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Adjacent Vertex Reducible Edge Coloring of Selected Graph Classes

N Yathavan [1*], N Paramaguru [2], D Manirathnam [3]

[1] Mathematics, Annamalai University,
Chidambaram - 608002, INDIA.
e-mail: yathavan1508@gmail.com

[2] Mathematics, Annamalai University,
(Deputed to Government Arts College for Women),
Krishnagiri-635002, INDIA.
e-mail: npguru@gmail.com

[1,3] Mathematics, Panimalar Engineering College, Anna University,
Chennai-600123, INDIA.
e-mail: manirathnam10@gmail.com

Abstract

If there is a function $f : E(G) \to \{1, 2, 3, \ldots, K\}$ such that for any two adjacent vertices u and v of the same degree, the sets $S(u)$ and $S(v)$ are equal, where $S(u) = \{f(uv), uv \in E(G)\}$, then a graph $G = (V, E)$ is said to have an adjacent vertex reducible edge coloring. The term "chromatic number of adjacent vertex reducible edge coloring" refers to the highest value of K. The purpose of this study is to ascertain the chromatic number for adja-

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cent vertex reducible edge coloring of specific graphs, including mirror graphs, alternate triangular cycles, prism wheel graphs, double wheel graphs, antiprism graphs, crown (or sun) graphs, degree splitting graphs, and H-graph.

Math. Subject Classification: 05C15
Key Words and Phrases: Edge coloring, AVREC chromatic number, degree splitting graphs, doble wheel graphs. H-graph.

1 Introduction

The most important source of inspiration for coloring is the four-color conjecture problem [1], [2]. A.C. Burries and associates came up with the innovative edge coloring method known as vertex-distinguishing edge coloring [3], [4]. This strategy is founded on the fundamental concepts of the four-color conjecture, which serve as the foundation for the invention. The reducible edge coloring was developed based on the idea of vertex distinguishing edge coloring, which was developed by Zhong Fu Zhang [5], [6]. Jing Wen Li and colleagues have been working on this for a long time. They have published on the neighbouring vertex sum reducible edge coloring of random graphs and several kinds of joint graphs [7]. For a range of joint graph modifications, they introduced adjacent vertex reducible edge coloring in their work [8]. Furthermore, we examine how the edges of certain associative graphs with a distance of two are coloured when they are reducible [9]. Also, we discussed reducible coloring concept for corona product graphs, splitting graphs and cartesian product graphs [10]- [12]. One of the most well-known difficulties in graph theory is figuring out the chromatic number, which can be challenging for some types of graphs. Graph operations, including products, are currently attracting a lot of attention from scholars. The present study aims to investigate this chromatic parameter for various families of graphs. Specifically, we examine its behavior in double vertex graphs, degree splitting graphs, prism wheel graphs, antiprism graphs, crown (or sun) graphs, dou-

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ble wheel graphs, alternate triangular cycles, H-graphs and mirror graphs. Through systematic analysis, we establish the reducible coloring characteristics of these graph classes and provide exact values or tight bounds for their chromatic numbers. The results contribute to a deeper understanding of edge-coloring variations in structured graphs and highlight new directions for future research in reducible coloring problems. This is how the rest of the paper is structured: The formal definition of adjacent vertex reducible edge colouring and exact definitions of the previously stated graphs are given in Section 2. Theorems on their chromatic characteristics are presented in Section 3 together with thorough proofs. A thorough discussion and closing thoughts based on the findings are provided in Section 4.

2 Preliminaries

Let $G = (V, E)$ be a finite, simple, and undirected graph, where $V(G)$ and $E(G)$ denote the vertex set and edge set, respectively. The maximum degree of G is represented by $\Delta(G)$, and the edge chromatic index of G is denoted by $\chi'(G)$. Unless otherwise stated, all graphs considered are connected.

