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EXACT PENDANT EQUITABLE DOMINATION IN GRAPHS

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Abstract

Let G be any graph. An exact equitable dominating set S in G is said to be an Exact pendant equitable dominating set if induced sub graph of $< S >$ contains at least one pendant vertex. The least cardinality of Exact pendant equitable set in G is called the Exact pendant equitable domination number of G, denoted by $\gamma_{epee}(G)$. In this article, we study the exact pendant domination number for some well known standard graph and also bounds for Exact pendant equitable domination number.

Keywords: exact dominating set, Pendant dominating set, Equitable dominating, exact pendant dominating set, Exact pendant equitable dominating set.

Introduction

Domination in graphs has become an important area of research in graph theory, as evidenced by the many results contained in the two books by Bondy, Haynes, Hedetniemi [3] [5] and also different kinds of domination parameters are studied with bounds on standard graphs.

Let $G = (V, E)$ be any graph with $V(G) = n$ vertices and $E(G) = m$ edges then n, m are respectively called the order and the size of the graph G. For each vertex $v \in V$, the open neighbourhood of v is the set $N(v)$ containing all the vertices of u adjacent to v and closed neighbourhood of v is the set $N[v]$ containing v and all the vertices u adjacent to v.

By a graph $G = (V, E)$ we mean a finite, undirected graph without loops or multiple edges. A subset D of V is a dominating set of G if every vertex of $V - S$ is adjacent to a vertex of S. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating of G . For a given graph $G = (V, E)$, a vertex of degree one is called pendant vertex and an edge incident to a pendant vertex is called pendant edge. Tree is a connected, undirected graph with no cycles. A dominating set S is called an exact dominating set if for every vertex $u \in S$, $|N(v) \cap S| = 1$ for every $v \in V - S$ and $|N(u) \cap S| \leq 1$ for every $u \in S$. A leaf of a tree is a vertex of degree 1, while a support vertex is a vertex adjacent to a leaf. In complete bipartite graph every vertex in one set is connected to every vertex in the other set. Ladder graph can be defined as $L_n = P_2 \times P_n$, where P_n is a path graph and it is equivalent to the $2 \times n$ grid graph. Lollipop graph is the graph obtained by joining a complete graph $K_m$to a path graph P_n with a

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bridge, denoted by $L_{m,n}$, A Barbell graph is a graph formed by joining of two complete graphs with a singleton edge.

A dominating set S in G is called a Pendant dominating set if the set S contains atleast one pendant vertex. The minimum cardinality of a Pendant dominating set is called the Pendant domination number $\gamma_{pe}(G)$ [6]. A set $S \subseteq V$ is said to be an exact dominating set if $i.$ for every $v \in V - S$ is adjacent to exactly one vertex of S or $u \in S$ is an isolate of S. That is, $|N(v) \cap S| = 1$ for every $v \in V - S$ and $|N(u) \cap S| \leq 1$ for every $u \in S$. An Exact dominating set $S \subseteq V$ is said to be a minimal dominating set if no proper subset is an exact dominating set. The exact domination number $\gamma_e(G)$ of a graph G is the cardinality of a minimum exact dominating set [2].

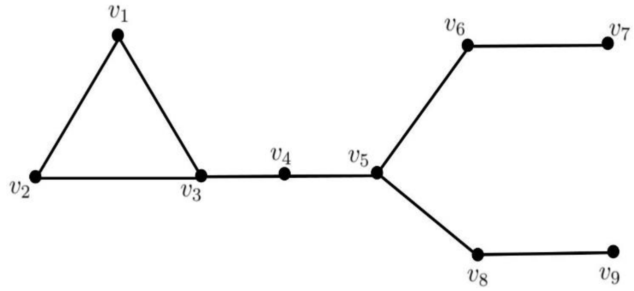
An exact dominating set S in G is called a Exact Pendant dominating set if the set S contains atleast one pendant vertex. The minimum cardinality of a Exact Pendant dominating set is called the Exact Pendant domination number $\gamma_{epe}(G)$ [7].

