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A STUDY ON LOWER LEVEL SUBSETS OF AN NEUTROSOPHIC ANTI-FUZZY SUBSEMIRING OF A SEMIRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of lower level subsets of an neutrosophic anti-fuzzy subsemiring (NAFSSR) of a semiring (SR) and we introduce the some theorems in lower level subsets of an neutrosophic anti-fuzzy subsemiring of a semiring (SR).

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Key Words: fuzzy set, neutrosophic fuzzy set, anti-fuzzy subsemiring, neutrosophic anti-fuzzy subsemiring, homomorphism, anti-homomorphism, lower level subset.

INTRODUCTION

Various notions in universal algebra expand upon the structure of an associative ring (Ꞧ; +, .). Among these, nearrings and different forms of semirings have proven to be highly valuable. An algebra (Ꞧ; +, .) is defined as a semiring (SR) when (Ꞧ; +) and (Ꞧ; .) form semigroups and adhere to the distributive laws ϰ.(ɏ + ʐ) = ϰ.ɏ + ϰ.ʐ and (ɏ + ʐ).ϰ = ɏ.ϰ + ʐ.ϰ universally for ϰ, ɏ, and ʐ in Ꞧ. A SRꞦ is deemed additively commutative if ϰ + ɏ = ɏ + ϰ is true universally for ϰ, ɏ in Ꞧ. Furthermore, a semiring Ꞧ might include an identity element 1, where 1.ϰ = ϰ = ϰ.1, and a zero element 0, where 0 + ϰ = ϰ = ϰ + 0 and ϰ.0 = 0 = 0.ϰ hold universally for ϰ in Ꞧ. After L.A. Zadeh introduced fuzzy sets in his work [14], scholars began exploring extensions of this idea. Smarandache.F [12,13], in publications from 2002 and 2005, put forth Neutrosophy as an innovative philosophical perspective. Later, S. Abou Zaid [1] introduced the notions of fuzzy subnearrings and ideals. In this paper, we introduce the some theorems in lower level subsets of an neutrosophic anti-fuzzy subsemiring of a semiring (NAFSSR).

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1.PRELIMINARIES:

1.1 Definition: Imagine X as a non-empty collection. A fuzzy subset (FS) Đ of X is characterized as a mapping Đ : X → [0, 1].

1.2 Definition: A neutrosophic fuzzy set (NFS) Đ over a universal set X is defined through a truth membership function $\check{T}_Đ(\varkappa)$, an indeterminacy function $\hat{I}_Đ(\varkappa)$, and a falsity membership function $F_Đ(\varkappa)$, represented as Đ = { < $\varkappa$, $\check{T}_Đ(\varkappa)$, $\hat{I}_Đ(\varkappa)$, $F_Đ(\varkappa)$ > ; $\varkappa \in X$ }, where $\check{T}_Đ, \hat{I}_Đ, F_Đ$: X → [0, 1] and the condition $0 \leq \check{T}_Đ(\varkappa) + \hat{I}_Đ(\varkappa) + F_Đ(\varkappa) \leq 3$ holds.

1.3 Definition: Let Ɽ be a SR. An FS Đ of Ɽ is identified as an "anti-fuzzy subsemiring" (AFSSR) of Ɽ provided it adheres to:

(i) $\mu_Đ(\varkappa + y) \leq \max\{ \mu_Đ(\varkappa), \mu_Đ(y) \}$,

(ii) $\mu_Đ(\varkappa y) \leq \max\{ \mu_Đ(\varkappa), \mu_Đ(y) \}$, for every $\varkappa$ and y in Ɽ.

1.4 Definition: Assume Ɽ is an SR. An FS Đ of Ɽ is termed a "Neutrosophic anti-fuzzy subsemiring" (NAFSSR) of Ɽ if it satisfies these criteria:

(i) (a) $\check{T}_Đ(\varkappa + y) \leq \max\{ \check{T}_Đ(\varkappa), \check{T}_Đ(y) \}$,

(b) $\hat{I}_Đ(\varkappa + y) \leq \max\{ \hat{I}_Đ(\varkappa), \hat{I}_Đ(y) \}$,

(c) $F_Đ(\varkappa + y) \geq \min\{ F_Đ(\varkappa), F_Đ(y) \}$,

(ii) (a) $\check{T}_Đ(\varkappa y) \leq \max\{ \check{T}_Đ(\varkappa), \check{T}_Đ(y) \}$,

(b) $\hat{I}_Đ(\varkappa y) \leq \max\{ \hat{I}_Đ(\varkappa), \hat{I}_Đ(y) \}$,

(c) $F_Đ(\varkappa y) \geq \min\{ F_Đ(\varkappa), F_Đ(y) \}$, universally for $\varkappa$ and y in Ɽ.

1.5 Definition: View (Ɽ, +, .) and (Ɽ^{I}, +, .) as two SRs. For, f :Ɽ → Ɽ^{I} is classified as a "semiring homomorphism" (SR Hom) if $f(\varkappa + y) = f(\varkappa) + f(y)$ and $f(\varkappa y) = f(\varkappa)f(y)$ hold universally for $\varkappa$ and y in Ɽ.

1.6 Definition: Consider (Ɽ, +, .) and (Ɽ^{I}, +, .) as two SRs. For, f :Ɽ → Ɽ^{I} is labeled a "semiring anti-homomorphism" (SRA Hom) if $f(\varkappa + y) = f(y) + f(\varkappa)$ and $f(\varkappa y) = f(y)f(\varkappa)$ are true universally for $\varkappa$ and y in Ɽ.

