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Pythagorean Fuzzy Open Maps and Homeomorphisms Pythagorean Fuzzy e-open sets in topological spaces

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Abstract

In this paper, we present the notions of Pythagorean fuzzy e-open and e-closed mappings within the framework of Pythagorean fuzzy topological spaces and explore several of their fundamental properties. The concepts of e-open and e-closed maps are formally defined and supported with illustrative examples. Furthermore, the study is extended to cover Pythagorean e-homeomorphism, Pythagorean e-completely homeomorphism, and Pythagorean $eT_{\frac{1}{2}}$-spaces, where a number of related results and properties are established.

Keywords and phrases: pythagorean fuzzy e-open maps, pythagorean fuzzy e-closed maps, $PFeT_{\frac{1}{2}}$-space, Pythagorean e-homeomorphism, Pythagorean e-C homeomorphism pythagorean fuzzy topological space.

1 INTRODUCTION

All branches of mathematics deals with the concept of fuzzy set that deals with membership values that ranges from[0,1] which has been introduced by zadeh[8]. As the extension of fuzzy sets. Then notion of intuitionistic fuzzy set theory that deals with non-membership values. Was proposed by Atanassov[5] . The concept of fuzzy topological space has need studied briefly by chang[1] on depending the concept of fuzzy set also some of open closed and continuity sets are been studied in topological space the cokor[2]. developed topological space. In intuitionistic fuzzy set Binshanhan[14] investigated the notions of pre-open ans pre-closed sets in fuzzy sets. In general concept of topology the notion of e-open set was developed by Ekici[11]. The fuzzy e-open sets concept in topological space as well as the e-continuity fuzzy. Was developed by Seenivasan et.al[12]. On studying the concept of fuzzy e-open set Vadivel et.al[19] established the concept in intuitionistic fuzzy set. To express the valuable information for any cryptic framework. The concept of pythagorean fuzzy sets was enriched by Yager[9,10,11]. properties and examples. There consist of . In this paper the concept of pythagorean fuzzy topological space. Is introduced by defining. e-continuity in section to the basic definition needed for the study on examples. e-open set are examine in topological space. The focus of this article is to introduce the idea of PFe-open and PFe-clsoed mappings in pythagorean topological spaces and also te work is extended to

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PFe-homeomorphism, PFe-C homeomorphism and PFe$T_{\frac{1}{2}}$-space, in pythagorean topological space and obtain some of its basic properties

2 PRELIMINARIES

In this paper, we present the notions of Pythagorean fuzzy e-open and e-closed mappings within the framework of Pythagorean fuzzy topological spaces and explore several of their fundamental properties. The concepts of e-open and e-closed maps are formally defined and supported with illustrative examples. Furthermore, the study is extended to cover Pythagorean e-homeomorphism, Pythagorean e-completely homeomorphism, and Pythagorean e$T_{\frac{1}{2}}$-spaces, where a number of related results and properties are established.

Definition 2.1 [11] Let (Y) be an universal set A fuzzy set A over (Y) can be represented as A={(Y, $\psi_A(Y)$) y ∈ (Y)}, where ψ_A:(Y) −→[0,1]denotes the membership function associated with the fuzzy set A. For each element y ∈ Y, the value ψ_A(Y) indicates te level of membership of y in A, ranging between 0 and 1.

Definition 2.2 [6] A intuitionistic fuzzy set A on a universe of discourse (Y) is represented as: A={(Y), ψ_A(Y), φ_A(Y)|$y \in$ Y}, ψ_A(Y), φ_A(Y) ∈ [0, 1], $0 \leq \psi_A(Y)^p + \varphi_A(Y)^p \leq 1.$ where ψ_A(Y) denotes the membership degree of the element (Y) in the set A. φ_A(Y) represents the non-membership degree of Y in A. The condition $0 \leq \psi_A(Y)^p + \varphi_A(Y)^p \leq 1.$ ensures that the intuitionistic fuzzy requirements are satisfied.

Definition 2.3 [9] The are defined on a laden set Y as objects having the form $P = \{(Y, \psi_q(y), \varphi_q(y)) : y \in Y\}$ where $\psi_q(y) : y \longrightarrow [0, 1]$ and $\varphi_q(y) : y \longrightarrow [0, 1]$ denote the degree membership and the degree of non-membership each element $y \in$ Y to the set q, respectively and $0 \leq (\psi_q(y))^2 + (\varphi_q(y))^2 \leq 1$ for all $y \in$ Y.

Definition 2.4 [14] Let Y be a non-empty set and τ be a family of Pythagorean fuzzy subsets of Y. If

(i) $1_Y, 0_Y \in \tau$,

(ii) for any $A_1, A_2 \in \tau$ we have $A_1 \cap A_2 \in \tau$,

(iii) for any$\{A_i\}_{i \in I} \subset \tau$, we have $\cup_{i \in I} F_{i \in \tau}$ given that denotes any index set $\tau^{\sim}$ is termed a pythagorean fuzzy topology on.

Definition 2.5 *Let Y be a non-empty set* & *the pythagorean* Y & Y *in the form* Y = $\{< y, \mu(y), \psi(y) >: y \in Y\}$,

Definition 2.6 [11]
Let q and fuzzy set of the forms.

