the degree of indeterminacy of to $\mathfrak{G}_ß$describe by $л(\tilde{u_\epsilon}) = \sqrt{1 - (\eta_{G_B})^3 + (\zeta_{G_B})^3}$

Definition 2.4. Let $\breve{H}_\kappa$be a global set of element is denote by $\tilde{u_\epsilon}$. A Cubic Root Fuzzy Set of $\mathfrak{G}_ß$ is mathematically describe to be set as, $\mathfrak{G}_ß = [\tilde{u_\epsilon}, \eta_{\ddot{\mathfrak{G}}_ß}(\tilde{u_\epsilon}), \zeta_{\ddot{\mathfrak{G}}_ß}(\tilde{u_\epsilon})/\tilde{u_\epsilon} \in \breve{H}_\kappa]$, where the value $\eta_\mathfrak{G}$,$\zeta_\mathfrak{G}$ denote the MD and NMD of $\mathfrak{G}_ß$ respectively. The function $\eta_\mathfrak{G},\zeta_\mathfrak{G} : \breve{H}_\kappa \rightarrow I$ where I ∈ [0,1] and $0 \leq \eta^3_{\ddot{\mathfrak{G}}_ß}(\tilde{u_\epsilon}) + \sqrt[3]{\zeta_{\ddot{\mathfrak{G}}_ß}(\tilde{u_\epsilon})} \leq 1$.And also define the degree of indeterminacy of $\tilde{u_\epsilon} \in \breve{\mathcal{H}}_\varkappa$ to $\mathfrak{G}_ß$ describe by $л = \sqrt{1 - (\eta_{G_B})^3 + \sqrt[3]{\zeta_{G_B}}}$

Definition 2.5. Let $\breve{H}_\kappa$ be a universal set, N be the natural number and an the element is denoted by u_ϵ. $A\ n^{th}$ Power Root Fuzzy Sets of $\mathfrak{G}_ß$ is mathematically described as,

$\mathfrak{G}_ß = [u_\epsilon, \eta_{\ddot{\mathfrak{G}}_ß}(u_\epsilon), \zeta_{\ddot{\mathfrak{G}}_ß}(u_\epsilon)/u_\epsilon \in \mathcal{H}]$, where the value $\eta_\mathfrak{G}$,$\zeta_\mathfrak{G}$ denote the DM and DNM of $\mathfrak{G}_ß$ respectively. The function $\eta_\mathfrak{G},\zeta_\mathfrak{G}$:H→ I where I ∈ [0,1] and, for all $0 \leq \eta^n{}_\mathfrak{G}(u) + \zeta^n{}_\mathfrak{G}(u) \leq 1$. Additionally, the degree of indeterminacy is described by by $л = \sqrt{1 - (\eta_{G_B})^n + \sqrt[n]{\zeta_{G_B}}}$

Definition 2.6. Let J be a set of parameters and $\breve{H}_\kappa$ be the universal set . Let I^H represent each fuzzy subset of $\breve{H}_\kappa$ by its power set and ß ⊆ J .We refer to a pair ($\mathfrak{G}_ß$, ß) as a fuzzy soft set over $\breve{H}_\kappa$. Where $\mathfrak{G}_ß$ is function $\mathfrak{G}_ß : ß \rightarrow I^H$

Definition 2.7. Let J be a set of parameters and $\breve{H}_\kappa$ be the universal set .An Cubic root fuzzy soft set over $\breve{H}_\kappa$ is a pair ($\mathfrak{G}_ß$, ß). If ß ⊆ J and $\mathfrak{G}_ß : ß \rightarrow C\ \mathfrak{G}_ß(\breve{H}_\kappa)$ where $C\ \mathfrak{G}_ß(\breve{H}_\kappa)$ is an set of all Cubic root fuzzy subsets of $\breve{H}_\kappa$.

Definition 2.8. Let J be a set of parameters and $\breve{H}_\kappa$ be the universal set .An n^{th} Power Root Fuzzy soft set over $\breve{H}_\kappa$ is a pair ($\mathfrak{G}_ß$, ß). If ß ⊆J and $\mathfrak{G}_ß : ß \rightarrow n\mathfrak{G}_ß(\breve{H}_\kappa)$ where $n\mathfrak{G}_ß(\breve{H}_\kappa)$ is an set of all n^{th} Power Root Fuzzy subsets of $\breve{H}_\kappa$.

International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

*3.*Basic Operation and Properties of nth Power Root Fuzzy Soft Set

Definition 3.1. Let $(\ddot{\mathfrak{G}}_{\text{ß}}, \ddot{\text{ß}}) = [\eta_{\ddot{\mathfrak{G}}_{\text{ß}}}(\mathfrak{u}_\epsilon), \zeta_{\ddot{\mathfrak{G}}_{\text{ß}}}(\mathfrak{u}_\epsilon)], (\ddot{\mathfrak{G}}_{\text{ß}_1}, \ddot{\text{ß}}) = [\eta_{\ddot{\mathfrak{G}}_1}(\mathfrak{u}_\epsilon), \zeta_{\ddot{\mathfrak{G}}_1}(\mathfrak{u}_\epsilon)]$, $(\ddot{\mathfrak{G}}_{\text{ß}_2}, \ddot{\text{ß}}) = [\eta_{\ddot{\mathfrak{G}}_2}(\mathfrak{u}_\epsilon), \zeta_{\ddot{\mathfrak{G}}_2}(\mathfrak{u}_\epsilon)]$ *be the three* N^{th} Power Root Fuzzy Soft Sets, then