1.7 Definition: Let Đ be a neutrosophic fuzzy subset (NFS) of X. For α,β,γ in [0, 1], with α+β+γ ≤ 3 the lower level subset of Đ is the set $Đ_{(\alpha,\beta,\gamma)}=\{x \in X : \check{T}_Đ(x) \leq \alpha, \hat{I}_Đ(x) \leq \beta, F_Đ(x) \geq \gamma\}$.

2.2 LOWER LEVEL SUBSETS OF AN NEUTROSOPHIC ANTI-FUZZY SUBSEMIRING OF A SEMIRING

2.2.1 Theorem: Let Đ be an NAFSSR of a SR Ɽ. Then for α,β,γ in [0,1], $Đ_{(\alpha,\beta,\gamma)}$ is a lower level subsemiring of Ɽ.

Proof: For all $\varkappa$ and y in $Đ_{(\alpha,\beta,\gamma)}$, we have, $\check{T}_Đ(\varkappa) \leq \alpha$, $\check{T}_Đ(y) \leq \alpha$, $\hat{I}_Đ(\varkappa) \leq \beta$, $\hat{I}_Đ(y) \leq \beta$ and $F_Đ(\varkappa) \geq \gamma$, $F_Đ(y) \geq \gamma$. Now, (i a) $\check{T}_Đ(\varkappa + y) \leq \max\{ \check{T}_Đ(\varkappa), \check{T}_Đ(y) \} \leq \max\{ \alpha, \alpha \} = \alpha$. Which implies that $\check{T}_Đ(\varkappa+y) \leq \alpha$. (i b) $\hat{I}_Đ(\varkappa+ y) \leq \max\{\hat{I}_Đ(\varkappa), \hat{I}_Đ(y)\} \leq \max\{ \beta, \beta \} = \beta$.

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Which implies that $\hat{I}_{Đ}(\varkappa + ɏ) \leq \beta$. (i c) $Ƒ_{Đ}(\varkappa + ɏ) \geq \min\{Ƒ_{Đ}(\varkappa), Ƒ_{Đ}(ɏ)\} \geq \min\{\gamma, \gamma\} = \gamma$. Which implies that $Ƒ_{Đ}(\varkappa + ɏ) \geq \gamma$. And, (ii a) $\check{T}_{Đ}(\varkappa ɏ) \leq \max\{\check{T}_{Đ}(\varkappa), \check{T}_{Đ}(ɏ)\}$ $\leq \max\{\alpha, \alpha\} = \alpha$. Which implies that $\check{T}_{Đ}(\varkappa ɏ) \leq \alpha$. (ii b) $\hat{I}_{Đ}(\varkappa ɏ) \leq \max\{\hat{I}_{Đ}(\varkappa), \hat{I}_{Đ}(ɏ)\}$ $\leq \max\{\beta, \beta\} = \beta$. Which implies that $\hat{I}_{Đ}(\varkappa ɏ) \leq \beta$. (ii c) $Ƒ_{Đ}(\varkappa ɏ) \geq \min\{Ƒ_{Đ}(\varkappa), Ƒ_{Đ}(ɏ)\}$ $\geq \min\{\gamma, \gamma\} = \gamma$. Which implies that $Ƒ_{Đ}(\varkappa ɏ) \geq \gamma$. Therefore, $\check{T}_{Đ}(\varkappa+ɏ) \leq \alpha$, $\check{T}_{Đ}(\varkappa ɏ) \leq \alpha$, $\hat{I}_{Đ}(\varkappa+ɏ) \leq \beta$, $\hat{I}_{Đ}(\varkappa ɏ) \leq \beta$ and $Ƒ_{Đ}(\varkappa+ɏ) \geq \gamma$, $Ƒ_{Đ}(\varkappa ɏ) \geq \gamma$ Therefore $\varkappa + ɏ$ and $\varkappa ɏ$ in $Đ_{(\alpha,\beta,\gamma)}$.

Hence $Đ_{(\alpha,\beta,\gamma)}$ is a lower level subsemiring of a SR Ɽ.

2.2.2 Theorem: Let Đ be an NAFSSR of a SR Ɽ. Then two lower level subsemiring $Đ_{\alpha 1}$, $Đ_{\alpha 2}$, $Đ_{\beta 1}$, $Đ_{\beta 2}$, $Đ_{\gamma 1}$, $Đ_{\gamma 2}$ and $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2$ are in [0,1] with $\alpha_1 < \alpha_2$, $\beta_1 < \beta_2$, $\gamma_1 > \gamma_2$ of Đ are equal iff there is no $\varkappa$ in Ɽ such that $\alpha_2 > \check{T}_{Đ}(\varkappa) > \alpha_1$, $\beta_2 > \hat{I}_{Đ}(\varkappa) > \beta_1$ and $\gamma_2 < Ƒ_{Đ}(\varkappa) < \gamma_1$.

