

RADIO LABELING OF MYCIELSKIAN GRAPH OF CERTAIN GRAPHS

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Abstract

For a connected graph G , an injective function $f: V(G) \rightarrow \mathbb{N}$ such that for every distinct vertices p and q of G , $d(p, q) + |f(p) - f(q)| \geq 1 + \text{diam}(G)$ is called a radio labeling of G . The radio number of f , $rn(f)$ is the highest number assigned to any vertex of G . The radio number of G , $rn(G)$ is the minimum value of $rn(f)$ taken over all radio labeling f of G . The radio number for Mycielskian graph of Path graph (P_n), Mycielskian graph of Cycle graph (C_n) and Mycielskian graph of Star graph ($K_{1,n}$) are obtained.

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Key Words and Phrases: Radio labeling; Diametre of graph; Mycielskian graph of Path graph (P_n); Mycielskian graph of Cycle graph (C_n); Mycielskian graph of Star graph ($K_{1,n}$).

Introduction

In this paper we consider finite, simple and connected graphs. The graph G has vertex set $V(G)$ and Edge set $E(G)$. Cardinality of vertex set is number vertices in G and Cardinality of Edge set is number of edges in G . The idea of channel assignment problem by Hale's [6] in 1980, is an inspiration to introduce the radio labeling in 2001 and Chartrand et al [4] defined the concept of radio labeling. If we are given a set of radio stations, the aim of this problem was to assign a channel or frequencies (Natural numbers) to each station (or transmitters) without interference of their signals. This was crucial task in wireless networks, as the interference level is intricately linked to the transmitter's geographical position. The major interference occur when the stations are very nearer, minor interference occur when stations are little far away and there is no interference occur when stations far away. The interference was depends on geographical distance. To formulate this problem the researchers have developed a graph. The object of radio labeling is to find valid label and minimized the range of the channels without interference [3,5,7]. The radio labeling of different graphs are discussed in [1,3,8,11].

In [2,9,10], Jan mycielski introduced an interesting construction that transforms again graph into a new large one, prescribing its triangle-free property. For a graph G , Let $V(G)$, $E(G)$ denoted the vertex set and edge set of G respectively. The open neighborhood $N(u)$ of a point u in $V(G)$ is the set of points adjacent to u . $N(u) = \{v/uv \in E(G); \forall v \in V(G)\}$. For each $u_i \in V(G)$, a new point u'_i is introduced and the resulting set of points is $V_1(G)$. Mycielskian graph $\mu(G)$ of graph G is defined as the graph having point set $V(G) \cup V_1(G) \cup u$ and edge set of $\mu(G)$

is edge of graph G with every vertex v'_i of $V_1(G)$ is adjacent to every element of open neighborhood $N(v_i)$ and x is adjacent to every element of $V_1(G)$ i.e edge set is $E(G) \cup \{u_i u'_j / u_i u_j \in E(G)\} \cup \{u u'_j : \forall u'_j \in V_1(G), \forall j\}$

In this paper we deals with radio labeling of mycielskian graph of paths, cycles and star graphs. And also, the radio number of these graphs has been obtained. Throughout this paper we consider simple, finite and connected graphs.

Main Results

Theorem 1. Diameter of every mycielskian graph of path and cycle is less than are equal to 4.

Theorem 2. Diameter of every mycielskian graph of star graphs $(K_{1,n})$ is always 2.

Theorem 3. The Mycielskian graph of Path graph P_n (Path of n number of vertices) $n \geq 8$ admits the radio labeling with radio number is $4n + 2$.

Proof. The vertices of $\mu(P_n)$, $n \geq 8$ are labeled as we see in figure 1. We know that $\text{diam}(\mu(P_n)) = 4$, $n \geq 8$. define a mapping $f: V(\mu(P_n)) \rightarrow \mathbb{N}$ as follows $f(u) = 1$, $f(v_i) = 4i$,

$$1 \leq i \leq n f\left(v'_{\left[\frac{n}{2}\right]+i}\right) = 2 + 4i, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(v'_i) = 2 + 4\left\lfloor \frac{n}{2} \right\rfloor + 4i, \quad 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$

Claim: The mapping f is a radio labeling and we prove that $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$ holds for all pair of points (p, q) , where $p \neq q$ and $p, q \in V(\mu(P_n))$.

