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MATHEMATICAL MODELING AND OPTIMIZATION OF CIRCULAR ECONOMY PROCESSES

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Abstract

The transition from linear production and consumption systems to circular economy processes requires rigorous quantitative tools to support sustainable decision-making. This study proposes a unified mathematical modeling and multi-objective optimization framework to improve economic performance, environmental impact, and material circularity in circular economy systems. The model explicitly represents forward and reverse material flows and incorporates recovery, capacity, and environmental constraints within an integrated optimization structure. A multi-objective solution strategy is applied to balance total operational cost, emissions, and recovery levels. The applicability of the framework is demonstrated through a numerical case study inspired by waste management and material recovery systems. Results indicate that the optimized circular configuration achieves a recovery level of 68 % of total demand, leading to a reduction of carbon emissions from 30,600 kilograms to 21,360 kilograms, while total system cost increases from 232,000 United States dollars to 248,400 United States dollars. Compared to a linear baseline scenario, disposal volumes are reduced by 68 %, highlighting the effectiveness of recovery-oriented decision-making. The findings confirm that integrating circularity as a primary optimization objective enables decision-makers to identify balanced solutions that support long-term sustainability goals while maintaining economic feasibility. The proposed framework offers a generalizable and scalable approach that can be adapted to diverse circular economy applications, providing valuable insights for managers and policymakers seeking to operationalize circular economy strategies.

Keywords: Circular economy; Mathematical modeling; Multi-objective optimization; Closed-loop systems; Sustainability

1. Introduction

The increasing pressure on natural resources, rising waste generation, and intensifying environmental degradation have exposed the structural inefficiencies of traditional linear economic systems. In response, the circular economy (CE) has emerged as a promising paradigm aimed at decoupling economic growth from resource consumption by promoting reuse, recycling, remanufacturing, and closed material loops. Rather than treating waste as an end product, CE frameworks emphasize value retention across product life cycles, thereby enhancing resource efficiency and environmental sustainability. However, despite its conceptual appeal, the operationalization of circular economy

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principles remains a complex challenge, particularly in systems involving multiple stakeholders, uncertain flows, and competing economic and environmental objectives (Lozano-Oviedo et al., 2024).

A critical aspect of implementing circular economy strategies lies in effective decision-making across interconnected processes such as production, recovery, recycling, and disposal. These processes are inherently interdependent and constrained by limited capacities, fluctuating demand, recovery efficiencies, and regulatory requirements. As such, intuitive or purely qualitative approaches are insufficient for designing and managing circular systems at scale. Mathematical modeling and optimization provide a rigorous foundation for analyzing these complexities, enabling decision-makers to evaluate trade-offs, test scenarios, and identify optimal configurations of circular processes (Morgan, 2024). Quantitative models allow the explicit representation of material flows, costs, and environmental impacts, making them indispensable tools for advancing CE from conceptual frameworks to actionable systems.

Recent research has increasingly focused on closed-loop supply chains as a key structural embodiment of the circular economy. These systems integrate forward and reverse logistics to recover value from end-of-life products while meeting customer demand. Nevertheless, existing studies often address specific components of circular systems in isolation, such as recycling network design or remanufacturing planning, rather than offering unified mathematical formulations that capture the full circular flow structure (Lozano-Oviedo et al., 2024). Moreover, many models rely on single-objective formulations, typically minimizing cost, while treating environmental or circularity considerations as secondary constraints. This limits their ability to reflect the inherently multi-objective nature of circular economy decision-making.

Another important limitation in the current literature relates to the treatment of uncertainty and system dynamics. Circular economy processes are subject to uncertain demand, variable return rates, and fluctuating recovery efficiencies, especially in post-disruption contexts such as the COVID-19 era. Although some studies have begun incorporating uncertainty into closed-loop supply chain models, these efforts remain fragmented and are often confined to specific case settings without generalized modeling structures (Abbasi et al., 2025). As a result, there is a lack of broadly applicable mathematical frameworks that can systematically optimize circular flows under realistic operational conditions.

