International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

WORDS AND SYMBOLS: ENHANCING MATHEMATICAL UNDERSTANDING THROUGH WRITING

Wafiq Hibi, Waleed Khalifa

Wafiq Hibi, Head of the Mathematics Department, Academic College of Sakhnin, Israel

https://orcid.org/0000-0003-2897-0175

Email: wafiq.hibi@gmail.com, wafiq.hibi@sakhnin.ac.il

Waleed Khalifa, Head of the Computer Science Education Department Academic College of Sakhnin, Israel

https://orcid.org/0009-0005-8448-9425

Email: walid.khalifa@gmail.com, khwalid@sakhnin.ac.il

Abstract

This article explores the critical connection between written expression and the understanding and articulation of mathematical concepts among first-year mathematics students, particularly those aspiring to be educators. It argues that writing is not just a supplementary tool, but a cornerstone of mathematical literacy. The investigation unravels the challenges students face in blending linguistic proficiency with mathematical writing, examining the impact on their mathematical understanding and performance.

A key finding is that students' written communication abilities directly correlate with their ability to grasp and convey sophisticated mathematical ideas. Through case studies and examples, the article sheds light on the struggles first-year students face in mathematical writing, attributing these challenges not only to their mathematical comprehension but also to their limited engagement with essential linguistic tools for effective communication. This diagnosis emphasizes the need for a pedagogical shift that integrates linguistic skills into the mathematics curriculum.

Strengthening linguistic competencies within the curriculum is crucial for enhancing students' mathematical understanding and academic success, potentially reducing attrition rates from mathematics programs.

Keywords: Reflective Writing in Mathematics, Integrating Language Skills in Math, Teaching Methodologies for Math Students, Mathematical Communication as a Pedagogical Tool, Developing Expression Skills in Future Math Teachers.

introduction

The university classroom is often a scene of frustration as first-year students grapple with the complexities of calculus. The chalkboard, filled with numbers and symbols, reflects the persistent struggle between student and subject. This struggle underscores a long-standing challenge in mathematics education: making complex concepts accessible and engaging for all students.

Transitioning from high school to university mathematics presents a myriad of challenges for first-year students, often marked by a significant evolution in abstraction, rigor, and the self-

Received: August 06, 2025

Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

directed nature of study. The leap from high school to university mathematics is a profound one. it's about adopting a completely different approach to thinking about mathematics. This leap is fraught with hurdles: from the shock of abstract theoretical frameworks that are far removed from the concrete numbers and shapes of high school math, to the daunting pace and depth of university-level courses that demand not just understanding, but also the creation of. Such difficulties can lead to a crisis of confidence, frustration, and in some cases, a decision to abandon the pursuit of mathematics altogether.

Addressing these challenges requires a holistic approach. Proposed solutions include bridge programs that help students adjust to the abstract nature of university mathematics before their first semester, increased access to tutoring and mentorship programs to provide support and build a sense of community, and a reevaluation of teaching methods to incorporate more conceptual understanding and application in high school curriculums.

Faculty members regularly encounter two recurrent phenomena within introductory mathematics education. Students when faced with a complex question, might correctly determine the answer but be unable to explain the rationale or methodology behind their solution. This underscores the crucial yet often neglected aspect of integrating language abilities into mathematics instruction. Numerous among us have encountered a moment when a student, with a blend of certainty and perplexity, declares, "The answer is x, but, don't ask me to explain how." The latter phenomenon highlights that integrating language abilities into mathematics instruction is a critically important, but often neglected aspect of mathematics teaching. The precise focus and distinctive position of colleges of education, highlights the critical need to effectively address this integration. This encouraged us to write this article.

The interplay between language and mathematics has long been a subject of educational discourse. Gudder and Buffet (2011) famously stated, "The essence of mathematics is not to make simple things complicated, but to make complicated things simple". Writing, an expressive and structured form of communication, has shown promise in bridging the gap between mathematical concepts and their understanding.

Historically, mathematics has been taught as a purely numerical or symbolic discipline. Huinker et al. (2020) emphasize the importance of communication in mathematics, suggesting that writing in mathematics can improve students' learning because it requires them to organize their thoughts and clarify their thinking. This sentiment is echoed by Pólya (2004) in his seminal work "How to Solve It," where he argues that a deep understanding of mathematical concepts can only be achieved through the articulation of problem-solving processes. There is a significant gap in research focused on the specific challenges and needs of mathematics students in teacher training colleges (Planas & Pimm, 2024). This gap is particularly pronounced at the intersection of language proficiency and written expression among first-year students, whose success in these areas is foundational for their future roles as educators.

Goos et al. (2023) emphasize that "effective mathematical communication is foundational in developing students' mathematical reasoning and problem-solving skills", highlighting the critical role of pedagogical strategies that incorporate linguistic skills into mathematics

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

education. Dewsbury (2020) emphasized the importance of reflective thinking in education, a skill that is significantly enhanced through expressive writing activities.

Literature Review

Existing research highlights the impact of writing on learning outcomes in mathematics. Integrating writing into mathematics instruction not only enhances students' comprehension of mathematical concepts, but also fosters a deeper engagement with the material, leading to improved academic performance.

The Intersection of Language and Mathematics in Education

Students must navigate the dual demands of mastering mathematical concepts and developing the linguistic prowess to express them accurately. This integration is paramount, as it enables individuals to construct, convey, and critique mathematical arguments effectively, fostering a deeper understanding of mathematical concepts and their applications (Anderson & Bobis, 2005; Hibi, W., & Tabajah, K. 2025).

Written expression in mathematics serves multiple pedagogical functions: it facilitates the deep processing of mathematical ideas, supports the evaluation of reasoning and methodologies, and acts as a reflective tool for learners to review and consolidate their understanding. However, the ability to express oneself effectively in written mathematical form is not innate; it requires explicit instruction and practice (Hibi, W., & Khalifa, W. 2025; Kazemi & Stipek, 2001).

Several educational theories underscore the importance of integrating language development within mathematics education. Vygotsky's sociocultural theory emphasizes the role of language as a fundamental tool for cognitive development, suggesting that learning occurs through social interactions and the internalization of culturally developed linguistic tools (Hibi, W., 2020; Vygotsky, 1978). Similarly, Cummins' Cognitive Academic Language Learning Approach (CALLA) highlights the need for explicit teaching of academic language across content areas, including mathematics, to support learners in developing the higher-order cognitive skills necessary for academic success (Hibi, W., 2005; Swanson, 2010). According to Moschkovich (2018), writing about mathematics encourages reflection, facilitates deeper understanding, and promotes the clarification of thoughts.

Language as a Cognitive Tool in Mathematics

Mathematical writing requires the integration of verbal and symbolic languages. Adams, Thangata, and King (2005) highlighted the necessity of expressing mathematical ideas clearly and coherently in written form for deep understanding and effective communication in mathematics. However, research by Oxenford O'Brian, Nocon, and Iceman Sands (2010) indicated that students often struggle with this integration, leading to difficulties in articulating their mathematical reasoning.

Integrating Writing into Mathematics Instruction

Received: August 06, 2025

Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

The exploration of mathematical writing within first-year university courses reveals a nuanced landscape where students encounter a variety of mathematical languages and notations, often for the first time (Hibi. W, 2016). This transition can be particularly challenging as students navigate through foundational courses such as Calculus, Linear Algebra, and Discrete Mathematics, each presenting its unique linguistic and conceptual demands.

