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ON PAIRED DOUBLE DOMINATION NUMBERS OF SOME GRAPHS

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Abstract

In this paper , we continue the study of paired double domination in graphs . A paired – double dominating set of a graph G with no isolated vertex is a double dominating sets of vertices whose induced subgraph has a perfect matching . We establish Paired double domination number of graphs (i) identifying vertices of Union Cycle and Path say D (ii) identifying vertices of two cycles say H with same order and different order.

MSC : 05C69

Key words: Double domination number, Paired - double domination number, Locating paired - double domination number.

1 INTRODUCTION

Let G = (V, E) be graph with vertex set V and edge set E. We obtain with some terminology. For a vertex v of a graph G, the open neighborhood of a vertex $v \epsilon V$ is $N(u) = \{ v/uv \in E \}$ and closed neighborhood of vertex $N[v] = N(v) \cup \{v\}$.

A subset $S \subseteq V$ is a dominating set of G, if for every vertex $v\epsilon V, |N[v] \cap S| \geq 1$. The domination number is the minimum cardinality of a dominating set of G. A subset S of V is double dominating set of G if for every vertex $v \epsilon V, |N[v] \cap s| \geq 2$, that is $v\ is\ in\ S$ and has at least one neighbor in S and v is in V-S has at least two neighbors in S [4].

A set S is called paired – dominating set if it dominates V and < S > contains at least one perfect matching. A paired –dominating set S with matching M is a dominating set $S = \{v_1, v_2, v_3, \dots, v_{2t-1}v_{2t}\}$ with independent edge set $M = \{e_{1,}e_2, \dots, e_t\}$ where each edge e_j joins two elements of S, that is M is perfect matching of < S > . If $v_jv_k = e_i \in M$, we say that v_j and v_k are paired in in S [5]. A set S is called a paired – double dominating set if it is a double dominating set and < S > contains at least one perfect matching. The double domination number $\gamma_{dd}(G)$ is the minimum cardinality of double dominating set of G, the paired – domination number $\gamma_{pr}(G)$ is the minimum cardinality of paired dominating set of G and paired double domination number $\gamma_{prdd}(G)$ is the minimum cardinality of a paired double dominating set of G. Identifying vertices means combining them into one vertex so that all edges incident to those vertices are now incident to the new single vertex.

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In this paper, we characterize Paired double domination number of identifying vertices of Union Cycle and Path say D and identifying vertices of two cycles say H with same order and different order.

Theorem 1.1. [7] For any path P_n , $\gamma_{prdd}(P_n) = \begin{Bmatrix} 2 & if n = 2 \\ does\ not\ exist & if\ n = 3 \\ 2\left\lfloor\frac{n}{3}\right\rfloor + 2 & other\ wise \end{Bmatrix}$

Theorem 1.2. [7] For any cycle C_n $\gamma_{prdd}(C_n)$= $2\left\lceil\frac{n}{3}\right\rceil$

Theorem 1.3. [7] For any path P_n , n≠3 , $\gamma_{dd}(P_n) \leq \gamma_{prdd}(P_n)$

Theorem 1.4. [7] For any cycle C_n , $\gamma_{dd}(C_n) \leq \gamma_{prdd}(C_n)$

Theorem 1.5. [7] If n = 3k+2 where k ϵ N, then$\gamma_{prdd}(P_n) = \gamma_{prdd}(C_n)$.

2. MAIN RESULTS

In this paper, we find paired double domination numbers of graph D and D'.

Theorem 2.1 If D is a connected graph of order $n \geq 3$ and which has a PDDS, then $\gamma_{prdd}(D) = \begin{Bmatrix} 4\left\lfloor\frac{n}{3}\right\rfloor + 2\ if\ n \equiv 1,2\ (mod\ 3) \\ 4\left\lceil\frac{n}{3}\right\rceil \quad if\ n \equiv 0\ (mod\ 3) \end{Bmatrix}$

Proof:

Let $v_1, v_2, v_3, \ldots . v_{n-1}, v_n$ be the vertices of C_n and $u_1, u_2, u_3, \ldots . u_{n-1}, u_n$ be the vertices of P_n. Consider a graph $C_n \cup P_n$. Let D be a graph obtained from $C_n \cup P_n$ with first vertices coincide that is $u_1 = v_1 = u$. By our construction D has 2n-1 vertices and 2n-1 edges. Let S be the γ_{prdd} set of D. The proof has following three cases.