In parallel, advances in optimization techniques and data-driven decision support systems have opened new opportunities for improving circular economy performance. The integration of multi-objective optimization enables simultaneous consideration of economic efficiency, environmental impact, and material recovery, which aligns closely with the core principles of CE. Emerging research highlights the potential of intelligent optimization frameworks, supported by analytical and computational tools, to enhance green decision-making in complex circular systems (Gong, 2025). The full potential of these approaches has yet to be realized due to insufficient emphasis on transparent, equation-based modeling structures that can be adapted across sectors.

Furthermore, industrial symbiosis and process integration have been identified as key enablers of circular economy implementation, particularly in manufacturing and process industries. Optimization-based models play a crucial role in identifying synergistic exchanges of materials and energy among interconnected processes, thereby reducing waste and improving system-level efficiency (Misrol et al., 2023). Despite this recognition, there remains a methodological gap between high-level CE concepts and mathematically rigorous optimization models that can guide real-world implementation decisions.

Motivated by these gaps, this study focuses on the development of a unified mathematical modeling and optimization framework for circular economy processes. The proposed approach emphasizes

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explicit formulation of material flows, recovery mechanisms, and sustainability-oriented objectives within a multi-objective optimization setting. By doing so, the study aims to contribute both theoretically and practically to the growing body of circular economy research, offering a structured and scalable modeling framework capable of supporting informed decision-making.

Research Objectives

1. To develop a unified mathematical model that explicitly represents circular economy processes through equation-based formulations of material flows, recovery, and capacity constraints
2. To design and apply a multi-objective optimization framework that simultaneously balances economic efficiency, environmental performance, and circularity outcomes

2. Literature Review and Research Positioning

The growing adoption of circular economy (CE) principles has stimulated extensive research on modeling and optimizing circular processes, particularly within supply chain and industrial systems. Existing studies largely conceptualize circular economy implementation through closed-loop supply chains, industrial symbiosis, and resource recovery networks, where material flows are redirected from disposal toward reuse and recycling. Quantitative modeling has been increasingly employed to support such transitions, enabling systematic evaluation of circular strategies under operational and sustainability constraints (Aldás & Mula, 2024).

A significant stream of literature focuses on mathematical optimization models for circular and sustainable supply chains. These models typically aim to integrate forward and reverse logistics decisions, including production, collection, remanufacturing, recycling, and disposal. Multi-objective formulations are particularly prominent, reflecting the need to balance economic and environmental goals. For instance, Poonia et al. (2024) developed a multi-objective fuzzy mathematical model that incorporates leasing as a circular strategy, highlighting the role of uncertainty and flexibility in CE decision-making. Similarly, Elfarouk et al. (2022) proposed a stochastic multi-objective optimization model for multi-echelon closed-loop supply chains, emphasizing trade-offs between cost efficiency and environmental performance.

Objective functions in existing CE optimization models can broadly be classified into three categories: economic, environmental, and sustainability-oriented objectives. Economic objectives typically involve minimizing total system costs, including production, transportation, recovery, and disposal costs. Environmental objectives often focus on minimizing carbon emissions, waste generation, or energy consumption, sometimes driven by regulatory mechanisms such as carbon pricing (Fareeduddin et al., 2015). Sustainability-oriented objectives extend beyond cost and emissions to include resource recovery rates, circularity levels, and long-term system resilience, thereby aligning more closely with CE principles (Morgan, 2024).

Constraints in these models commonly represent material balance, capacity limitations, demand satisfaction, and recovery feasibility. In more advanced formulations, uncertainty-related constraints have been introduced to capture variability in demand, return rates, and processing yields. Robust and stochastic optimization techniques have therefore gained attention in recent years. Jafarzadeh Ghoushchi et al. (2025) proposed a robust multi-objective optimization framework for agile closed-loop supply chains under uncertainty, demonstrating improved system adaptability in circular contexts. Such approaches are often computationally intensive and tailored to specific applications, limiting their generalizability.

