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A 2-D DIFFUSION-BASED POPULATION GROWTH MODEL USING CUBIC SPLINE NUMERICAL APPROXIMATION

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Abstract

This study presents a 2D diffusion model for population growth using a heat equation like partial differential equation. Population density changes over space and time are solved numerically with an explicit cubic spline interpolation scheme, giving smoother spatial estimates and better stability than basic finite differences. The model is validated with Surat city data using the 2001 census as the initial condition and comparing predictions to 2011 figures. Evaluation with RMSE and MAPE shows reasonable agreement and the framework can be extended to include spatially varying growth, migration and reaction diffusion effects.

Keywords: Mathematical Modelling, Population Growth, 2-D Heat Equation, 2-D region, Spline Interpolation Method, Numerical Approach.

1. Introduction

Population growth and how people move within cities are very important for urban planning, building infrastructure and using resources wisely. Fast urbanization in developing countries has made population distribution more uneven because of migration, industrial growth and economic differences. Because of this, accurate models that show how population changes in space and time are essential for making good policies and planning cities for the future.

Mathematical models are often used to study population change. Simple ordinary differential equation models like exponential and logistic growth help explain how population changes over time, but they do not show spatial differences. To model how people move across areas, we need spatial models that let population density change by place and time using partial differential equations.

Diffusion-based models, which are like the heat equation are a natural way to describe how population spreads. These models assume people move from crowded areas to less crowded ones, similar to heat flowing from hot to cold. Such models have been used in ecology and urban studies to capture overall spatial patterns.

Solving 2-D population PDEs poses significant computational challenges. Common methods like finite difference and finite element can cause numerical diffusion, produce fewer smooth results or need strict stability conditions. These problems can make it hard to capture sharp spatial changes in population.

Spline-based methods give smoother and more continuous spatial approximations, so they are a good alternative for diffusion problems. In this study, a 2-D diffusion model for population growth is solved using cubic spline interpolation. The model is tested with population data from parts of Surat city, India, and checked against census records. The main contribution is

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showing that combining a diffusion model with a cubic spline numerical method works well for predicting urban population distribution. [1][2][3][4].

Literature Review

Early mathematical studies of population dynamics primarily focused on temporal growth using ordinary differential equations. The Malthusian exponential model and the logistic growth model established foundational principles for population analysis but lacked spatial resolution. Consequently, these models were insufficient for describing spatial population redistribution.

The inclusion of spatial effects led to the development of partial differential equation based population models. Pioneering work by Fisher and by Kolmogorov, Petrovsky, and Piskunov introduced reaction–diffusion equations to model population spread forming the basis of modern spatial population dynamics. These models demonstrated how diffusion processes could generate spatial patterns and traveling waves in biological and demographic systems. [5][6]

Subsequent research expanded these models to include nonlinear reaction terms, advection components and heterogeneous environments. Reaction–diffusion population models have been extensively applied in ecology, epidemiology and urban studies to account for birth & death processes, carrying capacity and migration. Such models have proven effective in capturing complex spatial temporal population behaviour. [7][8][9]

From a numerical point of view, finite difference and finite element methods have been the dominant approaches for solving population PDEs. Although these methods are relatively simple to implement they often require fine spatial grids to achieve acceptable accuracy, increasing computational cost. Furthermore, numerical diffusion and limited smoothness can adversely affect solution quality, particularly in 2-D regions.

Spline-based numerical methods have gained attention due to their superior smoothness and approximation properties. Cubic spline interpolation, in particular offers continuous second-order derivatives and improved stability characteristics. Previous studies have demonstrated the effectiveness of spline methods in solving diffusion and boundary-value problems. However, their application to two-dimensional population diffusion models remains limited with most existing work focusing on one-dimensional problems or traditional discretization techniques.

This gap in the literature motivates the present study, which applies cubic spline interpolation to a 2-D diffusion-based population growth model and validates the approach using real census data.

Scope of Research:

This research focuses on formulating, numerically solving and validating a 2-D diffusion model for urban population growth. The goal is to predict how population is distributed in space over time using a reliable and efficient computational approach.

The work covers:

- Developing a 2-D population model governed by a diffusion type partial differential equation.
- Building an explicit numerical scheme based on cubic spline interpolation to solve the equation.
- Analyzing stability, convergence and computational cost of the method.
- Applying the model to real data from subregions of Surat city, India.
- Validating predictions quantitatively using error metrics such as RMSE and MAPE.

