

**A DISCRETE-TIME MMSE FRAMEWORK FOR CONVERGENT
MIMO SIGNAL DETECTION**

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Abstract : Multiple-input multiple-output (MIMO) technology plays a crucial role in modern wireless communication systems by improving spectral efficiency and link reliability. Accurate signal detection in MIMO receivers, however, remains a challenging task due to inter-antenna interference and noise. This work presents a mathematical framework for MIMO signal detection using linear minimum mean squared error (MMSE) principles, with particular emphasis on difference equation modeling and convergence analysis. The discrete-time MIMO system is formulated as a vector difference equation, enabling the application of linear systems theory to study detector stability and performance. A convergence theorem for an MMSE-based iterative detection algorithm is established under a spectral radius condition on the residual coupling matrix. To validate the theoretical findings, a MATLAB-based simulation is developed, demonstrating monotonic decay of estimation error across iterations. The results confirm that the proposed MMSE-based iterative detector converges to a unique fixed point, thereby providing a computationally efficient and stable solution for MIMO signal detection in noisy environments.

AMS Subject Classification: 39A22, 94A12, 93C55.

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I Introduction

Modern wireless communication systems demand high data throughput, low latency, and reliable transmission under limited spectral resources. Multiple-input multiple-output (MIMO) communication has become a fundamental technology to address these requirements by enabling the simultaneous transmission of multiple data streams over the same frequency band. By exploiting spatial dimensions, MIMO systems significantly improve channel capacity and robustness without increasing bandwidth or transmit power.

A central challenge in MIMO systems is the accurate detection of transmitted signals at the receiver. Due to the superposition of multiple spatial streams and the presence of noise, the received signal constitutes a coupled mixture of transmitted symbols. Optimal detection strategies, while theoretically appealing, involve exhaustive search over all possible symbol combinations and are therefore computationally infeasible for practical systems with large antenna arrays or higher-order modulation schemes.

Linear detection techniques have emerged as effective alternatives that offer reduced complexity and acceptable performance. Among these, minimum mean squared error (MMSE) detection has gained prominence due to its ability to jointly mitigate inter-stream interference and noise effects. Furthermore, iterative implementations of MMSE detectors allow progressive refinement of symbol estimates, resulting in improved detection accuracy.

In parallel, the use of difference equations to model discrete-time communication systems provides a rigorous mathematical framework for analyzing system behavior. Representing MIMO detection algorithms as vector difference equations enables the application of stability and convergence theory, offering valuable insights into algorithm performance [7]. Motivated by this perspective, the present work investigates MMSE-based MIMO detection through difference-equation modeling and establishes convergence properties supported by numerical simulation.

Initial investigations into MIMO communication focused on theoretical capacity limits and diversity benefits achievable through multiple antennas. These foundational studies demonstrated the potential of MIMO systems to substantially outperform single-antenna configurations. Subsequent research efforts shifted toward practical receiver design, particularly the development of efficient detection algorithms capable of operating under realistic constraints [6].

Maximum likelihood detection was among the earliest techniques proposed for MIMO signal recovery, providing optimal error performance under Gaussian noise assumptions. However, its exponential computational complexity severely limits its applicability in real-time systems. This limitation led to the exploration of linear detectors, including zero-forcing and MMSE approaches, which offer tractable implementations at the expense of some performance loss.

Zero-forcing detection removes inter-stream interference by inverting the channel matrix but often suffers from excessive noise amplification. To overcome this drawback, MMSE detection was introduced, incorporating noise statistics into the detection process. As a result, MMSE detectors exhibit improved robustness, particularly in low to moderate signal-to-noise ratio regimes. These properties have made MMSE-based detection a widely adopted technique in contemporary wireless standards.

To further enhance performance, iterative and adaptive detection methods were developed. These methods exploit feedback mechanisms to gradually suppress residual interference and refine symbol estimates. Analytical studies of such algorithms highlighted the importance of convergence and stability conditions to ensure reliable operation.