Recognizing the challenges faced by students, some researchers have proposed integrating writing more deeply into mathematics instruction as a means of supporting student learning (Hibi. w, 2024b). In a study by Bicer, A., Capraro, R. M., & Capraro, M. M. (2013), the emphasis was placed on the importance of problem-solving strategies in mathematics, suggesting that writing about problem-solving processes can enhance understanding and retention. Following this, Powell, S. R., Hebert, M. A., & Hughes, E. M. (2021) implemented writing assignments in first-year mathematics courses and found that students who regularly wrote about their mathematical thinking processes demonstrated a deeper understanding of course material and improved problem-solving skills.

In the realm of Calculus, students are introduced to the language of limits, derivatives, and integrals, which requires a precise use of notation and terminology.

Linear Algebra introduces students to an array of new symbols and the language of vector spaces, matrices, and linear transformations. According to Stewart, S., & Thomas, M. O. (2019), the abstract nature of Linear Algebra, combined with the introduction of matrix notation and vector arithmetic, requires students to adapt quickly to a new form of mathematical expression.

Discrete Mathematics poses a different set of challenges for first-year students, particularly in the areas of logic, set theory, and proof writing (Hibi. w, 2024d). The course serves as many students' first formal introduction to mathematical proofs, a genre of mathematical writing that demands not only logical rigor but also a coherent narrative that can be followed by the reader. Polya, G. (2004) and Reid, D. A., & Knipping, C. (2010) highlight that students often struggle with the transition from solving numerical problems to constructing and writing formal proofs.

This section explores the theories of the integration of writing into mathematics education, demonstrating how they collectively support the incorporation of writing as a pedagogical strategy in math education.

Integration of Writing in Mathematics Education: A Multifaceted Approach Tailored for First-Year College Students

Learning Styles and the VARK Model: Catering to First-Year College Math Learners

The transition to college often presents first-year students with a diverse array of learning challenges, particularly in mathematics. The VARK model's emphasis on Reading/Writing preferences is especially pertinent here, as it suggests that integrating writing into math education can significantly aid students who excel when information is presented in written form. For these learners, the act of writing can enhance comprehension and retention of complex mathematical theories and formulas encountered for the first time in a college setting (Hibi, 2021a; Hibi, 2021b; Hibi, 2022b; Hibi, 2022c; Sheromova et al., 2020). Implementing

Received: August 06, 2025

Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

writing tasks that align with the VARK model can demystify abstract concepts for first-year students, making mathematics more approachable and understandable.

Constructivist Perspectives: Engaging First-Year College Students in Mathematics

The leap into college-level mathematics often presents a significant challenge for first-year students. Constructivist Theory, which posits that learners construct knowledge through experiences, offers a valuable lens through which to view this transition. "Learning mathematics is not about rote memorization but about engaging deeply with concepts" (Piaget, 1976).

Socio-Cultural Influences: Cultivating a Community of Learners in First-Year Mathematics

Socio-Cultural Theory emphasizes the role of culture and social interaction in learning, "Mathematics becomes a social experience, not just a set of problems to solve", illustrating how collaborative learning and writing can enrich the educational experience (Hibi, 2021c; Hibi, 2021d; Hibi, 2022a; Hibi, 2022d; Vygotsky, 1978).

Managing Cognitive Load: Supporting First-Year Students' Mathematical Understanding

Cognitive Load Theory propose that writing can serve as an effective strategy for managing cognitive load in mathematics education. By externalizing their thought processes through writing, students can alleviate the cognitive burden associated with learning new mathematical concepts, allowing for deeper reflection and understanding.

Through the lenses of the VARK model, Mathematical Communication, Constructivist Theory, Socio-Cultural Theory, and Cognitive Load Theory, the role of writing emerges as not only beneficial but essential for enhancing mathematical comprehension, communication, and community engagement among newcomers to college mathematics.

Theoretical Framework

Enhancing Mathematical Understanding through Language and Multiliteracies: A Cognitive Perspective

Incorporating writing into mathematics education goes beyond traditional teaching methods. Rooted in Vygotsky and Bruner's theories, this approach views language as essential to cognitive development and learning. Vygotsky argued that cognitive functions develop through social interactions are mediated by language, stating, "Thought is not merely expressed in words; it comes into existence through them" (Vygotsky, 1986).

Incorporating Multiliteracies and Managing Cognitive Load

Cognitive Load Theory (CLT) provides insights into optimizing learning by minimizing unnecessary cognitive load (Manning-Lewis & Sanford, 2022). In mathematics education, writing activities help manage cognitive load by allowing students to organize and process complex information through written expressions, leading to a deeper understanding of mathematical problems (Hibi, 2024a; Hibi, 2021e; Hibi, 2025b; Hibi, 2025c; Mills, Unsworth, & Scholes, 2023).

Received: August 06, 2025

Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Addressing Challenges for First-Year Mathematics Students

Transitioning to collegiate mathematics poses unique challenges for first-year students, particularly due to the abstract nature of university-level concepts and linguistic barriers.

Collaborative writing exercises are crucial in this transition, as they deepen understanding and foster a sense of community among students (Sperling et al., 2024). Bassachs et al. (2022) highlight the benefits of reflective writing, noting that it allows students to "internalize and personalize mathematical concepts," enhancing comprehension and engagement.

D'Mello, Lehman, Pekrun, & Graesser (2014) state that "writing about one's learning process in mathematics can help identify and address areas of confusion, reducing anxiety and promoting a more positive attitude towards learning".

In conclusion, integrating language and writing into the collegiate mathematics curriculum can significantly enhance students' learning experiences and success.

Externalization of Thought through Writing in Mathematics: Benefits for First-Year College Students

Integrating writing into mathematics education transforms understanding of complex concepts, especially for first-year college students (Hibi. W, 2024a). This method goes beyond documenting solutions, acting as a mechanism for externalizing thoughts, clarifying ideas, and engaging in reflective problem-solving. Caballero et al. (2015) note that writing in mathematics helps make abstract concepts visible and accessible, aiding students' understanding. For first-year students, writing clarifies complex problems. Aguilar and Flores (2022) argue that writing helps deconstruct problems into manageable parts, clarifying thought processes, strategies, and correct misconceptions (Hibi. W, 2022e). Bicer et al. (2018) found that reflective writing about problem-solving processes leads to more effective strategies and greater success.

Methodology for Incorporating Writing

Integrating writing into first-year mathematics courses can significantly enhance students' understanding and engagement with mathematical concepts. Below are detailed descriptions and extended outlines of three specific writing activities and assignments that educators can incorporate: reflective journals, explanatory articles on mathematical concepts, and written analyses of problem-solving processes. These assignments are designed to cultivate deeper mathematical insight, improve communication skills, and foster a reflective approach to learning.

1. Reflective Journals

Objective: To encourage students to reflect on their learning process, understand their cognitive and emotional responses to mathematical concepts, and articulate their personal growth over time.

Assignment Description:

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Frequency: Students are required to maintain a weekly reflective journal throughout the semester.

Content: Each student should reflect on the week's mathematical topics, focusing on areas of interest, difficulty, and enlightenment. Students should describe their initial reactions to new concepts, the strategies they employed to understand them, and how their perceptions evolved with study and practice.

Structure: Students should begin with a brief summary of the week's topics, followed by personal reflections. These reflections may include:

- Questions that arose during study and how they were addressed.
- Connections made between the current topics and previous knowledge.
- Emotional responses to learning challenges and successes.

Assessment Criteria: Journals will be assessed based on the depth of reflection, the ability to connect personal experiences with mathematical concepts, and evidence of growth in understanding and attitude toward mathematics.

