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NON-SPLIT GEODETIC DOMINATION NUMBER OF A SOME GRAPHS

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Abstract

A set of vertices S in a graph G is a dominating set of each vertex of G is determined by some vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. A set S of vertices is a geodetic set of G if $I[S] = V(G)$ and the minimum cardinality of such set is geodetic number and is denoted by $g(G)$. A subset S of $V(G)$ is a geodetic dominating set of G if S is a geodetic set and a dominating set of G. The minimum cardinality of geodetic dominating set is called the geodetic domination number of G and is denoted by $\gamma_g(G)$. A set $S \subseteq V(G)$ is called a non-split geodetic dominating set if S is a geodetic and a dominating set and also $< V(G) - S >$ is connected. The minimum cardinality of non-split geodetic dominating set of G is called the non-split geodetic domination number of G and is denoted by $\gamma_{nsg}(G)$. In this paper we determined the non-split geodetic domination number of some graphs.

Keywords: Geodetic Number, Domination Number, Geodetic Domination Number, Non-split Geodetic Domination Number.

Subject Classification Number: 05C12

1. Introduction

Let $G = (V, E)$ be a finite undirected connected graph without multiple edges or loops. The order and size of G are denoted by k and l respectively. For basic graph theoretic terminology we refer to Harary [1]. For vertices p and q in a connected graph G, the distance $d(p,q)$ is the length of a shortest $p - q$ path in G. An $p - q$ path of length $d(p,q)$ is called an $p - q$ geodesic. A vertex x is said to lie on an $p - q$ geodesic P' if x is a vertex of P' including the vertices of p and q. The neighborhood of a vertex x is the set $N(x)$ consisting of all vertices y which are adjacent with x. A vertex x is an extreme vertex of G if the subgraph induced by its neighbors is complete.

The closed interval $I[p,q]$ consists of all vertices lying on some $p - q$ geodesic of G, while for $S \subseteq V, I[S] = \cup_{p,q \in S} I[p,q]$. If $I[S] = V$, then a set S of vertices is a geodetic set and the minimum cardinality of a geodetic set is the geodetic number $g(G)$. A geodetic number of a graph was introduced in [2,3] and further studied in [4,5]

Domination in graphs is one of the interesting areas in graph theory which has wide applications in Engineering and Science. There are more than 300 domination parameters available in the literature. Around 1960 Berge and ore started the mathematical exploration of domination theory in graphs. There is a plethora of material on domination theory; we recommend readers outstanding books [7] on domination-related parameters.

In a Graph G of dominating set S where each node of G is either in S or adjoining to any node in S. The domination number $\gamma = \gamma$ (G) is the least cardinality of an arrangement of domination. [7]

A set of vertices S in a graph G is a nonsplit geodetic set if S is a geodetic set and the subgraph $G[V - S]$ induced by $V(G) - S$ isconnected. The minimum cardinality of a non split geodetic set, denoted $gns\,(G)$, is called the nonsplit geodetic number of G. [6]

Theorem 1.1 [1]

Let x be a vertex of a connected graph G. The following are equivalent.

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(i) x is a cut vertex of G.
(ii) There exists vertices y and z distinct from x such that x is on every $y - z$ path.
(iii) There exists a partition of the set of vertices $V - \{x\}$ into subsets U and W such that for any vertices $u \in U$ and $w \in W$, the vertex x is on every $u - w$ path.

Theorem 1.2 [3]

Every geodetic set of a connected graph G contains its extreme vertices.

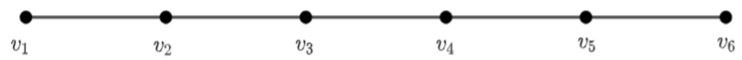
2. Non-Split Geodetic Domination Number of Graphs

Definition 2.1

A geodetic dominating set $S \subseteq V(G)$ is said to be a non-split geodetic donating set of G in $< V - S >$ is connected. The minimum cardinality of a non-split geodetic dominating set of G in $< V - S >$ is connected. The minimum cardinality of a non-split geodetic dominating set is the non-split geodetic domination number and is denoted by $\gamma_{nsg}(G)$.

Example 2.2

Consider the graph G shown in Figure 2.1.



G

$Figure\ 2.1$

Here $S = \{v_1, v_3, v_6\}$ is a minimum geodetic dominating set of G. But $< V - S >$ is not connected so that S itself is not a non-split geodetic dominating set of G.