Case (i) $n \equiv 0(mod\ 3)$. Take $n = 3k$ where $k \in N$. Let us remove the common vertex in D. Then the resulting graph is two paths of length $n-1$.Therefore Sis the union of PDDS of two P_{n-1}. Hence $\gamma_{prdd}(D) = 2\gamma_{prdd}(P_{n-1}) = 2\left(2\left\lfloor\frac{n-1}{3}\right\rfloor + 2\right) = 2\left(2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2\right) = 2(2(k-1) + 2) = 4k = 4\left\lceil\frac{n}{3}\right\rceil$.

Case (ii) $n \equiv 1(mod\ 3)$. Take $n = 3k + 1$ where $k \in N$. Then remove adjacent vertex of u in the path. Thus, we get a cycle C_n and the path of length $n-2$. Therefore, S is the union of PDDS of P_{n-2} and PDDS of C_n. Hence $\gamma_{prdd}(D) = \gamma_{prdd}(P_{n-2}) + \gamma_{prdd}(C_n) = 2\left\lfloor\frac{n-2}{3}\right\rfloor + 2 + 2\left\lceil\frac{n}{3}\right\rceil = 2\left\lfloor\frac{3k+1-2}{3}\right\rfloor + 2 + 2\left\lceil\frac{3k+1}{3}\right\rceil = 2(k-1) + 2 + 2(k+1) = 4k + 2 = 4\left(\frac{n-1}{3}\right) + 2 = 4\left\lfloor\frac{n}{3}\right\rfloor + 2$.

Case (iii) $n \equiv 2(mod\ 3)$. Take $n = 3k + 2$ where $k \in N$. Then remove adjacent vertices of common vertex of u in the cycle. Thus, we get a path P_n and the path of length $n-3$. Therefore, S is the union of PDDS of P_{n-3} and PDDS of P_n. Hence $\gamma_{prdd}(D) = \gamma_{prdd}(P_{n-3}) +$

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$\gamma_{prdd}(P_n) = 2\left\lfloor\frac{n-3}{3}\right\rfloor + 2 + 2\left\lfloor\frac{n}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k+1-2}{3}\right\rfloor + 2 + 2\left\lceil\frac{3k+1}{3}\right\rceil = 2(k-1) + 2 + 2(k+1) = 4k + 2 = 4\left(\frac{n-2}{3}\right) + 2 = 4\left\lfloor\frac{n}{3}\right\rfloor + 2.$

Theorem 2.2 If D is a connected graph of order $n \geq 3$ and which has a PDDS, then

i. If $n \equiv 0(mod 3)$, then

$$\gamma_{prdd}(D') = \begin{cases} 2\left\lfloor\dfrac{n+m}{3}\right\rfloor & if\ m \equiv 0,1\ (mod\ 3) \\ 2\left\lfloor\dfrac{n+m}{3}\right\rfloor + 2 & if\ m \equiv 2\ (mod\ 3) \end{cases}$$

ii. If $n \equiv 1(mod 3)$, then

$$\gamma_{prdd}(D') = \begin{cases} 2\left\lfloor\dfrac{n+m}{3}\right\rfloor & if\ m \equiv 2\ (mod\ 3) \\ 2\left\lfloor\dfrac{n+m}{3}\right\rfloor + 2 & if\ m \equiv 0,1\ (mod\ 3) \end{cases}$$

iii. If $n \equiv 2(mod 3)$, then

$$\gamma_{prdd}(D') = \begin{cases} 2\left\lfloor\dfrac{n+m}{3}\right\rfloor & if\ m \equiv 1,2\ (mod\ 3) \\ 2\left\lfloor\dfrac{n+m}{3}\right\rfloor + 2 & if\ m \equiv 0\ (mod\ 3) \end{cases}$$

Proof:

Let $v_1, v_2, v_3, \ldots. v_{n-1}, v_n$ be the vertices of C_n and $u_1, u_2, u_3, \ldots. u_{m-1}, u_m$ be the vertices of P_m. Consider a graph $C_n \cup P_m$. Let D' be a graph obtained from $C_n \cup P_m$ with first vertices coincide that is $u_1 = v_1 = u$. By our construction D' has n+m-1 vertices and n+m-1 edges. Let S be the γ_{prdd} set of D'.

Case (i) $n \equiv 0(mod\ 3)$.

The proof has following three cases.