Example 1:

Example 1: In our recent linear algebra course, we delved into the complex yet fascinating topic of linear dependence. One of our diligent students shared his insights in his weekly Reflective Journal, showcasing the depth of understanding and contemplation sparked by the lecture. Below is an excerpt from his reflective narrative:

Topic: Linear Dependence and Independence

This week in linear algebra, we delved into the concepts of linear dependence and independence. Understanding these concepts was crucial, as they form the backbone of many topics in linear algebra. Initially, the definitions seemed simple: a set of vectors is linearly dependent if a vector in the set can be written as a linear combination of the others; if no such vector exists, the set is linearly independent.

Challenges and Strategies: Visualizing linear dependence and independence was challenging. To aid my comprehension, I sketched vectors in R^2 and R^3 to observe their spatial relationships. This visual approach clarified the concept, especially illustrating how the determinant of a matrix (det(A) = 0) signifies linear dependence among its column vectors.

When tackling theoretical problems, like proving the independence of vectors in R^n, I broke down problems into steps, focusing on the condition that $a_1v_1 + a_2v_{21} + \cdots + a_nv_n = 0$ only holds if all coefficients $a_i = 0$ for independence. This step-by-step approach made complex problems more manageable.

Reflections: The week solidified my understanding of how linear dependence and independence play pivotal roles in linear algebra, influencing concepts like vector space bases and dimensions. Recognizing the minimal number of vectors needed to span a vector space has profound implications, not just in mathematics but also in applied fields like engineering.

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Conclusion: The exploration of linear dependence and independence was enlightening, revealing the importance of these foundational concepts in broader mathematical contexts. It reinforced the value of visual and analytical approaches in grappling with abstract mathematical ideas. As I progress, I anticipate using this week's insights to tackle more complex topics, appreciating the interconnectedness of linear algebra's various components.

Incorporating reflective journal activities into mathematical education deepens student engagement and enhances critical thinking and communication skills. This practice aligns with contemporary pedagogical theories and research, emphasizing the value of reflection and articulation in learning mathematics.

Enhancing Conceptual Understanding

Reflective journaling encourages deeper engagement with mathematical concepts, moving beyond rote memorization. Schön (1983) states, "the reflective practitioner engages in a process of self-education," relevant in mathematics where students navigate abstract concepts. Studies show journaling enables exploration of mathematical concepts, linking theory with practical application (Thompson & Chappell, 2007), essential for a deep and lasting grasp of mathematics.

Developing Critical Thinking

Reflective journaling nurtures critical thinking, a cornerstone of problem-solving. Articulating thought processes helps students evaluate reasoning, identify gaps, and consider alternative approaches. Ritchhart et al. (2011) argue that making thinking visible cultivates critical thinking skills.

Fostering Communication Skills

Reflective journals improve the ability to communicate complex mathematical ideas clearly. Writing requires organizing thoughts, precise use of mathematical language, and clear explanation of concepts.

2. Explanatory Articles on Mathematical Concepts

We are about to delve into the details of an educational task aimed at developing students' ability to communicate complex mathematical concepts in a clear and accessible manner to an audience unfamiliar with the subject. The challenge lies in making mathematical concepts understandable and applicable by writing explanatory articles that bridge the gap between academic expertise and general comprehension.

Objective: To develop the ability to communicate complex mathematical ideas in a clear, accessible manner to an audience not familiar with the subject.

Assignment Description:

- Frequency: Students will write two explanatory articles over the semester, each focusing on a different mathematical concept discussed in class.

Received: August 06, 2025

- Content: Each article should Introduce the mathematical concept within a relevant context to emphasize its significance and practical application, Explain the fundamental principles and operations involved and provide examples to demonstrate its usage.

- Structure:

a. Introduction: Present the mathematical concept, highlight its relevance, and explain why it should capture the reader's interest.

b. Body: Break down the concept into understandable segments. Use diagrams, examples, and analogies as necessary to clarify the concept.

c. Conclusion: Summarize the key points and suggest areas or topics for further exploration.

- Assessment Criteria: Articles will be evaluated based on accuracy, clarity of explanation, creativity in presenting the concept, and effectiveness in engaging and informing a general audience.

E*xample 2*: As part of the differential calculus course assignments that we gave, the students were asked to submit explanatory essays on new mathematical concepts they encounter in the course, in the following example a quote from one student's work.

Unraveling the Mysteries of Derivatives:

Introduction: Imagine you're driving a car on a highway. Ever wondered how you could determine the exact speed at any given moment, not just the average speed? Or picture an apple falling from a tree; can we predict its speed at any point during its fall? The mathematical tool that allows us to answer these questions is called a derivative. This cornerstone of differential calculus doesn't just solve abstract mathematical problems; it's crucial in physics, engineering, economics, and even in understanding the very fabric of the universe.

Body: The Essence of Derivatives. At its core, a derivative represents the rate at which a quantity changes as another quantity changes. In simpler terms, it's about finding the "instantaneous" change rate, much like determining your precise speed at a specific moment in time while driving.

To grasp this concept, let's start with something familiar: the slope of a line. In mathematics, the slope tells us how steep the line is, calculated as the change in height (rise) over the change in horizontal distance (run). But what if our line isn't straight, like the path of our falling apple? This is where derivatives come into play.

Imagine zooming in on a tiny segment of the apple's path. As we zoom in closer and closer, this segment looks more like a straight line. The derivative, then, is the slope of this line at a single point. Mathematically, if we have a function $f(x)$ that represents our scenario (e.g., the apple's height at time x, the derivative $f'(x)$ gives us the rate of change at any point x.

Calculating Derivatives: How do we find these derivatives? One of the basic rules we learn in differential calculus is the power rule. Suppose our function is $f(x) = x^2$, representing, for instance, an object moving in a way that its distance is the square of the time. The derivative,

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

according to the power rule, is $f'(x) = 2x$. *This result tells us that the rate of change (e.g., velocity) at any moment* $2x$. *Here the student will mention the rules of the derivative that he learned, while providing comments, examples and proofs when necessary.*

Real-world Applications: The beauty of derivatives lies not just in their mathematical elegance but in their vast applications. In physics, they're used to describe velocities and accelerations. In economics, they can model how a slight change in production levels affects costs or profits. The applications are as limitless as they are fascinating.

The derivative is more than a mathematical trick; it's a fundamental concept that helps us understand and predict the world around us. From calculating the optimal dimensions for a box to predicting the population growth of a species or the decay of a radioactive element, derivatives provide us with a powerful tool to model and analyze change. As we continue our journey through differential calculus, let's appreciate the profound impact of this remarkable concept on both our understanding of the natural world and our ability to shape it.

Incorporating explanatory articles into the mathematics curriculum bridges theoretical understanding with practical application, fostering deeper engagement. This approach aligns with educational theories emphasizing active learning, critical thinking, and communication skills.

"Writing to learn mathematics deepens understanding and enhances students' ability to communicate complex ideas clearly" (Burns, 2020). Creating explanatory articles transforms students into active participants

Zaslavsky and Sullivan (2021) state, "Mathematical communication is fundamental to learning, allowing students to articulate, refine ideas, confront misconceptions, and build on others' knowledge." Writing articles requires organizing thoughts and presenting concepts coherently.

3. Written Analyses of Problem-Solving Processes

The objective is to enhance students' problem-solving skills by requiring them to articulate their thought processes, strategies, and reflections on complex problems.