Q=$\{< \tilde{a} \,;\, \psi_q(\tilde{a}) > | a \in \tilde{Y}\}$ and

R=$\{< \tilde{a} \,;\, \psi_r(\tilde{a}) > | a \in \tilde{Y}\}$, Then

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a) Q≤R iff $\psi_q \leq \psi_r$ for all a∈ Y.

b) Q=R iff Q≤ R and R≤Q.

c) A^c={< $a, \psi_q(a)$ > | a ∈ Y}

d) Q ∩ R={< $a, \psi_q(a) \cap \psi_r(a), \psi_q(a) \cup \psi_q(a)$ > | a ∈ Y}

e) Q∪R={¡a,$\psi_q(a) \cup \psi_r(a), \psi_q(a) \cap \psi_q(a)$ > | a ∈ Y}

For the simplness, we shall use the notation $A =< a, \psi_q, \varphi_r >$ instead of $A = \{a, \psi_q(a), \varphi_r(a) > |a \in$ Y} also for the sake of simplicity we shall use the notation $A =< a, (Q/\psi_q, R/\psi_r), (Q/\varphi_q, R/\varphi_r.)$ The PF $0 = \{< a, 0, 1 > |a \in$ Y}, $1 = \{< a, 1, 0 > |a \in$ Y} are respectively the empty set of the whole set of y.

Definition 2.7 [9] Let(Y, τ) be a pythagorean fuzzy topological space and A = {< Y}, ψ_A(Y), φ_A(Y) > ; $y \in$ Y} be pythagorean fuzzy set in Y. Then the pythagorean fuzzy interior and closure of A are defined by

(i) $PFcl(A)$ = ∩{Q:Q is closed pythagorean fuzzy set in Y and Z⊂ Q}

(ii) $PFint(A)$ = ∪ {R:R is open fuzzy set in Y and pythagorean R⊂ Z}

Definition 2.8 [4] For any PF. A in a Fs (Y, τ). A is said to be PF pre-open set if

[Y] $A \leq PFint(PFcl(A))$

Definition 2.9 [16] A set L is known to be pythagorean fuzzy

(i) δ- pre open set (in short, PFδPOS) if L ⊆ PFSint(PFδcl(L)).

(ii) δ- semi open set (in short, PFδOS) if L ⊆ PFcl(PFδint(L)).

(iii) e-open set(in short, PFeOS) if (L) ⊆PFcl(PFδint(L)∪PFint(PFδcl(L)

(iv) e*-open set(in short, PFδ Se*OS) if(L) ⊆PFcl(PFint(PFδcl(L)).

Then complement of a PFe-open set (resp. PFδOS, PFδPOS, PFδSOS & PFe*) is said to be a pythagorean fuzzy e-(resp. δ, δ-pre, δ-semi e & e*) closed set (in short, PFeCS(resp. PFδCS, PFδPCS, PFδSCS, & PFe*CS)) in Y.

The family of all PFδPOS(resp. PFδCS, PFδSOS, PFδSCS, PFeCS, PFe*OS, & PFe*CS) of Y is denoted by PFδPOS(Y) , (resp. PFδPCS(Y), PFδSOS(Y), PFδSCS(Y), PFδeOS(Y), PFδeCS(Y), PFe*OS(Y), & PFe*CS(Y)).

Definition 2.10 [7] A mapping ϕ : (Y, τ) −→ (Z, σ) is said to be a e-continuous if ϕ^{-1}(M) is a fuzzy e-open in Y for every open set M in Y.

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Definition 2.11 [7] A mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is said to be a e-irresolute if $\phi^{-1}(M)$ is a fuzzy e-open in Y for every open set M in Y.

3 Pythagorean Fuzzy e-open mapping

Definition 3.1 A mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is pythagorean fuzzy (resp. e, δ, δ-semi, δ-pre, $e*$) open(in short, PFO(resp. PFeO, PFδO, PFδPO, & $e*$)) if the image of each PFOS of(Y, τ) is PFeO(resp. PFδOS, PFδSOS, PFδPOS, PFeOS, PFδO,and PFe∗O) set in (Z, σ).

Theorem 3.1 The statements holds but not the converse.

Every (i)PFδO(resp. (ii)PFSO, (iii)PFSO, (iv)PFδSO, (v)PFδPO & (vi)PFeO) mapping is a (i) PFO (resp. (ii)PFδSO, (iii)PFδPO, (v)PFeO, (v)PFeO & (vi)PFe∗O) mapping.

Proof. Consider the map $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$

(i) Let L be a PFOS in L. As ϕ is PFδO, ϕ(L) is PFδOS in Z. Since all PFδOS, are PFOS, ϕ(L) is PFOS in Z. Thus ϕ is a PFO.

(ii) Let L be a PFOS in Y. As ϕ is PFδO, ϕ(L) is PFδOS in Z. Since all PFδOS, are PFδSOS, ϕ(L) is PFδSOS in Z. Thus ϕ is a PFδSO.

(iii) Let L be a PFOS in Y. As ϕ is PFO, ϕ(L) is PFOS in Z. Since all PFOS, are PFδPOS, ϕ(L) is PFδPOS, in Z. Thus ϕ is a PFδPO.

(iv) Let L be a PFOS in Y. As ϕ is PFδSO, ϕ(L) is a PFδSOS in Z. Since all PFδOS, are PFeOS, $\phi(L) is a$PFeOS, in Z. Thus ϕ is a PFeO.

(v) Let L be a PFOS in Y. As ϕ is PFδPO mapping, ϕ(L) is a PFδPOS in Z. Since all PFδPOS, are PFeOS, ϕ(L) is a PFeOS, in Z. Thus ϕ is a PFeO.