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The current study is limited to diffusion driven population dynamics with a constant growth rate and does not include migration, socio economic factors or nonlinear reaction effects. However, the framework can be extended in the future to allow growth rates that vary across space include migration using advection terms and use reaction diffusion models to represent births, deaths and carrying capacity limits.

2. Proposed Methodology:

The system architecture of proposed mathematical model for prediction of population growth in 2-D region over a period of time is as shown in figure 1.

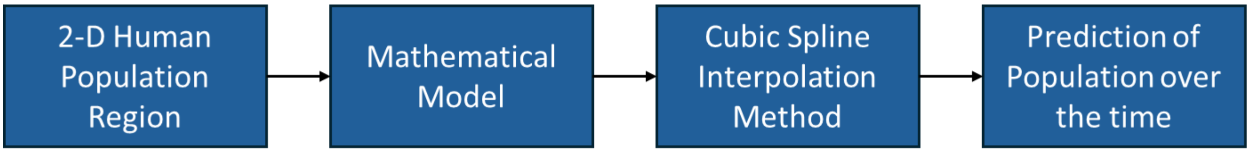

Figure 1: Proposed Methodology

2-D Region:

P1	**P2**	**P3**	**P4**	**P5**	**P6**
P7	**P8**	**P9**	**P10**	**P11**	**P12**
P13	**P14**	**P15**	**P16**	**P17**	**P18**
P19	**P20**	**P21**	**P22**	**P23**	**P24**
P25	**P26**	**P27**	**P28**	**P29**	**P30**
P31	**P32**	**P33**	**P34**	**P35**	**P36**
P37	**P38**	**P39**	**P40**	**P41**	**P42**

The proposed method is performed on 2-D region having sublocation P1,P2,…P42, are consider the subregion of some area of Surat(Gujarat, India), Also we consider that initial some population is living all subregion of Surat(Gujarat, India) for all P_i where i = 1,2,….,42. The growth rate is also be applied to 2-D region of all subregion over a specific time interval.

2.1 Spline Interpolation:

Spline interpolation is a powerful numerical technique for approximating functions with enhanced smoothness. In diffusion-dominated problems such as population spread modeling, smooth spatial approximations are essential to accurately capture gradual variations in population density. Cubic spline interpolation ensures continuity of the function and its first two derivatives, providing higher accuracy than conventional finite difference approximations.

In this study, spline interpolation is applied to discretize the spatial derivatives appearing in the governing population diffusion equation. This approach leads to a structured algebraic system that can be solved efficiently while maintaining numerical stability.

2.1.1 Cubic Splines:

Let the spatial domain be discretized using a uniform grid with spacing h. The population density is approximated using a cubic spline function along each spatial direction. The spline formulation enforces continuity of the population density and its first and second derivatives at the grid points, resulting in a smooth representation of spatial population distribution. The population density at the grid point x_i is denoted by p_i. In the cubic spline formulation, the second derivative of the spline approximation at x_i is represented by

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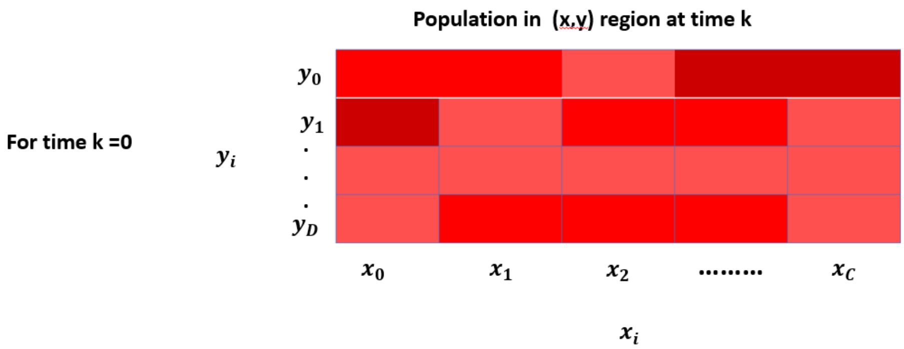
$$S''(x_i) = M_i \tag{1}$$

The cubic spline discretization yields a tridiagonal system of equations for the spline second derivatives, which can be expressed in the general form

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}(p_{i+1} - 2p_i + p_{i-1}) \; (i = 1, 2, \ldots, n-1). \tag{2}$$

where p_i denotes the discrete population density at the spatial grid point x_i, and $M_i = \frac{\partial^2 p}{\partial x^2}\Big|_{x=x_i}$ represents the second derivative of the cubic spline approximation evaluated at x_i. The grid spacing is uniform and denoted by h.