Sub case (i) Take $n = 3k$ and $m = 3k$ where $k \in N$. Let us remove the common vertex in D'. Then the resulting graph is a path of length $n-1$and another path of length m-1.Therefore Sis the union of PDDS of P_{n-1} and PDDS of P_{m-1}. Hence $\gamma_{prdd}(D') = \gamma_{prdd}(P_{n-1}) + \gamma_{prdd}(P_{m-1}) = 2\left\lfloor\frac{n-1}{3}\right\rfloor + 2 + 2\left\lfloor\frac{m-1}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2 + 2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2 = 2(k-1) + 2 + 2(k-1) + 2 = 2(k+k) = 2\left(\frac{n+m}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor.$

Sub case (ii) Take $n = 3k$ and $m = 3k+1$ where $k \in N$. Then remove adjacent vertex of u in the path. Thus, we get a cycle C_n and a path of length $m-2$. Therefore, S is the union of PDDS of P_{m-2} and PDDS of C_n. Hence $\gamma_{prdd}(D') = \gamma_{prdd}(C_n) + \gamma_{prdd}(P_{m-2}) = 2\left\lceil\frac{n}{3}\right\rceil + 2\left\lfloor\frac{m-2}{3}\right\rfloor + 2 = 2\left\lceil\frac{3k}{3}\right\rceil + 2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2 = 2k + 2(k-1) + 2 = 2(k+k) = 2\left(\frac{n+m-1}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor.$

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Sub case (iii) Take $n = 3k$ and $m = 3k + 2$ where $k \in N$. Then remove adjacent vertices of common vertex of u in the cycle. Thus, we get a path P_{m-1} and a path of length $n - 1$. Therefore, S is the union of PDDS of P_{m-1} and PDDS of P_{n-1}. Hence $\gamma_{prdd}(D) = \gamma_{prdd}(P_{n-1}) + \gamma_{prdd}(P_{m-1}) = 2\left\lfloor\frac{n-1}{3}\right\rfloor + 2 + 2\left\lfloor\frac{m-1}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2 + 2\left\lfloor\frac{3k+1}{3}\right\rfloor + 2 = 2(k-1) + 2 + 2k + 2 = 2\left(\frac{n+m-2}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor + 2.$

Case (ii) $n \equiv 1(mod\ 3)$.The proof has following three cases.

Sub case (i) Take $n = 3k + 1$ and $m = 3k$ where $k \in N$.

Then we get a cycle C_n and a path of length $m - 1$. Therefore, S is the union of PDDS of P_{m-1} and PDDS of C_n. Hence $\gamma_{prdd}(D') = \gamma_{prdd}(C_n) + \gamma_{prdd}(P_{m-1}) = 2\left\lceil\frac{n}{3}\right\rceil + 2\left\lfloor\frac{m-1}{3}\right\rfloor + 2 = 2\left\lceil\frac{3k+1}{3}\right\rceil + 2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2 = 2(k+1) + 2(k-1) + 2 = 2\left(\frac{n-1+m}{3}\right) + 2 = 2\left\lfloor\frac{n+m}{3}\right\rfloor + 2.$

Sub case (ii) Take $n = 3k + 1$ and $m = 3k + 1$where $k \in N$. Then remove adjacent vertex of u in the path . Then we get a cycle C_n and a path of length $m - 2$. Therefore, S is the union of PDDS of P_{m-2} and PDDS of C_n. Hence $\gamma_{prdd}(D') = \gamma_{prdd}(C_n) + \gamma_{prdd}(P_{m-2}) = 2\left\lceil\frac{n}{3}\right\rceil + 2\left\lfloor\frac{m-2}{3}\right\rfloor + 2 = 2\left\lceil\frac{3k+1}{3}\right\rceil + 2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2 = 2(k+1) + 2(k-1) + 2 = 2\left(\frac{n-1+m-1}{3}\right) + 2 = 2\left\lfloor\frac{n+m}{3}\right\rfloor + 2.$

Sub case (iii) Take $n = 3k + 1$ and $m = 3k + 2$ where $k \in N$. Then remove adjacent vertex of u in the cycle. Then we get a path P_m and a path of length $n - 2$. Therefore, S is the union of PDDS of P_{n-2} and PDDS of P_m. Hence $\gamma_{prdd}(D') = \gamma_{prdd}(P_m) + \gamma_{prdd}(P_{n-2}) = 2\left\lfloor\frac{m}{3}\right\rfloor + 2 + 2\left\lfloor\frac{n-2}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k+2}{3}\right\rfloor + 2 + 2\left\lceil\frac{3k-1}{3}\right\rceil + 2 = 2(k+1) + 2(k-1) + 4 = 2(k + k + 2) = 2\left(\frac{n-1+m}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor.$

Case (iii) $n \equiv 2(mod\ 3)$.

