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Stochastic Profit Modeling of a Two-Unit Parallel System with Bad Weather Effects and Activation Time

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ABSTRACT: *The present study investigates the stochastic performance and profit behavior of a two-unit non-identical parallel system operating under bad weather conditions. The system consists of unit A, which initially operates in active mode, and unit B, which remains in standby. Unit A becomes non-operative during bad weather conditions (BWC) and requires a random activation time to resume operation after the weather normalizes and necessary repairs are completed. To enhance its operational efficiency, unit A is subjected to preventive maintenance after a random interval. Priority is assigned to repair and activation activities over preventive maintenance to restore system operability, with unit A receiving higher priority than unit B. A single repairman is continuously available to attend to failed units. The failure times of both units are assumed to be independent and exponentially distributed, while the repair times follow general distributions. Employing the regenerative point technique, key measures of system effectiveness are derived, including transition probabilities, mean sojourn times, mean time to system failure (MTSF), system availability, repairman busy period, expected number of repairs performed, and the expected profit of the system.*

KEYWORD: *Bad weather condition, activation time, preventive maintenance, stochastic modeling, profit analysis*

1.1 INTRODUCTION

Repairable redundant systems have attracted the attention of several researchers working in the field of reliability. The incorporation of redundancy is one of the important techniques to improve the reliability of the system. The system models have been frequently analyzed by various authors including Goel, L.R et.al,[2,3,4] with different assumptions due to their vital existence in modern industries. Goel, L.R., et.al[2] provided the concept of cost analysis of a two unit cold standby system under different weather conditions. Malik S.C[7] studied stochastic modeling of a system subject to degradadtion and no functioning in abnormal weather condition. Shivani Chowdhary and Pawan Kumar[11] considered profit analysis of a non-redundant system with preventive maintenance and operative under favourable climatic conditions. S.C Malik and Barak M.S[8] developed reliability and economic analysis of a system operating under different weather conditions. Goel et.al [1] investigated cost analysis of a system with intermittent repair and inspection under abnormal weather. Neha Kumari and Pawan Kumar[10] developed a two non identical unit parallel system with preparation time for operation and proviso of rest.

In practice, a sort of repair called preventive maintenance is given to the system to improve its working capability and to delay its actual failure so that we can enhance the benefits made by the system. A few attempts have been made to study the models with the concepts of preventive maintenance Gupta, R. and Sharma, V.[4] analyzed two unit standby system with preventive

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maintenance and inverse Gaussian repair time distribution. Also, Kishan, R. and Kumar Manish [6] developed stochastic analysis of a two unit parallel system with preventive maintenance. Kapur and Kapur[5] analysed a two dissimilar unit redundant system with repair and preventive maintenance. Murari and Al-Ali[9] considered a one unit reliability system subject to random shocks and preventive maintenance. Gupta[3] introduced a two unit cold standby system with two phase repair and preventive maintenance. The provision of preventive maintenance has proved to be significant in improving system performance. But while providing preventive maintenance for the system, it should be kept in mind that the total cost of preventive maintenance should not be so high that it adversely effects the total profit. Thus a balance must be maintained between the cost of actual repair and cost of preventive maintenance (in view of failure and repair rates).

The purpose of the present paper is to analyze two non identical units system model with two units as A and B in which unit A is in operative mode and unit B is in standby mode. Here unit A does not work in bad weather condition and it takes some activation time before starting its operation after the disappearance of Bad Weather Condition and completion of repair. Also, Preventive maintenance is given to unit A after a random period of time to improve the efficiency of unit A. Repair, Activation and Unit A gets priority over preventive maintenance and unit B respectively to make system operative. A single repairman is always available with the system to repair the failed unit. The failures of the units are independent and the failure time distributions of the units are taken as Exponential and the repair time distribution of the unit are taken as general. All random variable are statistically independent.

The expression for some important performance measures of the system have been derived in steady state using semi- Markov process and regenerative point technique-

1. Transition Probabilities and Mean Sojourn times.
2. Reliability and Mean time to system failure (MTSF).
3. Availability of the system.
4. Busy period of repairman.
5. Expected number of repairs by repairman.
6. Net expected profit earned by the system during the interval (0,t) and in steady state.

1.2 MODEL DESCRIPTION AND ASSUMPTIONS

1. Initially unit A is in operative mode and unit B is in standby mode.

2. Unit A does not work in bad weather condition and it takes some activation time before starting its operation after disappearance of Bad Weather Condition and completion of repair.

3. Preventive maintenance is given to unit A after a random period of time to improve the efficiency of unit A.

4. Repair and Activation gets priority over Preventive Maintenance to keep system operative.

5. A single repairman is always available with the system.

6. The failures of the units are independent and the failure time distributions of the units are taken as Exponential.

7. The repair rates of the units are taken as general.

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1.3 NOTATIONS AND STATES OF THE SYSTEM

We define the following symbols for generating the various states of the system.

$A_o/B_o/B_s$: Unit A/ Unit B are in operative / unit B is in standby mode.

A_r/B_r : Unit A / unit B in total failure mode / under repair.

A_{pm}/A_{bwc} : Unit A under preventive maintenance/ inoperable due to bad weather condition.

A_a : Unit A in activation mode.

B_{wr} : Unit B in total failure mode and waiting for repair.

