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FINITE ELEMENT APPROACH TO SOLVE SEVENTH ORDER BOUNDARY VALUE PROBLEMS USING QUINTIC B-SPLINES

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Abstract

A finite element method involving Galerkin method with quintic B-splines as basis functions has been been solved the seventh order boundary value problem with boundary conditions. The basis functions are redefined into a new set of basis functions which vanish at the boundary where Dirichlet type of boundary conditions and Neumann boundary conditions, second derivative boundary conditions are prescribed. The proposed method was applied to solve several examples of sixth order linear and nonlinear boundary value problems. The solution of a nonlinear boundary value problem has been obtained as the limit of a sequence of solution of linear boundary value problems generated by quasilinearization technique. The obtained numerical results were found to be in good agreement with exact solutions available in the literature.

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Keywords: Galerkin method; Quintic B-spline; Basis function; Seventh order boundary value problem; Absolute error.

1. Introduction

In this paper, we consider a general sixth order linear boundary value problem given by

$$a_0(t)v^{(7)}(t)+a_1(t)v^{(6)}(t)+a_2(t)v^{(5)}(t)+a_3(t)v^{(4)}(t)+a_4(t)v'''(t)+a_5(t)v''(t)$$

$$+a_6(t)v'(t)+a_7(t)v(t)=b(t) \qquad l<t<m \qquad (1)$$

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subject to boundary conditions

$$v(l)=A_0, v(m)=C_0, v'(l)=A_1, v'(m)=C_1, v''(l)=A_2, v''(m)=C_2, v'''(l)=A_3 \qquad (2)$$

where $A_0, C_0, A_1, C_1, A_2, C_2$ and A_3 are finite real constants and $a_0(t)$, $a_1(t)$, $a_2(t)$, $a_3(t)$, $a_4(t)$, $a_5(t)$, $a_6(t)$, $a_7(t)$ and $b(t)$ are all continuous functions defined on the interval $[l, m]$.The boundary value problem is solved with the boundary conditions.

Generally, this type of sixth order boundary value problems arises in the various branches of pure and applied sciences including astrophysics, structural engineering, optimization and economics. These types of problems are special significance in astrophysics. The action of dynamo in some stars may be modelled by sixth order boundary value problems [1].

The brief details of the theorem which consist the conditions for the existence and uniqueness of the solution for these problems have been are given in Agarwal [1], but no numerical methods are contained therein. Solving such boundary value problems is analytically possible only in very rare cases. Currently, many research workers have been developed several method for solution of these types of boundary value problems.

In this paper, we try to present a simple finite element method which involves Gelerkin approach with quintic B-splines as basis functions to solve the seventh order two point boundary value problems of the type (1)-(2).This paper is organized as follows. Section 2, deals with the justification for using Galerkin Method. In Section 3, a description of Galerkin method with quintic B-splines as basis functions is explained. In particular we first introduce the basic concept of quintic B-splines and followed by the proposed method with the boundary conditions. In Section 4, the procedure to solve the nodal parameters has been presented. In section 5, the proposed method is tested on several linear and nonlinear boundary value problems. The solution to a nonlinear problem has been obtained as the limit of a sequence of solution of linear problems generated by the quasilinearization technique [3]. Finally, in the last section, the conclusion presented.

2 Justification for using Galerkin Method

For the few decades, the finite element method has become very powerful, useful tool to solve the boundary value problems in the complex dynamical systems. In finite element method (FEM) the approximate solution can be written as a linear combination of basis functions which constitute a basis for the approximation space under consideration.FEM involves variational methods like Rayleigh Ritz, Galerkin, Least Squares and Collocation etc.

In Galerkin method, the residual of approximation is made orthogonal to the basis functions. When we use Galerkin method, a weak form of approximation solution for a given differential equation exists and is unique under appropriate conditions [7, 9] irrespective of properties of a given differential operator.Further,a weak solution also tends to a classical solution of given differential equation, provided sufficient attention is given to boundary conditions [8]. That means the basis functions should vanish on the boundary where the Dirichlet type of boundary conditions are prescribed. Hence in this paper we employed the use

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of Galerkin method with quintic B-splines as basis functions to approximate the solution of seventh order boundary value problems.