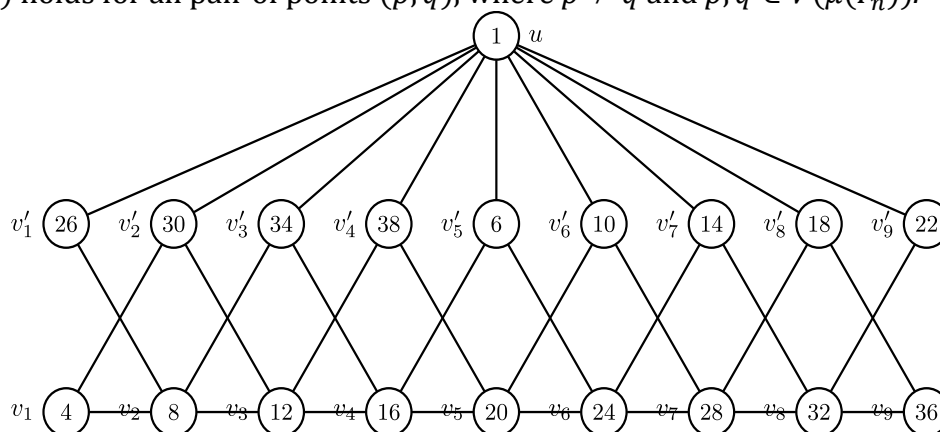


Figure 1: Radio Labeling of Mycielskian graph of path graph P_9

Case 1: Suppose $p = u$ and $q = v_i$, $1 \leq i \leq n$, then, $f(u) = 1$, $f(v_i) = 4i$, $1 \leq i \leq n$ and $d(u, v_i) = 2$

$$\begin{aligned} |f(u) - f(v_i)| + d(u, v_i) &\geq 1 + \text{diam}(\mu(P_n)) \\ &= |1 - 4i| + 2 \geq 1 + 4, 1 \leq i \leq n \end{aligned}$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$

Case 2: Suppose $p = u$ and $q = v'_{\left[\frac{n}{2}\right]+i}$, $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$, $n \geq 8$ then,

$$f(u) = 1 f\left(v'_{\left[\frac{n}{2}\right]+i}\right) = 2 + 4i, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ and } d\left(u, v'_{\left[\frac{n}{2}\right]+i}\right) = 1$$

$$\begin{aligned} \left|f(u) - f\left(v'_{\left[\frac{n}{2}\right]+i}\right)\right| + d\left(u, v'_{\left[\frac{n}{2}\right]+i}\right) &\geq 1 + \text{diam}(\mu(P_n)) \\ &= |1 - 2 - 4i| + 1 \geq 1 + 4, 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor, n \geq 8 \end{aligned}$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$ And the $p = u$ and $q = v'_i$ vertices are satisfied the above condition.

Case 3: Suppose $p = v_i$, $1 \leq i \leq n$ and $q = v'_{\lfloor \frac{n}{2} \rfloor + j}$, $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$ then,

$$f(v_i) = 4i, f\left(v'_{\lfloor \frac{n}{2} \rfloor + j}\right) = 2 + 4j, 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor \quad d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) \leq 3$$

Case i: $d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) = 3$

Case a: If $d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) = 3$ and $i = j$

$$\begin{aligned} |f(v_i) - f(v'_{\lfloor \frac{n}{2} \rfloor + j})| + d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) &\geq 1 + \text{diam}(\mu(P_n)) \\ &= |2 + 4j - 4i| + 3 \geq 1 + 4, 1 \leq i \leq n, 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor, n \geq 8 \end{aligned}$$

Case b: If $d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) = 3$ and $i \neq j$

$$\begin{aligned} |f(v_i) - f(v'_{\lfloor \frac{n}{2} \rfloor + j})| + d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) &\geq 1 + \text{diam}(\mu(P_n)) \\ &= |2 + 4j - 4i| + 3 \geq 1 + 4, 1 \leq i \leq n, 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor, n \geq 8 \end{aligned}$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$

Case ii: If $d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) < 3$ and $i \neq j$

Now we take minimum distance of $\mu(P_n)$ i.e $d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) = 1$

$$\begin{aligned} |f(v_i) - f(v'_{\lfloor \frac{n}{2} \rfloor + j})| + d(v_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) &\geq 1 + \text{diam}(\mu(P_n)) \\ &= |2 + 4j - 4i| + 1 \geq 1 + 4, 1 \leq i \leq n, 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor, n \geq 8 \end{aligned}$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$

Case 4: Suppose $p = v_i$, $1 \leq i \leq n$ and $q = v'_j$, $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$.
then,

$$f(v_i) = 4i, 1 \leq i \leq n \quad f(v'_j) = 4j, 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$$