A subset S of $V(G)$ is called an equitable dominating set if for every $v \in (V - S)$ there exists a vertex $u \in S$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of such an equitable dominating set is called equitable domination number of G and is denoted by $\gamma_e(G)$

If $u \in V$ such that $|deg(u) - deg(v)| \geq 2$ for every $v \in N(u)$ then u is in every equitable dominating set such points are called equitable isolates. I_e denotes the set of all equitable isolates. The equitable neighborhood of u denoted by $N_e(u)$ is defined as $N_e(u) = \{v \in V : |v \in N(u), |deg(u) - deg(v)| \leq 1\}$. The maximum and minimum equitable degree of a point in G are denoted by $\Delta_e(G)$ and $\delta_e(G)$ that is $\Delta_e(G) = max_{u \in V(G)}|N_e(u)|$ and $\delta_e(G) = min_{u \in V(G)}|N_e(u)|$. The open equitable neighborhood and closed equitable neighborhood of v are denoted by $N_e(v)$ and $N_e[v] = N_e(v) \cup \{v\}$ respectively. If $S \subseteq V$ then $N_e(S) = \cup_{v \in S} N_e(v)$ and $N_e[S] = N_e(S) \cup S$. For a detailed treatment of the equitable domination and pendant equitable domination reader may referred to [1], [4].

An equitable dominating set S is called an Exact Pendant equitable dominating set if $< S >$ contains at least one pendant vertex, $|N(v) \cap S| = 1$ for every $v \in V - S$ and and $|N(u) \cap S| \leq 1$ for every $u \in S$. The minimum cardinality of exact pendant equitable dominating set is an exact pendant equitable domination number denoted by $\gamma_{epee}(G)$.

The following is the example for a graph G having exact pendant equitable dominating set.



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The set $S = \{v_3, v_4, v_7, v_9\}$ is an exact pendant equitable dominating set. The set $S = \{v_3, v_4, v_6, v_8\}$ is not an exact pendant equitable dominating set because $v_5 \in V - S$ and $|N(v_5) \cap S| \neq 1$.

Observation 1.1: It is noticed that every exact pendant equitable dominating set is a equitable dominating set but the converse is not true. That is, every equitable dominating set need not be an exact pendant equitable dominating set.

Exact Pendant Equitable Domination number for some standard graphs

Theorem 2.1: Let $G = P_n$ be a path with n vertices then

$$\gamma_{epee}(G) = \begin{cases} \frac{n}{3} + 1, & if\ n \equiv 0\ (mod 3) \\ \left\lceil \frac{n}{3} \right\rceil, & if\ n \equiv 1\ (mod 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1, & if\ n \equiv 2 (mod 3) \end{cases}$$

Proof: Let $G \cong P_n$ be a path and let $V(P_n) = \{v_1, v_2, v_3, \dots, v_n\}$.

We consider the following possible cases.

Case (i): Suppose $n \equiv 0 (mod 3)$ i.e $n = 3k$ for some integer $k > 0$ then the set $S = \{v_2, v_{3i}/1 \leq i \leq k\}$ is an exact pendant equitable dominating set of G. Thus, $\gamma_{epee}(P_n) = \frac{n}{3} + 1$.

Case (ii): Suppose $n \equiv 1 (mod 3)$ then it is easy to check the exact pendant equitable dominating set. The pendant dominating set itself is an exact pendant equitable dominating set in G. Therefore, $\gamma_{epee}(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

Case (iii): Proof of this case is similar to case (i).

Theorem 2.2: Let G be a complete bipartite graph of order $2n$, then $\gamma_{epee}(G) = 2$.

Proof: Let G be a complete bipartite graph and $\{v_1, v_2, v_3, \dots, v_{2n}\}$ are vertices of G. Let us choose the set $S = \{v_1, v_{n+1}\}$ where v_1 and v_{n+1} are two adjacent vertices in G. Then the set S will be a minimal exact pendant equitable dominating set in G. Hence, $\gamma_{epee}(G) = 2$.

Proposition 2.3: Let G be a complete graph with n vertices. Then $\gamma_{epee}(G) = n$.

Theorem 2.4: Let G be a Ladder graph. Then the exact pendant dominating set $\gamma_{epee}(P_2 \times P_n) = 2 + \left\lfloor \frac{n-1}{2} \right\rfloor$.