(vi) Let L be a PFOS in Y. As ϕ is PFeO, ϕ(L) is a PFeOS in Z. Since all PFeOS,are PFe∗OS, ϕ(L) is a PFe∗OS, in Z. Thus ϕ is a PFe∗OS.

Example 3.1 Let $Y = \{a_1, a_2, \} = \{b_1, b_2\} = Z$ and the PF set $\{A_1, A_2, A_3, A_4, A_5\}$ be PFOS of Y are defined as,

$$A_1 =< (\frac{\psi a1}{0.4}, \frac{\varphi a1}{0.6}), (\frac{\psi a2}{0.5}, \frac{\varphi a2}{0.7}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.8}) >$$

$$A_2 =< (\frac{\psi a1}{0.3}, \frac{\varphi a1}{0.7}), (\frac{\psi a2}{0.4}, \frac{\varphi a2}{0.9}), (\frac{\psi a3}{0.1}, \frac{\varphi a3}{0.6}) >$$

$$A_3 =< (\frac{\psi a1}{0.4}), (\frac{\varphi a1}{0.6}), (\frac{\psi a2}{0.5}, \frac{\varphi a2}{0.7}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.6}) >$$

$$A_4 =< (\frac{\psi a1}{0.3}, \frac{\varphi a1}{0.7}), (\frac{\psi a2}{0.4}, \frac{\varphi a2}{0.9}), (\frac{\psi a3}{0.1}, \frac{\varphi a3}{0.8}) >.$$

$$A_5 =< (\frac{\psi a1}{0.2}, \frac{\varphi a1}{0.5}), (\frac{\psi a2}{0.9}, \frac{\varphi a2}{0.5}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.5}) >.$$

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Here we have $\tau = \{0_Y, 1_Y, A_1\}$ $\rho = \{0_z, 1_z, A_1, A_2, A_3, A_4, A_5\}$ be a $PFTs$ on Z. Let $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$be an identity mapping. Then

(i) ϕ is PFO but not $PF\delta OS$ because the set $\phi(A_1) = A_1$ is a$PFOS$ but not $PF\delta OS$.

(ii) ϕ is $PF\delta SO$ but not PFO because the set $\phi(A_2^c) = (A_2^c)$ is a $PF\delta SO$.

Example 3.2 Let $Y = \{a_1, a_2, \} = \{b_1, b_2\} = Z$ and the PF set $\{A_1, A_2, A_3, A_4, A_5\}$ be PFOS of Y are defined as,

$$A_1 =< (\frac{\psi a1}{0.4}, \frac{\varphi a1}{0.6}), (\frac{\psi a2}{0.5}, \frac{\varphi a2}{0.7}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.8}) >$$

$$A_2 =< (\frac{\psi a1}{0.3}, \frac{\varphi a1}{0.7}), (\frac{\psi a2}{0.4}, \frac{\varphi a2}{0.9}), (\frac{\psi a3}{0.1}, \frac{\varphi a3}{0.6}) >$$

$$A_3 =< (\frac{\psi a1}{0.4}), (\frac{\varphi a1}{0.6}), (\frac{\psi a2}{0.5}, \frac{\varphi a2}{0.7}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.6}) >$$

$$A_4 =< (\frac{\psi a1}{0.3}, \frac{\varphi a1}{0.7}), (\frac{\psi a2}{0.4}, \frac{\varphi a2}{0.9}), (\frac{\psi a3}{0.1}, \frac{\varphi a3}{0.8}) >.$$

$$A_5 =< (\frac{\psi a1}{0.2}, \frac{\varphi a1}{0.5}), (\frac{\psi a2}{0.9}, \frac{\varphi a2}{0.5}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.5}) >.$$

Here we have $\tau = \{0_Y, 1_Y, A_1\}$ $\rho = \{0_z, 1_z, A_1, A_2, A_3, A_4, A_5\}$ be a $PFTs$ on Z. Let $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$be an identity mapping.

Then (i) ϕ is PFO mapping in Y but not $PF\delta OS$ mapping in Y because the set $\phi(A_1) = A_1$ is a$PFOS$ inZ but not $PF\delta OS$.

(ii) ϕ is $PF\delta SO$ but not PFO because the set $\phi(A_2^c) = (A_2^c)$ is a $PF\delta SO$.

Example 3.3 Let $Y = \{a_1, a_2, \} = \{b_1, b_2\} = Z$ and the PF set $\{A_1, A_2, A_3, A_4, A_5\}$ be PFOS of Y are defined as,

$$A_1 =< (\frac{\psi a1}{0.4}, \frac{\varphi a1}{0.6}), (\frac{\psi a2}{0.5}, \frac{\varphi a2}{0.7}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.8}) >$$

$$A_2 =< (\frac{\psi a1}{0.3}, \frac{\varphi a1}{0.7}), (\frac{\psi a2}{0.4}, \frac{\varphi a2}{0.9}), (\frac{\psi a3}{0.1}, \frac{\varphi a3}{0.6}) >$$

$$A_3 =< (\frac{\psi a1}{0.4}), (\frac{\varphi a1}{0.6}), (\frac{\psi a2}{0.5}, \frac{\varphi a2}{0.7}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.6}) >$$

$$A_4 =< (\frac{\psi a1}{0.3}, \frac{\varphi a1}{0.7}), (\frac{\psi a2}{0.4}, \frac{\varphi a2}{0.9}), (\frac{\psi a3}{0.1}, \frac{\varphi a3}{0.8}) >.$$

$$A_5 =< (\frac{\psi a1}{0.2}, \frac{\varphi a1}{0.5}), (\frac{\psi a2}{0.9}, \frac{\varphi a2}{0.5}), (\frac{\psi a3}{0.6}, \frac{\varphi a3}{0.5}) >.$$