The proof has following three cases.

Sub case (i) Take $n = 3k + 2$ and $m = 3k$ where $k \in N$. Let us remove adjacent vertices of u in the path. Then the resulting graph is a cycle C_n and path of length $m - 2$. Therefore, Sis the union of PDDS of C_n and PDDS of P_{m-2}. Hence $\gamma_{prdd}(D') = \gamma_{prdd}(C_n) + \gamma_{prdd}(P_{m-2}) = 2\left\lceil\frac{n}{3}\right\rceil + 2\left\lfloor\frac{m-2}{3}\right\rfloor + 2 = 2\left\lceil\frac{3k+2}{3}\right\rceil + 2\left\lfloor\frac{3k-2}{3}\right\rfloor + 2 = 2(k+1) + 2(k-1) + 2 = 2\left(\frac{n-2+m}{3}\right) + 2 = 2\left\lfloor\frac{n+m}{3}\right\rfloor + 2$.

Sub case (ii) Take $n = 3k + 2$ and $m = 3k + 1$ where $k \in N$. Then remove adjacent vertex of u in the path. Thus, we get a cycle C_n and a path of length $m - 2$. Therefore, S is the union of PDDS of P_{m-2} and PDDS of C_n. Hence $\gamma_{prdd}(D') = \gamma_{prdd}(C_n) + \gamma_{prdd}(P_{m-2}) = 2\left\lceil\frac{n}{3}\right\rceil + 2\left\lfloor\frac{m-2}{3}\right\rfloor + 2 = 2\left\lceil\frac{3k+2}{3}\right\rceil + 2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2 = 2(k + k + 1) = 2\left(\frac{n-2+m}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor.$

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Sub case (iii) Take $n = 3k + 2$ and $m = 3k + 2$ where $k \in N$. Then remove adjacent vertices of u in the cycle. Thus, we get a path P_n and a path of length $m - 3$. Therefore, S is the union of PDDS of P_n and PDDS of P_{m-3}. Hence $\gamma_{prdd}(D) = \gamma_{prdd}(P_n) + \gamma_{prdd}(P_{m-3}) = 2\left\lfloor\frac{n}{3}\right\rfloor + 2 + 2\left\lfloor\frac{m-3}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k+2}{3}\right\rfloor + 2 + 2\left\lfloor\frac{3k-1}{3}\right\rfloor + 2 = 2(k+1) + 2(k-1) + 4 = 2(k+k+2) = 2\left(\frac{n-2+m}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

Theorem 2.3 If H is a connected graph of order $n \geq 3$ and which has a PDDS, then

$$\gamma_{prdd}(H) = \begin{cases} 2\left\lfloor\frac{2n}{3}\right\rfloor + 2 & if\ n \equiv 1\ (mod\ 3) \\ 2\left\lfloor\frac{2n}{3}\right\rfloor & if\ n \equiv 0,2\ (mod\ 3) \end{cases}$$

Proof:

Let $v_1, v_2, v_3, \ldots . v_{n-1}, v_n$ be the vertices of C_n and $u_1, u_2, u_3, \ldots . u_{m-1}, u_m$ be the vertices of C_n'. Consider a graph $C_n \cup C_n'$. Let H be a graph obtained from $C_n \cup C_n'$ with first vertices coincide that is $u_1 = v_1 = u$. By our construction H has 2n-1 vertices and 2n edges. Let S be the γ_{prdd} set of H. The proof has following three cases.

Case (i) $n \equiv 0(mod\ 3)$. Take $n = 3k$ where $k \in N$. Let us remove the adjacent vertex u in the cycleC_n'. Then the resulting graph is D' . Thus$D' = C_n \cup P_{n-1}$ Hence $\gamma_{prdd}(H) = \gamma_{prdd}(D') = 2\left\lfloor\frac{n+n-1}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k+3k-1}{3}\right\rfloor + 2 = 2(2k-1) + 2 = 2k = 2\left\lfloor\frac{2n}{3}\right\rfloor$

Case (ii) $n \equiv 1(mod\ 3)$. Take $n = 3k + 1$ where $k \in N$. Then remove adjacent vertex u in the cycleC_n'. Thus, we get a graph D'. Thus$D' = C_n \cup P_{n-1}$. Hence $\gamma_{prdd}(H) = \gamma_{prdd}(D') = 2\left\lfloor\frac{n+n-1}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k+1+3k+1-1}{3}\right\rfloor + 2 = 2(2k) + 2 = 2\left(\frac{2n-2}{3}\right) + 2 = 2\left\lfloor\frac{2n}{3}\right\rfloor + 2$.