3. Description of the method

Definition of quintic B-spline:

The quintic B-splines are defined in [4, 5, 6]. The existence of quintic spline interpolate $s(t)$ to a function in a closed interval $[l, m]$ for spaced knots (need not be evenly spaced) of a partition $l = t_0 < t_1 < t_2 < \dots < t_{n-1} < t_n = m$ is established by constructing it. The construction of $s(x)$ is done with the help of the quintic B-splines. Introduce ten additional knots t_{-5}, t_{-4}, t_{-3}, t_{-2}, t_{-1}, t_{n+1}, t_{n+2}, t_{n+3}, t_{n+4} and t_{n+5} such that

$t_{-5} < t_{-4} < t_{-3} < t_{-2} < t_{-1} < t_0$ and $t_n < t_{n+1} < t_{n+2} < t_{n+3} < t_{n+4} < t_{n+5}$.

Now the quintic B-splines $R_i(t)$'s are defined by

$$R_i(t) = \begin{cases} \sum\limits_{r=i-3}^{i+3} \dfrac{(t_r - t)_+^5}{\pi'(t_r)}, & t \in [t_{i-3}, t_{i+3}] \\ 0, & otherwise \end{cases}$$

where $$(t_r - t)_+^5 = \begin{cases} (t_r - t)^5, & if\ t_r \ge t \\ 0, & if\ t_r \le t \end{cases}$$

and $$\pi(t) = \prod_{r=i-3}^{i+3} (t - t_r)$$

where $\{ R_{-2}(t), R_{-1}(t), R_0(t), R_1(t), \dots, R_{n-1}(t), R_n(t), R_{n+1}(t), R_{n+2}(t) \}$ forms a basis for the space $S_5(\pi)$ of quintic polynomial splines. Schoenberg [6] has proved that quintic B-splines are the unique nonzero splines of smallest compact support with the knots at

$t_{-5} < t_{-4} < t_{-3} < t_{-2} < t_{-1} < t_0 < t_1 < \dots < t_{n+1} < t_{n+2} < t_{n+3} < t_{n+4} < t_{n+5}$.

To solve the boundary value problem (1) and (2) by the Galerkin method with quintic

B-splines as basis functions, we define the approximation for $v(t)$ as

$$v(t) = \sum_{j=-2}^{n+2} \alpha_j R_j(t) \qquad (3)$$

where α_j's are the nodal parameter to be determined. In Galerkin method the basis functions should vanish on the boundary where the Dirichlet types of boundary conditions are specified. In the set of quintic B-splines $\{ R_{-2}(t), R_{-1}(t), R_0(t), R_1(t), R_2(t), \dots, R_{n-1}(t), R_n(t), R_{n+1}(t), R_{n+2}(t$ $\}$ the basis functions $R_{-2}(t)$, $R_{-1}(t)$, $R_0(t)$, $R_1(t)$, $R_2(t)$, $R_{n-2}(t)$, $R_{n-1}(t)$, $R_n(t)$, $R_{n+1}(t)$, and $R_{n+2}(t)$ do not vanish at one of the boundary points. So, there is a necessity of redefining the basis functions into a new set of basis functions which vanish on the boundary where the Dirichlet type of boundary conditions are specified. The procedure for redefining is as follows. Using

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the quintic B-splines and the Dirichlet boundary conditions of (2), we get the approximate solution at the boundary points as

$$A_0 = v(l) = v(t_0) = \alpha_{-2}R_{-2}(t_0) + \alpha_{-1}R_{-1}(t_0) + \alpha_0 R_0(t_0) + \alpha_1 R_1(t_0) + \alpha_2 R_2(t_0) \tag{4}$$

$$C_0 = v(m) = v(t_n) = \alpha_{n-2}R_{n-2}(t_n) + \alpha_{n-1}R_{n-1}(t_n) + \alpha_n R_n(t_n) + \alpha_{n+1}R_{n+1}(t_n) + \alpha_{n+2}R_{n+2}(t_n) \tag{5}$$

Eliminating α_{-2} and α_{n+2} from the equations (3), (4) and (5), we get

$$v(t) = w_1(t) + \sum_{j=-1}^{n+1} \alpha_j S_j(t) \tag{6}$$

where $$w_1(t) = \frac{A_0}{R_{-2}(t_0)} R_{-2}(t) + \frac{C_0}{R_{n+2}(t_n)} R_{n+2}(t) \tag{7}$$

and $$S_j(t) = \begin{cases} R_j(t) - \dfrac{R_j(t_0)}{R_{-2}(t_0)} R_{-2}(t), & j = -1, 0, 1, 2 \\ R_j(t), & j = 3, \ldots, n-3 \\ R_j(t) - \dfrac{R_j(t_n)}{R_{n+2}(t_n)} R_{n+2}(t), & j = n-2, n-1, n, n+1. \end{cases} \tag{8}$$