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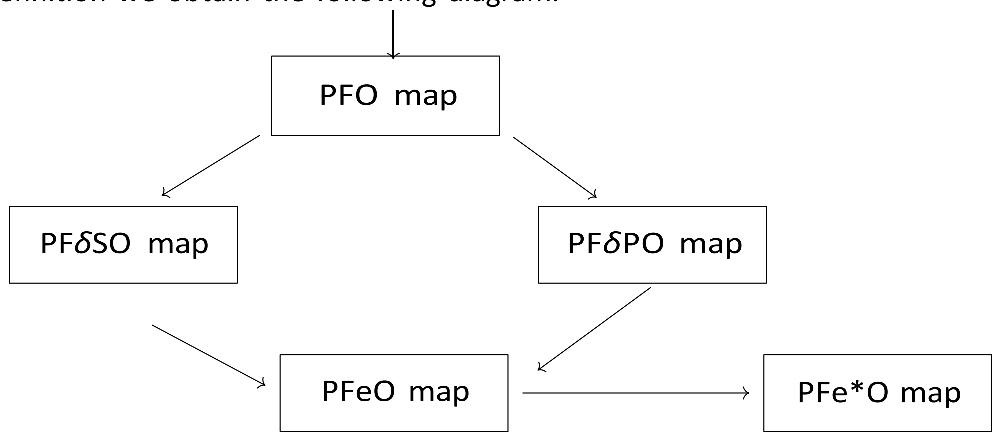
Here we have $\tau = \{0_Y, 1_Y, A_1\}$ $\rho = \{0_z, 1_z, A_1, A_2, A_3, A_4, A_5\}$ be a $PFTs$ on Z. Let $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$be an identity mapping.

Then (i) ϕ is $PF\delta SO$ mapping in Y but not PFO mapping in Y because the set $\phi(A_1) = A_1$ is a$PF\delta OS$ inZ but $PF\delta OS$.

(ii) ϕ is $PF\delta SO$ but not PFO because the set $\phi(A_2^c) = (A_2^c)$ is a $PF\delta POS.$

From the definition we obtain the following diagram.



Theorem 3.2 A mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is PFeO iff for every PFS L of (Y, τ), $\phi(PFint(L)) \subseteq PFeint(\phi(L))$.

Proof. Necessity: Assume ϕ is a PFeO mapping and M is a PFOS in (Y, τ).Now, PFint(L) $\subseteq \phi(L)$ implies $\phi(PFint(L)) \subseteq \phi(M)$. Since ϕ is a PFeO mapping, $\phi(PFint(L))$ is PFeOS in (Z, σ) such that $\phi(PFint(L)) \subseteq PFeint(\phi(L))$. Therefore $\phi(PFint(L)) \subseteq PFeint\phi(L)$.

Sufficiency: Assume M is a PFOS of (M, τ). Then $\phi(Z)= \phi(PFint(M)) \subseteq PFeint(\phi(M))$.But PFeint$(\phi(M)) \subseteq (\phi(M))$. S on $(\phi(M)) = PFeint(M)$ which implies $(\phi(M))$ is a PFeOS of (Z, σ) and therefore ϕ is a PFeO.

Theorem 3.3 $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is a PFeO mapping, then PFint$(\phi^{-1}(M)) \subseteq \phi^{-1}(PFeint)(M)$ for every PFSM of (Z, σ).

Proof. Assume M is a PFS of (Z, σ). We know that PFint$(\phi^{-1}(M)$ is a PFOS in $((Y, \tau)$. Since \$ is PFeO, $\phi(PFint(\phi^{-1}(M)))$ is PFeO in (M, σ) and so $\phi(PFint((\phi^{-1}(M))) \subseteq PFint((\phi^{-1}(M))) \subseteq PFeintM$. Thus PFint $(\phi^{-1}(M)) \subseteq \phi^{-1}(PFeint(M))$.

Theorem 3.4 A mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is PFeO iff for each PFS L of (Z, σ) and for each PFS M of (M, τ) containing $(\phi^{-1}(M))$, there is a PFeCS (H)of (Z, σ) s $(L) \subseteq (M)$ and $\phi^{-1}(H) \subseteq (M)$.

Proof. Necessity: Let ϕ be a PFeO mapping. Assume L is the PFCS of (Z, σ) and M is a PFCS (Y, τ) such that $\$^{-1}(M) \subseteq (M)$. Then H $=(\phi(M)^c)^c)$ is PFeCS of (M, σ) s $(M) and \phi(H) \subseteq (M)$. M

Sufficiency: Assume M is a PFOS of (Y, τ). We have $\phi^{-1}(\phi(M)^c) \subseteq))(M)^c$ and $(M)^c$ is PFCS in (M, τ). Then there is a PFeCS (M) of (M, σ) such that $((M))^c \subseteq M$ and $(\phi^{-1}(M)) \subseteq (M))^c)$ by presumption. Therefore $(M) \subseteq (\phi^{-1}(M))^c$. So$(M))^c \subseteq \phi(M) \subseteq \phi((\phi^{-1}(M))^c) \subseteq (M))^c$. Hence $(\phi(M)(\phi(M)^c)$. As $(M)^c$ is PFeOS of (M, σ), $(\phi(M)$ is PFeO in (M, σ) and hence ϕ is PFeO mapping.