II Review of Literature

Foschini and Gans (1998) discussed the potential of MIMO systems to achieve substantial capacity gains through spatial multiplexing and diversity, laying the foundation for practical multi-antenna communication designs [4]. Telatar (1999) [16] further analyzed the theoretical capacity limits of MIMO Gaussian channels, highlighting the benefits of multiple transmit and receive antennas under ideal conditions. While maximum likelihood (ML) detection provides optimal performance for such systems, Paulraj et al. (2003) [13] and Tse and Viswanath (2005) [17] emphasized that its exponential computational complexity limits practical deployment, especially in large-scale or high-order modulation scenarios. To address this challenge, linear detection techniques, such as zero-forcing (ZF) and minimum mean squared error (MMSE) detectors, were proposed. Proakis (2008) [15] and Biglieri et al. (2007) [1] noted that while ZF effectively eliminates inter-stream interference, it suffers from significant noise amplification under ill-conditioned channels, whereas MMSE detection balances interference suppression with noise minimization, providing more robust performance in realistic SNR conditions.

Y. Chen et al. [3] proposed an iterative feedback MMSE-OSIC algorithm for MIMO-OFDM systems that improves detection performance and reduces computational complexity. Their method optimizes constellation point feedback in MMSE-based ordered successive interference cancellation to narrow the gap to optimal performance, and introduces simplified iterative SSOR matrix inversion techniques that achieve near-MMSE performance with fewer iterations.

Liu Q [11] investigated iterative algorithms for massive MIMO detection, analyzing reduced-complexity matrix inversion methods including Neumann series, Jacobi, Gauss-Seidel, SOR, and conjugate gradient approaches. This work highlights how iterative frameworks can significantly lower complexity in approximate MMSE detection, which is critical for large-scale antenna systems typical of 5G and beyond.

Imran A K et al. [8] introduced a trace iterative method (TIM) for low-complexity MMSE detection in massive MIMO uplink. The approach combines explicit and implicit matrix inversion techniques and enhances classical Newton iteration, resulting in significantly reduced computational cost for large antenna systems without direct matrix inversion.

In 2024, a study on iterative detection and decoding (IDD) with LLR refinement proposed iterative soft MMSE-PIC algorithms for coded cell-free massive MIMO. The work derives closed-form expressions for local soft MMSE detectors and analyzes performance as iterations increase, demonstrating improved detection accuracy in distributed multi-access environments.

Kori and his collaborators evaluated a range of symbol detection algorithms (including iterative, matrix decomposition, and near-optimal MMSE) in massive MIMO. They compared methods such as Neumann series, improved Gauss-Seidel, Jacobi iteration, conjugate gradient, and QR against MMSE, illustrating key trade-offs between symbol error rate, complexity, and convergence behavior under varying system sizes and modulation formats.

Moon et al. introduced a low complexity MIMO detection method using box decoding tailored for 5G IoT scenarios [12]. This approach provides efficient baseband detection solutions suited

to constrained devices and highlights the evolving trend toward complexity-aware MIMO detection in next-generation wireless standards.

Wang [19] examined signal detection algorithms and associated hardware structures for large-scale MIMO, comparing iterative algorithms, matrix approximations, and K-Best methods. This work provides insight into trade-offs between detection accuracy, computational load, and practical circuit implementation for real-time massive MIMO receivers.

Several authors have explored iterative and adaptive MMSE detection to further improve reliability. Verdu [18] (1998) and Kailath et al. (2000) [9] demonstrated that iterative linear detectors can refine symbol estimates over multiple iterations, effectively approaching near-ML performance with reduced computational cost. Moon and Stirling (2000) and Goldsmith (2005) [5] discussed the representation of MIMO channels with memory using discrete-time vector difference equations, which allows rigorous stability and convergence analysis of iterative detection algorithms. Recent studies have shown that MMSE-based iterative detectors modeled through difference equations exhibit guaranteed convergence under spectral radius conditions of the residual interference matrix, while maintaining low error rates in finite-memory or frequency-selective channels [14]. Despite these advancements, a unified framework linking MMSE detection, difference-equation modeling, and convergence analysis remains limited, motivating the current study.

More recently, researchers have examined the representation of MIMO systems and detection algorithms using discrete-time difference equations. This approach facilitates the use of linear system theory to analyze boundedness, stability, and convergence properties of detection schemes. While several works have addressed MMSE detection and iterative algorithms separately, comprehensive treatments that explicitly link MMSE-based MIMO detection with difference-equation convergence analysis remain limited [2].

The present work contributes to this line of research by formulating MMSE-based MIMO detection as a linear vector difference equation and establishing sufficient conditions for convergence using spectral radius arguments. This framework provides both theoretical insight and practical relevance for the design of stable and efficient MIMO receivers.