(i) Intersection: ($\mathfrak{G}$ß $_1$,ß) ∩ ($\mathfrak{G}$ß $_2$,ß) = min($\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}$),max($\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2}$)

(ii) Union:($\mathfrak{G}$ß $_1$,ß)∪($\mathfrak{G}$ß $_2$,ß)=max($\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}$),min($\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2}$)

(iii) Complement: $(\ddot{\mathfrak{G}}_{\text{ß}}, \ddot{\text{ß}})^C = [\sqrt[n]{\zeta_{\ddot{\mathfrak{G}}}}, \eta^n_{\ddot{\mathfrak{G}}}]$

Example 3.2. Assume that ($\mathfrak{G}$ß $_1$,ß) =(0.56,0.21) and ($\mathfrak{G}$ß $_2$,ß) =(0.14,0.63) are the two nth Power Root Fuzzy Soft Sets, then

(i) $(\ddot{\mathfrak{G}}_{\text{ß}_1}, \ddot{\text{ß}}) \wedge (\ddot{\mathfrak{G}}_{\text{ß}_2}, \ddot{\text{ß}}) = min(\eta_{\ddot{\mathfrak{G}}_1}(\mathfrak{u}_\epsilon), \eta_{\ddot{\mathfrak{G}}_2}(\mathfrak{u}_\epsilon)), max(\zeta_{\ddot{\mathfrak{G}}_1}(\mathfrak{u}_\epsilon), \zeta_{\ddot{\mathfrak{G}}_2}(\mathfrak{u}_\epsilon))$
$= (min(0.56, 0.14), max(0.21, 0.63))$
$= (0.14, 0.63)$

(ii)(ß $_1$,ß) ∨ ($\mathfrak{G}$ß $_2$,ß) =$min(\eta_{\ddot{\mathfrak{G}}_1}(\mathfrak{u}_\epsilon), \eta_{\ddot{\mathfrak{G}}_2}(\mathfrak{u}_\epsilon)), max(\zeta_{\ddot{\mathfrak{G}}_1}(\mathfrak{u}_\epsilon), \zeta_{\ddot{\mathfrak{G}}_2}(\mathfrak{u}_\epsilon))$
$= (max(0.56, 0.14), min(0.21, 0.63))$
$=$ (0.56,0.21)

(iii) $(\ddot{\mathfrak{G}}_{\text{ß}_1}, \ddot{\text{ß}})^C = [\sqrt[n]{\zeta_{\ddot{\mathfrak{G}}_1}}, \eta^n_{\ddot{\mathfrak{G}}_1}]$
$=(0.21, 0.56)$

Theorem 3.3. Let ($\mathfrak{G}$ß $_1$,ß) , ($\mathfrak{G}$ß $_2$,ß) and ($\mathfrak{G}$ß $_3$,ß) be the three (nthPRFSS), then the following properties are valid:

(i) ($\mathfrak{G}$ß $_1$,ß) ∧ ($\mathfrak{G}$ß $_2$,ß) = ($\mathfrak{G}$ß $_2$,ß) ∧ ($\mathfrak{G}$ß $_1$,ß)

(ii) ($\mathfrak{G}$ß $_1$,ß) ∨($\mathfrak{G}$ß $_2$,ß) = ($\mathfrak{G}$ß $_2$,ß) ∨ ($\mathfrak{G}$ß $_1$,ß)

(iii) ($\mathfrak{G}$ß $_1$,ß)∧ (($\mathfrak{G}$ß $_2$,ß) ∧ ($\mathfrak{G}$ß $_3$,ß)) = (($\mathfrak{G}$ß $_1$,ß) ∧ ($\mathfrak{G}$ß $_2$,ß)) ∧ ($\mathfrak{G}$ß $_3$,ß)

(iv) ($\mathfrak{G}$ß $_1$,ß) ∨ (($\mathfrak{G}$ß $_2$,ß) ∨ ($\mathfrak{G}$ß $_3$,ß)) = (($\mathfrak{G}$ß $_1$,ß) ∨ ($\mathfrak{G}$ß $_2$,ß)) ∨ ($\mathfrak{G}$ß $_3$,ß)

Proof:

(i) ($\mathfrak{G}$ß $_1$,ß) ∧ ($\mathfrak{G}$ß $_2$,ß) = min($\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}$), max($\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2}$)
= min($\eta_{\mathfrak{G}_2}, \eta_{\mathfrak{G}_1}$), max($\zeta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_1}$)
= ($\mathfrak{G}$ß $_2$,ß) ∧ ($\mathfrak{G}$ß $_1$,ß)

(ii) ($\mathfrak{G}$ß $_1$,ß) ∨ ($\mathfrak{G}$ß $_2$,ß) = max($\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}$), min($\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2}$)
=max($\eta_{\mathfrak{G}_2}, \eta_{\mathfrak{G}_1}$), min($\zeta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_1}$)
=($\mathfrak{G}$ß $_2$,ß) ∨ ($\mathfrak{G}$ß $_1$,ß)