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Theorem 3.5 A mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is PFeO iff $\phi^{-1}(PFecl(M)) \subseteq PFecl(\phi^{-1}(M))$ for every PFS (M) of (Z, σ).

Proof. Necessity: Let ϕ be a PFeO mapping. For any PFS (M) of $(Z, \sigma).(\phi^{-1}(M)) \subseteq PFecl(\phi^{-1}(M))$. Therefore by Theorem 3.2.9, ∃ a PFeCS (M) in (Z, σ) s (L)) ⊆ (M) and $(\phi^{-1}(M) \subseteq PFecl(\phi^{-1}(M))$. Hence we get $(\phi^{-1}(PFecl(M)) \subseteq (\phi^{-1}(M)) \subseteq PFcl(\phi^{-1}(M))$.

Sufficiency: Let (M) be a PFS of (Z, σ) and (L) be a PFCS of (Y, τ) containing $(\phi^{-1}(M))$ put H=PFcl(M). Then $(M) \subseteq (H)$ and H is PFeC and $\phi^{-1}(H) \subseteq PFecl(\phi^{-1}(M)) \subseteq L$. Therefore by Theorem 3.2.9, ϕ is a PFeO map.

Theorem 3.6 Let $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$&Y : $(Z, \sigma) \longrightarrow (\tilde{Z}, \rho)$ be two PFS mappings & $Y \circ \phi$: $(Y, \tau) \longrightarrow (\tilde{Z}, \rho)$ is PFeO. If Y : $(Z, \sigma) \longrightarrow (\tilde{Z}, \rho)$ is PFeirr, then $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is PFeO mapping. **Proof.** Let (L) be a PFOS in (Y, τ). Since $Y\circ \phi$ is a PFeO map, $Y\circ \phi(L)$ is PFeOS of $(\tilde{Z}, \rho)$. As Y is PFeirr and $Y \circ \phi(Y)$ is PFeOS of $(\tilde{Z}, \rho)$, $Y^{-1}(Y \circ \phi(L)) = \phi(L)$ is PFeOS in (Z, σ).Hence ϕ is a PFeO map.

Theorem 3.7 Let $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is PFSO and Y : $(Z, \sigma) \longrightarrow (\tilde{Z}, \rho)$ be PFeO mappings. Then $Y \circ \phi : (Y, \tau) \longrightarrow (\tilde{Z}, \rho)$ is PFeO.

Proof. Assume (M) is a (Y, τ). Since ϕ is a PFO map, $\phi(L)$ is a PFOS in (Z, σ). As ϕ is PFeO, $Y(\phi(L)) = (Y \circ \phi(L))$ is a PFeOS of $(\tilde{Z}, \rho)$. Therefore $Y \circ f$ is a PFeO map.

4 Pythagorean fuzzy e-closed mapping

Definition 4.1 A mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is pythagorean fuzzy (resp. e, δ, δ-semi, δ-pre, $e*$) closed(in short, PFO(resp. PFeC, PFδSC, PFδPC, & PF$e*$C)) if the image of each PFCS of(Y, τ) is PFCS(resp. PFeCS, PFδSCS, PFδPCS, and PFe∗CS) in (Z, σ).

Theorem 4.1 The statements hold but not the converse. Every (i)PFδC (resp. (ii)PFSCS, (iii)PFC, (iv)PFδSC, (v)PFδPC & (vi)PFeCS) mapping is a (i) PFC (resp. (ii)PFδSC, (iii)PFδPC, (v)PFeC, (v)PFeC & (vi)PFe∗C) mapping.

Proof. Consider the map $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$

(i) Let M be a PFCS in Y. As ϕ is PFδC, $\phi(M)$ is PFδCS in Z. Since all PFδCS, are PFCS, $\phi(M)$ is PFCS in Z. Thus ϕ is a PFC.

(ii) Let M be a PFCS in Y. As ϕ is PFC, $\phi(M)$ is PFCS in Z. Since all PFCS are PFδSCS, $\phi(M)$ is PFδSCS in Z. Thus ϕ is a PFδSC.

(iii) Let M be a PFCS in Y. As ϕ is PFC, $\phi(M)$ is PFCS in Z. Since all PFCS, are PFδPCS, $\phi(M)$ is PFδPCS,in Z. Thus ϕ is a PFδPC.

(iv) Let M be a PFCS in Y. As ϕ is PFδSC, $\phi(M)$ is a PFδSCS in Z. Since all PFδCS, are PFeCS, $\phi(M)$PFeCS,in Z. Thus ϕ is a PFeC.

(v) Let M be a PFCS in Y. As ϕ is PFδPC mapping, $\phi(M)$ is a PFδPCS in Z. Since all PFδPCS,are PFeCS,$\phi(M)$ is a PFeCS, in Z. Thus ϕ is a PFeC.

(vi) Let M be a PFCS in Y. As ϕ is PFeC, $\phi(M)$ is a PFeCS in Z. Since all PFeCS,are PFe∗CS, $\phi(M)$ is a PFe∗CS, in Z. Thus ϕ is a PFe∗CS.