Proof: Let G be a Ladder graph with $n > 4$. Fix an edge $e = \{u_2, v_2\}$ of G. Then for any equitable dominating set of $P_2 \times P_{n-3}$ is the set S', the set $S = S' \cup \{u_2, v_2\}$ will be the minimum exact pendant equitable dominating set of G. Where S' is the minimum dominating set of $P_2 \times P_{n-3}$. Hence, $\gamma_{epe\,e}(P_2 \times P_n) = 2 + \left\lfloor \frac{n-1}{2} \right\rfloor$.

Theorem 2.5: Let G be a Lollipop graph then $\gamma_{epee}(G) = \left\lfloor \frac{n}{3} \right\rfloor + 2$.

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Proof: Let G be a Lollipop graph of the form $L_{m,n}$ where $m = 3$ and $n = 1,2,3, \dots$.

We have possible two cases.

Case (i): For $m = 3$ and $n = 3k$ where $k > 0$. The set $S = \{v_1, v_{3i}\}$ for $i = 1,2,3, \dots, \left\lceil\frac{n}{3}\right\rceil + 1$ will be the minimum exact pendant equitable dominating set.

Case (ii): For $m = 3$ and $n = 3k + 1, 3k + 2$ where $k > 0$. The set $S = \{v_3, v_{3i+1}\}$ for $i = 1,2,3, \dots, \left\lceil\frac{n}{3}\right\rceil$ will be the minimum exact pendant equitable dominating set.

Theorem 2.6: If G is a Barbell graph then, $\gamma_{epee}(G) = 2$.

Proof: A Barbell graph is a graph formed by joining of two complete graphs with vertices $(u_1, u_2, u_3, \dots, u_m)$ and $(v_1, v_2, v_3, \dots, v_n)$ by a singleton edge.

The set D={u,v} forms an exact pendant equitable dominating set for $B_{m,n}$. For every u_i, it is adjacent only to u; for every v_i, it is adjacent only to v; and u and v are adjacent only to each other. Thus, every vertex not in D is adjacent to exactly one vertex in D.

Definition 2.7: A pair of graphs G and H, where $O(G) = m$ and $O(H) = n$ will be considered. A graph called the corona product GoH of two graphs is created by taking one copy of G and $|V(G)| = m$ copies of H and connecting the $i - th$ vertex of G to each vertex in the $i - th$ copy of H.

Theorem 2.8: Let P_2 and K_n be two graphs with 2 and n vertices respectively, then $\gamma_{epee}(P_2 o K_n) = 2$.

Proof: Let $V(P_2) = \{v_1, v_2\}$ and $V(K_n) = \{u_1, u_2, \dots . u_n\}$ are the vertex set of the graphs P_2 and K_n respectively. The set $S = \{v_1, v_2\}$ will be the exact pendant equitable dominating set of the graph $P_2 o K_n$. The set of vertices of the graph P_2 are equitably dominated the vertices of K_n and each vertex in K_n are adjacent to exactly one vertex of P_2 and also the induced sub graph of S contains a pendant vertex.

Therefore, $\gamma_{epee}(P_2 o K_n) = |S| = 2$.

Bounds for Exact Pendant Equitable domination number $\gamma_{epee}(G)$

Theorem 3.1: Let S be a minimum exact pendant equitable dominating set in G then $|S| \leq |V - S|$.

Proof: Let S be a minimum pendant equitable dominating set in G. Take, $S = \{u_1, u_2, u_3, \dots, u_k\}$ and $|S| = k$. Note that, every vertex of S has at least one neighbor in $V - S$. Then, k vertices of S are adjacent to at least k vertices of $V - S$. That is, $V - S \geq k = |S|$. Therefore, $|S| \leq |V - S|$.

Theorem 3.2: For any connected graph G of order n then , $\gamma_{epee}(G) \leq \frac{n}{2}$

Proof : Let, $S = \{u_1, u_2, u_3, \dots, u_k\}$ be a minimum exact pendant equitable dominating set. Then, $\gamma_{epee}(G) = k$. For n number of vertices in a graph $n = |V - S| + |S| = |V - S| +$

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k. Therefore, $|V - S| = n - k$. We know that, if S is a minimum exact pendant equitable dominating set in G then $|S| \le |V - S|$. Therefore, $|S| \le n - k$. Since, $|S| = k$ implies $k \le n - k$. So, $k \le \frac{n}{2}$ yields that $\gamma_{epee}(G) \le \frac{n}{2}$.