Now we take minimum distance of $\mu(P_n)$ i.e. $d(v_i, v'_j) = 1$

$$\begin{aligned} |f(v_i) - f(v'_j)| + d(v_i, v'_j) &\geq 1 + \text{diam}(\mu(P_n)) \\ &= |2 + 4\left\lfloor \frac{n}{2} \right\rfloor + 4j - 4i| + 1 \geq 1 + 4, 1 \leq i \leq n, 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor \end{aligned}$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$

Case 5: Suppose $p = v_i$, $1 \leq i \leq n$ and $q = v_j$, $1 \leq j \leq n$. then, $f(v_i) = 4i$, $1 \leq i \leq f(v_j) = 4j$, $1 \leq j \leq n$

Now we take minimum distance of $\mu(P_n)$ i.e $d(v_i, v_j) = 1$

$$\begin{aligned} |f(v_i) - f(v_j)| + d(v_i, v_j) &\geq 1 + \text{diam}(\mu(P_n)) \\ &= |4i - 4j| + 1 \geq 1 + 4, 1 \leq i \leq n, 1 \leq j \leq n, n \geq 8 \end{aligned}$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$

Case 6: Suppose $p = v'_i, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$ and $q = v'_{\lfloor \frac{n}{2} \rfloor + j}, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor$

then,

$$\begin{aligned} f(v'_i) &= 2 + 4 \lfloor \frac{n}{2} \rfloor + 4i, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, f(v'_{\lfloor \frac{n}{2} \rfloor + j}) = 4j, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor \text{ and } d(v'_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) = 2 \\ |f(v'_i) - f(v'_{\lfloor \frac{n}{2} \rfloor + j})| + d(v'_i, v'_{\lfloor \frac{n}{2} \rfloor + j}) &\geq 1 + \text{diam}(\mu(P_n)) \\ &= |2 + 4 \lfloor \frac{n}{2} \rfloor + 4i - 4j| + 2 \geq 1 + 4, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, \\ &1 \leq j \leq \lfloor \frac{n}{2} \rfloor, n \geq 8 \\ &= |4(\lfloor \frac{n}{2} \rfloor + i - j)| + 2 \geq 1 + 4, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor, n \geq 8 \end{aligned}$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$ and $p = v'_i, q = v'_j$ vertices are satisfied the above condition.

Thus $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(P_n))$ for all $p, q \in V(\mu(P_n)), n \geq 8$. Thus six cases establish the claim that f is a radio labeling of $\mu(P_n)$. Since the vertex $v'_{\lfloor \frac{n}{2} \rfloor}$ receives the maximum label. The radio number of $\mu(P_n)$ is $4n + 2$. $\square$

Theorem 4. The Mycielskian graph of Cycle graph C_n (n number of vertices) $n \geq 8$ admits the radio labeling with radio number is $4n + 2$.

Proof. The proof is closely connect to the above theorem 3. Radio labeling of Mycielskian graph of Cycle C_8 (Cycle of 8 number of vertices) is shown in Figure 2. $\square$

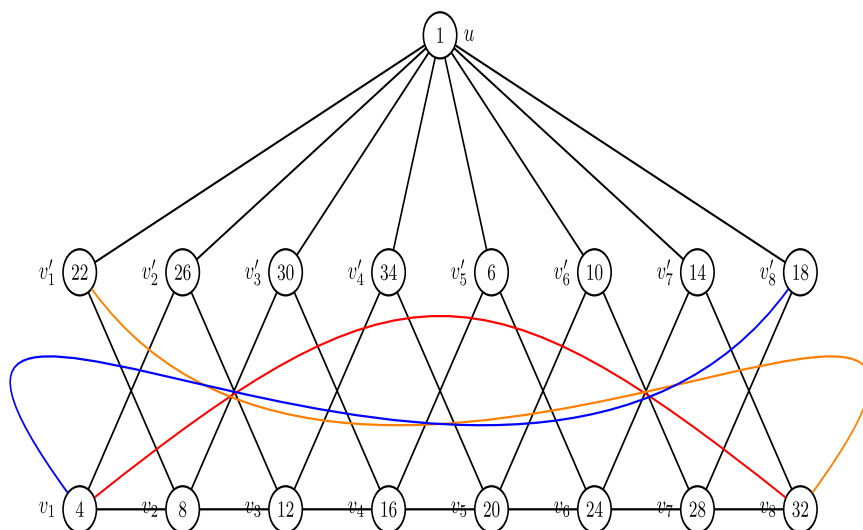


Figure 2. Radio labeling of Mycielskian graph of Cycle graph C_8

Theorem 5. The Mycielskian graph of Star graph $(K_{1,n}), n \geq 1$ admits the radio labeling with radio number is $2n + 3$.