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Despite these advances, several methodological gaps persist in the existing literature. First, many studies adopt problem-specific modeling structures that are difficult to extend or adapt across different circular economy domains. While detailed case-oriented models provide valuable insights, they often lack unified, equation-based frameworks that can serve as generic decision-support tools (Shahsavani & Goli, 2025). Second, although multi-objective optimization is widely used, the majority of studies emphasize economic and environmental objectives while treating circularity indicators, such as material recovery or reuse rates, as secondary considerations rather than core objectives.

Another limitation lies in the dominance of solution-focused research over modeling transparency. Metaheuristic algorithms, such as genetic algorithms and other evolutionary techniques, are frequently employed to solve complex CE optimization problems (Ehtesham Rasi & Sohanian, 2021; Faramarzi-Oghani et al., 2023). While these methods are effective for large-scale problems, they often overshadow the underlying mathematical structure of the model, making it difficult to interpret decision mechanisms or replicate results across contexts. This has led to calls for clearer articulation of model formulations and assumptions in CE optimization research.

From an application perspective, CE modeling studies have been conducted across diverse sectors, including e-waste management, construction, and agriculture. Ali (2025) presented a mathematical model for optimizing e-waste management in India, demonstrating the potential of optimization to improve recovery efficiency and environmental outcomes. In the construction sector, Ibe et al. (2025) highlighted the importance of material management strategies for circularity, emphasizing the need for quantitative tools to support implementation. These sector-specific studies underscore the versatility of mathematical modeling in CE but also reveal fragmentation in modeling approaches.

In addition, broader sustainability modeling frameworks, such as nexus-based approaches, have influenced CE optimization research by emphasizing system interdependencies and resource trade-offs. Although not exclusively focused on circular economy, such frameworks reinforce the importance of integrated modeling perspectives for sustainable decision-making (Hamidov & Helming, 2020). Strategic-level frameworks for CE roadmapping further highlight the need for analytical models that can translate high-level circular strategies into operational decisions (Abu-Bakar & Charnley, 2024).

Against this background, the present study positions itself as a response to the identified gaps in circular economy modeling and optimization research. Unlike many existing studies that focus on specific applications or solution techniques, this research emphasizes the development of a unified and transparent mathematical formulation that explicitly represents circular material flows, recovery mechanisms, and capacity constraints. Furthermore, the proposed approach treats circularity as a core optimization objective alongside economic and environmental considerations, rather than as a derived performance metric. By adopting a structured multi-objective optimization framework grounded in explicit equations, this study seeks to bridge the gap between conceptual circular economy principles and rigorous, generalizable decision-support models.

3. System Description and Modeling Assumptions

This study considers a generic circular economy system designed to capture the essential interactions between production, consumption, recovery, and disposal processes. The system integrates both forward and reverse material flows, enabling the transformation of a traditionally linear supply structure into a closed-loop configuration. In the forward flow, raw materials are processed into finished products and delivered to end users to satisfy market demand. In the reverse flow, end-of-life

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or post-consumer products are collected and directed toward recovery options such as recycling, remanufacturing, or final disposal, depending on their quality and feasibility of recovery.

3.1 Description of the Circular Economy System

The circular economy system comprises multiple interconnected entities, including production facilities, recovery centres, recycling units, and disposal sites. Products flow from production facilities to markets through conventional distribution channels, while used products are returned through collection mechanisms into the reverse network. Recovered materials are reintroduced into the production process as secondary inputs, thereby reducing reliance on virgin resources. The system is designed to explicitly represent circularity by linking recovery decisions to production planning, ensuring that material loops are quantitatively embedded within the system structure rather than treated as external processes. The main components of the circular economy system and their modeling representation are summarized in Table 1.

Table 1. Components and Modeling Characteristics of the Circular Economy System

System Component	Function in the System	Modeling Representation
Production facilities	Transform raw or recovered materials into finished products	Forward material flow variables and capacity constraints
Market / consumers	Generate demand and return end-of-life products	Demand parameters and return flows
Collection centers	Aggregate post-consumer products	Reverse flow linkage
Recovery facilities	Recycle or remanufacture returned products	Recovery decision variables and recovery-rate constraints
Disposal sites	Handle non-recoverable waste	Disposal flow variables

3.2 System Boundaries and Decision Levels

The system boundary is defined at the operational level, encompassing all activities directly involved in material transformation and circulation within the circular economy network. Upstream extraction of raw materials and downstream consumption behavior beyond demand fulfillment are excluded to maintain modeling tractability. The decision-making scope includes tactical and operational decisions, such as production quantities, allocation of materials across processing facilities, recovery levels, and disposal quantities over a finite planning horizon. Strategic decisions, such as facility location or long-term technology investment, are not explicitly considered but can be incorporated as extensions of the proposed framework.