Thus considering the above symbols in view of the assumptions stated, the possible states of the system are as follows-

Up states:

$S_0 = (A_oB_s)$ $S_1 = (A_{pm}B_s)$ $S_2 = (A_{bwc}B_o)$

$S_3 = (A_rB_0)$ $S_4 = (A_aB_o)$ $S_8 = (A_0B_r)$

Failed states:

$S_5 = (A_{bwc}B_r)$ $S_6 = (A_aB_{wr})$ $S_7 = (A_rB_{wr})$

b) NOTATIONS

E : Set of regenerative states
$= \{S_0, S_1, S_2, S_3, S_4, S_8\}$

$\overline{E}$: Set of non − regenerative states
$= \{S_5, S_6, S_7\}$

α_1 : Rate of Preventive Maintenance of unit A.

α_2 : Failure rate of unit A.

α_3 : Appearance rate of bad weather conditions.

θ_1 : Failure rate of unit B.

β_1 : Disappeareance rate of bad weather conditions.

β_2 : Activation rate of unit A .

$G_1(.)$: Cdf of repair time of unit A .

$G_2(.)$: Cdf of repair time of unit B.

$H(.)$: Cdf of repair time of unit A after preventive maintenance.

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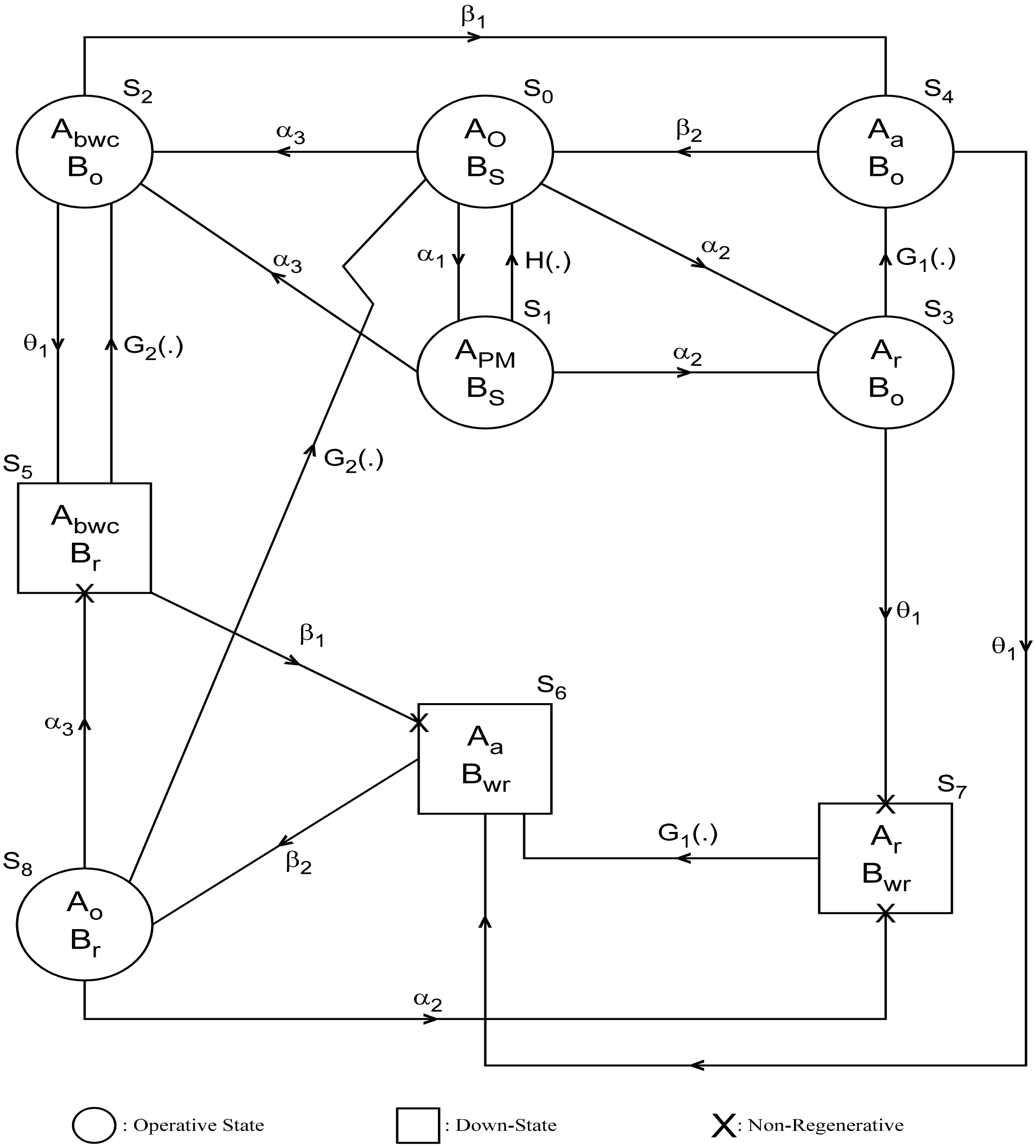

Fig 1.1

1.4 TRANSITION PROBABLITIES

A) STEADY STATE PROBABILITIES

Let X_n denotes the state visited at epoch T_{n^+} just after the transition at T_n, where T_1, T_2 .... represents the regenerative epochs, then$\{X_n, T_n\}$ constitute a Markov-Renewal process with state space E and

$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$

is the semi Markov kernel over E.