Proof: Assume that $Đ_{\alpha 1} = Đ_{\alpha 2}$, $Đ_{\beta 1} = Đ_{\beta 2}$ and $Đ_{\gamma 1} = Đ_{\gamma 2}$. Suppose there exists $\varkappa$ in Ɽ such that $\alpha_2 > \check{T}_{Đ}(\varkappa) > \alpha_1$, $\beta_2 > \hat{I}_{Đ}(\varkappa) > \beta_1$ and $\gamma_2 < Ƒ_{Đ}(\varkappa) < \gamma_1$. Then $Đ_{(\alpha 1, \beta 1, \gamma 1)} \subseteq Đ_{(\alpha 2, \beta 2, \gamma 2)}$ implies $\varkappa$ belongs to $Đ_{\alpha 2}$, $Đ_{\beta 2}$, $Đ_{\gamma 2}$ but not in $Đ_{\alpha 1}$, $Đ_{\beta 1}$, $Đ_{\gamma 1}$. This is contradiction to $Đ_{\alpha 1} = Đ_{\alpha 2}$ $Đ_{\beta 1} = Đ_{\beta 2}$ and $Đ_{\gamma 1} = Đ_{\gamma 2}$. Therefore there is no $\varkappa \in$ Ɽ such that $\alpha_2 > \check{T}_{Đ}(\varkappa) > \alpha_1$, $\beta_2 > \hat{I}_{Đ}(\varkappa) > \beta_1$ and $\gamma_2 < Ƒ_{Đ}(\varkappa) < \gamma_1$. Conversely if there is no $\varkappa \in$ Ɽ such that $\alpha_2 > \check{T}_{Đ}(\varkappa) > \alpha_1$, $\beta_2 > \hat{I}_{Đ}(\varkappa) > \beta_1$ and $\gamma_2 < Ƒ_{Đ}(\varkappa) < \gamma_1$. Then $Đ_{\alpha 1} = Đ_{\alpha 2}$ $Đ_{\beta 1} = Đ_{\beta 2}$ and $Đ_{\gamma 1} = Đ_{\gamma 2}$ (by the definition of lower level set).

2.2.3 Theorem: Let Ɽ be a SR and Đ be a NFS of Ɽ such that $Đ_{(\alpha,\beta,\gamma)}$ be a subsemiring of Ɽ. If α,β,γ in [0,1], then Đ is an NAFSSR of Ɽ.

Proof: Let Ɽ be a SR and $\varkappa$ and $ɏ$ in Ɽ. Let $\check{T}_{Đ}(\varkappa) = \alpha_1$, $\check{T}_{Đ}(ɏ) = \alpha_2$, $\hat{I}_{Đ}(\varkappa) = \beta_1$, $\hat{I}_{Đ}(ɏ) = \beta_2$ and $Ƒ_{Đ}(\varkappa) = \gamma_1$, $Ƒ_{Đ}(ɏ) = \gamma_2$

Case (i): If $\alpha_1 < \alpha_2$, $\beta_1 < \beta_2$, $\gamma_1 > \gamma_2$, then $\varkappa, ɏ \in Đ_{(\alpha 2,\beta 2,\gamma 2)}$. As $Đ_{(\alpha 2,\beta 2,\gamma 2)}$ is a subsemiring of Ɽ, $\varkappa + ɏ$ and $\varkappa ɏ$ in $Đ_{(\alpha 2,\beta 2,\gamma 2)}$. Now, (i a) $\check{T}_{Đ}(\varkappa+ɏ) \leq \alpha_2 = \max\{\alpha_1,\alpha_2\} = \max\{\check{T}_{Đ}(\varkappa), \check{T}_{Đ}(ɏ)\}$ which implies that $\check{T}_{Đ}(\varkappa+ɏ) \leq \max\{\check{T}_{Đ}(\varkappa),\check{T}_{Đ}(ɏ)\}$, for all $\varkappa$ and $ɏ$ in Ɽ. (i b) $\hat{I}_{Đ}(\varkappa + ɏ) \leq \beta_2 = \max\{\beta_1,\beta_2\} = \max\{\hat{I}_{Đ}(\varkappa),\hat{I}_{Đ}(ɏ)\}$ which implies that $\hat{I}_{Đ}(\varkappa+ɏ) \leq \max\{\hat{I}_{Đ}(\varkappa),\hat{I}_{Đ}(ɏ)\}$, for all $\varkappa$ and $ɏ$ in Ɽ. (i c) $Ƒ_{Đ}(\varkappa+ɏ) \geq \gamma_1 = \min\{\gamma_1,\gamma_2\} = \min\{Ƒ_{Đ}(\varkappa), Ƒ_{Đ}(ɏ)\}$ which implies that $Ƒ_{Đ}(\varkappa + ɏ) \geq \min\{Ƒ_{Đ}(\varkappa),Ƒ_{Đ}(ɏ)\}$, for all $\varkappa$ and $ɏ$ in Ɽ. Also, (ii a) $\check{T}_{Đ}(\varkappa ɏ) \leq \alpha_2 = \max\{\alpha_1,\alpha_2\} = \max\{\check{T}_{Đ}(\varkappa),\check{T}_{Đ}(ɏ)\}$ which implies that $\check{T}_{Đ}(\varkappa ɏ) \leq \max\{\check{T}_{Đ}(\varkappa),\check{T}_{Đ}(ɏ)\}$, for all $\varkappa$ and $ɏ$ in Ɽ. (ii b) $\hat{I}_{Đ}(\varkappa ɏ) \leq \beta_2 = \max\{\beta_1,\beta_2\} = \max\{\hat{I}_{Đ}(\varkappa),\hat{I}_{Đ}(ɏ)\}$ which implies that $\hat{I}_{Đ}(\varkappa ɏ) \leq \max\{\hat{I}_{Đ}(\varkappa),\hat{I}_{Đ}(ɏ)\}$, for all $\varkappa$ and $ɏ$ in Ɽ. (ii c) $Ƒ_{Đ}(\varkappa ɏ) \geq \gamma_2 = \min\{\gamma_1,\gamma_2\} = \min\{Ƒ_{Đ}(\varkappa), Ƒ_{Đ}(ɏ)\}$ which implies that $Ƒ_{Đ}(\varkappa ɏ) \geq \min\{Ƒ_{Đ}(\varkappa), Ƒ_{Đ}(ɏ)\}$, for all $\varkappa$ and $ɏ$ in Ɽ.