Now it is clear that $S_1 = \{v_1, v_2, v_5, v_6\}$ is a minimum non-split geodetic domating set of G and hence $\gamma_{nsg}(G) = 4$. Here $\gamma_g(G) = 3$.

Thus, the geodetic domination number and the non-split geodetic domination number are different.

Remark 2.3

For the complete graph G, the set of all vertices form a unique geodetic dominating set of G. In this case $< V - S >$ is empty. Therefore, there does not exist a non-split geodetic dominating set of G.

Throughout this paper we consider G as a non-complete connected graph.

Observation 2.4

For any connected graph G of order $n, 2 \leq g(G) \leq \gamma_{nsg}(G) \leq n - 1$.

Theorem 2.5

Every non-split geodetic dominating set of G contains its extreme vertices.

Proof.

Let x be an extreme vertex of G and let S be a non-split geodetic dominating set of G. To prove $x \in S$. Suppose $x \notin S$, then S is not a geodetic set. This implies that S is not a non-split geodetic dominating set of G. Because every non-split geodetic dominating set is a geodetic set. Thus, every non-split geodetic dominating set contains its extreme vertices.

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Theorem 2.6

Let G be a connected graph with a cut vertex x. Then every non-split geodetic dominating set S of G contains at least one vertex from each component of $G - \{x\}$.

Proof.

Let G be a connected graph with a cut vertex x. Let S be a non-split geodetic dominating set of G. Suppose there exists a component G_1 of $G - \{x\}$ such that G_1 contains no vertex of S. By Theorem 2.5, S contains extreme vertices of G. Thus G_1 does not contain any extreme vertex of G. Let $y \in G_1$. Since S is a non-split geodetic dominating set of G, there exists pair of vertices $u, v \in S$ such that $y \in I[u, v] \subseteq I[S]$ and $y \in N[S]$. Let the $u - v$ geodetic in G be $P: u = x_0, x_1, \dots, x_y, \dots, x_n = v$ in G with $y \neq u, v$. Since x is a cut vertex of G, by Theorem 1.1, the $u - y$ subpath P_1 of P and $y - v$ subpath P_2 of P both contains x, it follows that P is not a path, which is a contradiction. Hence, S contains at least one element from each component of $G - \{x\}$.

Theorem 2.7

For the star graph $G = K_{1,n} (n \geq 2), \gamma_{nsg}(G) = n$.

Proof.

Let $G = K_{1,n} (n \geq 2)$. Let $V(G) = \{v, v_1, v_2, \dots, v_n\}$ where v is support vertex of G and $\deg(v) = n$. Let $S = \{v\}$. Then clearly S is the dominating set of G. But it is not a non-split geodetic dominating set of G. Now, let S represents the collection of all end vertices of G. By Theorem 2.5, S belong to every non-split geodetic dominating set of G. Clearly S is a geodetic dominating set of G and $< V - S >$ is connected. Therefore S is the minimum non-split geodetic dominating set of G. Hence, $\gamma_{nsg}(G) = |S| = n$.

Theorem 2.8

Let T represents any tree and the collection of all end vertices of T dominates every internal vertex of T. Then $\gamma_{nsg}(G) = |end(G)|$.

Proof.

Let S represent the collection of all end vertices of T. That is $|end(G)| = |S|$. Clearly, each end vertices belong to every non-split geodetic dominating set of T. Therefore, $\gamma_{nsg}(G) \geq |S|$. Converse dis obvious. Hence $\gamma_{nsg}(G) = |S| = |end(G)|$.

Theorem 2.9

For the Bistar graph $G = B_{m,n}, (m, n \geq 1), \gamma_{nsg}(G) = m + n$.

Proof.

Let $G = B_{m,n}$ with the vertex set $V(G) = \{x, y, x_1, x_2, \dots, x_m, y_1, y_2, \dots, y_n\}$ and edge set $E(G) = \{xy, xx_i, yy_i; 1 \leq i \leq m, 1 \leq j \leq n\}$.

Let d$S = \{x_1, x_2, \dots, x_m, y_1, y_2, \dots, y_n\}$ be the collection of all end vertices of G. Then by Theorem 2.5, S belong to each non-split geodetic dominating set of G and so $\gamma_{nsg}(G) \geq |S| = m + n$. On the other hand no non-split geodetic dominating set exists with cardinality less than $|S|$. Thus $\gamma_{nsg}(G) = m + n$.