3.3 Key Modeling Assumptions

To ensure analytical tractability while preserving practical relevance, several assumptions are adopted. Market demand for products is assumed to be known and deterministic within each planning period. Recovery rates are assumed to be bounded and represent the maximum feasible proportion of

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products that can be recovered through recycling or remanufacturing processes. Processing and recovery capacities at each facility are assumed to be finite and known in advance, reflecting realistic operational constraints. Material flows are assumed to be divisible and continuous, allowing the use of continuous decision variables in the mathematical formulation. Furthermore, all recovered materials are assumed to meet minimum quality standards required for reintegration into the production process.

3.4 Conceptual Framework of Material Loops

The conceptual framework of the proposed system emphasizes the closure of material loops through explicit feedback mechanisms. Products delivered to customers eventually re-enter the system as returns, which are then evaluated for recovery or disposal. Recovered materials are routed back to production facilities, forming a closed-loop structure that directly links reverse flows with forward production decisions. This framework enables the model to capture trade-offs between recovery intensity, production costs, and system capacity utilization, thereby reflecting the core principles of circular economy operations. Figure 1 illustrates the conceptual framework of forward and reverse material flows within the proposed circular economy system.

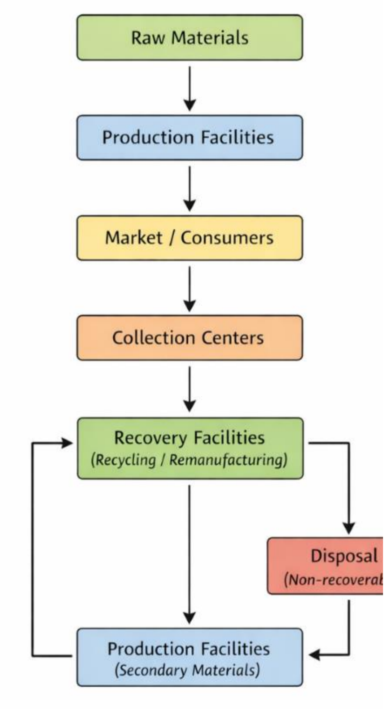

Figure 1. Conceptual Framework of Forward and Reverse Material Flows in the Circular Economy System

This figure illustrates the vertically structured circular economy system integrating forward and reverse material flows. Recovered materials from recycling and remanufacturing processes are reintegrated into production, forming a closed-loop system, while non-recoverable materials are directed to disposal.

3.5 Modeling Scope and Limitations

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While the proposed system framework is designed to be generic and adaptable across sectors, certain limitations are acknowledged. The model does not explicitly account for quality degradation across multiple recovery cycles, nor does it capture behavioral uncertainties in consumer return decisions. Additionally, environmental impacts are represented through aggregate indicators rather than detailed life-cycle assessments. Despite these limitations, the modeling scope is sufficient to analyze fundamental circular economy trade-offs and to support optimization-based decision-making in complex circular systems.

4. Mathematical Model Formulation

This section presents a unified mathematical model for optimizing circular economy processes by explicitly representing forward and reverse material flows within a closed-loop system. The formulation adopts a multi-objective optimization structure to simultaneously address economic efficiency, environmental performance, and circularity enhancement, consistent with quantitative modeling approaches used in circular production and closed-loop supply systems (Vimal et al., 2019).