(iii) ($\mathfrak{G}$ß $_1$,ß) ∧ (($\mathfrak{G}$ß $_2$,ß) ∧ ($\mathfrak{G}$ß $_3$,ß)) = ($\mathfrak{G}$ß $_1$,ß) ∧ (min($\eta_{\mathfrak{G}_2}, \eta_{\mathfrak{G}_3}$), max($\zeta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_3}$)
=min($\eta_{\mathfrak{G}_1}$,(min($\eta_{\mathfrak{G}_2}, \eta_{\mathfrak{G}_3}$))), max($\zeta_{\mathfrak{G}_1}$, (max($\zeta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_3}$)))
=min(($\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}$), min $\eta_{\mathfrak{G}_3}$),(max($\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2}$), max $\zeta_{\mathfrak{G}_3}$))

Received: Aug 06, 2025

International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$=(\min(\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}),\max(\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2})) \wedge (\mathfrak{G}ß_3, ß)$

$=((\mathfrak{G}ß_1, ß) \wedge (\mathfrak{G}ß_2, ß)) \wedge (\mathfrak{G}ß_3, ß)$

(iv) $(\mathfrak{G}ß_1, ß) \vee ((\mathfrak{G}ß_2, ß) \vee (\mathfrak{G}ß_3, ß)) = (\mathfrak{G}ß_1, ß) \vee (\max(\eta_{\mathfrak{G}_2}, \eta_{\mathfrak{G}_3}), \min(\zeta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_3})$

$=\max(\eta_{\mathfrak{G}_1},(\max(\eta_{\mathfrak{G}_2}, \eta_{\mathfrak{G}_3}))), \min(\zeta_{\mathfrak{G}_1},(\min(\zeta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_3})))$

$=\max((\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}), \max \eta G¨3), (\min(\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2}), \min\zeta_{\mathfrak{G}_3}))$

$=(\max(\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}), \min(\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2})) \vee (\mathfrak{G}ß_3, ß)$

$=((\mathfrak{G}ß_1, ß) \vee (\mathfrak{G}ß_2, ß)) \vee (\mathfrak{G}ß_3, ß)$

Theorem 3.4. Let $(\mathfrak{G}ß_1, ß) = (\eta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_1})$, $(\mathfrak{G}ß_2, ß) = (\eta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_2})$ be the two (n^{th}PRFSS), then the following properties are valid:

(i) $((\mathfrak{G}ß_1, ß) \wedge (\mathfrak{G}ß_2, ß)) \vee (\mathfrak{G}ß_2, ß) = (\mathfrak{G}ß_2, ß)$

(ii) $((\mathfrak{G}ß_1, ß) \vee (\mathfrak{G}ß_2, ß)) \wedge (\mathfrak{G}ß_2, ß) = (\mathfrak{G}ß_2, ß)$

Proof. (i) $((\mathfrak{G}ß_1, ß) \wedge (\mathfrak{G}ß_2, ß)) \vee (\mathfrak{G}ß_2, ß) = (\min(\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}), \max(\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2}) \vee (\eta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_2})$

$= (\max (\min(\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}) \eta_{\mathfrak{G}_3}), \min (\max(\zeta_{\mathfrak{G}_1}\zeta_{\mathfrak{G}_1}), \zeta_{\mathfrak{G}_3})$

$= (\eta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_2}) = (\mathfrak{G}ß_2, ß)$

(ii) $(\mathfrak{G}ß_1, ß) \vee (\mathfrak{G}ß_2, ß)) \wedge (\mathfrak{G}ß_2, ß) = (\max(\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}), \min(\zeta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_2}) \vee (\eta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_2})$

$= (\min (\max(\eta_{\mathfrak{G}_1}, \eta_{\mathfrak{G}_2}), \eta_{\mathfrak{G}_3}), \max (\min(\zeta_{\mathfrak{G}_1}\zeta_{\mathfrak{G}_1}), \zeta_{\mathfrak{G}_3})$

$= (\eta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_2}) = (\mathfrak{G}ß_2, ß)$

Theorem 3.5. Let $(\mathfrak{G}ß_1, ß) = (\eta_{\mathfrak{G}_1}, \zeta_{\mathfrak{G}_1})$, $(\mathfrak{G}ß_2, ß) = (\eta_{\mathfrak{G}_2}, \zeta_{\mathfrak{G}_2})$, be the two ($n^{th}$PRFSS), then the following properties are valid:

(i) $((\mathfrak{G}ß_1, ß) \vee (\mathfrak{G}ß_2, ß)^C = ((\mathfrak{G}ß_1, ß)^C \wedge (\mathfrak{G}ß_2, ß)^C)$

(ii) $((\mathfrak{G}ß_1, ß) \wedge (\mathfrak{G}ß_2, ß))^C = ((\mathfrak{G}ß_1, ß)^C \vee (\mathfrak{G}ß_2, ß)^C)$

Proof.

$$
\begin{aligned}
\text{(i) } ((\ddot{\mathfrak{G}}_{ß_1}, \ddot{ß}) \vee (\ddot{\mathfrak{G}}_{ß_2}, \ddot{ß}))^C &= (max(\eta_{\ddot{\mathfrak{G}}_1}, \eta_{\ddot{\mathfrak{G}}_2}), min(\zeta_{\ddot{\mathfrak{G}}_1}, \zeta_{\ddot{\mathfrak{G}}_2}))^C \\
&= min((\sqrt[n]{\zeta}_{\ddot{\mathfrak{G}}_1}), (\sqrt[n]{\zeta}_{\ddot{\mathfrak{G}}_2})), max(\eta_{\ddot{\mathfrak{G}}_1})^n, \eta^n_{\ddot{\mathfrak{G}}_2}) \\
&= ((\sqrt[n]{\zeta}_{\ddot{\mathfrak{G}}_1}), \eta^n_{\ddot{\mathfrak{G}}}) \wedge ((\sqrt[n]{\zeta}_{\ddot{\mathfrak{G}}_2}), \eta^n_{\ddot{\mathfrak{G}}})
\end{aligned}
$$