Definition 1. [13] Assume that the graph $G(V, E)$ is simple. For any two adjacent vertices $u, v \in V(G)$, if the degree of u equals the degree of v, there are color sets $S(u) = S(v)$. Where $S(u) = \bigcup_{uw \in E(G)} \{f(uw)\}$, then f is the adjacent vertex reducible edge colouring(AVREC) . This is achieved by mapping $f : E(G) \to \{1, 2, 3, \ldots, K\}$. The adjacent vertex reducible edge coloring chromatic number, or $\chi'_{\text{avrec}}(G) = \max\{K \mid K\text{-AVREC of } G\}$, is the maximum value of K.

Definition 2. [14] The H-graph of path P_n is constructed by taking two identical copies of P_n with vertex sets $\{v_1, v_2, v_3, \ldots, v_n\}$ and $\{u_1, u_2, u_3, \ldots, u_n\}$ and then adding exactly one extra edge between the two paths. This connecting edge joins: $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$, if

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n is odd and $v_{\frac{n}{2}+1}$ to $u_{\frac{n}{2}}$, if n is even.

Definition 3. [15] For $n \geq 3$, let $\{v_0, v_1, v_2, \ldots, v_n\}$ be the vertices of W_n with hub vertex v_0 and $W'_n = \{u_0, u_1, u_2, \ldots, u_n\}$ be a copy of W_n. Define prism(W_n), called the prism of W_n, by $P(W_n) = K_2 \times W_n$, that is joining v_0 of W_n to the corresponding vertex u_0 of W'_n for all $i \in \{1, 2, 3, \ldots, n\}$. Thus,

$$E(P(W_n)) = E(W_n) \cup E(W'_n) \cup \{(u_i v_i) \mid i \in \{1, 2, 3, \ldots, n\}\} \cup \{u_0 v_0\}.$$

Definition 4. [16] The double wheel graph $W_{(n,n)}$ has two cycles of n vertices, and both cycles are connected to a common hub. Let $\{v_1, v_2, \ldots, v_n\}$ be the vertices of one wheel and $\{u_1, u_2, \ldots, u_n\}$ be the vertices of the other wheel, with v_0 as the hub. The edges of the graph are $v_0 v_i$ and $v_i v_{i+1}$ for $i = 1, 2, \ldots, n$ from the first wheel, and $v_0 u_i$ and $u_i u_{i+1}$ for $i = 1, 2, \ldots, n$ from the second wheel. Hence, the graph has $p = 2n + 1$ vertices and $q = 4n$ edges.

Definition 5. [17] The degree splitting graph $DS(G)$ is made by adding a new vertex w_i for each division v_i that has at least two vertices and connecting it to each vertex of v_i.

Definition 6. [18] The crown (or sun) CW_n is a corona of form $C_n \circ K_1$ where $n \geq 3$. That is crown is a helm without central vertex.

Definition 7. [19] An alternate triangular cycle $A(C_{2n})$ is obtained from an even cycle $C_{2n} = \{v_1, u_1, v_2, u_2, \ldots, u_n, v_n\}$ by joining v_i and u_i to a new vertex w_i. That is, every alternate edge of a cycle is replaced by C_3.

Definition 8. [20]-[21] Let $G = (V, E)$ be a graph, and let $G' = (V', E')$ denote an isomorphic copy of G. The mirror graph of G, denoted by $M(G)$, is defined as the graph obtained from the disjoint union of G and G' by adding the set of edges.

Definition 9. [22] The antiprism graph A_n, for $n \geq 3$, consists of inner cycle with vertices $\{u_1, u_2, u_3, \ldots, u_n\}$ and outer cycle with

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vertices $\{v_1, v_2, v_3, \ldots, v_n\}$. These two cycles are connected by the edges $v_i u_i$ and $u_i v_{(i+1)} \pmod{n}$ for $i = 1, 2, 3, \ldots, n$. The antiprism graph is a 4-regular graph with $2n$ vertices and $4n$ edges.

3 Main Results

Theorem 1. *For $n \geq 3$, the degree splitting graph of path (P_n) admits $\chi'_{avrec} DS(P_n) = 3$.*

Proof. Let u_n, $1 \leq i \leq n$, and v_1, v_2 be the degree splitting graph of path P_n. We use the f-coloring rule to color the edges of a path P_n in order to determine its chromatic number. The proof is split into two cases, and we construct a tight constraint using a methodical approach, as seen in the methodology below.