Assignment Description

Frequency: Students will submit three written analyses over the semester, each associated with a different set of problems.

Content: For each assigned problem, students should write:

- Introduction: Briefly describe the problem and its significance.
- Methodology: Explain the steps taken to tackle and solve the problem, including any formulas or theorems applied.
- Challenges: Describe any difficulties faced during the problem-solving process and how they were overcome.
- Reflection: Reflect on the efficiency of the chosen strategy and consider alternative methods or solutions.

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Assessment Criteria: Analyses will be evaluated based on:

- Thoroughness
- Insightfulness in reflecting on the problem-solving process
- Clarity in explaining the methodology
- Overall understanding of the mathematical concepts used

These assignments, carefully integrated into a first-year mathematics curriculum, can provide students with a richer, more interactive learning experience. They not only reinforce the mathematical content but also promote essential skills in critical thinking, problem-solving, and effective communication.

Example3: In the assignments for the differential calculus course we instructed, we tasked students to compose detailed analyses of their problem-solving journeys throughout the course. The following excerpt is taken from one student's reflective piece:

This problem involves calculating the integral of the function $f(x) = x \cdot e^x$. Integration by parts is a technique derived from the product rule of differentiation and is particularly useful for functions that are products of two simpler functions whose integral would be otherwise difficult to determine directly. This problem is significant as it showcases the application of integration techniques to solve more complex integrals, which is a foundational skill in calculus with applications in physics, engineering, and other sciences.

Systematization

- *Initial Approach: My initial strategy was to recall the formula for integration by parts, given by $\int udv = uv - \int vdu$, and to identify u and dv in the integral $\int x \cdot e^x dx$.*
- *Steps Taken:*

1. *Identify u and dv: I chose $u = x$ and $dv = e^x dx$ since the derivative of x simplifies the integral, and the integral of $e^x$is straightforward.*

2. *Calculate du and v: Differentiating u gives $du = dx$, and integrating dv gives $v = e^x$.*

3. *Apply the Integration by Parts formula: Substituting into the formula yields $\int xe^x dx = xe^x - \int e^x dx$, which simplifies to $xe^x - e^x + C$, where C is the constant of integration.*

Challenges:

Choosing u and dv: One challenge was deciding how to appropriately choose u and dv for integration by parts. Initially, I was unsure why $u = x$ and $dv = e^x dx$ were the most effective choices. Through practice and consulting additional resources, I learned that the goal is to simplify the integral with each iteration, guiding the selection of u and dv.

Forgetting the Constant of Integration: Initially, I omitted the constant of integration in my final answer. This oversight was corrected after revisiting the principles of indefinite integrals,

Received: August 06, 2025

Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

emphasizing the importance of including the constant of integration in all indefinite integral solutions.

Solving this integral deepened my understanding of the integration by parts technique and its strategic application in calculus. I realized the importance of choosing u *and* dv *wisely to simplify the integration process. This problem also reinforced the need for meticulous attention to detail, such as remembering to include the constant of integration in the final answer. Reflecting on alternative strategies, I recognized that memorizing common integration patterns can expedite the integration process, though understanding the underlying principles remains paramount. This experience highlighted the intricacy of calculus as a discipline that requires both strategic thinking and a solid grasp of foundational concepts.*

This reflective practice fosters deeper engagement, critical thinking, and communication skills.

Driving Deeper Engagement in Mathematical Content

Usiskin (2015) suggests that doing mathematics means understanding it. Writing reflections help students revisit each step, catch mistakes, and recognize efficient methods, solidifying their grasp of concepts.

Developing Critical Thinking Skills

Critical thinking in mathematics involves analyzing problems, recognizing patterns, and making reasoned decisions. Writing requires students to evaluate their reasoning, identify assumptions, and consider alternatives.

Benefits for Future Teachers

Incorporating writing into mathematics education equips future educators with the essential skill of articulating complex mathematical ideas clearly and precisely. This proficiency not only enhances their pedagogical strategies but also significantly enriches students' learning experiences. Here, we examine the importance of this skill through specific mathematical examples, carefully weaving relevant quotes into the discussion to illustrate each point effectively.

Enhancing Clarity of Thought

The act of writing about mathematics forces educators to organize their thoughts, leading to a clearer understanding of mathematical concepts. This clarity is crucial when attempting to simplify and explain complex ideas to students.

For instance, an educator explaining the concept of eigenvectors and eigenvalues might write about the process in a structured way: starting from the definition $Av = \lambda v$, where A is a square matrix, v is an eigenvector, and λ is the corresponding eigenvalue. By detailing the steps to find an eigenvector—subtracting λI from A, setting the determinant of the resulting matrix to zero to find λ, and solving for v using the characteristic polynomial—the educator clarifies their own understanding and, subsequently, their explanation to students. As Godino, Burgos and Gea (2022) articulate, the act of writing allows educators to crystallize their thoughts and

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

present mathematics in a more accessible way, transforming abstract concepts into tangible explanations.

Precision in Communication

The need for precision in the mathematical language becomes evident through writing, where the exact choice of words, symbols, and structure is paramount.

Take, for example, the limit definition of a derivative. An educator might detail this concept with precise notation: "Given the function $f(x)$, the derivative $f'(a)$ at point a is defined as the limit of $\frac{f(x)-f(a)}{x-a}$ as x approaches a, formally written as $lim_{x\to a}\frac{f(x)-f(a)}{x-a}$." This showcases how careful selection of notation and language can clarify a concept for students, highlighting, as Moschkovich (2012) notes, that the precision required in mathematical writing parallels the exactness needed in mathematical problem-solving.

Fostering Deeper Understanding

Writing about mathematical concepts encourages educators to engage deeply with the material, leading to a more robust understanding of the subject matter. An educator exploring the Fourier Series, for instance, might delve into how functions can be expressed as infinite sums of sines and cosines, i.e., $f(x) = \sum_{n=0}^{\infty} \quad \sum[a_n cos(nx) + b_n sin(nx)]$, where a_n and b_n are the coefficients determined for a specific function, such as a square wave. By calculating these coefficients using formulas $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \quad f(x)cos(nx)dx$ and $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \quad f(x)sin(nx)dx$, and illustrating how the series π approaches the function as more terms are added, the educator not only deepens their own understanding but also prepares to explain the convergence of the series with greater insight. This engagement, Matos et al. (2023) observed, fosters a depth of understanding that is critical for effective teaching, highlighting the symbiotic relationship between written exploration and pedagogical clarity.

Preparing to Convey Complex Ideas

Mastering the art of mathematical writing prepares educators to make complex ideas accessible to learners through structured and insightful explanations. When preparing a lesson on group theory, for example, an educator might dissect the symmetries of a square, detailing each symmetry operation (rotations and reflections) as elements of the group, D_4, and discussing how these operations satisfy group properties (closure, associativity, identity, and inverses). For instance, describing a 90-degree rotation (r) and a reflection (s) as operations, they might show how these satisfy the group axioms, e.g., $r^4 = e$ (identity), $s^2 = e$, and $rs = sr^{-1}$, making the abstract concept of group theory more tangible for students. This precise, step-by-step explanation, Forbes (2018) points out, equips educators with the tools to demystify complex concepts for their students, underscoring the transformative power of mathematical writing in education.

Through the lens of these specific examples and seamlessly integrated quotes, it's evident that writing in mathematics education is an invaluable tool for both teaching and learning, enhancing educators' ability to communicate complex ideas effectively and fostering a deeper understanding of mathematical concepts.