Then the transition probability matrix of the embedded Markov chain is

$\mathrm{P} = \mathrm{p_{ij}} = \mathrm{Q_{ij}}(\infty) = \mathrm{Q}(\infty)$

We obtain the following direct steady-state transition probabilities:

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$p_{01} = \alpha_1 \int e^{-(\alpha_1+\alpha_3+\alpha_2)} = \frac{\alpha_1}{(\alpha_1+\alpha_3+\alpha_2)}$ $\qquad$ $p_{02} = \frac{\alpha_3}{(\alpha_3+\alpha_1+\alpha_2)}$

$p_{03} = \frac{\alpha_2}{(\alpha_2+\alpha_1+\alpha_3)}$ $\qquad$ $p_{10} = \int dH(t)\, e^{-(\alpha_3+\alpha_2)u} = \widetilde{H}(\alpha_3+\alpha_2)$

$p_{12} = \frac{\alpha_3}{(\alpha_2+\alpha_3)} - \frac{\alpha_3\widetilde{H}(\alpha_3+\alpha_2)}{(\alpha_2+\alpha_3)}$ $\qquad$ $p_{13} = \frac{\alpha_2}{(\alpha_2+\alpha_3)} - \frac{\alpha_2\widetilde{H}(\alpha_3+\alpha_2)}{(\alpha_2+\alpha_3)}$

$p_{24} = \frac{\beta_1}{\beta_1+\theta_1}$ $\qquad$ $p_{25} = \frac{\theta_1}{\beta_1+\theta_1}$

$p_{34} = \widetilde{G}_1(\theta_1)$ $\qquad$ $p_{40} = \frac{\beta_2}{\beta_2+\theta_1}$

$p_{25} = \frac{\theta_1}{\beta_2+\theta_1}$ $\qquad$ $p_{52} = \widetilde{G}_2(\beta_1)$

$p_{56} = 1 - \widetilde{G}_2(\beta_1)$ $\qquad$ $p_{80} = \widetilde{G}_2(\alpha_3+\alpha_2)$

$$p_{87} = \frac{\alpha_2}{(\alpha_2+\alpha_3)} - \frac{\alpha_2\widetilde{G}_2(\alpha_3+\alpha_2)}{(\alpha_2+\alpha_3)}$$

The indirect transition probability may be obtained as follows:

$$p_{36}^{(7)} = \frac{\theta_1}{\theta_1}\int dG_1(v)\left(1-e^{-\theta_1 v}\right) = \int dG_1(v) - \int dG_1(v)\, e^{-\theta_1 v} = 1 - \widetilde{G}_1(\theta_1)$$

Similarly,

$$p_{82}^{(5)} = \frac{\alpha_3}{(\alpha_3+\alpha_2-\beta_1)}\widetilde{G}_2(\beta_1) - \frac{\alpha_3}{(\alpha_3+\alpha_2-\beta_1)}\widetilde{G}_2(\alpha_3+\alpha_2)$$

$$p_{86}^{(5)} = \frac{\alpha_3}{(\alpha_3+\alpha_2)} - \frac{\alpha_3}{(\alpha_3+\alpha_2-\beta_1)(\alpha_3+\alpha_2)}\left[(\alpha_3+\alpha_2)\widetilde{G}_2(\beta_1) - \beta_1\widetilde{G}_2(\alpha_3+\alpha_2)\right]$$

It can be easily verified that

$p_{01} + p_{02} + p_{03} = 1$ $\qquad$ $p_{10} + p_{12} + p_{13} = 1$ $\qquad$ $p_{24} + p_{25} = 1$

$p_{34} + p_{36}^{(7)} = 1$ $\qquad$ $p_{40} + p_{46} = 1$ $\qquad$ $p_{52} + p_{56} = 1$

$p_{80} + p_{82}^{(5)} + p_{86}^{(5)} + p_{87} = 1$ $\qquad$ $p_{68} = p_{76} = 1$

B) MEAN SOJOURN TIMES

The mean sojourn time in state S_i denoted by μ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time μ_i, in state S_i, we observe that as long as the system is in stateS_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time μ_i in state S_i is:

$\mu_i = E[T_i] = \int P(T_i > t)dt$

Therefore,

$$\mu_0 = \int e^{-\alpha_1 t}e^{-\alpha_2 t}e^{-\alpha_3 t}dt = \frac{1}{\alpha_1+\alpha_2+\alpha_3}$$

$$\mu_1 = \int e^{-\alpha_2 t}e^{-\alpha_3 t}\,\overline{H}(t)\,dt = \frac{1}{\alpha_2+\alpha_3} - \widetilde{H}(\alpha_2+\alpha_3)$$

$\mu_2 = \frac{1}{\theta_1+\beta_1}$ $\qquad$ $\mu_3 = \frac{1}{\theta_1} - \widetilde{G}_1(\theta_1)$

$\mu_4 = \frac{1}{\theta_1+\beta_2}$ $\qquad$ $\mu_5 = \frac{1}{\beta_1} - \widetilde{G}_2(\beta_1)$

$\mu_6 = \frac{1}{\beta_2}$ $\qquad$ $\mu_7 = \int \overline{G}_1(t)dt$

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$$\mu_8 = \frac{1}{\alpha_2 + \alpha_3} - \widetilde{G}_2(\alpha_2 + \alpha_3)$$

1.5 ANALYSIS OF RELIABILITY AND MTSF

Let the random variable T_i be the time to system failure when system starts up from state $S_i \in E$. Then the reliability of the system is given by

$$R_i(t) = P[T_i > t]$$

To obtain $R_i(t)$, we consider failed states as absorbing states.