2.1.2 Population growth in a region based on 2-D heat equation with Cubic spline interpolation



The spatial–temporal evolution of population density is modeled using a two-dimensional diffusion equation analogous to the heat equation:

$$\frac{\partial p}{\partial t} = m\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right), \tag{3}$$

where m is the population diffusion coefficient.

Using cubic spline interpolation, the second-order spatial derivatives are approximated by the spline curvature terms:

$$\frac{\partial^2 p}{\partial x^2} \approx M^x_{i,j}, \frac{\partial^2 p}{\partial y^2} \approx M^y_{i,j} \tag{4}$$

Applying a forward difference approximation in time and substituting the spline-based spatial derivatives and considering the region $\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial y^2}$, the governing equation becomes

$$\frac{p_{i,j,k+1} - p_{i,j,k}}{\Delta t} = m\left(2\frac{\partial^2 p}{\partial x^2}\right) \tag{5}$$

Now with help of (2) and the value of (5) obtained from (3) we get,

$$\frac{p_{i-1,j,k+1} - p_{i-1,j,k}}{2m\Delta t} + \frac{p_{i,j,k+1} - p_{i,j,k}}{2m\Delta t} + \frac{p_{i+1,j,k+1} - p_{i+1,j,k}}{2m\Delta t} = \frac{6}{h^2}(p_{i-1,j,k} - 2p_{i,j,k} + p_{i+1,j,k}) \tag{6}$$

After simplifying we get,

$$p_{i-1,j,k+1} + 4p_{i,j,k+1} + p_{i+1,j,k+1} = (1 + 12\alpha)p_{i-1,j,k} + (4 - 24\alpha)p_{i,j,k} + (1 + 12\alpha)p_{i+1,j,k} \quad for\ i,j = 1,2,,\ldots..n-1 \tag{7}$$

Where $\alpha = \frac{m\Delta t}{h^2}$ these set of $(n-1) \times (n-1)$ equation in $(n-1) \times (n-1)$ unknowns can be solved by any well-known method. The above set of simultaneous equations gives square matrix. The (7) is known as cubic spline explicit formula to solve the (3).

The growth rate m is constant over a period of time.

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$p(x, y, t_0) = f(x, y)$, where $t_0 = 0$.
$p(x_0, y, t) = f(y, t)$ at any time t.
$p(x_n, y, t) = f(y, t)$, at any time t.
Now considering the explicit formula,

$$p_{i-1,j,k+1} + 4p_{i,j,k+1} + p_{i+1,j,k+1} = (1 + 12\alpha)p_{i-1,j,k} + (4 - 24\alpha)p_{i,j,k} + (1 + 12\alpha)p_{i+1,j,k}$$

For k^{th} level, where initial at k =0,
for j = 1 and i = 1 we get

$$p_{0,1,1} + 4p_{1,1,1} + p_{2,1,1} = (1 + 12\alpha)p_{0,1,0} + (4 - 24\alpha)p_{1,1,0} + (1 + 12\alpha)p_{2,1,0}$$

for j = 1 and i = 2, we get

$$p_{1,1,1} + 4p_{2,1,1} + p_{3,1,1} = (1 + 12\alpha)p_{1,1,0} + (4 - 24\alpha)p_{2,1,0} + (1 + 12\alpha)p_{3,1,0}$$

for j = 1 and i = 3, we get

$$p_{2,1,1} + 4p_{3,1,1} + p_{4,1,1} = (1 + 12\alpha)p_{2,1,0} + (4 - 24\alpha)p_{3,1,0} + (1 + 12\alpha)p_{4,1,0}$$

for j = 1 and i = 4, we get

$$p_{3,1,1} + 4p_{4,1,1} + p_{5,1,1} = (1 + 12\alpha)p_{3,1,0} + (4 - 24\alpha)p_{4,1,0} + (1 + 12\alpha)p_{5,1,0}$$

Similar for j = 2 and for i = 1 to 4 we get other equations. Also, for j = 3 and for i = 1 to 4 we get the other remaining equations.