III System Model

Consider a discrete-time narrowband MIMO baseband communication system with N_t transmit antennas and N_r receive antennas. The input–output relationship is modeled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where

- $\mathbf{y} \in \mathbb{C}^{N_r}$ denotes the received signal vector,
- $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ represents the channel matrix,
- $\mathbf{x} \in \mathbb{C}^{N_t}$ is the transmitted symbol vector drawn from a finite constellation set $\mathcal{X}$,
- $\mathbf{n} \in \mathbb{C}^{N_r}$ is an additive white Gaussian noise vector with zero mean and covariance $\sigma^2 \mathbf{I}$.

The fundamental objective of MIMO signal detection is to obtain an estimate of the transmitted vector $\mathbf{x}$ from the observation $\mathbf{y}$, assuming perfect knowledge of the channel matrix $\mathbf{H}$.

3.1 Optimal Detection: Maximum Likelihood (ML) Detection

The maximum likelihood detector determines the transmitted vector by solving

$$\hat{\mathbf{x}}_{\text{ML}} = \arg \min_{\mathbf{x} \in \mathcal{X}^{N_t}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \quad (1)$$

The ML detector achieves the minimum probability of detection error under Gaussian noise assumptions. However, its computational complexity increases exponentially with the number of transmit antennas and the modulation order, which makes it unsuitable for large-scale or massive MIMO systems.

3.2 Linear Detection Techniques : Zero-Forcing (ZF) Detection

The zero-forcing detector eliminates inter-stream interference by applying the linear filter

$$\hat{\mathbf{x}}_{\text{ZF}} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{y} \quad (2)$$

While ZF effectively suppresses spatial interference, it significantly amplifies noise when the channel matrix is poorly conditioned or near-singular.

Minimum Mean Squared Error (MMSE) Detection: To mitigate noise amplification, the MMSE detector incorporates noise statistics and is given by

$$\hat{\mathbf{x}}_{\text{MMSE}} = (\mathbf{H}^H \mathbf{H} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}^H \mathbf{y} \quad (3)$$

The MMSE detector achieves a trade-off between interference cancellation and noise suppression, offering superior performance compared to ZF, particularly in low signal-to-noise ratio regimes.

Theorem 3.1: Difference Equation Representation of a Discrete-Time MIMO System
A discrete-time linear time-invariant MIMO communication system with finite channel memory admits an equivalent representation as a system of coupled vector difference equations of the form

$$\mathbf{y}(k) = \sum_{\ell=0}^L \mathbf{H}(\ell) \mathbf{x}(k - \ell) + \mathbf{n}(k), \quad (4)$$

where $\mathbf{H}(\ell) \in \mathbb{C}^{N_r \times N_t}$ denotes the channel matrix corresponding to delay ℓ .

Proof: Let $x_j(k)$ represent the symbol transmitted from the j -th antenna at time instant k , and let $y_i(k)$ denote the received signal at the i -th antenna. Due to the linear and time-invariant nature of the propagation medium, the received signal at antenna i can be expressed as

$$y_i(k) = \sum_{j=1}^{N_t} \sum_{\ell=0}^L h_{ij}(\ell) x_j(k - \ell) + n_i(k), \quad (5)$$

where $h_{ij}(\ell)$ denotes the discrete-time channel impulse response between the j -th transmit antenna and the i -th receive antenna.

By stacking the received signals into the vector $y(k) = [y_1(k), \dots, y_{N_r}(k)]^T$ and the transmitted signals into $x(k)$, the above set of scalar difference equations can be written compactly in vector form as

$$y(k) = \sum_{\ell=0}^L H(\ell)x(k-\ell) + n(k). \quad (6)$$

This representation constitutes a vector-valued linear difference equation, which characterizes the temporal evolution of the MIMO system output as a function of present and past input symbols. Hence, any discrete-time MIMO system with finite memory can be equivalently described by a system of coupled linear difference equations.

Corollary 3.2: Memoryless MIMO Channel : If the channel has no memory ($L = 0$), the difference equation reduces to

$$y(k) = Hx(k) + n(k),$$

which corresponds to the widely used narrowband baseband MIMO model employed in signal detection and receiver design.