Case (ii): If $\alpha_1 > \alpha_2$, $\beta_1 > \beta_2$, $\gamma_1 < \gamma_2$ then $\varkappa$ and $ɏ$ in $Đ_{(\alpha 1,\beta 1,\gamma 1)}$. As $Đ_{(\alpha 1,\beta 1,\gamma 1)}$ is a subsemiring of R, $\varkappa+ɏ$ and $\varkappa ɏ$ in $Đ_{(\alpha 1,\beta 1,\gamma 1)}$. Now, (ia) $\check{T}_{Đ}(\varkappa+ɏ) \leq \alpha_1 = \max\{\alpha_2,\alpha_1\} = \max\{\check{T}_{Đ}(ɏ),\check{T}_{Đ}(\varkappa)\}$ which implies that $\check{T}_{Đ}(\varkappa+ɏ) \leq \max\{\check{T}_{Đ}(\varkappa),\check{T}_{Đ}(ɏ)\}$, for all $\varkappa$ and $ɏ$ in Ɽ.

(ib) $\hat{I}_{Đ}(\varkappa+ɏ) \leq \beta_1 = \max\{\beta_2,\beta_1\} = \max\{\hat{I}_{Đ}(ɏ),\hat{I}_{Đ}(\varkappa)\}$ which implies that $\hat{I}_{Đ}(\varkappa+ɏ) \leq \max\{\hat{I}_{Đ}(\varkappa), \hat{I}_{Đ}(ɏ)\}$, for all $\varkappa$ and $ɏ$ in Ɽ. (i c) $Ƒ_{Đ}(\varkappa + ɏ) \geq \gamma_1 = \min\{\gamma_2, \gamma_1\} = \min\{Ƒ_{Đ}(ɏ), Ƒ_{Đ}(\varkappa)\}$

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which implies that $Ƒ_Đ(ϰ + ꞥ) \geq$ min $\{ Ƒ_Đ(ϰ), Ƒ_Đ(ꞥ) \}$, for all ϰ and ꞥ in Ɍ. Also, (ii a) $Ť_Đ(ϰꞥ) \leq \alpha_1=$ max $\{ \alpha_2 , \alpha_1 \}=$ max $\{ Ť_Đ(ꞥ), Ť_Đ(ϰ) \}$ which implies that $Ť_Đ(ϰꞥ) \leq$ max $\{ Ť_Đ(ϰ), Ť_Đ(ꞥ) \}$, for all ϰ and ꞥ in Ɍ.(ii b) $Î_Đ(ϰꞥ) \leq \beta_1=$ max $\{ \beta_2 , \beta_1 \} =$ max $\{ Î_Đ(ꞥ), Î_Đ(ϰ) \}$

which implies that $Î_Đ(ϰꞥ) \leq$ max $\{ Î_Đ(ϰ), Î_Đ(ꞥ) \}$, for all ϰ and ꞥ in Ɍ. (ii c) $Ƒ_Đ(ϰꞥ) \geq \gamma_2$

$=$ min $\{ \gamma_2 , \gamma_1 \} =$ min $\{ Ƒ_Đ(ꞥ), Ƒ_Đ(ϰ) \}$ which implies that $Ƒ_Đ(ϰꞥ) \geq$ min $\{ Ƒ_Đ(ϰ), Ƒ_Đ(ꞥ) \}$, for all ϰ and ꞥ in Ɍ.

Case (iii): If $\alpha_1 = \alpha_2$. It is trivial.

In all the cases, Đ is an NAFSSR of a SR Ɍ.

2.2.4 Theorem: Let Đ be an NAFSSR of a SR Ɍ. If any two lower level subsemirings of Đ belongs to Ɍ, then their intersection is also lower level subsemiring of Đ in Ɍ.

Proof: Let $\alpha_1, \alpha_2 \in [0,1]$.

Case (i): If $\alpha_1 < Ť_Đ(ϰ) < \alpha_2, \beta_1 < Î_Đ(ϰ) < \beta_2, \gamma_1 > Ƒ_Đ(ϰ) > \gamma_2$ then $Đ_{(\alpha1,\beta1,\gamma1)} \subseteq Đ_{(\alpha2,\beta2,\gamma2)}$.

Therefore, $Đ_{(\alpha1,\beta1,\gamma1)} \cap Đ_{(\alpha2,\beta2,\gamma2)} = Đ_{(\alpha1,\beta1,\gamma1)}$, but $Đ_{(\alpha1,\beta1,\gamma1)}$ is a lower level subsemiring of Đ.

Case (ii): If $\alpha_1 > Ť_Đ(ϰ) > \alpha_2, \beta_1 > Î_Đ(ϰ) > \beta_2, \gamma_1 < Ƒ_Đ(ϰ) < \gamma_2$ then $Đ_{\alpha2} \subseteq Đ_{(\alpha1,\beta1,\gamma1)}$.

Therefore, $Đ_{(\alpha1,\beta1,\gamma1)} \cap Đ_{(\alpha2,\beta2,\gamma2)} = Đ_{(\alpha2,\beta2,\gamma2)}$, but $Đ_{(\alpha2,\beta2,\gamma2)}$ is a lower level subsemiring of Đ.