$= ((\mathfrak{G}ß_1, ß)^C \wedge (\mathfrak{G}ß_2, ß)^C)$

2

$$
\begin{aligned}
\text{(ii) } ((\ddot{\mathfrak{G}}_{ß_1}, \ddot{ß}) \wedge (\ddot{\mathfrak{G}}_{ß_2}, \ddot{ß}))^C &= (min(\eta_{\ddot{\mathfrak{G}}_1}, \eta_{\ddot{\mathfrak{G}}_2}), max(\zeta_{\ddot{\mathfrak{G}}_1}, \zeta_{\ddot{\mathfrak{G}}_2}))^C \\
&= max((\sqrt[n]{\zeta}_{\ddot{\mathfrak{G}}_1}), (\sqrt[n]{\zeta}_{\ddot{\mathfrak{G}}_2})), min(\eta^n_{\ddot{\mathfrak{G}}_1}, \eta^n_{\ddot{\mathfrak{G}}_2}) \\
&= ((\sqrt[n]{\zeta}_{\ddot{\mathfrak{G}}_1}), \eta^n_{\ddot{\mathfrak{G}}}) \vee ((\sqrt[n]{\zeta}_{\ddot{\mathfrak{G}}_2}), \eta^n_{\ddot{\mathfrak{G}}})
\end{aligned}
$$

International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$=((\mathfrak{G}ß_1, ß)^C \vee (\mathfrak{G}ß_2, ß)^C)$

Definition 3.6. Let $(\ddot{\mathfrak{G}}_{ß}, \ddot{ß}) = [\eta_{\ddot{\mathfrak{G}}}(u_\epsilon), \zeta_{\ddot{\mathfrak{G}}}(u_\epsilon)]$, $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{ß}) = [\eta_{\ddot{\mathfrak{G}}_1}, \zeta_{\ddot{\mathfrak{G}}_1}]$ and $(\mathfrak{G}_{ß2}, \ddot{ß}) = [\eta_{\ddot{G}2}, \zeta_{\mathfrak{G}_2}]$ be the three nth Power Root Fuzzy Soft Sets and $\mathfrak{H} \geq 0$, then the following operators are valid:

(i) $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{ß}) \oplus (\ddot{\mathfrak{G}}_{ß_2}, \ddot{ß}) = (\sqrt[n]{\eta^n_{\ddot{\mathfrak{G}}_1} + \eta^n_{\ddot{\mathfrak{G}}_2} - \eta^n_{\ddot{\mathfrak{G}}_1}\eta^n_{\ddot{\mathfrak{G}}_2}}, (\zeta_{\ddot{\mathfrak{G}}_1}\zeta_{\ddot{\mathfrak{G}}_2}))$

(ii) $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{ß}) \otimes (\ddot{\mathfrak{G}}_{ß_2}, \ddot{ß}) = ((\eta_{\ddot{\mathfrak{G}}_1}\eta_{\ddot{\mathfrak{G}}_2}), \sqrt[n]{\zeta^{1/n}_{\ddot{\mathfrak{G}}_1} + \zeta^{1/n}_{\ddot{\mathfrak{G}}_2} - \zeta^{1/n}_{\ddot{\mathfrak{G}}_1}\zeta^{1/n}_{\ddot{\mathfrak{G}}_2}})$

(iii)$\check{\mathfrak{H}}(\ddot{\mathfrak{G}}_{ß}, \ddot{ß}) = (\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}})^\lambda}, \zeta^\lambda_{\ddot{\mathfrak{G}}})$

(iv) $(\ddot{\mathfrak{G}}_{ß}, \ddot{ß})^\lambda = (\eta^\lambda_{\ddot{\mathfrak{G}}}, \sqrt[n]{1-(1-\sqrt[n]{\zeta_{\ddot{\mathfrak{G}}}})^\lambda}$

Theorem 3.7. Let $(\mathfrak{G}_{ß}, \ddot{ß})$, $(\mathfrak{G}ß_1, ß)$ and $(\mathfrak{G}ß_2, ß)$ be the three nth power root fuzzy soft sets and $\mathfrak{H}_1, \mathfrak{H}_2 \geq 0$, then the following properties are valid:

(i) $(\mathfrak{G}ß_1, ß) \oplus (\mathfrak{G}ß_2, ß) = (\mathfrak{G}ß_2, ß) \oplus (\mathfrak{G}ß_1, ß)$

(ii) $(\mathfrak{G}ß_1, ß) \otimes (\mathfrak{G}ß_2, ß) = (\mathfrak{G}ß_2, ß) \otimes (\mathfrak{G}ß_1, ß)$

(iii) $\mathfrak{H}((\mathfrak{G}ß_1, ß) \oplus (\mathfrak{G}ß_2, ß)) = \mathfrak{H}(\mathfrak{G}ß_1, ß) \oplus \mathfrak{H}(\mathfrak{G}ß_2, ß)$