Case (iii) $n \equiv 2(mod\ 3)$. Take $n = 3k + 2$ where $k \in N$. Then remove adjacent vertices of common vertex u in the cycle C_n'.Thus, we get a path P_{n-3} and the cycle C_n . Therefore, S is the union of PDDS of P_{n-3} and PDDS of C_n.Hence $\gamma_{prdd}(H) = \gamma_{prdd}(P_{n-3}) + \gamma_{prdd}(C_n) = 2\left\lfloor\frac{n-3}{3}\right\rfloor + 2 + \left\lceil\frac{n}{3}\right\rceil = 2\left\lfloor\frac{3k+2-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{3k+1}{3}\right\rceil = 2(k-1) + 2 + 2(k+1) = 2k + 2k + 2 = 2\left(\frac{2n-4+3}{3}\right) = 2\left\lfloor\frac{2n}{3}\right\rfloor$.

Let C_n be the cycle of n vertices and n edges and C_m be the cycle of m vertices and m edges. Let $v_1, v_2, v_3, \ldots . v_{n-1}, v_n$ be the vertices of C_n and $u_1, u_2, u_3, \ldots . u_{m-1}, u_m$ be the vertices of C_m. Consider a graph $C_n \cup C_m$. Let H' be a graph obtained from $C_n \cup C_m$ with first vertices coincide that is $u_1 = v_1 = u$.

Theorem 2.4 If H' is a connected graph of order $n \geq 3$ and which has a PDDS, then

i. If $n \equiv 0,2(mod3)$, then

$$\gamma_{prdd}(H') = 2\left\lfloor\frac{n+m}{3}\right\rfloor \quad if\ m \equiv 0,1,2\ (mod\ 3)$$

ii. If $n \equiv 1(mod3)$, then

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$$\gamma_{prdd}(H') = \begin{cases} 2\left\lfloor\frac{n+m}{3}\right\rfloor & if\ m \equiv 0,2\ (mod\ 3) \\ 2\left\lfloor\frac{n+m}{3}\right\rfloor + 2 & if\ m \equiv 1\ (mod\ 3) \end{cases}$$

Proof: By our construction H' has 2n-1 vertices and 2n edges. Let S be the γ_{prdd} set of H'. The proof has following three cases.

Case (i) $n \equiv 0(mod\ 3)$.

The proof has following three cases

Sub case (i) $m \equiv 0(mod\ 3)$. Take $m = 3k$ where $k \in N$. Let us remove the adjacent vertex u in the cycle C_m. Then the resulting graph is D'.Thus $H' = D' = C_n \cup P_{m-1}$ Hence $\gamma_{prdd}(H') = \gamma_{prdd}(D') = 2\left\lfloor\frac{n+m-1}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k+3k-1}{3}\right\rfloor + 2 = 2(2k-1) + 2 = 2(k+k) = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

Sub Case (ii) $m \equiv 1(mod\ 3)$. Take $m = 3k+1$ where $k \in N$. Let us remove adjacent vertex u in the cycle C_n'. Thus, we get a graph D'. Thus $H' = D' = C_n \cup P_{m-1}$. Hence $\gamma_{prdd}(H') = \gamma_{prdd}(D') = 2\left\lfloor\frac{n+m-1}{3}\right\rfloor = 2\left\lfloor\frac{3k+3k+1}{3}\right\rfloor = 2\left(2k+\frac{1}{3}\right) = 2\left(\frac{n+m-1}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

Sub Case (iii) $m \equiv 2(mod\ 3)$. Take $n = 3k+2$ where $k \in N$. Let us remove adjacent vertices of common vertex u in the cycle C_n'.Thus, we get a path P_{m-3} and the cycle C_n . Therefore, S is the union of PDDS of P_{m-3} and PDDS of C_n.Hence $\gamma_{prdd}(D') = \gamma_{prdd}(P_{m-3}) + \gamma_{prdd}(C_n) = 2\left\lfloor\frac{m-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{n}{3}\right\rceil = 2\left\lfloor\frac{3k+2-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{3k}{3}\right\rceil = 2(k-1) + 2 + 2(k) = 2\left(\frac{n+m-2}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

Case (ii) $n \equiv 1(mod\ 3)$.