For odd n:

$$f\{u_i v_2\} = 1, i = 2, 3, 4, \ldots, n-1$$

$$f\{u_i u_{(i+1)}\} = \begin{cases} 2, i = 1, 3, 5, \ldots \\ 3, i = 2, 4, 6, \ldots \end{cases}$$

$$f\{u_1 v_n\} = 2$$

$$f\{u_1 v_1\} = 3$$

For even n:

$$f\{u_i v_2\} = 1, i = 2, 3, 4, \ldots, n-3$$

$$f\{u_i u_{(i+1)}\} = \begin{cases} 2, i = 1, 3, 5, \ldots, n-3 \\ 3, i = 2, 4, 6, \ldots, n-4 \end{cases}$$

$$f\{u_{(n-1)} u_n\} = 3$$

$$f\{u_{(n-2)} v_2\} = 3$$

$$f\{u_{(n-2)} u_{(n-1)}\} = 1$$

$$f\{u_{(n-1)} v_2\} = 2$$

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$$f\{u_1v_1\} = 3$$
$$f\{u_nv_1\} = 2$$

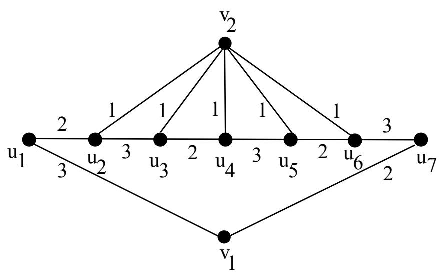

Figure 1: $\chi'_{avrec}DS(P_7) = 3$

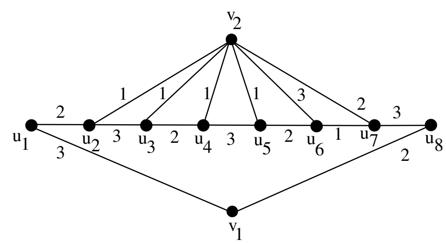

Figure 2: $\chi'_{avrec}DS(P_8) = 3$

The degree splitting graph, $DS(P_n)$, has a maximum degree of $\Delta = n-2$. The vertices of degree three in this graph are next to one another. As a result, the color set $\{1, 2, 3\}$ must be applied to all 3-degree vertices. Additionally, the 2-degree vertices are adjacent to these 3-degree vertices. It is not necessary to add more colors for the 2-degree vertices because the colors $\{1, 2, 3\}$ are already assigned. Thus, $\chi'_{avrec}DS(P_n) = 3$. □

Theorem 2. *For $n \geq 3$, the degree splitting graph of cycle C_n admits $\chi'_{avrec}DS(C_n) = 3$.*

Proof. Let u_i, $1 \leq i \leq n$, and v_1 be the degree splitting graph of cycle C_n. We use the f-coloring rule to colour the edges of a cycle C_n in order to determine its chromatic number. The proof is split into two cases, and we construct a tight constraint using a methodical approach, as seen in the methodology below.

For even n:

$$f\{u_iv_1\} = 1, i = 1, 2, 3, \ldots, n$$
$$f\{u_iu_{(i+1)}\} = \begin{cases} 2, i = 1, 3, 5, \ldots \\ 3, i = 2, 4, 6, \ldots \end{cases}$$

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For odd n:

$$f\{u_i v_1\} = 1, i = 1, 2, 3, \ldots, n-1$$

$$f\{u_i u_{(i+1)}\} = \begin{cases} 2, i = 1, 3, 5, \ldots, n-2 \\ 3, i = 2, 4, 6, \ldots, n-1 \end{cases}$$

$$f\{u_{(n-1)} v_n\} = 3$$

$$f\{u_1 v_1\} = 3$$

$$f\{u_n v_1\} = 2$$

$$f\{u_n u_1\} = 3$$

Only two types of degree vertices which are 3-degree and one-degree, all the 3-degree vertices are adjacent. We should make only 3-degree vertices colors are adjacent. Hence $\chi'_{avrec} DS(C_n) = 3$. □

Theorem 3. $\chi'_{avrec}(K_1 + W_n) = 5$, $n \geq 4$.