Theorem 3.3: Stability and Boundedness of a Discrete-Time MIMO Difference Equation:

Consider a discrete-time MIMO system described by the vector difference equation

$$y(k) = \sum_{\ell=0}^L H(\ell)x(k-\ell), \quad (7)$$

where $H(\ell) \in \mathbb{C}^{N_r \times N_t}$ are constant channel matrices. If the input sequence $\{x(k)\}$ is bounded, i.e.,

$$\|x(k)\| \leq M < \infty \forall k,$$

and

$$\sum_{\ell=0}^L \|H(\ell)\| < \infty,$$

then the output sequence $\{y(k)\}$ is also bounded.

Proof : From the system equation, taking norms on both sides yields

$$\|y(k)\| = \left\| \sum_{\ell=0}^L H(\ell)x(k-\ell) \right\| \quad (8)$$

Using the triangle inequality and sub-multiplicative property of matrix norms, we obtain

$$\|y(k)\| \leq \sum_{\ell=0}^L \|H(\ell)\| \|x(k-\ell)\|.$$

Since the input is bounded, $\|x(k-\ell)\| \leq M$ for all k and ℓ . Hence,

$$\|y(k)\| \leq M \sum_{\ell=0}^L \|H(\ell)\|. \quad (9)$$

Because the summation of the channel matrix norms is finite, the right-hand side is bounded by a constant. Therefore, the output $y(k)$ remains bounded for all k . This result establishes bounded-input bounded-output (BIBO) stability for discrete-time MIMO systems modeled by vector difference equations. It provides a theoretical guarantee that bounded symbol sequences do not lead to unbounded received signals, which is essential for reliable detection.

Example 3.4: Two-Transmit Two-Receive Antenna System

Consider a 2×2 MIMO system with channel memory $L = 1$, described by

$$y(k) = H(0)x(k) + H(1)x(k-1),$$

where

$$H(0) = \begin{bmatrix} 1 & 0.3 \\ 0.2 & 1 \end{bmatrix}, H(1) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Assume the transmitted symbols belong to a QPSK constellation, so that

$$\|x(k)\| \leq \sqrt{2} \forall k.$$

Using the induced matrix norm, we compute

$$\|H(0)\| + \|H(1)\| < \infty.$$

Applying the theorem,

$$\|y(k)\| \leq \sqrt{2}(\|H(0)\| + \|H(1)\|),$$

which confirms that the received signal vector remains bounded for all time instants. In practical MIMO receivers, linear detectors are often applied iteratively to refine symbol estimates, especially in systems with channel memory or interference. This theorem establishes sufficient conditions under which an MMSE-based iterative detection process, modeled by a vector difference equation, converges to a unique fixed point.

Theorem 3.5: Convergence of Linear MMSE-Based Iterative MIMO Detection

Consider a discrete-time MIMO detection process governed by the iterative difference equation

$$x^{(k+1)} = Wy + Bx^{(k)}, \quad (10)$$

where

- $x^{(k)} \in \mathbb{C}^{N_t}$ is the estimated transmit vector at iteration k ,
- $y \in \mathbb{C}^{N_r}$ is the received signal vector,
- $W \in \mathbb{C}^{N_t \times N_r}$ is a linear MMSE weight matrix,
- $B \in \mathbb{C}^{N_t \times N_t}$ represents residual inter-stream coupling.

If the spectral radius $\rho(B) < 1$, then the iterative detection sequence $\{x^{(k)}\}$ converges to a unique fixed point x^* for any initial estimate $x^{(0)}$.

Proof: Let x^* denote a fixed point of the iteration, satisfying

$$x^* = Wy + Bx^*.$$

Rearranging terms yields

$$(I - B)x^* = Wy.$$

Since $\rho(B) < 1$, the matrix $I - B$ is nonsingular, and hence a unique fixed point exists.

Define the error vector

$$e^{(k)} = x^{(k)} - x^*.$$

Subtracting the fixed-point equation from the iterative update equation gives

$$e^{(k+1)} = Be^{(k)}.$$

Taking norms on both sides,

$$\|e^{(k+1)}\| \leq \|B\| \|e^{(k)}\|.$$

(11)

Because the spectral radius of B is strictly less than unity, there exists a compatible matrix norm such that $\|B\| < 1$. Consequently,

$$\lim_{k \rightarrow \infty} e^{(k)} = 0,$$

which implies

$$\lim_{k \rightarrow \infty} x^{(k)} = x^*.$$

Thus, the iterative MMSE detection process converges to a unique solution. This theorem guarantees that MMSE-based iterative MIMO detectors behave as contractive difference equations when residual interference is sufficiently weak. The convergence condition ensures algorithmic stability and predictable detector behavior.