The recursive relations among $R_i(t)$ can be developed on the basis of probabilistic arguments. Taking their Laplace Transform and solving the resultant set of equations for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad \textbf{(1.5.1)}$$

Where

$$N_1(s) = Z_0^* + Z_1^* + (q_{12}^* + q_{02}^*)Z_2^* + (q_{13}^* + q_{03}^*)Z_3^* + (q_{12}^*q_{24}^* + q_{02}^*q_{24}^* + q_{13}^*q_{34}^* + q_{03}^*q_{34}^*)Z_4^*$$

$$D_1(s) = 1 - [(q_{01}^* + q_{10}^*) + (q_{12}^* + q_{02}^*)q_{24}^*q_{40}^* + (q_{13}^* + q_{03}^*)q_{34}^*q_{40}^*]$$

Taking the Inverse Laplace Transform of **(1.5.1)**, one can get the reliability of the system.

To get MTSF, we use the well known formula

$$E(T_0) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) = N_1(0)/D_1(0)$$

where,

$$N_1(s) = \mu_0 + \mu_1 + (p_{12} + p_{02})\mu_2 + (p_{13} + p_{03})\mu_3 + [(p_{12} + p_{02})p_{24} + (p_{13} + p_{03})p_{34}]\mu_4$$

$$D_1(s) = 1 - [(p_{10} + p_{01}) + (p_{12} + p_{02})p_{24}p_{40} + (p_{13} + p_{03})p_{34}p_{40}]$$

Since, we have $q_{ij}^*(0) = p_{ij}$ and $\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \mu_i$

1.6 AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is available at epoch t, when it initially starts from $S_i \in E$. Using the regenerative point technique and the tools of L.T., one can obtain the value of above probabilities in terms of their L.T. i.e$A_i^*(s)$.Solving the resultant set of equations and simplifying for $A_0^*(s)$, we have

$$A_0^*(s) = N_2(s)/D_2(s) \quad \textbf{(1.6.1)}$$

Where

$$N_2(s) = (1 - q_{52}q_{25})\{Z_0 + Z_1q_{01} + (q_{01}q_{13} + q_{03})[Z_3 + q_{34}Z_4]\} + (q_{01}q_{12} + q_{02})[Z_2 + q_{24}Z_4 + q_{25}Z_5] + \frac{\left[(q_{01}q_{12}+q_{02})(q_{25}q_{56}+q_{24}q_{46})+(q_{01}q_{13}+q_{03})\left[(1-q_{52}q_{25})\left(q_{34}q_{46}+q_{36}^{(7)}\right)\right]\right]}{\left(1-q_{76}q_{87}q_{68}-q_{86}^{(5)}\right)(1-q_{52}q_{25})-q_{68}q_{82}^{(5)}(q_{25}q_{56}+q_{24}q_{46})}\left\{(1 - q_{52}q_{25})q_{68}Z_8 + q_{68}q_{82}^{(5)}[Z_2 + q_{24}Z_4 + q_{25}Z_5]\right\}$$

And

$$D_2(s) = (1 - q_{52}q_{25}) - (1 - q_{52}q_{25})[q_{01}(q_{10} + q_{13}q_{34}q_{40}) + q_{03}q_{34}q_{40}] - q_{24}q_{40}(q_{01}q_{12} + q_{02}) + \frac{\left[(q_{01}q_{12}+q_{02})(q_{25}q_{56}+q_{24}q_{46})+(q_{01}q_{13}+q_{03})\left[(1-q_{52}q_{25})\left(q_{34}q_{46}+q_{36}^{(7)}\right)\right]\right]}{\left(1-q_{76}q_{87}q_{68}-q_{86}^{(5)}\right)(1-q_{52}q_{25})-q_{68}q_{82}^{(5)}(q_{25}q_{56}+q_{24}q_{46})}\left\{(1 - q_{52}q_{25})q_{68}q_{80} + q_{68}q_{82}^{(5)}q_{24}q_{40}\right\}$$

(1.6.2)

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The steady state availability is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2(0)}$$

As we know that, $q_{ij}(t)$ is the pdf of the time of transition from state S_i to S_j and $q_{ij}(t)dt$ is the probability of transition from state S_i to S_j during the interval $(t, t+dt)$, thus

$\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \mu_i$ and $q_{ij}^*(s) = q_{ij}^*(0) = p_{ij}$, we get

Therefore,

$$N_2(0) = (1 - p_{52}p_{25})\{\mu_0 + \mu_1 p_{01} + (p_{01}p_{13} + p_{03})[\mu_3 + p_{34}\mu_4]\} + (p_{01}p_{12} + p_{02})[\mu_2 + p_{24}\mu_4 + p_{25}\mu_5] + \frac{\left[(p_{01}p_{12}+p_{02})(p_{25}p_{56}+p_{24}p_{46})+(p_{01}p_{13}+p_{03})\left[(1-p_{52}p_{25})\left(p_{34}p_{46}+p_{36}^{(7)}\right)\right]\right]}{\left(1-q_{76}q_{87}q_{68}-q_{86}^{(5)}\right)(1-q_{52}q_{25})-q_{68}q_{82}^{(5)}(q_{25}q_{56}+q_{24}q_{46})}\left\{(1 - p_{52}p_{25})p_{68}\mu_8 + p_{68}p_{82}^{(5)}[\mu_2 + p_{24}\mu_4 + p_{25}\mu_5]\right\} \quad \textbf{(1.6.3)}$$