The proof has following three cases

Sub case (i) $m \equiv 0(mod\ 3)$. Take $m = 3k$ where $k \in N$. Let us remove the adjacent vertex u in the cycle C_n. Then the resulting graph is D'.Thus $H' = D' = C_m \cup P_{n-1}$ Hence $\gamma_{prdd}(H') = \gamma_{prdd}(D') = 2\left\lfloor\frac{m+n-1}{3}\right\rfloor = 2(k+k) = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

Sub Case (ii) $m \equiv 1(mod\ 3)$. Take $m = 3k+1$ where $k \in N$. Let us remove adjacent vertex u in the cycle C_n . Thus, we get a graph D'. Thus $H' = D' = C_m \cup P_{n-1}$. Hence $\gamma_{prdd}(H') = \gamma_{prdd}(D') = 2\left\lfloor\frac{m+n-1}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k+1+3k+1-1}{3}\right\rfloor + 2 = 2\left(k+k+\frac{1}{3}\right) + 2 = 2\left(\frac{n-1+m-1}{3}\right) + 2 = 2\left\lfloor\frac{n+m}{3}\right\rfloor + 2$.

Sub Case (iii) $m \equiv 2(mod\ 3)$. Take $n = 3k+2$ where $k \in N$. Let us remove adjacent vertex u in the cycle C_n . Thus, we get a graph D'. Thus $H' = D' = C_m \cup P_{n-1}$. Hence $\gamma_{prdd}(H') = \gamma_{prdd}(D') = 2\left\lfloor\frac{m+n-1}{3}\right\rfloor + 2 = 2\left\lfloor\frac{3k+1+3k+2-1}{3}\right\rfloor + 2 = 2\left\lfloor k+k+\frac{2}{3}\right\rfloor + 2 = 2(k+k) + 2 = 2\left(\frac{n-1+m-2}{3}\right) + 2 = 2\left\lfloor\frac{n+m}{3}\right\rfloor - 2 + 2 = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

Case (iii) $n \equiv 2(mod\ 3)$.

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The proof has following three cases

Sub Case (i) $m \equiv 0(mod\ 3)$. Take $m = 3k$ where $k \in N$. Let us remove adjacent vertices of common vertex u in the cycleC_n.Then, we get a path P_{n-3} and the cycle C_m . Therefore, S is the union of PDDS of P_{n-3} and PDDS of C_m.Hence $\gamma_{prdd}(\,D') = \gamma_{prdd}(P_{n-3}) + \gamma_{prdd}(C_m) = 2\left\lfloor\frac{n-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{m}{3}\right\rceil = 2\left\lfloor\frac{3k+2-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{3k}{3}\right\rceil = 2(k-1) + 2 + 2(k) = 2\left(\frac{n+m-2}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

Sub Case (ii) $m \equiv 1(mod\ 3)$. Take $m = 3k + 1$ where $k \in N$. Let us remove adjacent vertices of common vertex u in the cycleC_n.Then, we get a path P_{n-3} and the cycle C_m . Therefore, S is the union of PDDS of P_{n-3} and PDDS of C_m.Hence $\gamma_{prdd}(\,D') = \gamma_{prdd}(P_{n-3}) + \gamma_{prdd}(C_m) = 2\left\lfloor\frac{n-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{m}{3}\right\rceil = 2\left\lfloor\frac{3k+2-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{3k+1}{3}\right\rceil = 2(k-1) + 2 + 2(k+1) = 2\left(\frac{n+m-3}{3}\right) + 2 = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

Sub Case (iii) $m \equiv 2(mod\ 3)$. Take $m = 3k + 2$ where $k \in N$. Let us remove adjacent vertices of common vertex u in the cycleC_n.Then, we get a path P_{n-3} and the cycle C_m . Therefore, S is the union of PDDS of P_{n-3} and PDDS of C_m.Hence $\gamma_{prdd}(\,D') = \gamma_{prdd}(P_{n-3}) + \gamma_{prdd}(C_m) = 2\left\lfloor\frac{n-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{m}{3}\right\rceil = 2\left\lfloor\frac{3k+2-3}{3}\right\rfloor + 2 + 2\left\lceil\frac{3k+2}{3}\right\rceil = 2(k-1) + 2 + 2(k+1) = 2\left(\frac{n+m-1}{3}\right) = 2\left\lfloor\frac{n+m}{3}\right\rfloor$.

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