For $k = 0$	$x = 0$	1	2	3	4	5
$y = 1$	$p_{0,1}$	$p_{1,1}$	$p_{2,1}$	$p_{3,1}$	$P_{4,1}$	$P_{5,1}$
2	$p_{0,2}$	$p_{1,2}$	$p_{2,2}$	$p_{3,2}$	$p_{4,2}$	$P_{5,2}$
3	$p_{0,3}$	$p_{1,3}$	$p_{2,3}$	$p_{3,3}$	$p_{4,3}$	$P_{5,3}$
4	$p_{0,4}$	$p_{1,4}$	$p_{2,4}$	$p_{3,4}$	$p_{4,4}$	$P_{5,4}$
5	$p_{0,5}$	$p_{1,5}$	$p_{2,5}$	$p_{3,5}$	$p_{4,5}$	$P_{5,5}$
6	$p_{0,6}$	$p_{1,6}$	$p_{2,6}$	$p_{3,6}$	$p_{4,6}$	$P_{5,6}$
7	$p_{0,7}$	$p_{1,7}$	$p_{2,7}$	$p_{3,7}$	$p_{4,7}$	$P_{5,7}$

Here x = 0 and x = 5 columns are known values from initial and boundary condition.
For k = 1 level.

y	Coefficient Matrix [A]	Constant matrix [B]
1	$\begin{bmatrix} 4p_{1,1} & p_{2,1} & 0 & 0 \\ p_{1,1} & 4p_{2,1} & p_{3,1} & 0 \\ 0 & p_{2,1} & 4p_{3,1} & p_{4,1} \\ 0 & 0 & p_{3,1} & 4p_{4,1} \end{bmatrix}$	$\begin{bmatrix} (1 + 12\alpha))p_{0,1,0} + (4 - 24\alpha)p_{1,1,0} + (1 + 12\alpha)P_{2,1,0} - p_0 \\ (1 + 12\alpha)p_{1,1,0} + (4 - 24\alpha)p_{2,1,0} + (1 + 12\alpha)p_{3,1,0} \\ (1 + 12\alpha)p_{2,1,0} + (4 - 24\alpha)p_{3,1,0} + (1 + 12\alpha)p_{4,1,0} \\ (1 + 12\alpha)p_{3,1,0} + (4 - 24\alpha)p_{4,1,0} + (1 + 12\alpha)p_{5,1,0} - p_{5,} \end{bmatrix}$
2	$\begin{bmatrix} 4p_{1,2} & p_{2,2} & 0 & 0 \\ p_{1,2} & 4p_{2,2} & p_{3,2} & 0 \\ 0 & p_{2,2} & 4p_{3,2} & p_{4,2} \\ 0 & 0 & p_{3,2} & 4p_{4,2} \end{bmatrix}$	$\begin{bmatrix} (1 + 12\alpha)p_{0,2,0} + (4 - 24\alpha)p_{1,2,0} + (1 + 12\alpha)P_{2,2,0} - p_{0,2} \\ (1 + 12\alpha)p_{1,2,0} + (4 - 24\alpha)p_{2,2,0} + (1 + 12\alpha)p_{3,2,0} \\ (1 + 12\alpha)p_{2,2,0} + (4 - 24\alpha)p_{3,2,0} + (1 + 12\alpha)p_{4,2,0} \\ (1 + 12\alpha)p_{3,2,0} + (4 - 24\alpha)p_{4,2,0} + (1 + 12\alpha)p_{5,2,0} - p_{5,2} \end{bmatrix}$
3	$\begin{bmatrix} 4p_{1,3} & p_{2,3} & 0 & 0 \\ p_{1,3} & 4p_{2,3} & p_{3,3} & 0 \\ 0 & p_{2,3} & 4p_{3,3} & p_{4,3} \\ 0 & 0 & p_{3,3} & 4p_{4,3} \end{bmatrix}$	$\begin{bmatrix} (1 + 12\alpha)p_{0,3,0} + (4 - 24\alpha)p_{1,3,0} + (1 + 12\alpha)P_{2,3,0} - p_{0,3} \\ (1 + 12\alpha)p_{1,3,0} + (4 - 24\alpha)p_{2,3,0} + (1 + 12\alpha)p_{3,3,0} \\ (1 + 12\alpha)p_{2,3,0} + (4 - 24\alpha)p_{3,3,0} + (1 + 12\alpha)p_{4,3,0} \\ (1 + 12\alpha)p_{3,3,0} + (4 - 24\alpha)p_{4,3,0} + (1 + 12\alpha)p_{5,3,0} - p_{5,3} \end{bmatrix}$