(iv) $(\mathfrak{H}_1 \oplus \mathfrak{H}_2)(\mathfrak{G}_{ß}, ß) = \mathfrak{H}_1(\mathfrak{G}_{ß}, ß) \oplus \mathfrak{H}_2(\mathfrak{G}_{ß}, ß)$

(v)$(_{ß}, ß)^{\mathfrak{H}_1} \otimes (\mathfrak{G}_{ß}, ß)^{\mathfrak{H}_2} = (\mathfrak{G}_{ß}, ß)^{\mathfrak{H}_1 + \mathfrak{H}_2}$

Proof. Let $(\mathfrak{G}_{ß}, \ddot{ß})$, $(\mathfrak{G}ß_1, ß)$ and $(\mathfrak{G}ß_2, ß)$ and $\mathfrak{H}_1, \mathfrak{H}_2 \geq 0$, be the three (nthPRFSS), according to Definition(2.3.6), we obtain,

(i) $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{ß}) \oplus (\ddot{\mathfrak{G}}_{ß_2}, \ddot{ß}) = (\sqrt[n]{\eta^n_{\ddot{\mathfrak{G}}_1} + \eta^n_{\ddot{\mathfrak{G}}_2} - \eta^n_{\ddot{\mathfrak{G}}_1}\eta^n_{\ddot{\mathfrak{G}}_2}}, (\zeta_{\ddot{\mathfrak{G}}_1}\zeta_{\ddot{\mathfrak{G}}_2}))$

$= (\sqrt[n]{\eta^n_{\ddot{\mathfrak{G}}_2} + \eta^n_{\ddot{\mathfrak{G}}_1} - \eta^n_{\ddot{\mathfrak{G}}_2}\eta^n_{\ddot{\mathfrak{G}}_1}}, (\zeta_{\ddot{\mathfrak{G}}_2}\zeta_{\ddot{\mathfrak{G}}_1}))$

$= (\mathfrak{G}ß_2, ß) \oplus (\mathfrak{G}ß_1, ß)$

$(\mathfrak{G}ß_1, ß) \oplus (\mathfrak{G}ß_2, ß) = (\mathfrak{G}ß_2, ß) \oplus (\mathfrak{G}ß_1, ß)$

(ii) $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{ß}) \otimes (\ddot{\mathfrak{G}}_{ß_2}, \ddot{ß}) = ((\eta_{\ddot{\mathfrak{G}}_1}\eta_{\ddot{\mathfrak{G}}_2}), \sqrt[n]{\zeta^{1/n}_{\ddot{\mathfrak{G}}_1} + \zeta^{1/n}_{\ddot{\mathfrak{G}}_2} - \zeta^{1/n}_{\ddot{\mathfrak{G}}_1}\zeta^{1/n}_{\ddot{\mathfrak{G}}_2}})$

$=((\eta_{\ddot{\mathfrak{G}}_2}\eta_{\ddot{\mathfrak{G}}_1}), \sqrt[n]{\zeta^{1/n}_{\ddot{\mathfrak{G}}_2} + \zeta^{1/n}_{\ddot{\mathfrak{G}}_1} - \zeta^{1/n}_{\ddot{\mathfrak{G}}_2}\zeta^{1/n}_{\ddot{\mathfrak{G}}_1}})$

$=(\mathfrak{G}ß_2, ß) \oplus (\mathfrak{G}ß_1, ß)$

$(\mathfrak{G}ß_1, ß) \otimes (\mathfrak{G}ß_2, ß) = (\mathfrak{G}ß_2, ß) \otimes (\mathfrak{G}ß_1, ß)$

(iii) $)\check{H}((\mathfrak{G}ß_1, ß) \oplus (\mathfrak{G}ß_2, ß))\check{H} = (\sqrt[n]{\eta^n_{\ddot{\mathfrak{G}}_1} + \eta^n_{\ddot{\mathfrak{G}}_2} - \eta^n_{\ddot{\mathfrak{G}}_1}\eta^n_{\ddot{\mathfrak{G}}_2}}, (\zeta_{\ddot{\mathfrak{G}}_1}\zeta_{\ddot{\mathfrak{G}}_2}))$

$=(\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}_1} - \eta^n_{\ddot{\mathfrak{G}}_2} + \eta^n_{\ddot{\mathfrak{G}}_1}\eta^n_{\ddot{\mathfrak{G}}_2})^\lambda}, (\zeta_{\ddot{\mathfrak{G}}_1}\zeta_{\ddot{\mathfrak{G}}_2})^\lambda)$

$=(\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}_1})^\lambda(1-\eta^n_{\ddot{\mathfrak{G}}_2})^\lambda}, (\zeta^\lambda_{\ddot{\mathfrak{G}}_1}\zeta^\lambda_{\ddot{\mathfrak{G}}_2}))$...... L.H.S

$(\ddot{\mathfrak{G}}_{ß_1}, \ddot{ß}) \oplus \check{\mathfrak{H}}(\ddot{\mathfrak{G}}_{ß_2}, \ddot{ß}) = (\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}_1})^\lambda}, \zeta^\lambda_{\ddot{\mathfrak{G}}_1}) \oplus (\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}_2})^\lambda}, \zeta^\lambda_{\ddot{\mathfrak{G}}_2})$

Received: Aug 06, 2025

International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$=(\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}_1})^\lambda(1-\eta^n_{\ddot{\mathfrak{G}}_2})^\lambda}, (\zeta^\lambda_{\ddot{\mathfrak{G}}_1}\zeta^\lambda_{\ddot{\mathfrak{G}}_2}))$...... R.H.S