Using the Neumann boundary conditions of (2) to the approximate solution $v(t)$ in (6), we get

$$A_1 = v'(l) = v'(t_0) = w_1'(t_0) + \alpha_{-1}S_{-1}'(t_0) + \alpha_0 S_0'(t_0) + \alpha_1 S_1'(t_0) + \alpha_2 S_2'(t_0) \tag{9}$$

$$C_1 = v'(m) = v'(t_n) = w_1'(t_n) + \alpha_{n-2}S_{n-2}'(t_n) + \alpha_{n-1}S_{n-1}'(t_n) + \alpha_n S_n'(t_n) + \alpha_{n+1}S_{n+1}'(t_n) \tag{10}$$

Eliminating α_{-1} and α_{n+1} from the equations (6), (9) and (10), we get approximation for $v(t)$ as

$$v(t) = w_2(t) + \sum_{j=0}^{n} \alpha_j T_j(t) \tag{11}$$

where $$w_2(t) = w_1(t) + \frac{A_1 - w_1'(t_0)}{S_{-1}'(t_0)} S_{-1}(t) + \frac{C_1 - w_1'(t_n)}{S_{n+1}'(t_n)} S_{n+1}(t) \tag{12}$$

and $$T_j(t) = \begin{cases} S_j(t) - \dfrac{S_j'(t_0)}{S_{-1}'(t_0)} S_{-1}(t), & j = 0, 1, 2 \\ S_j(t), & j = 3, \ldots, n-3 \\ S_j(t) - \dfrac{S_j'(t_n)}{S_{n+1}'(t_n)} S_{n+1}(t), & j = n-2, n-1, n. \end{cases} \tag{13}$$

Using the second derivative boundary conditions of (2) to the approximate solution $v(t)$ in (11), we get

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$$A_2 = v''(l) = v''(t_0) = w_2''(t_0) + \alpha_0 T_0''(t_0) + \alpha_1 T_1''(t_0) + \alpha_2 T_2''(t_0) \tag{14}$$

$$C_2 = v''(m) = v''(t_n) = w_2''(t_n) + \alpha_{n-2} T_{n-2}''(t_n) + \alpha_{n-1} T_{n-1}''(t_n) + \alpha_n T_n''(t_n) \tag{15}$$

Eliminating α0 and α*n* from the equations (11), (14) and (15), we get approximation for *v*(*t*) as

$$v(t) = w(t) + \sum_{j=1}^{n-1} \alpha_j \tilde{R}_j(t) \tag{16}$$

where $$w(t) = w_2(t) + \frac{A_2 - w_2''(t_0)}{T_0''(t_0)} T_0(t) + \frac{C_2 - w_2''(t_n)}{T_n''(t_n)} T_n(t) \tag{17}$$

and $$\tilde{R}_j(t) = \begin{cases} T_j(t) - \dfrac{T_j''(t_0)}{T_0''(t_0)} T_0(t), & j = 1,2 \\ T_j(t), & j = 3,\ldots,n-3 \\ T_j(t) - \dfrac{T_j''(t_n)}{T_n''(t_n)} T_n(t), & j = n-2, n-1. \end{cases} \tag{18}$$

Now the new set of basis functions for the approximation *v*(*t*) is $\{\tilde{R}_j(t)$, *j*=1, 2,..., $n-1\}$. Applying the Galerkin method to (1) with the new set of basis functions, we get

$$\int_{t_0}^{t_n} \Big[a_0(t)v^{(7)}(t) + a_1(t)v^{(6)}(t) + a_2(t)v^{(5)}(t) + a_3(t)v^{(4)}(t) + a_4(t)v'''(t) + a_5(t)v''(t) + a_6(t)v'(t)$$

$$+ a_7(t)v(t)\Big]\tilde{R}_i(t)\,dt = \int_{t_0}^{t_n} b(t)\tilde{R}_i(t)dt \qquad for \quad i = 1,2,\ldots\ldots,n-2,n-1 \tag{19}$$