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Theorem 4.2 *A mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is PFeC iff for each PFys (M) of (Z, σ) and for each PFOS (M)of (Y, τ) containing $(\phi^{-1}(M))$, there is a PFeCS (H) and $(\phi^{-1}(H)) \subseteq (M)$.*

Proof. *Necessity: Consider a PFeC mapping ϕ. Assume M is the PFCS of (Z, σ) and (L) is a PFOS of (Y, τ)such that $(\phi^{-1}(M)) \subseteq M$. Then (H)=1- $(\phi^{-1}(M)^c)$ is PFeOS of (Z, σ) such that $(\phi^{-1}(H)) \subseteq (M)$.*

Sufficiency: Assume (L) is a PFCS of (Y, τ). Then $(\phi(M))^c$ is a PYs of (Z, σ) and $(M)^c$ is PFOS in (Y, τ) such that $(\phi^{-1}(\phi(M))^c \subseteq (M)^c$. There exists a PFeOS (H) of (Z, σ) such that $(\phi(M))^c \subseteq (H)$ by presumption and $(\phi^{-1}(H)) \subseteq (M)^c$. Therefore $(Y \subseteq (\phi^{-1}(H))^c$. Hence $(H)^c \subseteq (\phi(H)) \subseteq \phi((\phi^{-1}(H))^c) \subseteq (H)^c$ which implies $(\phi(M)) = (H)^c$. Since $(H)^c$ is PFeCS of (Z, σ), $(\phi(M))$ is PFeC in $(\phi(M))$ and hence (ϕ) is PFeC mapping.

Theorem 4.3 *If $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is PFC and $Y : (Z, \sigma) \longrightarrow (\tilde{Z}, \rho)$ is PFeC, then $Y \circ \phi : (Y, \tau) \longrightarrow (\tilde{Z}, \rho)$ is PFeC.*

Proof. *Consider a PFCS (M) in (Y, τ). As ϕ is a PFC map,$(\phi(M))$ is PFCS of (Z, σ). As (ϕ) is a PFeC map, $Y \circ \phi)(Y) = Y(\phi(M))$ is PFeCS in $(\tilde{Z}, \rho)$. Hence $Y \circ f$ is PFeC mapping.*

Theorem 4.4 *If $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is PFeC map, hen $PFecl(\phi(L)) \subseteq \phi(PFcl(L))$.*

Proof. *Let (ϕ) be a PFeC mapping and (L) be a PFCS in (Y, τ). Now, $(L) \subseteq PFcl(L)$ implies $(\phi(L)) \subseteq \phi(PFcl(L))$. Since ϕ is a PFeC mapping,$\phi(PFcl(L))$ is PFeCS in (Z, σ)* s *$(\phi(L)) \subseteq \phi(PFcl(L))$. Therefore, $PFecl\phi(L) \subseteq \phi(PFcl(L))$.*

Theorem 4.5 *Let $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$&$Y : (Z, \sigma) \longrightarrow (\tilde{Z}, \rho)$ be PFeC mappings. If each PFeCS of (Z, σ) is PFCS, then $Y \circ \phi)(Y) = Y(\phi(M))$ is PFeC.*

Proof. *Consider a PFCS (Y)in (Y, τ). Asϕ is a PFeC map,$(\phi(L))$ is PFeCS of (Z, σ). Then $(\phi(L))$ is PFCS of (Z, σ) by presumption. AsY is a PFeC map, $Y(\phi(L)) = (Y \circ \phi)(L)$ is PFeCS in (Q, ρ). Hence $Y \circ \phi)$is PFeC mapping.*

Theorem 4.6 Consider a bijective mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$. Then the following statements are equivalent:

(i) ϕ is a PFeO mapping.

(ii) ϕ is a PFeC mapping.

(iii) ϕ^{-1} is a PFeCts mapping.

Proof. $(i) \Rightarrow (ii)$: Consider a PFeO mapping ϕ. If (Y) is a PFOS in (Y, τ), by presumption $(\phi(L))$ is a PFeOS in (Z, σ). But now, (Y) is PFCS in (Y, τ). It implies 1-(Y) is a PFOS in (Y, τ). By assumption $\phi(1 - (Y))$ is a PFeOS in (Z, σ). Thus, $1 - (\phi(1 - Y))$ is a PFeCS in (Y, τ). Hence, ϕ is a PFeC mapping.

(ii)$\Rightarrow$ (iii) : Consider a PFCS L in (Y, τ). By assumption, $(\phi(L))$ is a PFeCS in (Z, σ). Therefore, $(\phi(L)) = (\phi^{-1})^{-1}(L)$. Hence $(\phi^{-1})$is PFeCts.

(ii)$\Rightarrow$ (iii) : Consider a PFOS L in (Y, τ). By assumption, $(\phi^{-1})^{-1}(L) = (\phi(L))$ is a PFeO mapping.

5 Pythagorean fuzzy e-homeomorpism

Definition 5.1 A bijection $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ is called a PyF homeomorphism (resp. PYFe-

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homeomorphism)(in short,PF$\tilde{H}om$($resp.PFe\tilde{H}om$))if ϕ and ϕ^{-1} are PFCts (resp. PFeCts)mappings.

Theorem 5.1 Each PF$\tilde{H}$ om is a PFe$\tilde{H}$ om. But the converse not true. **Proof.**Assume ϕ is PF$\tilde{H}$ om. Then ϕ and ϕ^{-1} are PFCts. But each PFCts function is PFeCts. Therefore,ϕ and ϕ^{-1} are PFCts. Hence, ϕ is a PF$\tilde{H}$ om.