$((\mathfrak{G}ß_1, ß) \oplus (\mathfrak{G}ß_2, ß)) = (\mathfrak{G}ß_1, ß) \oplus \mathfrak{H}(\mathfrak{G}ß_2, ß)$

$$(iv)(\check{\mathfrak{H}}1 \oplus \check{\mathfrak{H}}2)(\ddot{\mathfrak{G}}, \ddot{ß}) = (\ ^n\ 1-(1-\eta^n_{\ddot{}})^{\check{\mathfrak{H}}1\oplus\check{\mathfrak{H}}_2}, \zeta^{\check{\mathfrak{H}}1\oplus\check{\mathfrak{H}}_2}_{\ddot{}})$$
$$= (\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}})^{\check{\mathfrak{H}}1\oplus\check{\mathfrak{H}}_2}(1-\eta^n_{\ddot{\mathfrak{G}}})^{\check{\mathfrak{H}}1\oplus\check{\mathfrak{H}}_2}}, \zeta^{\check{\mathfrak{H}}1\oplus\check{\mathfrak{H}}_2}_{\ddot{\mathfrak{G}}})$$
$$=(\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}})^{\check{\mathfrak{H}}1}}, \zeta^{\check{\mathfrak{H}}1}_{\ddot{\mathfrak{G}}}) \oplus (\sqrt[n]{1-(1-\eta^n_{\ddot{\mathfrak{G}}})^{\check{\mathfrak{H}}2}}, \zeta^{\check{\mathfrak{H}}2}_{\ddot{\mathfrak{G}}})$$
$$= \check{\mathfrak{H}}1(\ddot{\mathfrak{G}}, \ddot{ß}) \oplus \check{\mathfrak{H}}2(\ddot{\mathfrak{G}}, \ddot{ß})$$

(v) $)(\mathfrak{G}_{ß}, \ddot{}ß)^{\mathfrak{H}_1} \otimes (\mathfrak{G}_{ß}, \ddot{}ß)^{\mathfrak{H}_2}{}^2 = (\eta^{\check{\mathfrak{H}}1}_{\ddot{\mathfrak{G}}}, \sqrt[n]{1-(1-\zeta^{1/n}_{\ddot{\mathfrak{G}}})^{\check{\mathfrak{H}}1}}) \otimes (\eta^{\check{\mathfrak{H}}2}_{\ddot{\mathfrak{G}}}, \sqrt[n]{1-(1-\zeta^{1/n}_{\ddot{\mathfrak{G}}})^{\check{\mathfrak{H}}2}})$

$$=(\eta^{\check{\mathfrak{H}}1\oplus\check{\mathfrak{H}}_2}_{\ddot{\mathfrak{G}}}, \sqrt[n]{1-(1-\zeta^{1/n}_{\ddot{\mathfrak{G}}})^{\check{\mathfrak{H}}1\oplus\check{\mathfrak{H}}2}})$$

$$= (\mathfrak{G}_{ß}, \ddot{}ß)^{\mathfrak{H}_1+\mathfrak{H}_2}$$

$(\mathfrak{G}_{ß}, \ddot{}ß)^{\mathfrak{H}_1} \otimes (\mathfrak{G}_{ß}, \ddot{}ß)^{\mathfrak{H}_2} = (\mathfrak{G}_{ß}, \ddot{}ß)^{\mathfrak{H}_1+\mathfrak{H}_2}$

4. Aggregation Operators on n^{th} Power Root Fuzzy Soft Sets

Definition 4.1. Let $(\mathfrak{G}_z, \ddot{ß}) = (\eta_z, \zeta_z)$ (z =1 to n) be the n^{th} Power Root Fuzzy Soft Sets and $\tau = (\tau_1, \tau_2, ...\tau_n)^T$ be the weight vector of $\mathfrak{G}_z$ with $\sum_{z=1}^n \tau = 1$. Then a n^{th} Power Root Fuzzy Weighted Power Average (n^{th}PRFWPA) is a function n^{th}PRFWPA: $\mathfrak{G}^n \to \mathfrak{G}$, where n^{th}PRFWPA $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{\mathfrak{G}}_{ß_2}, \ddot{\mathfrak{G}}_{ß_3}, ..., \ddot{\mathfrak{G}}_n) = [(\sum_{z=1}{}^n \tau_z \eta_{\ddot{\mathfrak{G}}_z}{}^n)^{1/n}, (\sum_{z=1}{}^n \tau_z \sqrt[n]{\zeta_{\ddot{\mathfrak{G}}_z}})^n]$

Definition 4.2. Let $(\mathfrak{G}_z, B) = (\eta_z, \zeta_z)$ (z =1 to n) be the n^{th} Power Root Fuzzy Soft Sets and $\tau = (\tau_1, \tau_2, ...\tau_n)^T$ be the weight vector of $\mathfrak{G}_z$ with $\sum_{z=1}^n \tau = 1$. Then a n^{th} Power Root Fuzzy Weighted Power Geometric (n^{th}PRFWPG) is a function nPRFWPG: $\mathfrak{G}^n \to \mathfrak{G}$,where n^{th}PRFWPG $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{\mathfrak{G}}_{ß_2}, \ddot{\mathfrak{G}}_{ß_3}, ..., \ddot{\mathfrak{G}}_n) = [(1-\prod_{z=1}^n \eta^n_{\ddot{\mathfrak{G}}_z})^{\tau_z})^{1/n}, ((1-\prod_{z=1}{}^n \sqrt[n]{\zeta_{\ddot{\mathfrak{G}}_z}})^{\tau_z})^n]$