and

$$D_2(0) = (1 - p_{52}p_{25}) - (1 - p_{52}p_{25})[p_{01}(p_{10} + p_{13}p_{34}p_{40}) + p_{03}p_{34}p_{40}] - p_{24}p_{40}(p_{01}p_{12} + p_{02}) + \frac{\left[(p_{01}p_{12}+p_{02})(p_{25}p_{56}+p_{24}p_{46})+(p_{01}p_{13}+p_{03})\left[(1-p_{52}p_{25})\left(p_{34}p_{46}+p_{36}^{(7)}\right)\right]\right]}{\left(1-q_{76}q_{87}q_{68}-q_{86}^{(5)}\right)(1-q_{52}q_{25})-q_{68}q_{82}^{(5)}(q_{25}q_{56}+q_{24}q_{46})}\left\{(1 - p_{52}p_{25})p_{68}p_{80} + p_{68}p_{82}^{(5)}p_{24}p_{40}\right\}$$

(1.6.4)

The steady state probability that the system will be up in the long run is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) \lim_{s\to 0} \frac{sN_2(s)}{D_2(s)} = \lim_{s\to 0} N_2(s) \lim_{s\to 0} \frac{s}{D_2(s)}$$

as $s \to 0$, $D_2(s)$ becomes zero.

Therefore, by L' Hospital's rule, A_0 becomes

$$A_0 = N_2(0)/D_2'(0) \quad \textbf{(1.6.5)}$$

where

$$D_2'(0) = \mu_0(1 - p_{52}p_{25}) + \mu_1(1 - p_{52}p_{25})p_{01} + \mu_2(p_{01}p_{12} + p_{02}) + \mu_3(1 - p_{52}p_{25})(p_{01}p_{13} - p_{03}) + \mu_4\{(p_{01}p_{12} + p_{02})p_{24} + (p_{01}p_{13} + p_{03})p_{34}(1 - p_{52}p_{25})\} + \mu_5(p_{01}p_{12} + p_{02})p_{24}(p_{01}p_{12} + p_{02})p_{25} + (\mu_6 + \mu_7)(1) + \mu_8\left\{(p_{01}p_{12} + p_{02})(p_{25}p_{56} + p_{24}p_{46}) + (p_{01}p_{13} + p_{03})\left[(1 - p_{52}p_{25})\left(p_{34}p_{46} + p_{36}^{(7)}\right)\right]\right\}$$

(1.6.6)

Using the results **(1.6.3)** and **(1.6.6)** in **(1.6.5),** we get the expressions for A_0.

The expected up (operative) times of the system during (0, t] are given by

$$\mu_{up}(t) = \int_0^t A_0(u)du$$

So that,

$$\mu_{up}^*(s) = \frac{A_0^*(s)}{s}$$

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1.7 BUSY PERIOD OF REPAIRMAN

Let $B_i(t)$ be the probability that the repairman is busy in the repair of failed unit at epoch t, when the system initially starts operation from state $S_i \in E$. Developing the recursive relations among $B_i(t)'s$ and solving the resultant set of equations and simplifying for $B_0^*(s)$, we have

$$B_0^*(s) = N_3(s)/D_2(s) \qquad \textbf{(1.7.1)}$$

where

$$N_3(s) = (1 - q_{25}^* q_{52}^*)[q_{01}^* Z_1^* + (q_{01}^* q_{13}^* + q_{03}^*)(Z_3^* + q_{34}^* Z_4^*) + (q_{01}^* q_{12}^* + q_{02}^*)(q_{24}^* Z_4^* + q_{25}^* Z_5^*)] + \frac{(q_{01}^* q_{12}^* + q_{02}^*)(q_{24}^* q_{46}^* + q_{25}^* q_{56}^*) + (q_{01}^* q_{13}^* + q_{03}^*)(1 - q_{25}^* q_{52}^*)\left(q_{34}^* q_{46}^* + q_{36}^{(7)*}\right)}{(1 - q_{25}^* q_{52}^*)\left(1 - q_{87}^* q_{76}^* - q_{86}^{(5)*}\right) - q_{68}^* q_{82}^{(5)*}(q_{24}^* q_{46}^* + q_{25}^* q_{56}^*)}\left\{(1 - q_{25}^* q_{52}^*)(Z_6^* + q_{68}^* Z_8^* + q_{87}^* q_{68}^* Z_7^*) + q_{68}^* q_{82}^{(5)*}(q_{24}^* Z_4^* + q_{25}^* Z_5^*)\right\}$$

In the long run, the expected fraction of time for which the server is busy in the repair of failed unit is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} B_0^*(s) = \frac{N_3(0)}{D_2'(0)} \qquad \textbf{(1.7.2)}$$

Where

$$N_3(0) = (1 - p_{52} p_{25})[p_{01}\mu_1 + (p_{01} p_{13} + p_{03})(\mu_3 + p_{34}\mu_4) + (p_{01} p_{12} + p_{02})(p_{24}\mu_4 + p_{25}\mu_5)] + \frac{(p_{01} p_{12} + p_{02})(p_{24} p_{46} + p_{25} p_{56}) + (p_{01} p_{13} + p_{03})(1 - p_{52} p_{25})\left(p_{34} p_{46} + p_{36}^{(7)}\right)}{(1 - p_{52} p_{25})\left(1 - p_{87} - p_{86}^{(5)}\right) - p_{36}^{(7)}(p_{24} p_{46} + p_{25} p_{56})}\left\{(1 - p_{52} p_{25})(\mu_6 + \mu_8 + p_{87}\mu_7) + p_{82}^{(5)}(p_{24}\mu_4 + p_{25}\mu_5)\right\} \qquad \textbf{(1.7.3)}$$

and $D_2'(0)$ is same as given by **(1.6.6)**.