Case (iii): If $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_1 = \gamma_2$ then $Đ_{(\alpha1,\beta1,\gamma1)} = Đ_{(\alpha2,\beta2,\gamma2)}$.

In all cases, intersection of any two lower level subsemirings is a lower level subsemiring of Đ.

2.2.5 Theorem: Let Đ be an NAFSSR of a SR Ɍ. If $\alpha_i, \beta_i, \gamma_i \in [0,1]$, and $Đ_{(\alpha i,\beta i,\gamma i)}$, $i \in I$ is a collection of lower level subsemirings of Đ, then their intersection is also a lower level subsemiring of Đ.

Proof:

It is trivial.

2.2.6 Theorem: Let Đ be an NAFSSR of a SR Ɍ. If any two lower level subsemirings of Đ belongs to Ɍ, then their union is also a lower level subsemiring of Đ in Ɍ.

Proof:

Let $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2 \in [0,1]$.

Case (i): If $\alpha_1 < Ť_Đ(ϰ) < \alpha_2, \beta_1 < Î_Đ(ϰ) < \beta_2, \gamma_1 > Ƒ_Đ(ϰ) > \gamma_2$ then $Đ_{(\alpha1,\beta1,\gamma1)} \subseteq Đ_{(\alpha2,\beta2,\gamma2)}$.

Therefore, $Đ_{(\alpha1,\beta1,\gamma1)} \cup Đ_{(\alpha2,\beta2,\gamma2)} = Đ_{(\alpha2,\beta2,\gamma2)}$, but $Đ_{(\alpha2,\beta2,\gamma2)}$ is a lower level subsemiring of Đ.

Case (ii): If $\alpha_1 > Ť_Đ(ϰ) > \alpha_2, \beta_1 > Î_Đ(ϰ) > \beta_2, \gamma_1 < Ƒ_Đ(ϰ) < \gamma_2$ then $Đ_{(\alpha2,\beta2,\gamma2)} \subseteq Đ_{(\alpha1,\beta1,\gamma1)}$.

Therefore, $Đ_{(\alpha1,\beta1,\gamma1)} \cup Đ_{(\alpha2,\beta2,\gamma2)} = Đ_{(\alpha1,\beta1,\gamma1)}$, but $Đ_{(\alpha1,\beta1,\gamma1)}$ is a lower level subsemiring of Đ.

Case (iii): If $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_1 = \gamma_2$ then $Đ_{(\alpha1,\beta1,\gamma1)} = Đ_{(\alpha2,\beta2,\gamma2)}$.

In all cases, union of any two lower level subsemiring is also a lower level subsemiring of Đ.

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2.2.7 Theorem: Let Đ be an NAFSSR of a SR Ɽ. If α_i, β_i, $\gamma_i \in[0,1]$, and $Đ_{(\alpha i,\beta i,\gamma i)}$, $i\in I$ is a collection of lower level subsemirings of Đ, then their union is also a lower level subsemiring of Đ.

Proof:

It is trivial.

2.2.8 Theorem: The homomorphic image of a lower level subsemiring of an NAFSSR of a SR Ɽ is a lower level subsemiring of an NAFSSR of a SR Ɽ^{l}.