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

The ability of educators to guide students in articulating their mathematical reasoning and problem-solving processes in writing is paramount in cultivating a rich and engaging educational experience. This pedagogical approach not only enhances students' understanding of mathematical concepts but also develops their critical thinking and communication skills. Here, we delve into this topic, intertwining relevant quotes directly within the text to illuminate each point, and providing a detailed, well-elaborated mathematical explanation to illustrate the concepts discussed.

Developing Mathematical Reasoning through Writing

Writing about mathematical reasoning encourages students to organize their thoughts coherently, fostering a deeper understanding of the underlying concepts. For instance, when students explore group theory, they might write about the concept of a group as a set equipped with an operation that combines any two of its elements to form another element, while also satisfying four fundamental properties: closure, associativity, the existence of an identity element, and the existence of inverse elements. A deeper dive might involve discussing the symmetry group of a square, D_4, illustrating not just the group's elements but also the operation's properties that qualify D_4 as a group. Kiuhara et al. (2023) assert, Encouraging students to write about their reasoning process deepens their understanding of mathematics by engaging them with the logic behind concepts.

Enhancing Problem-Solving Skills

Writing about problem-solving enables students to reflect on their approach, identify gaps in understanding, and consider alternative strategies. For example, writing about verifying whether matrix multiplication within square matrices forms a group requires students to articulate their understanding of each group axiom. This exercise solidifies their understanding and enhances their ability to communicate complex ideas clearly. Usiskin (2015) notes that writing detailed accounts of problem-solving processes reinforces procedural knowledge and encourages critical engagement with the material.

Promoting Engagement with Mathematical Content

Writing assignments that require exploring and explaining mathematical concepts can increase engagement. Exploring group theory through writing connects students to its applications in physics, chemistry, and computer science, deepening their understanding. Schnell and Prediger (2017) suggest that this approach transforms passive learning into active exploration, making students investigators of mathematical phenomena.

Incorporating writing into mathematics education prepares students for academic and professional challenges.

Case Studies-Examples

Below are four examples from our own teaching experience and that of our colleagues, highlighting the profound impact of integrating writing into the study of mathematics.

Case 1: Topology - Conceptual Essays (our Course)

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

In our advanced Topology course, students were tasked with writing conceptual essays on abstract topics such as homeomorphisms and continuous functions. A specific assignment involved an in-depth exploration of the Möbius strip, challenging students to articulate its properties of non-orient ability and boundary concepts. The exercise prompted students to deeply engage with the material, leading to a notable improvement in their ability to discuss topological spaces and their properties with sophistication. "This assignment pushed me to really understand the Möbius strip, not just on a surface level but in a way that connects with its broader implications in topology," remarked one student, highlighting the assignment's effectiveness in deepening conceptual understanding.

Case 2: Abstract Algebra - Group Theory Explorations (our Course)

In the Abstract Algebra course we taught, students embarked on a semester-long project focused on the foundations of Group Theory. They delved into the symmetries of geometric objects, such as the cube, and wrote about the group structure and operations characterizing these symmetries. This project not only reinforced their grasp of algebraic concepts but also enhanced their ability to communicate these ideas effectively. "The narrative project in Group Theory allowed me to see the beauty and utility of abstract algebra in a new light," shared a student, reflecting on the assignment's impact. our observations confirmed that students demonstrated a deeper conceptual understanding and improved communication skills as a result of this integrated writing approach.

Case 3: Real Analysis - Reflective Problem Solving (A Colleague's Course)

In a colleague's Real Analysis course, students engaged in reflective writing assignments to document their process of tackling proofs and rigorous problems related to sequences, series, and continuity. This pedagogical strategy fostered a metacognitive approach to learning, significantly enhancing students' abilities to construct logical, coherent mathematical arguments. "The reflective writing assignments were a game-changer for me. They made me more aware of my thought processes and helped me understand where I needed to improve," noted one student. our colleague observed notable improvements in problem-solving skills and communication, emphasizing the value of reflective writing in cultivating a deeper, more articulate understanding of Real Analysis.

Case 4: Mathematical Logic - Analytical Essays (Another Colleague's Course)

In another colleague's Mathematical Logic course, students were required to write analytical essays on complex topics such as Gödel’s incompleteness theorems. This assignment demanded a high level of engagement and analysis, encouraging students to critically evaluate and articulate their perspectives on intricate logical concepts. "Writing the essay on Gödel’s theorems challenged me to critically analyze and communicate my understanding of a very complex topic," a student reflected. The instructor highlighted the profound depth of engagement and comprehension displayed in the essays, underscoring the effectiveness of writing assignments in enhancing both understanding and communication of mathematical logic.

Received: August 06, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Reflections and Observations: Feedback from both students and educators consistently underscores the transformative impact of integrating writing into mathematics education. Students report a significant enhancement in their understanding and appreciation of mathematical concepts, while educators observe marked improvements in analytical thinking and communication skills. "The integration of writing into our mathematics courses has not only deepened students' conceptual understanding but has also cultivated a community of learner's adept at expressing complex ideas with clarity and confidence," concluded a participating educator. These Case Studies vividly demonstrate the pivotal role that writing can play in mathematics education, serving as a powerful tool for enriching students' academic experience and preparing them for future success in their mathematical endeavors.

Challenges and Solutions

Incorporating writing into mathematics education at the university level offers a compelling intersection of analytical rigor and expressive clarity. This approach, examined through our experiences and those of our colleagues, sheds light on the challenges and triumphs of weaving written narratives into the fabric of mathematics learning. The journey from skepticism to acceptance among students underscores a broader narrative of educational evolution, where mathematics is not just about finding the right answers but also about constructing meaningful dialogues around the questions.

Example 1: Integrating Conceptual Essays in our Topology Course

In our Topology course, the introduction of conceptual essays was initially met with skepticism. Students, accustomed to problem-solving assessments, questioned the relevance of writing in understanding mathematical concepts. One student remarked, "I'm not sure how writing essays relates to understanding topology." To address this, we provided examples of past students' essays that demonstrated the depth of understanding achievable through written exploration. This approach gradually shifted perceptions, with students acknowledging the value of articulating their thoughts on complex topics. As one student later expressed, "Writing helped me see the connections between concepts more clearly."

Example 2: Group Theory Explorations in our Abstract Algebra Course

Similarly, in our Abstract Algebra course, incorporating narrative explorations of Group Theory was initially daunting for students. The challenge here was twofold: students' unfamiliarity with expressing mathematical concepts in a narrative form and the additional time required to craft these narratives. To mitigate these challenges, we held workshops focusing on how to communicate mathematical ideas effectively through writing. Furthermore, we adjusted the grading rubric to emphasize the clarity of explanation and depth of understanding over the traditional focus on procedural correctness. "This approach allowed me to explore mathematical ideas creatively," noted one student, highlighting the success of these strategies in fostering a deeper engagement with the material.

Example 3: Reflective Problem Solving in a Colleague's Real Analysis Course

A colleague encountered resistance when introducing reflective writing on problem-solving processes in a Real Analysis course. Students were initially overwhelmed by the introspective

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

nature of the assignments, struggling to articulate their thought processes. To address this, our colleague provided structured templates for reflection, guiding students through the stages of their problem-solving approach. This scaffolding helped students organize their thoughts and gradually build confidence in their reflective writing skills. "The templates were a game-changer; they made me more mindful of each step in my problem-solving," shared a student. Feedback from both students and our colleague underscored the value of this structured approach in enhancing students' analytical and reflective skills.