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4	$\begin{bmatrix} 4p_{1,4} & p_{2,4} & 0 & 0 \\ p_{1,4} & 4p_{2,4} & p_{3,4} & 0 \\ 0 & p_{2,4} & 4p_{3,4} & p_{4,4} \\ 0 & 0 & p_{3,4} & 4p_{4,4} \end{bmatrix}$	$\begin{bmatrix} (1+12\alpha)p_{0,4,0}+(4-24\alpha)p_{1,4,0}+(1+12\alpha)P_{2,4,0}-p_{0,} \\ (1+12\alpha)p_{1,4,0}+(4-24\alpha)p_{2,4,0}+(1+12\alpha)p_{3,4,0} \\ (1+12\alpha)p_{2,4,0}+(4-24\alpha)p_{3,4,0}+(1+12\alpha)p_{4,4,0} \\ (1+12\alpha)p_{3,4,0}+(4-24\alpha)p_{4,4,0}+(1+12\alpha)p_{5,4,0}-p_{5,} \end{bmatrix}$
5	$\begin{bmatrix} 4p_{1,5} & p_{2,5} & 0 & 0 \\ p_{1,5} & 4p_{2,5} & p_{3,5} & 0 \\ 0 & p_{2,5} & 4p_{3,5} & p_{4,5} \\ 0 & 0 & p_{3,5} & 4p_{4,5} \end{bmatrix}$	$\begin{bmatrix} (1+12\alpha)p_{0,5,0}+(4-24\alpha)p_{1,5,0}+(1+12\alpha)P_{2,5,0}-p_{0,} \\ (1+12\alpha)p_{1,5,0}+(4-24\alpha)p_{2,5,0}+(1+12\alpha)p_{3,5,0} \\ (1+12\alpha)p_{2,5,0}+(4-24\alpha)p_{3,5,0}+(1+12\alpha)p_{4,5,0} \\ (1+12\alpha)p_{3,5,0}+(4-24\alpha)p_{4,5,0}+(1+12\alpha)p_{5,5,0}-p_{5,} \end{bmatrix}$
6	$\begin{bmatrix} 4p_{1,6} & p_{2,6} & 0 & 0 \\ p_{1,6} & 4p_{2,6} & p_{3,6} & 0 \\ 0 & p_{2,6} & 4p_{3,6} & p_{4,6} \\ 0 & 0 & p_{3,6} & 4p_{4,6} \end{bmatrix}$	$\begin{bmatrix} (1+12\alpha)p_{0,6,0}+(4-24\alpha)p_{1,6,0}+(1+12\alpha)P_{2,6,0}-p_{0,} \\ (1+12\alpha)p_{1,6,0}+(4-24\alpha)p_{2,6,0}+(1+12\alpha)p_{3,6,0} \\ (1+12\alpha)p_{2,6,0}+(4-24\alpha)p_{3,6,0}+(1+12\alpha)p_{4,6,0} \\ (1+12\alpha)p_{3,6,0}+(4-24\alpha)p_{4,6,0}+(1+12\alpha)p_{5,6,0}-p_{5,} \end{bmatrix}$
7	$\begin{bmatrix} 4p_{1,7} & p_{2,7} & 0 & 0 \\ p_{1,7} & 4p_{2,7} & p_{3,7} & 0 \\ 0 & p_{2,7} & 4p_{3,7} & p_{4,7} \\ 0 & 0 & p_{3,7} & 4p_{4,7} \end{bmatrix}$	$\begin{bmatrix} (1+12\alpha)p_{0,7,0}+(4-24\alpha)p_{1,7,0}+(1+12\alpha)P_{2,7,0}-p_{0,} \\ (1+12\alpha)p_{1,7,0}+(4-24\alpha)p_{2,7,0}+(1+12\alpha)p_{3,7,0} \\ (1+12\alpha)p_{2,7,0}+(4-24\alpha)p_{3,7,0}+(1+12\alpha)p_{4,7,0} \\ (1+12\alpha)p_{3,7,0}+(4-24\alpha)p_{4,7,0}+(1+12\alpha)p_{5,7,0}-p_{5,} \end{bmatrix}$

To find the population the at k = 1 we use $p_{i,j} = A^{-1}.B\ for\ each\ row\ y.$

2.1.3 Numerical Stability Analysis and Convergence:

The explicit cubic spline scheme introduces the parameter $\alpha = \frac{m\Delta t}{h^2}$, where σ represents the diffusion coefficient (growth parameter), Δt is the time step, and h is the spatial grid size.