4.1 Sets, Indices, and Parameters

Let the following sets and indices be defined:

$i \in I$	Set of products or materials
$j \in J$	Set of processing and recovery facilities
$t \in T$	Set of planning periods

Key parameters are defined as:

D_{it}	Demand for product i in period t
C_{ij}	Unit processing cost of product i at facility j
E_{ij}	Environmental impact coefficient of i at j
R_i	Maximum recovery rate of product i
U_{jt}	Capacity of facility j in period t

Decision variables include:

x_{ijt}	Quantity of product i processed at facility j in t
y_{it}	Recovered quantity of product i in t
z_{it}	Disposed quantity of product i in t

4.2 Objective Functions

4.2.1 Economic Cost Minimization

The first objective minimizes the total operational cost associated with production, processing, and recovery activities across the circular system:

$$\min\ Z_1 = \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} C_{ij}\, x_{ijt}$$

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This objective aligns with cost-based optimization structures commonly used in circular production system design (Vimal et al., 2019).

4.2.2 Environmental Impact Minimization

The second objective minimizes the aggregated environmental impact generated by processing and recovery activities:

$$\min\ Z_2 = \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} E_{ij}\, x_{ijt}$$

Environmental minimization objectives are widely incorporated in sustainable supply chain models to capture regulatory and ecological considerations (Gharibi & Abdollahzadeh, 2025).

4.2.3 Circularity Maximization

The third objective maximizes the total quantity of recovered materials reintroduced into the system, thereby enhancing circularity:

$$\max\ Z_3 = \sum_{t \in T} \sum_{i \in I} y_{it}$$

This objective explicitly promotes material recovery as a primary decision criterion, consistent with recent circular economy modeling frameworks (Vimal et al., 2022).

4.3 Model Constraints

4.3.1 Demand Satisfaction Constraint

$$\sum_{j \in J} x_{ijt} = D_{it} \forall i, t$$

This constraint ensures that customer demand is fully satisfied within each planning period.

4.3.2 Material Balance and Recovery Constraints

$$y_{it} + z_{it} = \sum_{j \in J} x_{ijt}\ \forall i, t$$

$$y_{it} \le R_i \sum_{j \in J} x_{ijt}\ \forall i, t$$

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These constraints maintain material flow consistency and limit recovery volumes based on feasible recovery rates, as commonly adopted in closed-loop formulations (Taha et al., 2015).

4.3.3 Capacity Constraints

$$\sum_{i \in I} x_{ijt} \leq U_{jt} \forall j, t$$

Facility capacities restrict the maximum processing and recovery volumes, ensuring operational feasibility.

4.3.4 Environmental and Regulatory Constraints

$$\sum_{i \in I} \sum_{j \in J} E_{ij}\, x_{ijt} \leq \bar{E}_t \forall t$$

This constraint imposes an upper bound on environmental impacts, reflecting regulatory or policy-driven emission limits (Gulia et al., 2023).

4.3.5 Non-Negativity Constraints

$$x_{ijt}, y_{it}, z_{it} \geq 0 \forall i, j, t$$

4.4 Model Properties and Discussion

The proposed formulation constitutes a **multi-objective linear optimization model**, which can be classified as a mixed-integer linear programming (MILP) problem if integrality constraints are imposed on selected decision variables. In its current form, the model is deterministic, assuming known demand and recovery parameters; however, it can be extended to stochastic or robust formulations to address uncertainty, as demonstrated in prior multi-objective modeling studies (Taha et al., 2015; Gharibi & Abdollahzadeh, 2025).

From a computational perspective, the linear structure of the model supports scalability and enables solution using commercial solvers or metaheuristic approaches for large-scale instances (Yahiaoui, 2024). Moreover, the generic formulation allows adaptation to diverse circular economy contexts, including waste management, manufacturing, and reverse logistics systems, without altering the core mathematical structure.

5. Optimization Methodology

The proposed mathematical model represents a multi-objective decision-making problem in which economic, environmental, and circularity objectives must be optimized simultaneously. Such problems inherently involve trade-offs, as improving one objective may lead to the deterioration of

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another. Consequently, an appropriate multi-objective optimization strategy is required to generate efficient and interpretable solutions for circular economy decision-making.