Integrating by parts terms the first three terms on the left hand side of (19), we get term after applying the Neumann, second derivative boundary conditions prescribed in (2), we get

$$\begin{aligned} \int_{t_0}^{t_n} a_0(t)\tilde{R}_i(t)v^{(7)}(t)\,dt &= -\frac{d^3}{dt^3}\big[a_0(t)\tilde{R}_i(t)\big]v'''(t)\big|_{t_n} + A_3\frac{d^3}{dt^3}\big[a_0(t)\tilde{R}_i(t)\big]\big|_{t_0} \\ &+ \int_{t_0}^{t_n} \frac{d^4}{dt^4}\big[a_0(t)\tilde{R}_i(t)\big]v'''(t)\,dt \end{aligned} \tag{20}$$

$$\int_{t_0}^{t_n} a_1(t)\tilde{R}_i(t)v^{(6)}(t)\,dt = -\int_{t_0}^{t_n} \frac{d}{dt}\big[a_1(t)\tilde{R}_i(t)\big]v^{(5)}(t)\,dt \tag{21}$$

$$\int_{t_0}^{t_n} a_2(t)\tilde{R}_i(t)v^{(5)}(t)\,dt = -\int_{t_0}^{t_n} \frac{d}{dt}\big[a_2(t)\tilde{R}_i(t)\big]v^{(5)}(t)\,dt \tag{22}$$

Substituting (20)-(22) in (19) and using the approximation for $v(t)$ given in (16),and after rearranging the terms for resulting equations, we get a system of equations in the matrix form as

$$\mathbf{A}\alpha=\mathbf{B} \tag{23}$$

Where $\mathbf{A}=[a_{ij}]$;

$$a_{ij}=\int_{t_0}^{t_n}\{-\frac{d}{dt}[a_1(t)\tilde{R}_i(t)]\tilde{R}_j^{(5)}(t)+[a_3(t)\tilde{R}_i(t)-\frac{d}{dt}[a_2(t)\tilde{R}_i(t)]\tilde{R}_j^{(4)}(t)+[a_4(t)\tilde{R}_i(t)+$$
$$\frac{d^4}{dt^4}(a_0(t)\tilde{R}_i(t)]\tilde{R}_j'''(t)+a_5(t)\tilde{R}_i(t)\tilde{B}_j''(t)+a_6(t)\tilde{R}_i(t)\tilde{R}_j'(t)+a_7(t)\tilde{R}_i(t)\tilde{R}_j(t)\}dt$$
$$-\frac{d^3}{dt^3}[a_0(t)\tilde{R}_i(t)]\tilde{R}_j'''(t)\,|_{t_n} \qquad \text{for}\quad \mathrm{i}=1,2,3,\ldots,n-2,\mathrm{n}-1;\mathrm{j}=1,2,3,\ldots,\mathrm{n}-2,n-1. \tag{24}$$

$\mathbf{B}=[b_i]$;

$$b_i=\int_{t_0}^{t_n}\{b(t)\tilde{R}_i(t)-\frac{d}{dt}[a_1(t)\tilde{R}_i(t)]\tilde{R}_j^{(5)}(t)+[a_3(t)\tilde{R}_i(t)-\frac{d}{dt}(a_2(t)\tilde{R}_i(t))]\tilde{R}_j^{(4)}(t)$$
$$+[a_4(t)\tilde{R}_i(t)+\frac{d^4}{dt^4}(a_0(t)\tilde{R}_i(t))]\tilde{R}_j'''(t)+a_5(t)\tilde{R}_i(t)\tilde{R}_j''(t)+a_6(t)\tilde{R}_i(t)\tilde{R}_j'(t)$$
$$+a_7(t)\tilde{R}_i(t)\tilde{R}_j(t)\}dt+\frac{d^3}{dt^3}[a_0(t)\tilde{R}_i(t)]\tilde{R}_j'''(t)\,|_{t_n}-A_3\frac{d^3}{dt^3}[a_0(t)\tilde{R}_i(t)]\,|_{t_0}$$
$$\text{for}\quad \mathrm{i}=1,2,3,\ldots,\mathrm{n}-1. \tag{25}$$