Proof: Let (Ɽ, +, .) and (Ɽ^{l} , +, .) be any two SR's and f : Ɽ → Ɽ^{l} be a homomorphism. That is, f(ϰ + ɏ) = f(ϰ) + f(ɏ) and f(ϰɏ) = f(ϰ)f(ɏ), for all ϰ and ɏ in Ɽ. Let V = f(Đ), where Đ is an NAFSSR of a SR Ɽ. Clearly V is an NAFSSR Ɽ^{l}. Let ϰ and ɏ in Ɽ, implies f(ϰ) and f(ɏ) in Ɽ^{l}. Let $Đ_{(\alpha,\beta,\gamma)}$ is a lower level subsemiring of Đ. That is, Ť$_{Đ}$(ϰ) ≤ α and Ť$_{Đ}$(ɏ)≤α; Ť$_{Đ}$(ϰ+ɏ)≤α, Ť$_{Đ}$(ϰɏ)≤α, Î$_{Đ}$(ϰ)≤β and Î$_{Đ}$(ɏ)≤β; Î$_{Đ}$(ϰ+ɏ)≤β, Î$_{Đ}$(ϰɏ) ≤ β and Ƒ$_{Đ}$(ϰ)≥γ and Ƒ$_{Đ}$(ɏ)≥γ; Ƒ$_{Đ}$(ϰ+ɏ)≥γ, Ƒ$_{Đ}$(ϰɏ)≥γ. We have to prove that f ($Đ_{(\alpha,\beta,\gamma)}$) is a lower level subsemiring of V. Now, Ť$_{V}$(f(ϰ))≤Ť$_{Đ}$(ϰ)≤α, consequently Ť$_{V}$(f(ϰ))≤α; Î$_{V}$(f(ϰ))≤ Î$_{Đ}$(ϰ) ≤ β, consequently Î$_{V}$(f(ϰ)) ≤ β; Ƒ$_{V}$(f(ϰ)) ≥ Ƒ$_{Đ}$(ϰ) ≥ γ, consequently Ƒ$_{V}$(f(ϰ))≥γ; and Ť$_{V}$(f(ɏ))≤Ť$_{Đ}$(ɏ)≤α, consequently Ť$_{V}$(f(ɏ))≤α; Î$_{V}$(f(ɏ))≤Î$_{Đ}$(ɏ)≤ β, consequently Î$_{V}$(f(ɏ))≤β;Ƒ$_{V}$(f(ɏ))≥Ƒ$_{Đ}$(ɏ)≥γ, consequently Ƒ$_{V}$(f(ɏ))≥γ and (ia) Ť$_{V}$(f(ϰ)+f(ɏ))=Ť$_{V}$(f(ϰ+ɏ))≤Ť$_{Đ}$(ϰ+ɏ)≤α, consequently Ť$_{V}$(f(ϰ)+f(ɏ))≤α.(ib) Î$_{V}$(f(ϰ)+f(ɏ)) = Î$_{V}$(f(ϰ+ɏ))≤Î$_{Đ}$(ϰ+ɏ)≤β, consequently Î$_{V}$(f(ϰ)+f(y))≤β. (i c) Ƒ$_{V}$(f(ϰ) + f(y))= Ƒ$_{V}$(f(ϰ + y)) ≥ Ƒ$_{Đ}$(ϰ + y) ≥ γ, consequently Ƒ$_{V}$(f(ϰ) + f(y)) ≥ γ. Also, (ii a) Ť$_{V}$(f(ϰ)f(y))=Ť$_{V}$(f(ϰy))≤Ť$_{Đ}$(ϰy)≤α, consequently Ť$_{V}$(f(ϰ)f(y))≤α.(ii b) Î$_{V}$(f(ϰ)f(y))= Î$_{V}$(f(ϰy))≤ Î$_{Đ}$(ϰy) ≤ β, consequently Î$_{V}$(f(ϰ)f(y)) ≤ β. (ii c) Ƒ$_{V}$(f(ϰ)f(y)) = Ƒ$_{V}$(f(ϰy)) ≥ Ƒ$_{Đ}$(ϰy) ≥ γ, consequently Ƒ$_{V}$(f(ϰ)f(y)) ≥ γ. Therefore, Ť$_{V}$(f(ϰ) + f(y)) ≤ α, Ť$_{V}$(f(ϰ) f(y)) ≤ α; Î$_{V}$(f(ϰ) + f(y)) ≤ β, Î$_{V}$(f(ϰ) f(y)) ≤ β; Ƒ$_{V}$(f(ϰ) + f(y)) ≥ γ, Ƒ$_{V}$(f(ϰ) f(y)) ≥ γ. Hence f ($Đ_{(\alpha,\beta,\gamma)}$) is a lower level subsemiring of an NAFSSR V of a SR R^{l}.

2.2.9 Theorem: The homomorphic pre-image of a lower level subsemiring of an NAFSSR of a SR Ɽ^{l} is a lower level subsemiring of an NAFSSR of a SR Ɽ.

Proof: Let (Ɽ, +, .) and (Ɽ^{l} , +, .) be any two SR's and f : Ɽ → Ɽ^{l} be a homomorphism. That is, f(ϰ + ɏ) = f(ϰ) + f(ɏ) and f(ϰɏ) = f(ϰ)f(ɏ) for all ϰ and ɏ in Ɽ. Let V = f(Đ), where V is an NAFSSR of a SR Ɽ^{l}. Clearly Đ is an NAFSSR of a SR Ɽ. Let f(ϰ) and f(ɏ) in R^{l}, implies ϰ and ɏ in Ɽ. Let f($Đ_{(\alpha,\beta,\gamma)}$) is a lower level subsemiring of V. That is, Ť$_{V}$(f(ϰ)) ≤ α and Ť$_{V}$(f(ɏ)) ≤ α; Î$_{V}$(f(ϰ)) ≤ β and Î$_{V}$(f(ɏ)) ≤ β; Ƒ$_{V}$(f(ϰ)) ≥ γ and Ƒ$_{V}$(f(ɏ)) ≥ γ consequently Ť$_{V}$(f(ϰ) + f(ɏ)) ≤ α, Ť$_{V}$(f(ϰ) f(ɏ)) ≤ α; Î$_{V}$(f(ϰ)+f(ɏ))≤ β, Î$_{V}$(f(ϰ) f(ɏ)) ≤ β; Ƒ$_{V}$(f(ϰ) + f(ɏ)) ≥ γ, Ƒ$_{V}$(f(ϰ) f(ɏ)) ≥ γ. We have to prove that $Đ_{(\alpha,\beta,\gamma)}$ is a lower level subsemiring of Đ. Now, Ť$_{Đ}$(ϰ) = Ť$_{V}$(f(ϰ)≤α, implies that Ť$_{Đ}$(ϰ)≤α; Î$_{Đ}$(ϰ)= Î$_{V}$(f(ϰ))≤β, implies that Î$_{Đ}$ (ϰ) ≤ β; Ƒ$_{Đ}$(ϰ) = Ƒ$_{V}$(f(ϰ)) ≥ γ, implies that Ƒ$_{Đ}$(ϰ) ≥ γ and Ť$_{Đ}$(ɏ)=Ť$_{V}$(f(ɏ))≤α, implies that Ť$_{Đ}$(ɏ) ≤ α; Î$_{Đ}$(ɏ) = Î$_{V}$(f(ɏ)) ≤ β, implies that Î$_{Đ}$(ɏ) ≤ β; Ƒ$_{Đ}$(ɏ)=Ƒ$_{V}$(f(ɏ))≥γ, implies that Ƒ$_{Đ}$(ɏ)≥γ. (i a) Ť$_{Đ}$(ϰ+ɏ)=Ť$_{V}$(f(ϰ+ɏ))=Ť$_{V}$(f(ϰ)+f(ɏ))≤ α, consequently Ť$_{Đ}$(ϰ + ɏ) ≤ α. (i b) Î$_{Đ}$(ϰ + ɏ) = Î$_{V}$(f(ϰ + ɏ) = Î$_{V}$(f(ϰ) + f(ɏ)) ≤ β, consequently Î$_{Đ}$(ϰ+ɏ)≤β. (i c) Ƒ$_{Đ}$(ϰ+ɏ)=Ƒ$_{V}$(f(ϰ+ɏ))=Ƒ$_{V}$(f(ϰ) + f(ɏ)) ≥ γ, consequently Ƒ$_{Đ}$(ϰ + ɏ) ≥ γ. Also, (ii a) Ť$_{Đ}$(ϰɏ)=Ť$_{V}$(f(ϰɏ))=Ť$_{V}$(f(ϰ)f(ɏ))≤