Thus using **(1.7.3)** and **(1.6.6)** in **(1.7.2)**, we get the expression for B_0.

The expected busy period of repairman due to repair of failed unit during the time interval (0,t] is given by

$$\mu_b(t) = \int_0^t B_0(u)\,du$$

So that

$$\mu_b^*(s) = \frac{B_0^*(s)}{s}$$

1.8 EXPECTED NUMBER OF REPAIRS BY REPAIRMAN

Let $V_i^r(t)$be the expected number of repairs by the repairman in (0,t] given that the system entered the regenerative state S_i at t=0. The solution for $\tilde{V}_0^r(s)$ can be written in the following form:

$$\tilde{V}_0^r(s) = \frac{N_4(s)}{D_2(s)} \qquad \textbf{(1.8.1)}$$

where,

$$N_4 = \left[(\tilde{Q}_{01}\tilde{Q}_{13} + \tilde{Q}_{03})(1 - \tilde{Q}_{25}\tilde{Q}_{52}) + (\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02})\tilde{Q}_{25}\tilde{Q}_{52}\right] + (1 - \tilde{Q}_{25}\tilde{Q}_{52})[\tilde{Q}_{01}(\tilde{Q}_{10} + \tilde{Q}_{13}\tilde{Q}_{34}\tilde{Q}_{40}) + \tilde{Q}_{03}\tilde{Q}_{34}\tilde{Q}_{40}] +$$

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$\frac{(\tilde{Q}_{01}\tilde{Q}_{12}+\tilde{Q}_{02})(\tilde{Q}_{25}\tilde{Q}_{56}+\tilde{Q}_{24}\tilde{Q}_{46})+(\tilde{Q}_{01}\tilde{Q}_{13}+\tilde{Q}_{03})(1-\tilde{Q}_{25}\tilde{Q}_{52})\left(\tilde{Q}_{34}\tilde{Q}_{46}+\tilde{Q}_{36}^{(7)}\right)}{(1-\tilde{Q}_{25}\tilde{Q}_{52})\left(1-\tilde{Q}_{87}\tilde{Q}_{76}-\tilde{Q}_{86}^{(5)}\right)-\tilde{Q}_{82}^{(5)}\tilde{Q}_{68}(\tilde{Q}_{25}\tilde{Q}_{56}+\tilde{Q}_{24}\tilde{Q}_{46})}\left\{(1-\tilde{Q}_{25}\tilde{Q}_{52})\tilde{Q}_{68}\left(\tilde{Q}_{80}+\right.\right.$
$\left.\left.\tilde{Q}_{82}^{(5)}\right)+\tilde{Q}_{68}\tilde{Q}_{87}\tilde{Q}_{76}+\tilde{Q}_{82}^{(5)}\tilde{Q}_{25}\tilde{Q}_{52}\tilde{Q}_{68}\right\}$

In steady-state per-unit of time expected number of repairs by repairman is given by

$$V_0^r=\lim_{t\to\infty}\frac{V_0^r(t)}{t}=\lim_{s\to 0}\tilde{V}_0^r(s)=\frac{N_4(0)}{D_2'(0)} \qquad \textbf{(1.8.2)}$$

$N_4=[(p_{01}p_{13}+p_{03})(1-p_{25}p_{52})+(p_{01}p_{12}+p_{02})p_{25}p_{52}]+(1-p_{25}p_{52})[p_{01}(p_{10}+$
$p_{13}p_{34}p_{40})+p_{03}p_{34}p_{40}]+\frac{(p_{01}p_{12}+p_{02})(p_{25}p_{56}+p_{24}p_{46})+(p_{01}p_{13}+p_{03})(1-p_{25}p_{52})\left(p_{34}p_{46}+p_{36}^{(7)}\right)}{(1-p_{25}p_{52})\left(1-p_{87}p_{76}-p_{86}^{(5)}\right)-p_{68}p_{82}^{(5)}(p_{25}p_{56}+p_{24}p_{46})}\left\{(1-\right.$
$\left.p_{25}p_{52})p_{68}\left(p_{80}+p_{82}^{(5)}\right)+p_{87}p_{76}p_{68}+p_{82}^{(5)}p_{25}p_{52}p_{68}\right\}$

(1.8.3)

and $D_2'(0)$ is same as given by **(1.6.6).**

Here we have used $\tilde{Q}_{ij}(0)=p_{ij}$

Thus using **(1.8.3)** and **(1.6.6)** in **(1.8.2),** we get the expression for V_0^r.

1.9 PROFIT FUNCTION ANALYSIS

Two profit functions $P_1(t)$ and $P_2(t)$ can be easily obtained for the system model under study with the help of characteristics obtained earlier. The expected total profits incurred during (0,t] are:

$P_1(t)=$ Expected total revenue in $(0,t]$ $-$ Expected total expenditure in $(0,t]$

$$=K_0\mu_{up}(t)-K_1\mu_b(t) \qquad \textbf{(1)}$$

Similarly,

$$P_2(t)=K_0\mu_{up}(t)-K_2V_0(t) \qquad \textbf{(2)}$$

where,

K_0 is revenue per unit up time of the system.