Stability Condition:

For explicit diffusion-type schemes, numerical stability requires that the time step be sufficiently small relative to the spatial discretization. For the proposed cubic spline explicit formulation, stability is ensured when $\alpha \leq \frac{1}{12}$.

If this condition is violated, numerical oscillations and divergence may occur. In the present study, Δt and h are chosen to satisfy this restriction, ensuring stable numerical evolution.

Convergence Discussion:

The cubic spline interpolation provides fourth-order spatial accuracy under uniform grid spacing, while the temporal discretization using a forward difference scheme yields first-order accuracy in time. As both $\Delta t \rightarrow 0$ and $h \rightarrow 0$, the numerical solution converges to the exact solution of the governing PDE, provided the stability condition is satisfied. The smoothness of spline functions ensures convergence with reduced numerical diffusion compared to standard finite difference methods.

Computational Complexity:

At each time step, the numerical scheme requires solving a tridiagonal linear system for each row of the spatial grid. Using the Thomas algorithm, each system can be solved in $O(N)$ time, where N is the number of interior grid points in one spatial direction. For an $N \times N$ grid, the total computational complexity per time step is $O(N^2)$. This makes the method computationally efficient and scalable for moderate grid resolutions.

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3. Experimental results:

To simulate the proposed population growth model over 2-D region researcher has taken a portion of 2-D region of Surat city (Gujarat, India) classified into 2-D sub regions as shown in Table 1 and 2.

Researcher has considerations the nature of initial and boundary condition with respect time & space and the value of population growth rate m. [21] [22]

Table 1: Area of Surat

Ved	Katargam	Amroli & Utran	Fulpada	Kapodara	Puna
Dambholi & Singanpore	Katargam	Katargam	Navagam	Karanj	Puna & Saroli
Rander	Rander & Tunki	Old city	Old city	Umarwada	Dumbhal & Magob
Adajan	Adajan	Old city	Old city	Anajana	Limbayat
Pal & Adajan	Adajan	Athwa	khatodara	Udhana	Udhana
Umara	Umara & Athwa	Bhatar	Majura	Udhana	Bedvad & Dindoli
Piplod & Rundh	Vesu	Bharthana & Althan	Althan & Bhimrad	Bamroli	Pandesara

Table 2: Area wise Population of Surat in 2001

9540	80825	112785	159226	99011	70886
15771	28832	86796	93191	310061	80688
55022	47891	132571	132571	91237	100885
54713	43668	132571	132571	122683	187475
28850	43668	43090	61190	66225	77861
19616	34189	44363	43111	79416	52155
16024	21332	20757	30602	55096	25253

Table 2 shows the initial population of 2-D subregion of the Surat city (Gujarat, India) in 2001.

Furthermore, the growth rate of Surat city in the year 2001 taken as 5.95%. and initial and boundary condition with respect and time and space are derived on the past data of Surat subregion from 1961 to 2001 by using multi polynomial regression method are defined as below. Such that the population growth model is designed as equation eq (3.1)

$$\frac{\partial p}{\partial t} = 0.0595\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) \; where \; 0 < x < 6, 0 < y < 7, t \geq 0 \tag{8}$$

where $\Delta x = 1, \Delta y = 1 \; and \; \Delta t = 1$.