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α,consequently Ť_Đ(ϰy)≤α. (ii b) Î_Đ(ϰy)=Î_V(f(ϰy))= Î_V(f(ϰ)f(y)) ≤ β, consequently Î_Đ(ϰy) ≤ β. (ii c) Ƒ_Đ(ϰy) = Ƒ_V(f(ϰy))= Ƒ_V(f(ϰ)f(y))≥ γ, consequently Ƒ_Đ(ϰy) ≥ γ. Therefore, Ť_V(f(ϰ)+f(y))≤α,Ť_V(f(ϰ)f(y))≤ α; Î_V(f(ϰ)+f(y))≤β, Î_V(f(ϰ)f(y))≤β; Ƒ_V(f(ϰ)+f(y)) ≥ γ, Ƒ_V(f(ϰ)f(y)) ≥ γ. Hence, $Đ_{(α,β,γ)}$ is a lower level subsemiring of an NAFSSR Đ of R.

2.2.10 Theorem: The anti-homomorphic image of a lower level subsemiring of an NAFSSR of a SR Ꞧ is a lower level subsemiring of an NAFSSR Ꞧ^{l}.

Proof: Let (Ꞧ, +, .) and (Ꞧ^{l} , +, .) be any two SR's and f: Ꞧ→Ꞧ^{l} be an anti-homomorphism. That is, f(ϰ + y) = f(y) + f(ϰ) and f(ϰy) = f(y)f(ϰ), for all ϰ and y in Ꞧ.Let V = f(Đ), where Đ is an NAFSSR of Ꞧ. Clearly V is an NAFSSR of Ꞧ^{l}. Let ϰ and y in Ꞧ, implies f(ϰ) and f(y) in Ꞧ^{l}.Let $Đ_{(α,β,γ)}$ is a lower level subsemiring of Đ. That is, Ť_Đ(ϰ) ≤ α and Ť_Đ(y) ≤ α; Î_Đ(ϰ) ≤ β and Î_Đ(y)≤β; Ƒ_Đ(ϰ)≥ γ and Ƒ_Đ(y)≥γ. Ť_Đ(y+ϰ)≤α , Ť_Đ(yϰ)≤α; Î_Đ(y+ϰ)≤β , Î_Đ(yϰ)≤β; Ƒ_Đ(y+ϰ)≥γ , Ƒ_Đ(yϰ)≥γ. We have to prove that f ($Đ_{(α,β,γ)}$) is a lower level subsemiring of V. Now, Ť_V(f(ϰ))≤ Ť_Đ(ϰ)≤α, consequently Ť_V(f(ϰ))≤α; Î_V(f(ϰ))≤ Î_Đ(ϰ)≤β, consequently Î_V(f(ϰ))≤β; Ƒ_V(f(ϰ))≥Ƒ_Đ(ϰ) ≥ γ, consequently Ƒ_V(f(ϰ))≥γ. Ť_V(f(y))≤Ť_Đ(y)≤ α, consequently Ť_V(f(y))≤α; Î_V(f(y))≤ Î_Đ(y) ≤ β, consequently I_V(f(y))≤β; Ƒ_V(f(y))≥Ƒ_Đ(y)≥γ, consequently Ƒ_V(f(y))≥γ. Now, (i a) Ť_V(f(ϰ)+f(y)) =Ť_V(f(ϰ)+f(y))= Ť_V(f (y+ϰ))≤Ť_Đ(y+ϰ)≤ α, which implies that, Ť_V(f(ϰ)+f(y))≤α. (i b) Î_V(f(ϰ) + f(y))= Î_V(f(ϰ)+f(y) = Î_V(f (y+ϰ))≤ Î_Đ(y+ϰ)≤β, which implies that, Î_V(f(ϰ)+f(y))≤ β. (i c) Ƒ_V(f(ϰ) + f(y)) = Ƒ_V(f(ϰ)+f(y)) = Ƒ_V(f (y+ϰ))≥ Ƒ_Đ(y+ϰ)≥γ, which implies that, Ƒ_V(f(ϰ)+f(y)) ≥ γ. Also, (ii a) Ť_V(f(ϰ)f(y))=Ť_V(f(ϰ)f(y))=Ť_V(f(yϰ))≤Ť_Đ(yϰ)≤α, which implies that, Ť_V(f(ϰ)f(y)≤ α. (ii b) Î_V(f(ϰ)f(y)) = Î_V(f(ϰ)f(y)) = Î_V(f (yϰ))≤ Î_A(yϰ) ≤ β, which implies that, Î_V(f(ϰ)f(y)) ≤β. (ii c) Ƒ_V(f(ϰ)f(y))=Ƒ_V(f(ϰ)f(y))=Ƒ_V(f(yϰ))≥Ƒ_Đ(yϰ)≥γ, which implies that, Ƒ_V(f(ϰ)f(y))≥γ. Therefore, Ť_V(f(ϰ)+f(y))≤ α and Ť_V(f(ϰ)f(y))≤ α; Î_V(f(ϰ)+f(y)) ≤ β and Î_V(f(ϰ)f(y))≤ β; Ƒ_V(f(ϰ) + f(y)) ≥ γ and Ƒ_V(f(ϰ)f(y)) ≥ γ. Hence f($Đ_{(α,β,γ)}$) is a lower level subsemiring of an NAFSSR V of Ꞧ^{l}.