Proof. Let the vertices of K_1, W_n be u_0 and $u_1, u_2, u_3, \ldots, u_n$ respectively. Now color the edges of $K_1 + W_n$ by using f coloring rule we get the chromatic number.

$$f\{u_0 u_1\} = 5, \quad f\{u_2 u_n\} = 2,$$

$$f\{u_2 u_0\} = 1,$$

$$f\{u_1 u_i\} = 3, i = 2, 3, 4, \ldots, n$$

$$f\{u_i u_0\} = 2, i = 3, 4, 5, \ldots, n-1$$

$$f\{u_n u_0\} = \begin{cases} 1, & \text{if } n \text{ is even} \\ 4, & \text{if } n \text{ is odd} \end{cases}$$

$$f\{u_i u_{(i+1)}\} = \begin{cases} 1, & i = 3, 5, 7, \ldots \\ 4, & i = 2, 4, 6, \ldots \end{cases}$$

The maximum degree of this graph is $\Delta = n + 1$, not adjacent and same degree vertices. 4-degree vertices are adjacent to each

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other to make same color sets of these vertices it is enough four colors which are $\{1, 2, 3, 4\}$. Also adding one color to the centre of the wheel graph to K_1. Hence $\chi'_{avrec}(K_1 + W_n) = 5$. $\square$

Theorem 4. *The Double wheel graph $W_{(n,n)}$, then* $\chi'_{avrec}(W_{(n,n)}) = 6$, $n \geq 4$.

Proof. We use the f-coloring rule to color the edges of $W_{(n,n)}$ in order to determine its chromatic number. The proof is split into two cases, and we construct a tight constraint using a methodical approach, as seen in the methodology below.

Case 1: For even n

$$f\{v_0 v_i\} = 1, \quad i = 1, 2, 3, \ldots, n$$
$$f\{v_0 u_i\} = 4, \quad i = 1, 2, 3, \ldots, n$$
$$f\{u_i u_{(i+1)}\} = \begin{cases} 5, & i = 1, 3, 5, \ldots \\ 6, & i = 2, 4, 6, \ldots \end{cases}$$
$$f\{v_i v_{(i+1)}\} = \begin{cases} 2, & i = 1, 3, 5, \ldots \\ 3, & i = 2, 4, 6, \ldots \end{cases}$$

Case 2: For odd n

$$f\{v_0 v_i\} = 1, \quad i = 1, 2, 3, \ldots, n-2$$
$$f\{v_0 u_i\} = 4, \quad i = 1, 2, 3, \ldots, n-2$$
$$f\{u_i u_{(i+1)}\} = \begin{cases} 5, & i = 1, 3, 5, \ldots, n-2 \\ 6, & i = 2, 4, 6, \ldots, n-3 \end{cases}$$
$$f\{v_i v_{(i+1)}\} = \begin{cases} 2, & i = 1, 3, 5, \ldots, n-2 \\ 3, & i = 2, 4, 6, \ldots, n-3 \end{cases}$$
$$f\{v_1 v_n\} = 3, \quad f\{v_0 v_n\} = 2,$$
$$f\{u_{(n-1)} u_n\} = 4, \quad f\{v_{(n-1)} v_n\} = 1,$$

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$$f\{u_1u_n\} = 6, \quad f\{u_0u_n\} = 5.$$

The vertex v_0 in the centre has the highest degree, however it is not an adjacent vertex with the same degree. Except for v_0, all other vertices have a degree of 3. It takes color sets $\{1, 2, 3\}$ for $v_1, v_2, v_3, \ldots, v_n$ to have 3-degree vertices that are next to each other. The vertices $u_1, u_2, u_3, \ldots, u_n$ are also 3-degree vertices that are next to one another, and they take color sets $\{4, 5, 6\}$. Because 3-degree vertices u_i are not adjacent to 3-degree vertices v_i, we conclude that $\chi'_{avrec}(W_{(n,n)}) = 6$.