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$$P(0,y,t) = 6705.32t + 4670.95t^2 + 15519.18y + 27110.65yt + 1304.20yt^2 + 24408.14y^2 + 7409.5y^2t + 101.55y^2t^2 + 7709.28y^3 + 538.92y^3t + 7467.41 + 681.97y^4$$

$$P(6,y,t)) = 30757.65t + 33337.17t^2 + 73765.48y + 22936.4yt + 11842.93yt^2 + 71241.08y^2 + 17699.81y^2t + 6480.15y^2t^2 + 8354.76y^3 + 4224.43y^3t + 650.08y^3t^2 + 2382.15y^4 + 298.35y^4t + 11014.16 + 356.440892y^5$$

Here $\alpha = 0.0595 \leq \frac{1}{12}$, Hence Stability criteria is satisfied

The proposed population growth model is solved by cubic Spline method and obtained result is tabulated as in table 3

Table 3: Predicted Population if subregion of Surat in year 2011

50272	76346	116460	135150	118040	198700
77553	46639	69043	156650	212000	253440
79345	82030	110000	121690	130870	274060
56779	78640	104850	136940	161070	207240
27352	42506	47087	61191	72823	85106
24927	36195	40314	56193	67000	68028
99737	33320	24483	19662	107860	487400

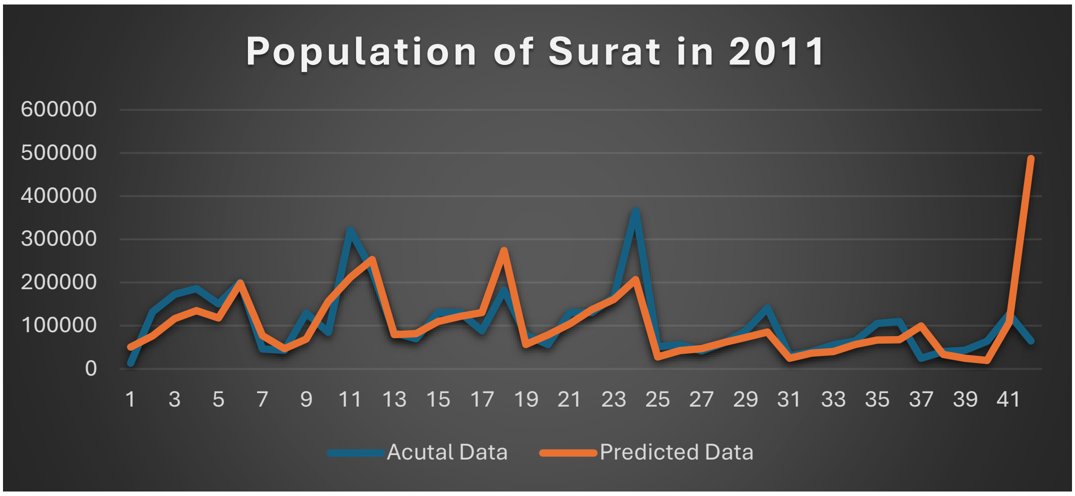

Figure 3: Comparison between Actual Data and Predicted data

Table 3 and figure 3 show that the population growth of subregion in year of 2011 of Surat city. It is observed that in some regions the population is increasing while in some regions population is decreased due to migration effect and other factors.

Error Analysis:

To quantitatively evaluate the predictive performance of the proposed model, Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) are computed by comparing the predicted population values with the actual census data of 2011.

MAPE is computed as: MAPE = $\left(\frac{1}{n}\right) \sum \left|\frac{A_i - P_i}{A_i}\right| \times 100$

RMSE is computed as: RMSE = $\sqrt{\left(\frac{1}{n}\right) \sum (A_i - P_i)^2}$

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where P_i and A_i denote the predicted and actual population values of the i^{th} subregion respectively, and n is the total number of subregions. For the Surat city case study, the computed values are:
RMSE $= 76{,}207.99$ and MAPE $= 54.36\%$.

The results show the model captures the general spatial trend of population growth but notable deviations occur in some subregions because of migration, industrial growth and the assumption of a constant diffusion coefficient. Adding migration and spatially varying growth rates should reduce prediction errors. The relatively high MAPE is likely due to simplifying assumptions such as a constant growth rate and the lack of explicit migration and socio-economic factors, while rapid urbanization and industrial development in parts of Surat produce uneven growth that a pure diffusion model cannot fully represent. Some areas show anomalously high predicted values, which may result from migration inflows, industrial or infrastructure expansion or the limits of a constant diffusion assumption, so future work should include migration terms and spatially varying parameters to address these issues.

4. Conclusion

A 2-D diffusion-based population growth model has been developed and numerically solved using a cubic spline interpolation scheme. The proposed method demonstrates stable, smooth, and accurate population prediction when applied to real census data. Error analysis confirms the effectiveness of the approach while identified limitations suggest promising directions for future research involving migration and reaction–diffusion dynamics.

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