Theorem 2.10

For the path graph $G = P_n (n \geq 3), \gamma_{nsg}(P_n) = \begin{cases} 2 \ if \ 3 \leq n \leq 4 \\ n - 2 \ if \ n \geq 5 \end{cases}$.

Proof.

Let $G = P_n$ with vertex set $V(G) = \{x_1, x_2, \dots, x_n\}$. We consider the following case:

Case (i) $n = 3,4$

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Let S be the set of all end vertices of G. Then $|S| = 2$. Clearly, S itself form a non-split geodetic dominating set of G and so $\gamma_{nsg}(G) = |S| = 2$.

Case (ii) $n \geq 5$

Let $S = \{x_1, x_n\}$. Then clearly, S is a geodetic set of G. But S is not a dominating set of G. Therefore that S is not a non-split geodetic dominating set of G. Now, let $S_1 = \{x_1, x_4, x_5, \dots x_n\}$. Clearly, S_1 is a geodetic dominating set of G and $< V - S_1 >$ is connected. Therefore, that S_1 is a non-split geodetic dominating set of G. Next we show that S_1 is a minimum non-split geodetic dominating set G. Let $W = S_1 - \{x_k; 5 \leq k \leq n-1\}$. Then W is not a non-split geodetic dominating set G. Because $< V - W >$ is not connected. Suppose $W = S_1 - \{x_k\}, k = 1$ or $k = n$. Then W is not a geodetic set of G. If $W = S_1 - \{x_4\}$, then W is not a dominating set of G. Thus all the cases W is not a non-split geodetic dominating set of G. Hence, $\gamma_{nsg}(G) = n - 2$.

Theorem 2.11

For the cycle graph $G = C_n, (n \geq 4), \gamma_{nsg}(G) = \begin{cases} n-1 \ for \ n = 4,5 \\ n-2 \ for \ n > 5 \end{cases}$.

Proof.

Let G be a cycle graph for $n \geq 4$.

Case (i) $n = 4$

Let $G = C_4: x_1, x_2, x_3, x_4, x_1$. Let $S = \{x_1, x_5\}$. Clearly, S is a geodetic set of G. Let S is not a non-split geodetic dominating set of G. Now, consider $S_1 = S \cup \{x_i\}$ for $i = 2 \ or \ 4$. Then $< V - S_1 >$ is connected and hence, S_1 is a minimum non-split geodetic dominating set of G. Thus, $\gamma_{nsg}(G) = 4 - 1 = 3 = n - 1$.

Case (ii) $n = 5$

Let $G = C_5: x_1x_2x_3x_4x_5x_1$. Let $S = \{x_1, x_3, x_4\}$. Then, S is not a non-split geodetic dominating set of G. Let $S_1 = S \cup \{x_i\}$, where $i = 2 \ or \ i = 5$. Then $< V - S_1 >$ is connected and hence S_1 is a minimum non-split geodetic dominating set of G. Thus, $\gamma_{nsg}(G) = 5 - 1 = 4 = n - 1$.

Case (iii) $n > 5$.

Let $G = C_n: x_1x_2 \dots x_nx_1$. Let $S = \{x_1, x_2, \dots, x_{n-2}\}$. Clearly S is a geodetic dominating set of G and $< V - S >$ is connected. Therefore S is a non-split geodetic dominating set of G. Next we claim S is a minimum non-split geodetic dominating set of G. Suppose not, there exists a non-split geodetic dominating set W, such that $|W| < |S|$. Let $W = S - \{x_k; 6 \leq k \leq n-2\}$. Then W is not a non-split geodetic dominating set of G. If $W = S - \{x_i\}$ where $i = 6 \ or \ i = n - 2$, then W is not a dominating set of G. This all the cases we obtained a contradiction. Hence S is a minimum non-split geodetic dominating set of G. Thus, $\gamma_{nsg}(G) = n - 2$.

Therefore, $\gamma_{nsg}(G) = \begin{cases} n-1 \ for \ n = 4,5 \\ n-2 \ for \ n \geq 6 \end{cases}$.

3. Conclusion

In this paper, we find the non-split geodetic domination number of graphs. This work can be extended to find non-split edge geodetic domination number and upper non-split geodetic domination number, etc. The findings united in this paper would support to the readers to develop various useful applications to Science and Technology.

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