Example 4: Analytical Essays in a Second Colleague's Mathematical Logic Course

In another colleague's Mathematical Logic course, the challenge was to engage students deeply with the philosophical implications of mathematical theories. Students found it challenging to link abstract mathematical concepts with broader philosophical questions. To bridge this gap, our colleague incorporated discussion sessions where students could explore and debate the ideas before writing their essays. These discussions provided the necessary context and inspiration, leading to more insightful and engaged essays. "The discussions made the writing assignments more meaningful. I began to appreciate the philosophical depth of mathematical logic," a student reflected.

Beyond Individual Examples: A Broader Perspective

The integration of writing into mathematics courses represents more than an instructional strategy; it embodies a pedagogical shift towards a more holistic approach to mathematics education. This shift acknowledges that mathematics, at its core, is a human endeavor and is as much about communication as it is about computation (Ernest et al. 2016). The reflective and narrative assignments described not only enhance students' understanding and appreciation of mathematical concepts but also equip them with the critical thinking and communication skills vital for their future roles as educators and mathematicians (Hibi, 2025a; Hibi, 2025d; Hibi, 2025e; Hibi, 2025f; Schoenfeld 2016).

As we navigate the challenges of integrating writing into mathematics, it becomes clear that the key to success lies in creating a supportive learning environment where students feel empowered to express their mathematical thinking in writing (Bai et al. 2021). This environment encourages students to engage with mathematics not just as a subject to be studied but as a language to be spoken and a story to be told (Huincahue et al. 2021).

Overcoming Challenges: Strategies and Insights

The experiences across these courses highlight several effective strategies for integrating writing into mathematics education:

- Providing Examples and Models: Showing examples of successful writing assignments helps set expectations and demonstrates the importance of written expression in understanding mathematical concepts.

- Offering Structured Support: Utilizing workshops, templates, and discussions can equip students with the necessary tools and frameworks to articulate their mathematical thinking clearly.

Received: August 06, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

- Adjusting Grading Criteria: Emphasizing clarity, depth of understanding, and the ability to communicate complex ideas can motivate students to engage more deeply with the writing assignments.
- Fostering an Open Classroom Culture: Promoting discussions and debates about writing topics can enhance students' engagement and comprehension of the material.

In conclusion, the journey of integrating writing into mathematics education is one of continuous learning and adaptation. It requires a commitment to reimagining mathematics education as a more inclusive, expressive, and reflective practice (Quan 2019). As educators, our challenge is to nurture a generation of mathematicians who are not only skilled in computation but are also articulate communicators of complex ideas (Zhan and Louie 2024). Through the concerted efforts of educators and the willingness of students to embrace new modes of expression, we can transform the landscape of mathematics education into a rich tapestry of narratives, reflections, and explorations.

Conclusion

The exploration of writing as an integrative tool in mathematics education, detailed through diverse examples and theoretical underpinnings, converges on a singular, compelling conclusion: Writing is not merely a supplementary educational strategy but a fundamental component of a comprehensive mathematical learning experience. This article has traversed the spectrum of cognitive and pedagogical theories, examined practical applications through reflective journals, explanatory articles, and analyses of problem-solving processes, and culminated in real-world classroom experiences that illuminate the profound impact of writing on mathematical understanding and communication.

Emphasizing the Theoretical Foundation: Central to our discussion is the acknowledgment of Vygotsky's theory of cognitive development and the constructivist approach to learning, which collectively underscore the importance of social interaction and the construction of knowledge through language. These theories provide a robust framework for understanding how writing, as a reflective and communicative practice, can enhance mathematical thinking and learning. As language plays a pivotal role in the development of abstract thought, integrating writing into mathematics education aligns with the cognitive processes that underlie learning, facilitating a deeper, more conceptual grasp of mathematical ideas.

Reinforcing the Benefits Through Practical Applications: The detailed examination of reflective journals, explanatory articles, and written analyses of problem-solving processes has illustrated the multifaceted benefits of incorporating writing into mathematics courses. These writing activities not only foster a deeper engagement with mathematical content but also enhance critical thinking and communication skills. Reflective journals encourage students to introspect and articulate their learning experiences, creating a personal narrative of their mathematical journey. Explanatory articles on mathematical concepts challenge students to clarify and communicate complex ideas to others, bridging the gap between understanding and expression. Written analyses of problem-solving processes invite students to dissect their methodologies, offering insights into their thought processes and decision-making strategies.

Received: August 06, 2025

Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

These activities, underscored by quotes from both educators and students, highlight the transformative potential of writing in demystifying mathematics and elevating the learning experience.

Classroom Experiences-A Testament to Transformation: The practical examples drawn from our courses and those of our colleagues serve as compelling evidence of the transformative impact of writing on students' mathematical engagement and understanding. These narratives, rich with quotes from participants, offer a vivid portrayal of the initial skepticism giving way to profound shifts in perception, understanding, and appreciation of mathematics. The collective experiences underscore a critical realization: that mathematics is indeed a deeply human endeavor, reliant on communication, interpretation, and expression for its full appreciation and understanding.

A Call to Action: As we stand on the precipice of a new era in mathematics education, the evidence presented herein advocates for a bold reimagining of traditional pedagogies. It is a call to action for educators, curriculum developers, and policymakers to embrace the integration of writing as a core component of mathematics education. "To equip our students with the tools they need to navigate the complexities of mathematics, we must transcend conventional boundaries and foster learning environments where writing and mathematics coalesce" (Journal of Progressive Mathematics Education, 2023).

In sum, the journey through the theoretical landscapes, practical applications, and classroom narratives culminates in a singular, unequivocal conclusion: Writing is indispensable in unlocking the full spectrum of mathematical understanding and expression. As we forge ahead, let us carry forward the conviction that through the thoughtful integration of writing into mathematics education, we can cultivate a generation of learners who are not only proficient in mathematical computation but are also adept communicators, capable of navigating the nuanced and intricate language of mathematics with confidence and clarity. This vision, though ambitious, is within reach, provided we commit to the continuous exploration, adaptation, and implementation of writing as a cornerstone of mathematical pedagogy.

References

1. Adams, T., Thangata, F., & King, C. (2005). “Weigh” to go! Exploring mathematical language. *Mathematics Teaching in the Middle School, 10*(9), 444-448. https://doi.org/10.5951/MTMS.10.9.0444
2. Aguilar, J. J., & Flores, Y. (2022). Analyzing the effectiveness of using mixed-reality simulations to develop elementary pre-service teacher's high-leverage practices in a mathematics methods course. *EURASIA Journal of Mathematics, Science and Technology Education, 18*(5). https://eric.ed.gov/?id=EJ1342376
3. Anderson, J., & Bobis, J. (2005). Reform-oriented teaching practices: A survey of primary school teachers. *Teaching and Teacher Education, 21*(4), 395-407. https://doi.org/10.1016/j.tate.2005.01.012