Theorem 5.2 Consider a bijective mapping $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$. If ϕ is PFeCts, then the followings statements are equivalent:

(i) ϕ is a PFeC mapping.

(ii) ϕ is a PFeO mapping.

(iii) ϕ^{-1} is a PFe$\tilde{H}$ om.

Proof. $(i) \Rightarrow (ii)$: Let ϕ be a bijective and PFeC mapping. Therefore, ϕ^{-1} is PFeCts. As each PFOS in (Y, τ) is a PFeOS in (Z, σ), ϕ is a PFeO mappings.

$(ii) \Rightarrow (iii)$: Assume ϕ be a bijective and PFeO mapping. Also, ϕ^{-1} is PFeCts. Therefore, ϕ and ϕ^{-1} are PFeCts. Thus, ϕ is a PFe$\tilde{H}$ om).

$(iii) \Rightarrow (i)$: Assume ϕ is a PFe$\tilde{H}$ om). It follows that ϕ and ϕ^{-1} are PFeCts. As each PFCS in (Y, τ) is a PFeCS in (Z, σ), ϕ is a PFeC mapping.

Definition 5.2 A PFts (Y, τ) is called a pythagorean fuzzy set e$T_{\frac{1}{2}}$ (in short, PFe$T_{\frac{1}{2}}$)-space if each PFeCS is PFC in (Y, τ)

Theorem 5.3 Let $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ be a PFe$\tilde{H}$ om). Then ϕ is a PF$\tilde{H}$ om) if (Y, τ) and (Z, σ) are PFe$T_{\frac{1}{2}}$-space.

Proof. Let M be a PFCS in (Z, σ). So $\phi^{-1}(M)$ is a PFeCS in (Y, τ) is a PFe$T_{\frac{1}{2}}$-space, $\phi^{-1}(M)$ is a PFCS in (Y, τ). Hence, ϕ is PFCts. BY presumption, ϕ^{-1} is PFCts. Let M be a PFCS in (Y, τ). Then $(\phi^{-1})^{-1}(L) = (\phi(L))$ is a PFCS in (Z, σ), by presumption. Since (Z, σ) is a PFe$T_{\frac{1}{2}}$-space, $(\phi(L))$

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is a PFCS in (ϕ(L)). Hence,ϕ^{-1} is PFCts. Therefore, ϕ is a P F$\tilde{H}$ *om*).

Theorem 5.4 Let ϕ : (Y, τ) −→ (Z, σ) be a PFts. If (Z, σ) is a P F$\tilde{H}$ *om*)-space, then the following are equivalent:

(i) ϕ is a PFeC mapping.

(ii) If M is a PFOS in (Z, σ), then (ϕ(L)) is PFeOS in (Z, σ).

(iii) $\phi(PFint(L)) \subseteq PFcl(PFint(\phi(L)))$ for each PFS M in (Z, σ).

Proof. (*i*) ⇒ (*ii*) : Obvious. (*ii*) ⇒ (*iii*) : Let M be a PFS (Z, σ). Then, PFintM is a PFOS in (Y, τ). Then, $\phi(PFint(M))$ is a PFeOS in (Z, σ). Since (Z, σ) is a PFe$T_{\frac{1}{2}}$-space,so $\phi(PFint(M))$ is a PFOS in (Z, σ). Therefore, $\phi(PFint(M)) = PFint(\phi(PFint(M))) \subseteq PFcl(PFint(\phi(L)))$.

(*iii*) ⇒ (*i*) : Let M be a PFCS in (Y, τ). Then $(L)^c$ is a PFOS in (Y, τ). From, $\phi(PFint(L)^c) \subseteq PFcl(PFint(\phi(L)^c))$, $(\phi(L)^c))$

Theorem 5.5 Let ϕ : (Y, τ) −→ (Z, σ) and Y : (Z, σ) −→ ($\tilde{Z}$, ρ) be PFeC, where (Y, τ) and (Q,ρ) are two PFTS and (Z, σ) a PFe$T_{\frac{1}{2}}$-space, then the composition Y ∘ ϕ is PFeC.

Proof. Consider a PFCS L in (Z, σ). As ϕ is PFeC and (ϕ(L) is a PFeCS in (Z, σ), by assumption (ϕ(L) is a PFCS in (Z, σ). As Y is PFeC, Y(ϕ(L)) is PFeC in (Q,ρ)and Y(ϕ(L)) = Y ∘ ϕ(L).Hence, Y ∘ ϕ is PFeC.

Theorem 5.6 *Let* ϕ : (Y, τ) −→ (Z, σ) *and* Y : (Z, σ) −→ ($\tilde{Z}$, ρ) *be two PFTs, then the following hold:*

(i) If Y ∘ ϕ *is PFeO and* ϕ *is PFCts, then* Y *is PFeO.*

(ii) If Y ∘ ϕ *is PFeO and* Y *is PFCts, then* ϕ *is PFeO.*

Proof. *(i) Let* (Z) *be a PFOS in* (Z, σ)*.As* ϕ *is PFCts mapping,* (ϕ^{-1}(M)) *is PFOS in* (Y, τ)*. As* Y∘ ϕ *is PFeO mapping,* (Y∘ ϕ)(ϕ^{-1}(M)) = Y(M) *is PFeOS in (Q,ρ). Thus* Y *is PFeO mapping. The other case is similar.*

6 Pythagorean fuzzy e-C homeomorphism

Definition 6.1 A bijection ϕ : (Y, τ) −→ (Z, σ) is called a PFe-C homeomorphism (in short, P FeC$\tilde{H}$ *om*)if ϕ and (ϕ^{-1} are PFeirr mappings.