K_1 is repair cost per unit time for which the repairman is busy in repair of the failed unit.

K_2 is per unit repair cost.

Now the expected total profits per unit time, in steady state, is given by

$$P_1=\lim_{t\to\infty}\frac{P_1(t)}{t}=\lim_{s\to 0}s^2P_1^*(s)$$

and

$$P_2=\lim_{t\to\infty}\frac{P_2(t)}{t}=\lim_{s\to 0}s^2P_2^*(s)$$

so that

$$P_1=K_0A_0-K_1B_0 \qquad \textbf{(3)}$$

and

$$P_2=K_0A_0-K_2V_0 \qquad \textbf{(4)}$$

1.10 CONCLUSION

For a graphical representation, the following particular cases are considered

$$G_1(t)=1-e^{-\gamma_1 t}G_2(t)=1-e^{-\gamma_2 t}H_1(t)=1-e^{-\mu_1 t}$$

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To study the behavior of MTSF, availability and profit function through graphs w.r.t various parameters, curves are plotted for these characteristics w.r.t failure parameter α_3 in Fig.4.2 and Fig.4.3 respectively for three different values of repair rate $\beta_1 = (0.25, 0.50, 0.75)$ whereas other parameters are kept fixed $\alpha_1 = 0.5, \alpha_2 = 0.25, \beta_2 = 0.90, \theta_1 = 0.90, \gamma_1 = 0.35, \gamma_2 = 0.20, \mu_1 = 0.95, K_0 = 1000, K_1 = 600, K_2 = 500$;

Fig.1.2 depicts that MTSF decreases with the increase in failure rate α_3 and it shows an increase with the increase in repair rate β_1.So in order to increase the MTSF, we have to minimize the failure rate and maximize the repair rate.

Fig.1.3 indicates that Availability decreases with the increase in failure rate α_3 and it shows an increase with the increase in repair rate β_1.

Fig.1.4 represents the change in profit function P_1 and P_2 w.r.t α_3 for different values of repair rates β_1. From the graph it is seen that both profit functions decrease with the increase in failure rate α_3 and increase with the increase in β_1. So in order to make the system more efficient we have to maximize the repair rate and minimize the failure rate of the system. Also it can be seen from figure that profit function P_2 is always higher than profit function P_1 for some values of the failure and repair parameters.