and $\alpha=[\alpha_1,\alpha_2,\ldots\ldots,\alpha_{n-1}]^T$

4. Procedure to find solution for nodal parameters

A typical integral element in the matrix **A** is

$$\sum_{p=0}^{n-1} I_p$$

Where $I_p=\int_{t_p}^{t_{p+1}} r_i(t)r_j(t)Z(t)dt$ and $r_i(t)$, $r_j(t)$ are the quintic B-spline basis functions or their derivatives. It may be noted that $I_p=0$ if $(t_{i-3}, t_{i+3}) \cap (t_{j-3}, t_{j+3}) \cap (t_p, t_{p+1}) = \theta$. To evaluate each I_p, we employed 6-point Gauss-Legendre quadrature formula. Thus the stiff matrix **A** is a eleven diagonal band matrix. The nodal parameter vector α has been obtained from the system **A**α = **B** using a band matrix solution package. We have used FORTRAN-90 program to solve the boundary value problems (1)-(2) by the proposed method.

5. Numerical Results

To demonstrate the applicability of the proposed method for solving the seventh order bounary value problems of the types (1) and (2), we considered four linear boundary value problems and three nonlinear boundary value problems. Numerical results for each problem a represented in tabular forms and compared with the exact solutions available in the literature.

Example 1: Consider the linear boundary value problem

$$v^{(7)}+tv=-\left(35+13t+t^{3}\right)e^{t}, \qquad 0<t<1 \tag{26}$$

subject to $v(0)=v(1)=0,\ v'(0)\ =1,\ v'(1)=-e,\ v''(0)=0,\ v''(1)=-4e,\ v'''(0)=-3.$

The exact solution for the above problem is $v(t)=t(1-t)e^{t}$. The proposed method is tested on this problem where the domain [0, 1] is divided into10 equal subintervals. The obtained numerical results for this problem are given in Table 1. The maximum absolute error obtained by the proposed method is 5.036592×10^{-06}.

Example 2: Consider the linear boundary value problem

$$v^{(7)}+4v''+tv=(-43-16t+t^{2})\sin t+(12-15t-4t^{2}+t^{3}), \qquad -1<t<1 \tag{27}$$

subject to $v(-1)=0, v(1)=0, v'(-1)=-2cos1, v'(1)=2cos1, v''(-1)=2cos1-4sin1,$

$$v''(1)=2cos1-4sin1, v'''(-1)=6cos1+6sin1.$$

The exact solution for the above problem is $v(t)=(t^{2}-1)\cos t$. The proposed method is tested on this problem where the domain [–1,1] is divided into 10 equal subintervals. The obtained numerical results for this problem are given in Table 2. The maximum absolute error obtained by the proposed method is 5.722046×10^{-06}.

Table1: Numerical results for Example1

t	Exact solution	Absolute error by Proposed method
0.1	9.946539E-02	2.756715E-07
0.2	1.954244E-01	8.493662E-07
0.3	2.834704E-01	1.788139E-06
0.4	3.580379E-01	3.457069E-06
0.5	4.121803E-01	4.440546E-06
0.6	4.373085E-01	4.917383E-06
0.7	4.228881E-01	5.036592E-06
0.8	3.560865E-01	4.023314E-06
0.9	2.213642E-01	2.145767E-06

Table 2: Numerical results for Example 2

t	Exact solution	Absolute error by proposed method
-0.8	-2.508144E-01	9.834766E-07
-0.6	-5.282148E-01	9.536743E-07
-0.4	-7.736912E-01	2.920628E-06
-0.2	-9.408639E-01	5.722046E-06
0.0	-1.000000	4.291534E-06
0.2	-9.408639E-01	4.768372E-07
0.4	-7.736912E-01	7.748604E-07
0.6	-5.282148E-01	1.192093E-06
0.8	-2.508144E-01	9.536743E-07

Example 3: Consider the nonlinear boundary value problem

$$v^{(7)} = 6!\left[e^{-7v} - \frac{2}{(1+t)^7}\right], \qquad 0 < t < e^{\frac{1}{2}} - 1 \qquad (28)$$

subject to $v(0)=0, v(e^{\frac{1}{2}}-1)=\frac{1}{2}, v'(0)=1, v'(e^{\frac{1}{2}}-1)=e^{-\frac{1}{2}}, v''(0)=-1, v''(e^{\frac{1}{2}}-1)=\frac{-1}{e}, v'''(0)=2.$