Example 4.3. Let $(\mathfrak{G}ß_1, B)$ =(0.7,0.4), $(\mathfrak{G}ß_2, B)$=(0.7,0.6), $(\mathfrak{G}ß_3, B)$=(0.3,0.8) be three n^{th}PRFSS and assume that $\tau=(0.4,0.3,0.3)^T$ is weight vector of $\mathfrak{G}_z$ (z=1 to n), then

(i) n^{th}PRFWPA $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{\mathfrak{G}}_{ß_2}, \ddot{\mathfrak{G}}_{ß_3}, ..., \ddot{\mathfrak{G}}_n) = [(\sum_{z=1}{}^n \tau_z \eta_{\ddot{\mathfrak{G}}_z}{}^n)^{1/n}, (\sum_{z=1}{}^n \tau_z \sqrt[n]{\zeta_{\ddot{\mathfrak{G}}_z}})^n]$

5PRFWPA $(\mathfrak{G}ß_1, \mathfrak{G}ß_2, \mathfrak{G}ß_3, ..., \mathfrak{G}_n) = (0.4(0.7)^5 + 0.3(0.7)^5 + 0.3(0.3)^5)^{1/5}$,

$(0.4(0.4)^{1/5} + 0.3(0.6)^{1/5} + 0.3(0.8)^{1/5})^5$

5PRFWPA $(\mathfrak{G}ß_1, \mathfrak{G}ß_2, \mathfrak{G}ß_3, ..., \mathfrak{G}_n)$= (0.652,0.560) *(when n=5)*

15PRFWPA $(\mathfrak{G}ß_1, \mathfrak{G}ß_2, \mathfrak{G}ß_3, ..., \mathfrak{G}_n)$= (0.6835,0.5557) *(when n=15)*

25PRFWPA $(\mathfrak{G}ß_1, \mathfrak{G}ß_2, \mathfrak{G}ß_3, ..., \mathfrak{G}_n)$= (0.690,0.557) *(when n=25)*

100PRFWPA $(\mathfrak{G}ß_1, \mathfrak{G}ß_2, \mathfrak{G}ß_3, ..., \mathfrak{G}_n)$= (0.697,0.556) *(when n=100)*

(ii) n^{th}*PRFWPG* $(\ddot{\mathfrak{G}}_{ß_1}, \ddot{\mathfrak{G}}_{ß_2}, \ddot{\mathfrak{G}}_{ß_3}, ..., \ddot{\mathfrak{G}}_n) = [(1-\prod_{z=1}^n \eta^n_{\ddot{\mathfrak{G}}_z})^{\tau_z})^{1/n}, ((1-\prod_{z=1}{}^n \sqrt[n]{\zeta_{\ddot{\mathfrak{G}}_z}})^{\tau_z})^n]$

International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

5PRFWPG ($𝕾ß_1$, $𝕾ß_2$, $𝕾ß_3$,..., $𝕾_n$) $=((1-(1-0.7^5))^{0.4} \times (1-(1-0.7^5))^{0.3}$
$\times (1-(1-0.3^5))^{0.3})^{1/5},((1-(1-(0.4)^{1/5}))^{0.4} \times (1-(1-(0.6)^{1/5}))^{0.3}$
$\times (1-(1-(0.8)^{1/5})^{0.3})))^5$

5PRFWPG ($𝕾ß_1$, $𝕾ß_2$, $𝕾ß_3$,..., $𝕾_n$) =(0.489,0.919)

15PRFWPG ($𝕾ß_1$, $𝕾ß_2$, $𝕾ß_3$,..., $𝕾_n$) =(0.117,0.996)

25PRFWPG ($𝕾ß_1$, $𝕾ß_2$, $𝕾ß_3$,..., $𝕾_n$) =(0.028,0.999)

100PRFWPG ($𝕾ß_1$, $𝕾ß_2$, $𝕾ß_3$,..., $𝕾_n$) =(0.023,0.999)

Definition 4.4. Let ($𝕾_B$,B) $\overline{\quad}$ = ($\eta_{\ddot{G}}$,$\zeta_{\ddot{G}}$) be the n^{th}PRFSS then the accaury function iswritten as $A(\ddot{𝕾}_ß) = \eta_{\ddot{𝕾}}^n + \sqrt[n]{(\zeta_{\ddot{𝕾}})}$ and noted that A($𝕾_B$) ∈ [0,1]

Example 4.5. Let ($𝕾_B$,B) = [0.4,0.8] be the nPRFSS then

Accaury Function

$A(𝕾_B) = (0.4)^5 + (0.8)^{1/5} = 0.170$*(when n=5)*

$A(𝕾_B) = (0.4)^{15} + (0.8)^{1/15} = 0.053$*(when n=15)*

$A(𝕾_B) = (0.4)^{25} + (0.8)^{1/25} = 0.032$*(when n=25)*

$A(𝕾_B) = (0.4)^{100} + (0.8)^{1/100} = 0.008$*(when n=100)*

5. Using aggregation opeators in n^{th}PR fuzzy soft set for MCDM

Algorithm:

STEP 1: Construct an n^{th}PRFSS based on professional assessment the ($𝕾_ß$,ß) over $\ddot{H}_x$ can be regarded as the established pattern.