Proof. The vertices of $\mu(K_{1,n}), n \geq 1$ are labeled as we see in Figure 3. We know that $\text{diam}(\mu(K_{1,n})) = 2, n \geq 1$.

Define a mapping $f: V(\mu(K_{1,n})) \rightarrow \mathbb{N}$ as follows $f(v_i) = i + 2, 1 \leq i \leq n, f(v'_j) = n + 2 + j, 1 \leq j \leq n, f(u) = 1, f(x) = 2, f(u') = 2n + 3$

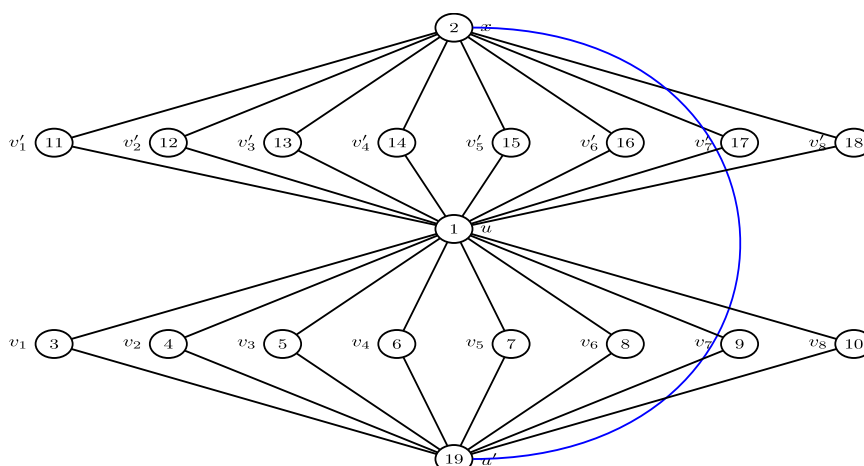


Figure 3. Radio labeling of Mycielskian graph of Star graph $K_{1,8}$

Claim: The mapping f is a radio labeling and we prove that $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$ holds for all pair of points (p, q) , where $p \neq q$ and $p, q \in V(\mu(K_{1,n}))$.

Case 1: Suppose $p = v_i$ and $q = v'_j$, $1 \leq i, j \leq n$ then,

$$f(v_i) = 2 + i, f(v'_j) = n + 2 + j \quad 1 \leq j \leq n \text{ and } d(v_i, v'_j) = 2$$

$$|f(v_i) - f(v'_j)| + d(v_i, v'_j) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |i + 2 - n - 2 - j| + 2 \geq 1 + 2 \quad 1 \leq i \leq n, \quad 1 \leq j \leq n$$

$$= |i - n - j| + 2 \geq 3 \quad 1 \leq i \leq n, \quad 1 \leq j \leq n$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$

Case 2: Suppose $p = v_i$ and $q = u$, $1 \leq i \leq n$.

then, $f(v_i) = 2 + i$ $f(u) = 1$ $1 \leq i \leq n$ and $d(v_i, u) = 1$

$$|f(v_i) - f(u)| + d(v_i, u) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |i + 2 - 1| + 1 \geq 1 + 2, \quad 1 \leq i \leq n,$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$

Case 3: Suppose $p = v_i$ and $q = x$, $1 \leq i \leq n$.

then, $f(v_i) = 2 + i$ $f(x) = 2$ $1 \leq i \leq n$ and $d(v_i, x) = 2$

$$|f(v_i) - f(x)| + d(v_i, x) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |i + 2 - 2| + 2 \geq 1 + 2 \quad 1 \leq i \leq n,$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$

Case 4: Suppose $p = v_i$ and $q = u'$, $1 \leq i \leq n$.

then, $f(v_i) = 2 + i$, $f(u') = 2n + 3$, $1 \leq i \leq n$ and $d(v_i, u') = 1$

$$|f(v_i) - f(u')| + d(v_i, u') \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |i + 2 - 2n + 3| + 1 \geq 1 + 2 \quad 1 \leq i \leq n,$$

$$= |2n + 1 - i| \geq 3 \quad 1 \leq i \leq n$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$

Case 5: Suppose $p = v_i$ and $q = v_j$, $i \neq j$, $1 \leq i \leq n$, $1 \leq j \leq n$.