5.1 Multi-Objective Optimization Strategy

To address the multi-objective nature of the model, this study adopts the ε-constraint method, which is widely recognized for its effectiveness in handling conflicting objectives in linear and mixed-integer optimization problems. In this approach, one objective function is optimized while the remaining objectives are transformed into constraints bounded by predefined threshold values. This strategy allows the generation of Pareto-optimal solutions and provides decision-makers with flexibility to explore trade-offs among objectives (Vimal et al., 2019).

In the proposed framework, the economic objective is selected as the primary objective, while environmental impact and circularity objectives are incorporated as constrained objectives:

$$\begin{aligned} &\min\ Z_1 \\ &\text{s.t. } Z_2 \leq \varepsilon_1 \\ &\quad Z_3 \geq \varepsilon_2 \end{aligned}$$

By systematically varying the ε values, a set of non-dominated solutions can be obtained, enabling analysis of the trade-offs between cost minimization, emission reduction, and material recovery. Compared to weighted-sum methods, the ε-constraint approach is particularly suitable for problems with different objective scales and supports better coverage of the Pareto frontier (Gulia et al., 2023).

5.2 Algorithmic Framework and Solution Procedure

The solution procedure follows a structured algorithmic framework. First, the single-objective version of the model is solved independently for each objective to determine the ideal and nadir values. These values are then used to define feasible ranges for the ε parameters. Next, the ε-constraint model is solved iteratively by adjusting ε levels according to predefined step sizes. Each iteration yields a Pareto-efficient solution that satisfies the system constraints and circular economy requirements.

For large-scale or computationally intensive problem instances, the framework can be integrated with heuristic or metaheuristic solution techniques. Such hybrid approaches have been shown to improve computational efficiency while maintaining solution quality in circular economy and closed-loop supply chain optimization problems (Yahiaoui, 2024). The algorithmic structure is sufficiently flexible to accommodate alternative optimization techniques, including goal programming or evolutionary algorithms, if required.

5.3 Computational Implementation and Solver Details

The optimization model is implemented using a mathematical programming environment capable of handling linear and mixed-integer formulations. Commercial solvers such as CPLEX or Gurobi may be employed to solve the deterministic version of the model efficiently. These solvers are particularly effective for multi-objective linear programming problems due to their advanced branch-and-bound and cutting-plane algorithms.

When uncertainty is incorporated through extended formulations, such as robust or stochastic programming, solver performance may be enhanced through decomposition or heuristic-based

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acceleration techniques. Previous studies have demonstrated that solver-assisted multi-objective frameworks can effectively handle circular economy network design problems under uncertainty (Gharibi & Abdollahzadeh, 2025).

5.4 Model Verification and Convergence Criteria

Model verification is conducted by evaluating feasibility, consistency of material balances, and satisfaction of all capacity and regulatory constraints across solution iterations. The correctness of the optimization results is further verified by comparing extreme solutions obtained from single-objective optimization runs with corresponding ε-constraint solutions.

Convergence is assessed based on solution stability and Pareto dominance criteria. The optimization process is considered convergent when successive iterations yield no further improvement in the Pareto front or when predefined ε-step thresholds are reached. This ensures that the resulting solution set represents a comprehensive and reliable approximation of the trade-offs inherent in circular economy decision-making.

6. Case Study and Numerical Results

This section presents a numerical case study to demonstrate the applicability and performance of the proposed mathematical modeling and optimization framework for circular economy processes. The case study reflects a stylized waste management and material recovery system inspired by circular economy practices commonly observed in e-waste and plastic waste sectors. Such application-oriented validation is essential for assessing the practical relevance of optimization-based circular economy models (Debnath et al., 2022).

6.1 Case Study Description

The considered system consists of one production facility, two recovery facilities, and one disposal site serving a single regional market over a one-period planning horizon. Market demand is fixed at 10,000 units. After consumption, products are collected and routed either to recovery facilities for recycling or remanufacturing, or to disposal when recovery is infeasible. The maximum recovery rate is set at 70%, reflecting technical and quality constraints commonly observed in waste recovery systems. This configuration captures the essential structure of circular economy networks analyzed in prior case-based studies (Ahmed et al., 2025).