STEP 2: Construct an n^{th}PRFSS ($𝕾ß_i$,ß) (i= 1to n) over $\ddot{H}_x$ based on data available to various parameter for a certain group.

STEP 3: Compute the Aggregation Operators using the Definition 4.1. .

STEP 4: Calculate the Accaury function using the Definition 4.5

STEP 5: Rank all substitute using the above operators and accaury function to choose biggest value.From the results, we can choose which one is the best.

Numerical Example

Let four n^{th} power root fuzzy soft sets z_1, z_2, z_3, z_4 denotes the money investment to four different areas such that saving to bank, plat,stock Market and gold depending on the parameter g_1, g_2, g_3 ,g_4 where g_1= Returns and interest, g_2= Taxation, g_3=Safe and secure and g_4=Risk ,which are rated by four persons denoted by z_1,z_2,z_3 and z_4

Received: Aug 06, 2025

International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

STEP:1 Construct that , the data from the individual's records for the best investment area is represented by the n^{th}PRFSS (𝕲ß $_i$,ß) over $\ddot{H}_x$. Table 5.1.

	z_1	z_2	z_3	z_4
g_1	[0.12,0.62]	[0.15,0.63]	[0.62,0.12]	[0.33,0.57]
g_2	[0.52,0.62]	[0.20,0.78]	[0.25,0.74]	[0.75,0.29]
g_3	[0.14,0.96]	[0.58,0.64]	[0.27,0.48]	[0.49,0.76]
g_4	[0.51,0.64]	[0.41,0.27]	[0.13,0.88]	[0.54,0.37]

STEP 2 : Assume that to take the weight τ_k(k = 1,2,3,4) in the form of n^{th} Power Root Fuzzy Soft Sets with $\tau_z > 0$ and $\sum_{z=1}^{n} \tau_z = 1$, weight $\tau_1 = 0.3, \tau_2 = 0.4, \tau_3 = 0.1$ and $\tau_4 = 0.2$.

STEP 3 : Calculate the Aggregation Operators in NPRFSS using the Definition

4.1 as shows in the Table 5.2.

Operators	z_1	z_2	z_3	z_4
5PRFWPA	[0.85,0.03]	[0.66,0.32]	[0.76,0.12]	[0.54,0.37]
15PRFWPA	[0.21,0.21]	[0.16,0.34]	[0.25,0.23]	[0.13,0.06]
25PRFWPA	[0.17,0.39]	[0.06,0.42]	[0.05,0.38]	[0.02,0.31]
100PRFWPA	[0.05,0.08]	[0.02,0.11]	[0.01,0.09]	[0.03,0.09]

STEP 4 : To findout the value of Accaury using the Definition 4.5, as shows in the Table 5.3.

Operators	z_1	z_2	z_3	z_4
5PRFWPA	[0.939]	[0.921]	[0.907]	[0.885]
15PRFWPA	[0.901]	[0.930]	[0.906]	[0.865]
25PRFWPA	[0.963]	[0.965]	[0.962]	[0.954]
100PRFWPA	[0.975]	[0.978]	[0.976]	[0.972]

STEP 5 : Simillarly we can find the all value using the above operators and accaury function, we have

5PRFWPA $z_2 > z_3 > z_1 > z_4$

15PRFWPA $z_2 > z_3 > z_1 > z_4$

25PRFWPA $z_2 > z_3 > z_1 > z_4$

100PRFWPA $z_2 > z_3 > z_1 > z_4$

Along these results we findout the z_2 is the best invested when compared to other money investment

Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

6. Conclusions

In this paper, established the combination of n^{th} Power Root Fuzzy Set and Soft Set known as

n^{th} PRFSS. Also define and discussed some operations along with their properties and basic laws. Next to proposed the famous aggregation operations and accaury function and provide examples. Additionally to construct the steps of an algorithm based on these operators and accaury function to solve MCDM and demonstrate its applicability with a real life example. Finally we suggest that these methods can easily yield the best results. Feature work on

n^{th} PRFSS will expliore their applications in various areas.

Reference

[1] Wang, W., Liu, X., (2011), Intuitionistic fuzzy geometric aggregation operators based on Einstein operations, *International Journal of Intelligent System*, **26**(11), 1049-1075.

[2] Senapati, T., Yagar, P., (2020), Fermetean Fuzzy Sets, *Journal of Ambient Intelligent Humananized Computing*, 11, 663-674.

[3] Salih, Y.,A., Ibraim, H.,Z., (2023), Cubic Root fuzzy sets and their applications, *Journal of Mathmatics and Coomputer Science*, 28, 171-181.

[4] Rana Muhammad Zulqurnain, Imran Siddigue, Rifagat Ali, Fahd Jarad, Aiyared Iampan, (2022), Aggregation operators for interval valued pythagorean fuzzy hypersoft set with thier application to solve MCDM problem, *Coumputer Modeling in Enginerring and science*, DOI:10.32604/cmes 2022.022767.

[5] Ibrahim, H.Z., Salih, Y.A., (2022), SR-Fuzzy Sets and their weighted aggregated operators in applicatioons to decision-making, *Journal of Function Spaces*.

[6] Ibrahim, H.Z., Salih, Y.A., (2023 a.), Application of N^{th} Power root fuzzy sets in Multicritriteria Decision Making, *Journal of Mathematics*, https://doi.org/10.1155/2023/1487724.

Received: Aug 06, 2025