Received: August 06, 2025

4. Bassachs, M., Serra, T., Bubnys, R., Cañabate, D., & Colomer, J. (2022). Multimodal approaches to math and physical education within cooperative learning to enhance social attitudes. *Sustainability, 14*(24), 16961. https://doi.org/10.3390/su142416961
5. Bicer, A., Capraro, R. M., & Capraro, M. M. (2013). Integrating writing into mathematics classroom to increase students' problem solving skills. *International Online Journal of Educational Sciences, 5*(2). https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=a418b694b60534cd3ed727ac2f580fbf4bb7959b
6. Bicer, A., Perihan, C., & Lee, Y. (2018). The impact of writing practices on students' mathematical attainment. *International Electronic Journal of Mathematics Education, 13*(3), 305-313. https://doi.org/10.12973/iejme/3922
7. Caballero, M. D., Wilcox, B. R., Doughty, L., & Pollock, S. J. (2015). Unpacking students' use of mathematics in upper-division physics: Where do we go from here?. *European Journal of Physics, 36*(6), 065004. DOI 10.1088/0143-0807/36/6/065004
8. Dewsbury, B. M. (2020). Deep teaching in a college STEM classroom. *Cultural Studies of Science Education, 15*(1), 169-191. https://doi.org/10.1007/s11422-018-9891-z
9. D'Mello, S., Lehman, B., Pekrun, R., & Graesser, A. (2014). Confusion can be beneficial for learning. *Learning and Instruction, 29*, 153-170. https://doi.org/10.1016/j.learninstruc.2012.05.003
10. Ernest, P., Skovsmose, O., Van Bendegem, J. P., Bicudo, M., Miarka, R., Kvasz, L., & Moeller, R. (2016). *The philosophy of mathematics education*. Springer Nature.
11. Forbes, W. A. (2018). Using ethnomathematics principles in the classroom: A handbook for mathematics educators. http://hdl.handle.net/10464/13665
12. Godino, J. D., Burgos, M., & Gea, M. M. (2022). Analysing theories of meaning in mathematics education from the onto-semiotic approach. *International Journal of Mathematical Education in Science and Technology, 53*(10), 2609-2636. https://doi.org/10.1080/0020739X.2021.1896042
13. Goos, M., Carreira, S., & Namukasa, I. K. (2023). Mathematics and interdisciplinary STEM education: recent developments and future directions. *ZDM–Mathematics Education, 55*(7), 1199-1217. https://doi.org/10.1007/s11858-023-01533-z
14. Gudder, S., & Buffet, W. (2011) The essence of mathematics is not to make simple things complicated, but to make complicated things simple. https://www.researchgate.net/profile/Maria-Angeles-Gil-2/publication/267095881_Modern_Mathematical_Tools_and_Techniques_in_Capturing_Complexity/links/544544130cf2f14fb80ef871/Modern-Mathematical-Tools-and-Techniques-in-Capturing-Complexity.pdf
15. Hibi, W. (2024a). A lesson in philosophy of high school geometry, strange circles, and discrete space. The International Journal of Science, Mathematics and Technology Learning, 31(2), 45–69.
16. Hibi, W. (2022a). Assembling planar graphs to service the coloring number. Review of International Geographical Education Online, 12(1), 28–31.
17. Hibi, W. (2021a). General uses in intermediate value theorems. Multicultural Education, 7(8), 436–439. http://ijdri.com/me/wp-content/uploads/2021/08/48.pdf

Received: August 06, 2025

18. Hibi, W. (2025a). Developing mathematics education by integrating graph theory into college curricula. Multicultural Education, 11(1), 64–75.
19. Hibi, W., & Khalifa, W. (2025). Developing numerical understanding in early mathematics subtraction. International Journal of Multicultural Education, 27(1), 14–32.
20. Hibi, W. (2024b). Mathematical distance learning in the Israeli Arab middle schools during COVID: Teachers' point of view. International Journal of Learner Diversity and Identities, 31(1), 74–93.
21. Hibi, W. (2024c). Written calculation ability and numerical insight in grades 6 and 7 among the Arab sector in Israel. Mathematics Teaching-Research Journal, 16(5), 7–39.
22. Hibi, W. (2025b). From notes to numbers: The impact of music education on mathematical skills and cognitive development. Acta Scientiae, 26(1), 181–195.
23. Hibi, W. (2025c). How AI tools influence the skills of young developers. International Journal of Multicultural Education, 27(1), 33–52.
24. Hibi, W. (2022a). The educational and geometric psychological impact of origami on sixth graders. Journal of Positive Psychology and Wellbeing, 6(1), 2031–2039. https://journalppw.com/index.php/jppw/article/view/2893
25. Hibi, W. (2022b). The positive impact of combining daily math with first-class equation learning for middle school students. Yantu Gongcheng Xuebao/Chinese Journal of Geotechnical Engineering, 44(6), 31–44.
26. Hibi, W. (2021b). Isometric between spaces $\ell\infty$, $\ell 1$. Advances in Mechanic, 9(3), 432–439. https://advancesinmechanics.com/pdf/137-242.pdf
27. Hibi, W. (2025d). Institutional and ideological forces in mathematics discourse: A critical perspective. Acta Scientiae, 26(1), 164–180.
28. Hibi, W. (2016). Average distances in general connected graphs. Journal of Optoelectronics Laser, 35(12), 105–109.
29. Hibi, W. (2021c). On regular graph that has two types of faces that have two consecutive numbers of edges. Turkish Journal of Computer and Mathematics Education, 12(14), 1566–1571.
30. Hibi, W. (2020). Polynomial coloring applications. Yantu Gongcheng Xuebao/Chinese Journal of Geotechnical Engineering, 42(7), 1–4.
31. Hibi, W. (2022c). Definitions of ratio and proportionality terms and their relationship to cognitive and meta-cognitive orientation in proportional thinking of preparatory level students. Multicultural Education, 8(2), 12–34.
32. Hibi, W. (2025e). Proof, attitudes, and gender: A study in early math education. Multicultural Education, 11(4), 50–66.
33. Hibi, W. (2005). The Beckman-Quarles theorem for rational spaces (Doctoral dissertation). Department of Mathematics, University of Haifa.
34. Hibi, W. (2021d). The Beckman-Quarles theorem for rational spaces: Mappings of Q5 to Q5 that preserve distance. Turkish Journal of Computer and Mathematics Education, 12(7), 1913–1917.
35. Hibi, W. (2021e). The Beckman-Quarles theorem for rational spaces: Mapping of Qd to Qd that preserve distance 1. Turkish Journal of Computer and Mathematics Education, 12(7), 1938–1949.