The exact solution for the above problem is $v(t) = lnt$. The nonlinear boundary value problem (28) is converted into a sequence of linear boundary value problems generated by quasilinearization technique [3] as

$$v_{(n+1)}^{(7)} + \left[7!e^{-7v_{(n)}}\right]v_{(n+1)} = \left[7!v_{(n)} + 6!\right]e^{-7v_{(n)}} - \frac{2\times 6!}{(1+t)^7}, \qquad n = 0,1,2,3,.... \qquad (29)$$

subject to
$$v_{(n+1)}(0)=0, v_{(n+1)}(e^{\frac{1}{2}}-1)=\frac{1}{2}, v'_{(n+1)}(0)=1, v'_{(n+1)}(e^{\frac{1}{2}}-1)=e^{-\frac{1}{2}}, v''_{(n+1)}(0)=-1,$$
$$v''_{(n+1)}(e^{\frac{1}{2}}-1)=\frac{-1}{e}, v'''_{(n+1)}(0)=2.$$

Here $v_{(n+1)}$ is the $(n+1)^{th}$ approximation for $v(t)$.The domain $[0, e^{\frac{1}{2}}-1]$ is divided into10 equal subintervals and the proposed method is applied to the sequence of a linear problems (29).The obtained numerical results for this problem are presented in Table 3.The maximum absolute error obtained by the proposed method is 2.503395×10^{-5}.

Example 4: Consider the nonlinear boundary value problem

$$v^{(7)} - e^{-t}v^2 = -e^{-t} - e^{-3t}, \qquad 0 < t < 1 \qquad (30)$$

subject to $v(0) = 1, v(1) = \frac{1}{e}, v'(0) = -1, v'(1) = \frac{-1}{e}, v''(0) = 1, v''(1) = \frac{1}{e}, v'''(0) = -1$

The exact solution for the above problem is $v(t) = e^{-t}$.The nonlinear boundary value problem (30) is converted into a sequence of linear boundary value problems generated by quasilinearization technique [3] as

$$v^{7}_{(n+1)} - \left[2v_n e^{-t}\right] v_{(n+1)} = -v^{2}_{n} e^{-t} - e^{-t} - e^{-3t}, \qquad n = 0,1,2,3\ldots\ldots \qquad (31)$$

subject to

$v_{(n+1)}(0) = 1, v_{(n+1)}(1) = \frac{1}{e}, v'_{(n+1)}(0) = -1, v'_{(n+1)}(1) = \frac{-1}{e}, v''_{(n+1)}(0) = 1, v''_{(n+1)}(1) = \frac{1}{e}, v'''_{(n+1)}(0) = -1$.

Here $v_{(n+1)}$ is the $(n + 1)^{th}$ approximation for $v(t)$. The domain [0,1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of a linear problems (31).The obtained numerical results for this problem are presented in Table 4.The maximum absolute error obtained by the proposed method is 8.404255×10^{-06}.

Table 3: Numerical results for Example 3

t	Exact solution	Absolute error by proposed method
6.487213E-02	6.285473E-02	5.960464E-08
1.297443E-01	1.219913E-01	2.264977E-06
1.946164E-01	1.778251E-01	8.061528E-06
2.594885E-01	2.307057E-01	1.654029E-05
3.243607E-01	2.809298E-01	2.306700E-05
3.892328E-01	3.287517E-01	2.503395E-05
4.541049E-01	3.743905E-01	2.178550E-05
5.189770E-01	4.180371E-01	1.126528E-05
5.838492E-01	4.598581E-01	3.725290E-06

Table 4: Numerical results for Example 4

t	Exact solution	Absolute error by proposed method
0.1	9.048374E-01	7.152557E-07
0.2	8.187308E-01	2.205372E-06

0.3	7.408182E-01	4.529953E-06
0.4	6.703200E-01	7.390976E-06
0.5	6.065307E-01	8.404255E-06
0.6	5.488116E-01	7.152557E-06
0.7	4.965853E-01	4.827976E-06
0.8	4.493290E-01	1.847744E-06
0.9	4.065697E-01	1.788139E-07

6. Conclusions

In this paper, we have deployed a Galerkin method with quintic B-splines as basis functions to solve a general seventh order boundary value problem with boundary conditions. The quintic B-spline basis set has been redefined into a new set of basis functions which vanish on the boundary where the Dirichlet and Neumann, secondary boundary conditions are prescribed. The proposed method has been tested on two linear and two nonlinear seventh order boundary value problems. The numerical results obtained by the proposed method are in good agreement with the exact solutions available in the literature. The objective of this paper is to present a simple, efficient method to solve sixth order boundary value problems.

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