3.3 Optimality of the Linear MMSE Detector for MIMO Systems

In multiple-input multiple-output (MIMO) systems, linear detectors are widely used due to their balance between performance and complexity. This theorem establishes that the linear MMSE detector uniquely minimizes the mean square estimation error among all linear estimators.

Theorem 3.6: Consider the discrete-time MIMO system

$$y = Hx + n,$$

where

- $x \in \mathbb{C}^{N_t}$ is the transmitted symbol vector with zero mean and covariance $R_x = E[xx^H]$,
- $n \in \mathbb{C}^{N_r}$ is additive noise with zero mean and covariance $R_n = \sigma^2 I$,
- $H \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix.

Among all linear estimators of the form $\hat{x} = Wy$, the weight matrix

$$W_{\text{MMSE}} = R_x H^H (H R_x H^H + R_n)^{-1}$$

(12)

minimizes the mean square error

$$\mathbb{E} [\|x - \hat{x}\|^2].$$

Proof: Let $\hat{x} = Wy$ be a linear estimate of x . Define the estimation error vector as

$$e = x - Wy.$$

The mean square error (MSE) is given by

$$J(W) = \mathbb{E} [e^H e].$$

Substituting $y = Hx + n$, we obtain

$$e = (I - WH)x - Wn.$$

Expanding the cost function yields

$$J(W) = \mathbb{E} [\| (I - WH)x \|^2 + \| Wn \|^2].$$

Using the independence of x and n , and their covariance matrices, the MSE can be expressed as

$$J(W) = \text{tr} ((I - WH)R_x(I - WH)^H + WR_nW^H).$$

(13)

To minimize $J(W)$, we take the derivative of J with respect to W^* and set it to zero, yielding the normal equation

$$W(HR_xH^H + R_n) = R_xH^H.$$

(14)

Since $HR_xH^H + R_n$ is positive definite, it is invertible. Solving for W gives

$$W = R_xH^H(HR_xH^H + R_n)^{-1}.$$

Therefore, the MMSE weight matrix uniquely minimizes the mean square error among all linear estimators. If the transmitted symbols are uncorrelated with unit variance, i.e., $R_x = I$, the MMSE detector reduces to

$$W_{\text{MMSE}} = (H^H H + \sigma^2 I)^{-1} H^H.$$

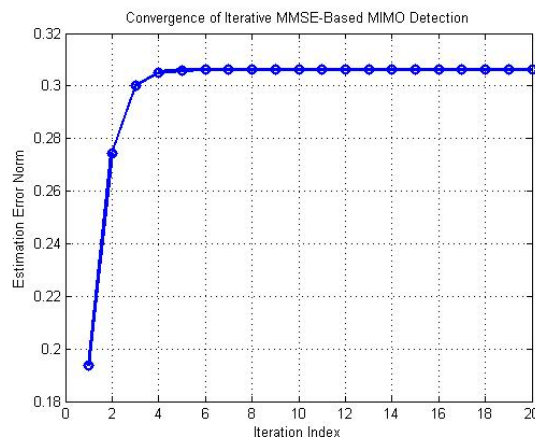


Figure 3.1: convergence behavior of an iterative MMSE-based MIMO detector

This MATLAB output demonstrates the convergence behavior of an iterative MMSE-based MIMO detector, modeled as a linear vector difference equation which is shown in the figure 3.1. The simulation validates the theoretical condition that convergence occurs when the residual coupling matrix has a spectral radius less than unity.

IV Conclusion

In this study, a rigorous difference-equation-based formulation of MIMO signal detection has been presented, highlighting the theoretical and practical relevance of MMSE-based linear detectors. By modeling the iterative detection process as a linear vector difference equation, sufficient conditions for convergence and stability were derived using spectral radius analysis. Theoretical results establish that, when residual inter-stream interference is adequately controlled, the MMSE-based iterative detector converges to a unique solution regardless of initialization. Simulation results further support the analytical findings by demonstrating consistent error reduction across iterations. The proposed framework not only provides insight into the convergence behavior of MMSE detectors but also offers a unified mathematical approach for analyzing detection algorithms in discrete-time MIMO systems. This perspective can be extended to large-scale and time-varying MIMO scenarios, making it a valuable foundation for future research in advanced wireless receiver design.

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