Theorem 6.1 Each P FeC$\tilde{H}$ *om* i a P Fe$\tilde{H}$ *om*. But e converse not true. **Proof.** Assume (M) is a PFCS in (Z, σ). This implies (M) is a PFeCS in (Z, σ). By hypothesis, (ϕ^{-1}(M)) is a PFeCS in (Y, τ). Thus, ϕ is a PFeCts mapping. Therefore, c and (ϕ^{-1}) are PFeCts ,mapping. So ϕ is a P Fe$\tilde{H}$ *om.*

Theorem 6.2 If ϕ : (Y, τ) −→ (Z, σ) is a P Fe$\tilde{H}$ *om,* then Pecl(ϕ^{-1}(M)) ⊆ $\phi^{-1}(PFcl(M))$for each PFs (M) in(Z, σ). **Proof.** Consider a PFs (M) in (Z, σ). Now, PFcl(M) is a PFCS in (Z, σ) and every PFCS is a PFeCSin (Z, σ). Assume ϕ is PFeirr and $\phi^{-1}(PFcl(M))$ is a PFeCS in (Y, τ). Then, PFcl($\phi^{-1}(PFcl(M))$) = ($\phi^{-1}(PFcl(M))$. Therefore, $(PFecl(\phi^{-1}(M))) \subseteq \phi^{-1}(PFcl(M))$ for every PFs (M) in (Z, σ).

Volume 38 No. 12s 2025,
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Theorem 6.3 Let $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ be a PFeC$\tilde{H}om$. Then $(PFecl(\phi^{-1}(M))) = \phi^{-1}(PFcl(M))$ for each PFs (M) in (Z, σ).

Proof. As ϕ is a PFeC$\tilde{H}om$, ϕ is a PFeirr mapping. Assume (M) is a PFs in (Z, σ). It is obvious that, PFecl(M) is a PFeCS in (M, σ). Then PFecl(M) is a PFeCS in (Z, σ).As $(\phi^{-1}(M)) \subseteq (\phi^{-1}(PFecl(M))$. PFecl$(\phi^{-1}(M)) \subseteq PFecl(\phi^{-1}(PFecl(M))) = (\phi^{-1}(PFecl(M)))$. Therefore, PFecl$(\phi^{-1}(M)) \subseteq (\phi^{-1}(PFecl(M))$. Assume ϕ is a PFeC$\tilde{H}om$. ϕ^{-1} is a PFeirr mapping. Consider a PFs $(\phi^{-1}(M))$ in $(\phi^{-1}(M))$, which implies PFecl$(\phi^{-1}(M))$ is a PFeCS in (Z, σ). Hence, PFecl$(\phi^{-1}(M))$ is a PFeCS in (Z, σ). Hence, $(\phi^{-1})^{-1}(PFecl(\phi^{-1}(M))) = \phi(PFecl(\phi^{-1}(M)))$ is a PFeCS in (Z, σ). Therefore,

$(M) = (\phi^{-1})^{-1}(\phi^{-1}(M)))$

$\subseteq (\phi^{-1})^{-1}(PFecl(\phi^{-1}(M)))$

$=\phi(PFecl(\phi^{-1}(M)))$

Hence, PFecl(M) $\subseteq PFecl(\phi(PFecl(\phi^{-1}(M)))$, because ϕ^{-1} is a PFeirr mapping. Thus,

$(\phi^{-1}(PFecl(M)) \subseteq \phi^{-1}(\phi(PFecl(\phi^{-1}(M))) = (PFecl(\phi^{-1}(M)))$. That is, $(\phi^{-1}(PFecl(M)) \subseteq PFecl(\phi^{-1}(M)))$. Therefore, PFecl$(\phi^{-1}(M)) = \phi^{-1}(PFecl(M))$.

Theorem 6.4 If $\phi : (Y, \tau) \longrightarrow (Z, \sigma)$ and $Y : (Z, \sigma) \longrightarrow (\tilde{Z}, \rho)$ are PFeC$\tilde{H}om$'s, then $Y \circ \phi$ is a PFeC$\tilde{H}om$.

Proof. Assume ϕ and Y are two PFeC$\tilde{H}om$'s. Let M be a PFeCS in (Q,ρ). So, $Y^{-1}(M)$ is a PFeCS in (Z, σ). It implies $\phi^{-1}(Y^{-1}(M))$ is a PFeCS in (Y, τ) by hypothesis. Thus, $Y \circ \phi$ is a PFeirr mapping. Consider a PFeCS(L)in (Y, τ). Then, by presumption, is a PFeCS in(Z, σ). Hence, $Y(\phi(Y))$ is a PFeCS in (Q,ρ). Therefore, $Y \circ \phi$ is a PFeC$\tilde{H}om$.

7 conclusion

In this chapter, the concepts of pythagorean fuzzy e-open and pythagorean fuzzy e-closed mappings in pythagorean fuzzy topological spaces were discussed. Furhermore, e work was extended to include pythagorean fuzzy homeomorphism, pythagorean fuzzy e-homeomorphism and pythagorean fuzzy e$T_{\frac{1}{2}}$-space. Moreover, the study demonstrated pythagorean fuzzy e-C homeomorphism and derived some of its related characteristics.

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