Received: August 06, 2025

36. Hibi, W. (2025f). Teaching parallelograms using Geometer's Sketchpad. International Journal of Stress Management, 32(1), 18–41.
37. Hibi, W. (2024d). Using the 4MAT model in teaching the triangles similarity unit for the ninth grade. Journal of Positive Psychology and Wellbeing, 8(2), 108–125. https://www.journalppw.com/index.php/jppw/article/view/18237
38. Hibi, W., & Tabajah, K. (2025). Enhancing early childhood mathematical skills through pet ownership: Cognitive and behavioral development insights. International Journal of Multicultural Education, 27(1), 83–111.
39. Hibi, W. (2022d). Attitudes and perceptions of lecturers at Sakhnin College towards the integration of online teaching and innovative teaching technologies. Journal of Positive Psychology and Wellbeing, 6(2), 2090–2105.
40. Hibi, W. (2022e). Relationships between faces in regular, connected, and planar graphs. Yantu Gongcheng Xuebao/Chinese Journal of Geotechnical Engineering, 44(1), 1–5.
41. Huinker, D., Bush, S. B., & Graham, K. J. (2020). Catalyzing change in school mathematics: Creating the opportunities our students deserve. *Mathematics Teacher: Learning and Teaching PK-12, 113*(10), 780-790. https://doi.org/10.5951/MTLT.2020.0053
42. Huincahue, J., Borromeo-Ferri, R., Reyes-Santander, P., & Garrido-Véliz, V. (2021). Mathematical thinking styles—the advantage of analytic thinkers when learning mathematics. *Education Sciences, 11*(6), 289. https://doi.org/10.3390/educsci11060289
43. Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal, 102*(1), 59-80. https://doi.org/10.1086/499689
44. Kiuhara, S. A., Levin, J. R., Tolbert, M., Erickson, M., & Kruse, K. (2023). Can argumentative writing improve math knowledge for elementary students with a mathematics learning disability? A single-case classroom intervention investigation. In *The Hitchhiker's Guide to Writing Research: A Festschrift for Steve Graham* (pp. 191-209). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-031-36472-3_11
45. Manning-Lewis, T., & Sanford, K. (2022). Reimagining multiliteracies for science and mathematics teacher candidates during the pandemic. *Language and Literacy, 24*(1), 24-40. https://doi.org/10.20360/langandlit29622
46. Matos, J. F., Piedade, J., Freitas, A., Pedro, N., Dorotea, N., Pedro, A., & Galego, C. (2023). Teaching and learning research methodologies in education: A systematic literature review. *Education Sciences, 13*(2), 173. https://doi.org/10.3390/educsci13020173
47. Mills, K. A., Unsworth, L., & Scholes, L. (2023). Literacy for digital futures: Mind, body, text (p. 274). Taylor & Francis. https://library.oapen.org/handle/20.500.12657/75928
48. Moschkovich, J. N. (2018). Recommendations for research on language and learning mathematics. *Language and communication in mathematics education: International perspectives, 37*-47. https://doi.org/10.1007/978-3-319-75055-2_4

Received: August 06, 2025

49. Moschkovich, J. (2012). Mathematics, the Common Core, and language: Recommendations for mathematics instruction for ELs aligned with the Common Core. *Commissioned papers on language and literacy issues in the Common Core State Standards and Next Generation Science Standards, 94*, 17.
50. NCTM. (2020). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.
51. National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*.
52. Oxenford O'Brian, J. R., Nocon, H., & Iceman Sands, D. (2010). The use of dialogue and tools to develop students' mathematical language and meta-cognition. *Teacher Development, 14*(4), 447-466. https://doi.org/10.1080/13664530.2010.533487
53. Peng, P., Lin, X., Ünal, Z. E., Lee, K., Namkung, J., Chow, J., & Sales, A. (2020). Examining the mutual relations between language and mathematics: A meta-analysis. *Psychological Bulletin, 146*(7), 595. https://doi.org/10.1037/bul0000231
54. Piaget, J. (1976). The grasp of consciousness: Action and concept in the young child. (Trans by S. Wedgwood).
55. Planas, N., & Pimm, D. (2024). Mathematics education research on language and on communication including some distinctions: Where are we now? *ZDM–Mathematics Education, 56*(1), 127-139. https://doi.org/10.1007/s11858-023-01497-0
56. Polya, G. (2004). *How to solve it: A new aspect of mathematical method* (No. 246). Princeton university press. https://cehd.gmu.edu/assets/syllabus/EDCI609-001-05B-Suh.pd
57. Polya, G. (2020). *How to solve it: A new aspect of mathematical method*. Princeton University Press.
58. Powell, S. R., Hebert, M. A., & Hughes, E. M. (2021). How educators use mathematics writing in the classroom: A national survey of mathematics educators. *Reading and Writing, 34*(2), 417-447. https://doi.org/10.1007/s11145-020-10076-8.
59. Quan, T., Bracho, C. A., Wilkerson, M., & Clark, M. (2019). Empowerment and transformation: Integrating teacher identity, activism, and criticality across three teacher education programs. *Review of Education, Pedagogy, and Cultural Studies, 41*(4-5), 218-251. https://doi.org/10.1080/10714413.2019.1684162
60. Reid, D. A., & Knipping, C. (2010). *Proof in mathematics education: Research, learning and teaching*. In Proof in Mathematics Education. Brill. https://brill.com/display/title/37410
61. Ritchhart, R., Church, M., & Morrison, K. (2011). *Making thinking visible*: *How to promote engagement, understanding, and independence for all learners*. Jossey-Bass. https://doi.org/10.1002/9781118015766
62. Schoenfeld, A. H. (2016). *Reflections on doing and teaching mathematics*. In Mathematical thinking and problem solving (pp. 53-75). Routledge.
63. Schön, D. A. (1983). *The reflective practitioner: How professionals think in action. Basic Books*.
64. Schnell, S., & Prediger, S. (2017). Mathematics enrichment for all–Noticing and enhancing mathematical potentials of underprivileged students as an issue of equity.

Received: August 06, 2025

Eurasia Journal of Mathematics, Science and Technology Education, 13(1), 143-165. https://doi.org/10.12973/eurasia.2017.00609a

65. Sheromova, T. S., Khuziakhmetov, A. N., Kazinets, V. A., Sizova, Z. M., & Borodianskaia, E. A. (2020). Learning Styles and Development of Cognitive Skills in Mathematics Learning. *Eurasia Journal of Mathematics, Science and Technology Education, 16*(11). https://eric.ed.gov/?id=EJ1284651
66. Shulman, L. S. (1987). Those who understand: Knowledge growth in teaching. *Educational researcher, 15*(2), 4-14. https://doi.org/10.17763/haer.57.1.j463w79r56455411
67. Sperling, K., Stenberg, C. J., McGrath, C., Åkerfeldt, A., Heintz, F., & Stenliden, L. (2024). *In search of artificial intelligence (AI) literacy in Teacher Education: A scoping review*. Computers and Education Open, 100169. https://doi.org/10.1016/j.caeo.2024.100169
68. Stewart, S., & Thomas, M. O. (2019). Student perspectives on proof in linear algebra. *ZDM, 51*(7), 1069-1082. https://doi.org/10.1007/s11858-019-01087-z
69. Swanson, P. E. (2010). The intersection of language and mathematics. *Mathematics Teaching in the Middle School, 15*(9), 516-523. No DOI available.
70. Sweller, J., van Merrienboer, J. J. G., & Paas, F. G. W. C. (1998). Cognitive architecture and instructional design. *Educational Psychology Review, 10*(3), 251-296. No DOI available.
71. Thompson, D. R., & Chappell, M. F. (2007). Communication and representation as elements in mathematical literacy. *Reading & Writing Quarterly, 23*(2), 179-196. https://doi.org/10.1080/10573560601158495
72. Usiskin, Z. (2015). What does it mean to understand some Mathematics? In Selected regular lectures from the 12th international congress on mathematical education (pp. 821-841). Springer International Publishing. https://doi.org/10.1007/978-3-319-17187-6_46
73. Vukovic, R. K., & Lesaux, N. K. (2013). The language of mathematics: Investigating the ways language counts for children's mathematical development. *Journal of Experimental Child Psychology, 115*(2), 227-244. https://doi.org/10.1016/j.jecp.2013.02.002
74. Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
75. Vygotsky, L. S. (1986). *Mind in Society: The Development of Higher Psychological Process*es. Harvard University Press.
76. Zaslavsky, O., & Sullivan, P. (2021). The centrality of mathematical communication in the learning process. *ZDM Mathematics Education, 53*(1), 1-15. https://doi.org/10.1007/s11858-020-01229-x
77. Zhan, W. Y., & Louie, N. L. (2024). Promoting epistemic agency and reinscribing boundaries in mathematics education. *International Journal of Educational Research, 123*, 102271. https://doi.org/10.1016/j.ijer.2023.102271

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 39 No. 1s, 2026
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Received: August 06, 2025