2.2.11 Theorem: The anti-homomorphic pre-image of a lower level subsemiring of an NAFSSR of a SR Ꞧ^{l} is a lower level subsemiring of an NAFSSR of a SR Ꞧ.

Proof: Let (Ꞧ, +, .) and (Ꞧ^{l} , +, .) be any two SR's and f : Ꞧ → Ꞧ^{l} be an anti-homomorphism. That is, f(ϰ + y) = f(y) + f(ϰ) and f(ϰy) = f(y)f(ϰ), for all ϰ and y in Ꞧ. Let V = f(Đ), where V is an NAFSSR of a SR Ꞧ^{l}. Clearly Đ is an NAFSSR of a SR Ꞧ. Let f(ϰ) and f(y) in R^{l}, implies ϰ and y in Ꞧ. Let f($Đ_{(α,β,γ)}$) is a lower level subsemiring of V. That is, Ť_V(f(ϰ)) ≤ α and Ť_V(f(y)) ≤ α; Î_V(f(ϰ)) ≤ β and Î_V(f(y)) ≤ β; Ƒ_V(f(ϰ)) ≥ γ and Ƒ_V(f(y)) ≥ γ. Ť_V(f(y)+f(ϰ))≤ α, Ť_V(f(y)f(ϰ))≤α; Î_V(f(y)+f(ϰ))≤β, Î_V(f(y)f(ϰ))≤β; Ƒ_V(f(y)+f(ϰ))≥ γ, Ƒ_V(f(y) f(ϰ)) ≥ γ. We have to prove that $Đ_{(α,β,γ)}$ is a lower level subsemiring of Đ. Now, Ť_Đ(ϰ)= Ť_V(f(ϰ))≤α, consequently Ť_Đ(ϰ)≤α; Î_Đ(ϰ)= Î_V(f(ϰ))≤β, consequently Î_Đ(ϰ)≤β; Ƒ_Đ(ϰ)= Ƒ_V(f(ϰ)) ≥ γ, consequently Ƒ_Đ(ϰ) ≥ γ. Ť_Đ(y)=Ť_V(f(y)) ≤ α, consequently Ť_Đ(y)≤α; Î_Đ(y) = I_V(f(y))≤ β, consequently Î_Đ(y) ≤ β; Ƒ_Đ(y) = Ƒ_V(f(y))≥γ, consequently Ƒ_Đ(y)≥ γ. Now, (i a) Ť_Đ(ϰ+y)=Ť_V(f(ϰ+y))=Ť_V(f(y)+f(ϰ))=Ť_V(f(y) + f(ϰ))≤ α, consequently Ť_Đ(ϰ+y)≤ α.(i b) Î_Đ(ϰ+y)=Î_V(f(ϰ+y))= Î_V(f(y)+f(ϰ))= Î_V(f(y) + f(ϰ)) ≤ β, consequently Î_Đ(ϰ+y)≤β. (i c) Ƒ_Đ(ϰ y)= Ƒ_V(f(ϰ+y))=Ƒ_V(f(y)+f(ϰ))=Ƒ_V(f(y)+f(ϰ))≥γ, consequently Ƒ_Đ(ϰ+y)≥γ. Also, (ii a)

$\check{T}_{Đ}(ϰɏ)=\check{T}_V(f(ϰɏ))=\check{T}_V(f(ɏ)f(ϰ))=\check{T}_V(f(ɏ)f(ϰ))\leq \alpha$, consequently $\check{T}_{Đ}(ϰɏ) \leq \alpha$.(ii b) $\hat{I}_{Đ}(ϰɏ)= \hat{I}_V(f(ϰɏ))= \hat{I}_V(f(ɏ)f(ϰ))=\hat{I}_V(f(ɏ)f(ϰ))\leq\beta$, consequently $I_A(ϰɏ)\leq\beta$. (ii c) $Ƒ_{Đ}(ϰɏ) = Ƒ_V(f(ϰɏ))= Ƒ_V(f(ɏ)f(ϰ))=Ƒ_V(f(ɏ)f(ϰ))\geq \gamma$ consequently $Ƒ_{Đ}(ϰɏ) \geq \gamma$. Therefore, $\check{T}_V(f(ϰ)+f(ɏ))\leq\alpha$ and $\check{T}_V(f(ϰ)f(ɏ))\leq\alpha$; $\hat{I}_V(f(ϰ)+f(ɏ))\leq\beta$ and $\hat{I}_V(f(ϰ)f(ɏ))\leq\beta$; $Ƒ_V(f(ϰ) + f(ɏ)) \geq \gamma$ and $Ƒ_V(f(ϰ) f(ɏ)) \geq \gamma$. Hence $Đ_{(\alpha,\beta,\gamma)}$ is a lower level subsemiring of an NAFSSR Đ of Ɽ.

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