then, $f(v_i) = 2 + i$, $f(v_j) = 2 + j$, $i \neq j$, $1 \leq i \leq n$, $1 \leq j \leq n$ and $d(v_i, v_j) = 2$

$$|f(v_i) - f(v_j)| + d(v_i, v_j) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |2 + i - 2 - j| + 2 \geq 1 + 2, i \neq j, 1 \leq i \leq n, 1 \leq j \leq n$$

$$= |i - j| + 2 \geq 3 \quad i \neq j \quad 1 \leq i \leq n \quad 1 \leq j \leq n$$

$$\text{Hence } |f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

Case 6: Suppose $p = v'_i$ and $q = u$, $1 \leq i \leq n$.

then, $f(v'_i) = n + 2 + i$, $f(u) = 1$, $1 \leq i \leq n$ and $d(v'_i, u) = 1$

$$|f(v'_i) - f(u)| + d(v'_i, u) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |n + 2 + i - 1| + 1 \geq 1 + 2 \quad 1 \leq i \leq n,$$

$$= |n + 1 + i| + 1 \geq 3 \quad 1 \leq i \leq n,$$

$$\text{Hence } |f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

Case 7: Suppose $p = v'_i$ and $q = x$, $n \geq 1$ $1 \leq i \leq n$.

Then, $f(v'_i) = n + 2 + i$, $f(x) = 2$, $1 \leq i \leq n$ and $d(v'_i, x) = 1$

$$|f(v'_i) - f(x)| + d(v'_i, x) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |n + 2 + i - 2| + 1 \geq 1 + 2 \quad 1 \leq i \leq n,$$

$$= |n + i| + 1 \geq 3, \quad 1 \leq i \leq n$$

$$\text{Hence } |f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

Case 8: Suppose $p = v'_i$ and $q = u'$, $n \geq 1$ $1 \leq i \leq n$.

Then,

$f(v'_i) = n + 2 + i$, $f(u') = 2n + 3$, $1 \leq i \leq n$ and $d(v'_i, u') = 2$

$$|f(v'_i) - f(u')| + d(v'_i, u') \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |n + 2 + i - 2n - 3| + 2 \geq 1 + 2 \quad 1 \leq i \leq n,$$

$$= |n + 1 - i| + 2 \geq 3 \quad 1 \leq i \leq n,$$

$$\text{Hence } |f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

Case 9: Suppose $p = v'_i$ and $q = v'_j$, $1 \leq i, j \leq n$.

Then,

$f(v'_i) = n + 2 + i$, $f(v'_j) = n + 2 + j$, $1 \leq i \leq n$, $1 \leq j \leq n$ and $d(v'_i, v'_j) = 2$

$$|f(v'_i) - f(v'_j)| + d(v'_i, v'_j) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |n + 2 + i - n - 2 - j| + 2 \geq 1 + 2, \quad 1 \leq i \leq n, \quad 1 \leq j \leq n$$

$$= |i - j| + 2 \geq 3 \quad 1 \leq i \leq n \quad 1 \leq j \leq n$$

$$\text{Hence } |f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

Case 10: Suppose $p = u$ and $q = x$.

Then, $f(u) = 1$ $f(x) = 2$ and $d(u, x) = 2$

$$|f(u) - f(x)| + d(u, x) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |1 - 2| + 2 \geq 1 + 2$$

$$\text{Hence } |f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$$

Case 11: Suppose $p = u$ and $q = u'$.

Then, $f(u) = 1$ $f(u') = 2n + 3$ and $d(u, u') = 2$

$$|f(u) - f(u')| + d(u, u') \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |1 - 2n - 3| + 2 \geq 1 + 2$$

$$= |2n + 2| + 2 \geq 3$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$

Case 12: Suppose $p = x$ and $q = u'$.

Then, $f(x) = 2$ $f(u') = 2n + 3$ $n \geq 1$ and $d(x, u') = 1$

$$|f(x) - f(u')| + d(x, u') \geq 1 + \text{diam}(\mu(K_{1,n}))$$

$$= |2 - 2n - 3| + 1 \geq 1 + 2$$

$$= |2n + 1| + 1 \geq 3$$

Hence $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n}))$

Thus $|f(p) - f(q)| + d(p, q) \geq 1 + \text{diam}(\mu(K_{1,n})) = 3$ for all $p, q \in V(\mu(K_{1,n}))$ $n \geq 1$. Thus twelve cases establish the claim that f is a radio labeling of $\mu(K_{1,n})$. Since the vertex u' receives the maximum label. The radio number of $\mu(K_{1,n})$ is $2n + 3$. $\square$

Conclusion

We have obtained Radio numbers for Mycielskian graph of Path(P_n), Mycielskian graph of Cycle(C_n) and Mycielskian graph of Star graph ($K_{1,n}$)

Conflict of interest. The authors declare that they have no conflicts of interest.

Acknowledgments. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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