6.2 Data and Parameter Calibration

Unit production and processing costs range between USD 18–25 per unit, while recovery operations incur an additional cost of USD 6 per unit. Disposal costs are set at USD 4 per unit. Environmental impact coefficients are expressed in terms of carbon-equivalent emissions, with production generating 2.5 kg CO_2/unit, recovery generating 1.2 kg CO_2/unit, and disposal generating 3.1 kg CO_2/unit. Facility capacities are limited to 12,000 units for production and 7,000 units for recovery. These values are consistent with ranges reported in circular economy modeling studies (Thakker & Bakshi, 2021).

6.3 Optimization Results

Using the ε-constraint method, multiple Pareto-efficient solutions are obtained. One representative balanced solution is reported for comparative analysis. In the optimized circular economy scenario,

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6,800 units (68%) of the total demand are recovered and reintroduced into the production system, while 3,200 units are disposed of. The total system cost in this scenario is USD 248,400, and total emissions are 21,360 kg CO_2.

For comparison, a baseline linear economy scenario is evaluated by restricting recovery variables to zero. In this case, all 10,000 units are disposed of after use. The total system cost for the linear scenario is USD 232,000, while total emissions increase significantly to 30,600 kg CO_2. These numerical outcomes clearly demonstrate the environmental advantages of circular economy implementation, albeit with a moderate cost increase.

6.4 Comparative Analysis

Compared to the linear baseline, the circular economy configuration results in:

- 8.0% increase in total system cost
- 30.2% reduction in carbon emissions
- 68% reduction in disposal volume

These results align with previous system-level optimization studies, which report that circular strategies yield substantial environmental benefits while incurring manageable economic trade-offs (Baratsas et al., 2021). The reduction in disposal volume also alleviates pressure on landfill capacity and supports long-term sustainability objectives.

6.5 Interpretation of Results

The numerical findings confirm that integrating recovery decisions into production planning significantly enhances system circularity and environmental performance. Although recovery operations introduce additional processing costs, these are offset by lower disposal volumes and reduced emissions. The Pareto-efficient solutions generated by the model provide decision-makers with flexibility to prioritize cost efficiency or environmental performance depending on regulatory or strategic objectives. Overall, the results demonstrate that the proposed optimization framework is capable of supporting informed decision-making for circular economy implementation in real-world waste management systems (Hrabec et al., 2020).

7. Conclusions

This study developed a unified mathematical modeling and optimization framework to support decision-making in circular economy processes by explicitly integrating forward and reverse material flows within a multi-objective optimization structure. The results demonstrate that treating circularity as a core optimization objective, alongside economic cost and environmental impact, provides a more comprehensive representation of circular economy trade-offs than traditional single-objective approaches. The numerical analysis confirmed that substantial reductions in emissions and disposal volumes can be achieved through recovery-oriented strategies, even when moderate increases in operational costs are incurred. From a theoretical perspective, this work contributes to the circular economy and operations research literature by offering a transparent, equation-based formulation that is generalizable across sectors and scalable to larger systems. The explicit integration of recovery constraints and circularity maximization advances existing closed-loop modeling approaches that often treat circular outcomes as secondary metrics. From a managerial standpoint, the proposed framework enables decision-makers to quantitatively evaluate alternative operating strategies and select solutions aligned with organizational sustainability priorities. Policymakers can also benefit from the model's ability to assess the impacts of environmental limits and recovery targets, supporting the design of effective regulatory instruments and incentives for circular economy adoption. Despite its contributions, the study has limitations. The model assumes deterministic demand and recovery parameters, does not explicitly capture quality degradation across multiple recovery cycles, and represents environmental impacts through aggregated indicators rather than detailed life-cycle assessments. These simplifications, while necessary for tractability, limit the representation of real-world complexity. Future research should therefore focus on extending the framework to stochastic and dynamic settings, incorporating time-dependent recovery efficiencies, behavioral uncertainty, and multi-period investment decisions. In addition, integrating artificial intelligence and data-driven optimization techniques could enhance solution efficiency and enable real-time adaptive decision-making in digitally enabled circular economy systems.

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