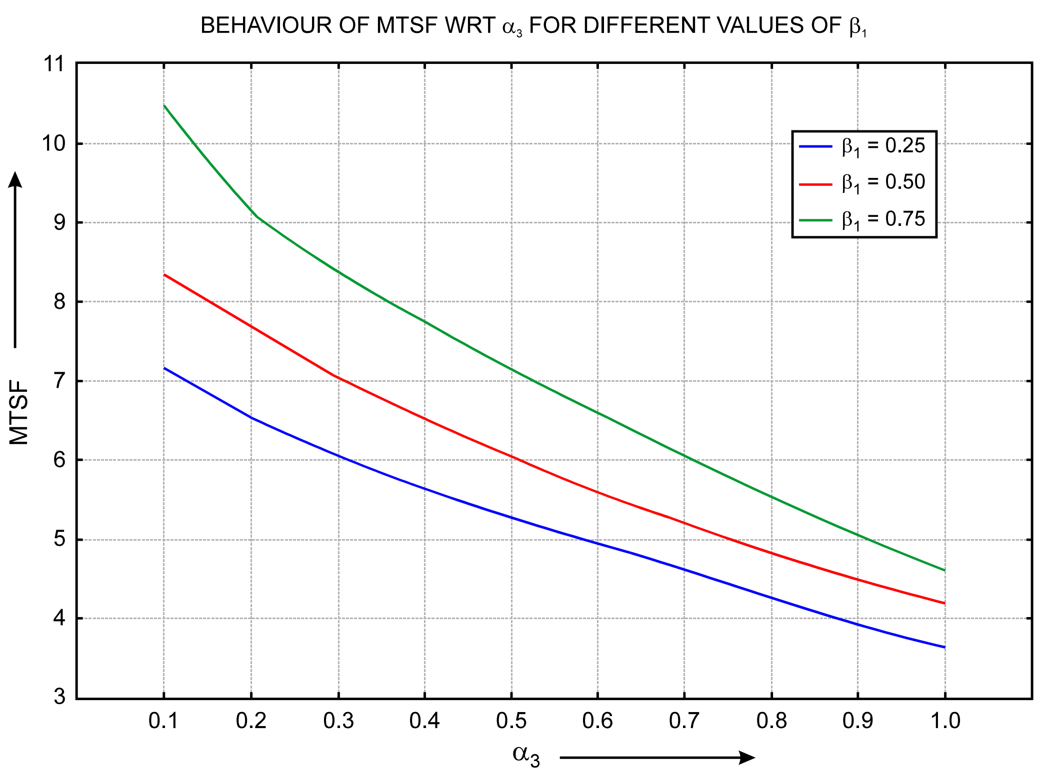

FIG 1.2

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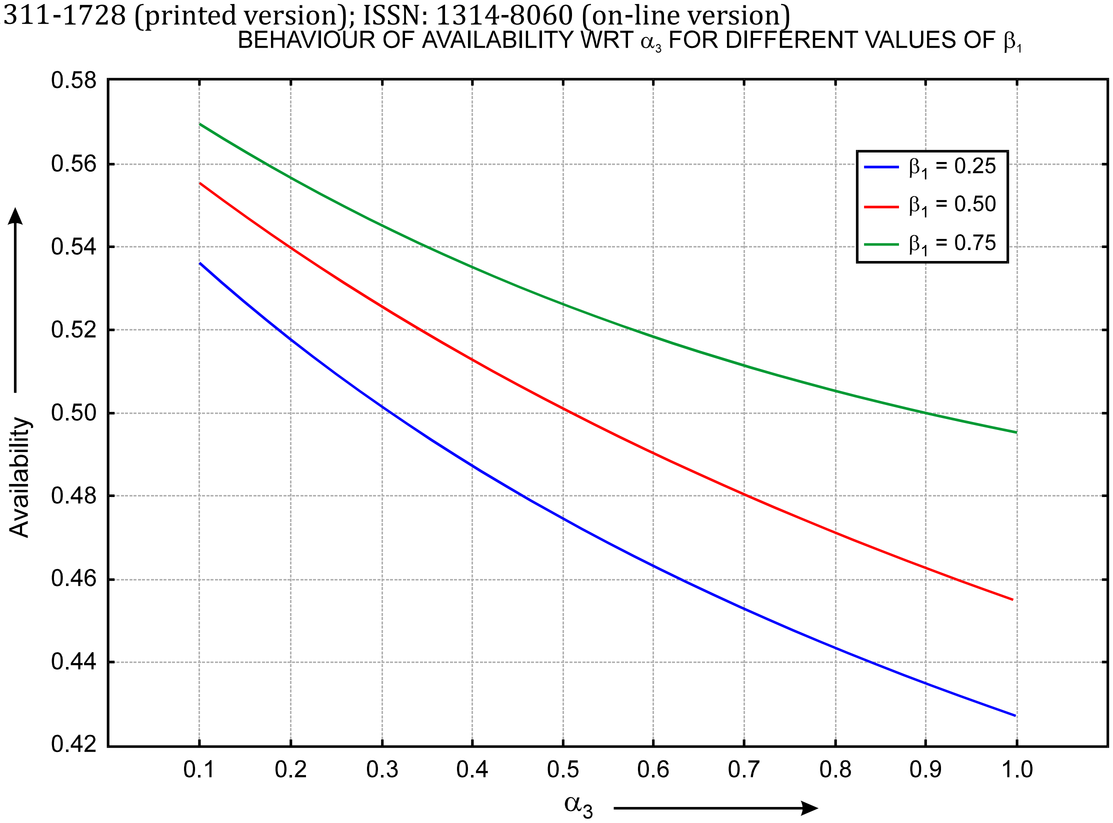

FIG. 1.3

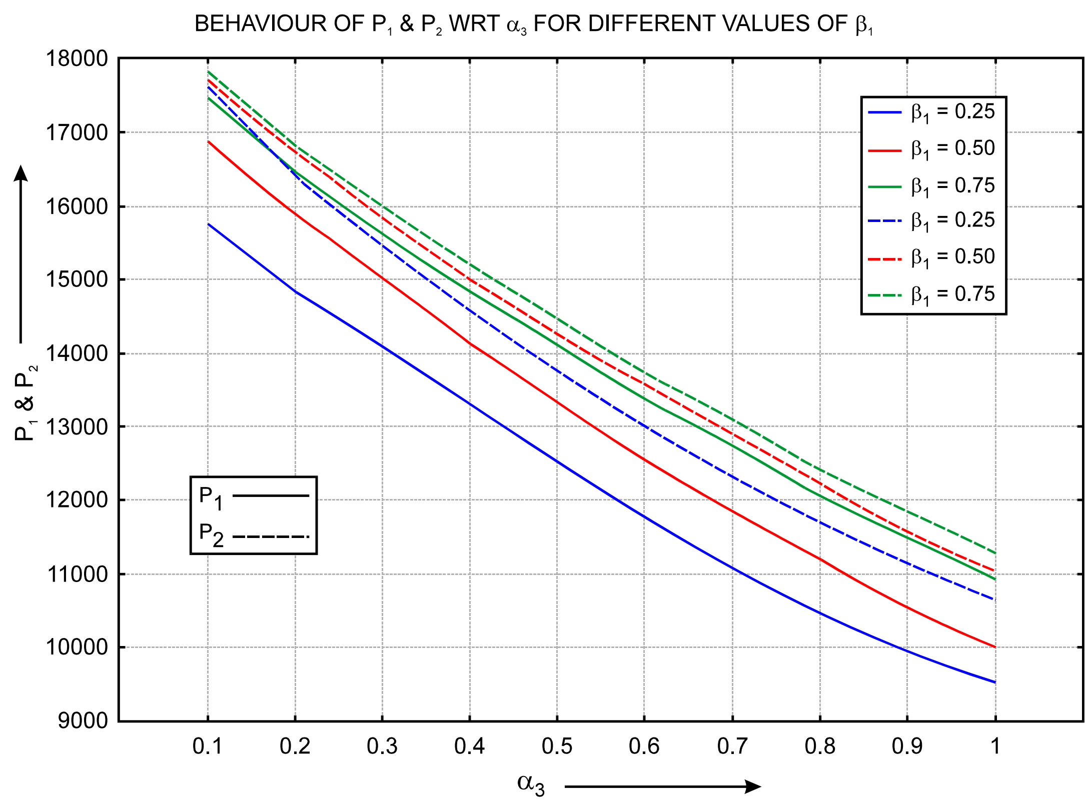

FIG. 1.4

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