Theorem 3.3: For any graph G of order n we have, $2 \le \gamma_e(G) \le \gamma_{epe}(G) \le \gamma_{eepe}(G) \le n$.

Further, $\gamma_{epee}(G) = 2$ if and only if G contains an edge of degree at least $n - 2$.

Proposition 3.4. Let G be any graph with n vertices. Then $\gamma_{epee}(G) = n$ if and only if G is a path P_2.

Proposition 3.5. Let G be a connected graph of order n. Then $\gamma_{epee}(G) = n - 1$ if and only if G is P_3

Let $\mathbb{G}$ be the collection of graphs of following types. A cycle and path graph each of order 4 and a path of order 5.

Theorem 3.6. Let G be a connected graph of order n. Then $\gamma_{epee}(G) = n - 2$ if and only if $G \in \mathbb{G}$.

Proof. Suppose $\gamma_{epee}(G) = n - 2$ and S is a $\gamma_{epee} - set$, then $\langle V - S \rangle$ either K_2 or $\overline{K_2}$.

We first prove that $n \le 5$. Clearly $V - S$ will be an equitable dominating set of G. Now if $\langle V - S \rangle = K_2$, then $V - S$ is itself a pendant equitable dominating set of G and hence $n \le 5$. Assume $\langle V - S \rangle = \overline{K_2}$ and assume $V(\overline{K_2}) = \{u, v\}$. Suppose u and v have at least two neighbors in S. Then the set $S' = (S - N_e(u, v)) \cup \{u, v\}$ is an equitable dominating set in G of cardinality less than $n - 3$. Hence G contains a pendant equitable dominating set of size less than $n - 2$, a contradiction. Therefore each vertex in $V - S$ has at most one neighbor in S, from which it follows that $|S| \le 3$ and consequently $n \le 5$. Thus G must be one among the graphs in $\mathbb{G}$. Converse is obvious.

Theorem 3.7. Let G be any graph, Then $\left\lceil \frac{n}{1+\Delta_e(G)} \right\rceil \le \gamma_{epee}(G) \le n - \Delta_e(G) + 1$.

Proof. Let G be any graph. Since any vertex in G can dominate at most $\Delta_e(G) + 1$ vertices including itself, it follows that $\left\lceil \frac{n}{1+\Delta_e(G)} \right\rceil \le \gamma_{epee}(G)$. On the other hand, let u be a vertex of maximum degree $\Delta_e(G)$ in G. Then for any vertex $v \in N_e(u)$, the set $(V - N_e(u)) \cup \{v\}$ will be a exact pendant equitable dominating set in G. Therefore $\gamma_{epee}(G) \le n - \Delta_e(G) + 1$.

Theorem 3.8: For any graph G of order n, $\gamma_{epee}(G) + \gamma_{epee}(\bar{G}) \le n + 2$.

Proof: Let G be any graph. Suppose G contains an isolated vertex then $\gamma_{epee}(G) \le n$ and $\gamma_{epee}(\bar{G}) \le 2$. Therefore, $\gamma_{epee}(G) + \gamma_{epee}(\bar{G}) \le n + 2$. Similarly, if $\bar{G}$ contains an isolated vertex, we obtain that, $\gamma_{epee}(G) + \gamma_{epee}(\bar{G}) \le n + 2$.

Suppose G and $\bar{G}$ contains no isolated vertices and also, we know that if a graph G has no isolated vertices, then $\gamma_{epee}(G) \le \frac{n}{2}$ also $\gamma_{epee}(\bar{G}) \le \frac{n}{2}$.

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Since, $\gamma_{epee}(G) \leq \gamma_{pe}(G) + 1$ always, it follows that $\gamma_{epee}(G) \leq \frac{n}{2} + 1$ and $\gamma_{epee}(\bar{G}) \leq \frac{n}{2} + 1$. Therefore, , $\gamma_{epee}(G) + \gamma_{epee}(\bar{G}) \leq 2\left(\frac{n}{2}\right) + 2 = n + 2$. Therefore, $\gamma_{epee}(G) + \gamma_{epee}(\bar{G}) \leq n + 2$.

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