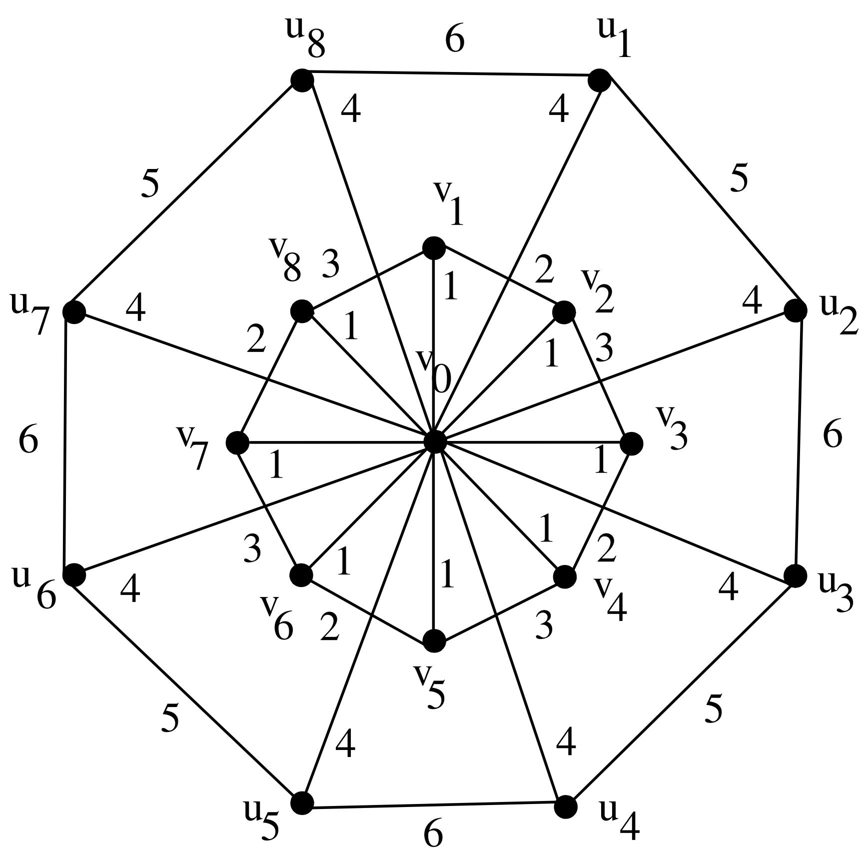

Figure 3: $\chi'_{avrec}(W_{8,8}) = 6$

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The Chromatic number of Prism wheel graphs:

For the Prism wheel graph $P(W_4)$, $\chi'_{avrec}(P(W_4)) = 4$.

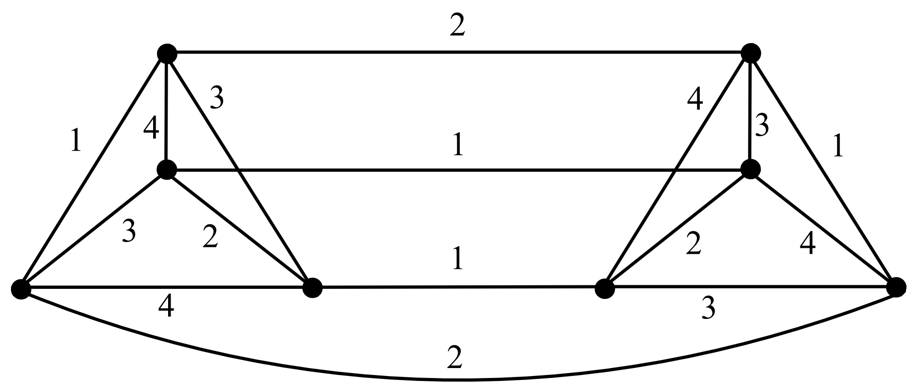

Figure 4: $\chi'_{avrec}(P(W_4)) = 4$

For the Prism wheel graph $P(W_5)$, $\chi'_{avrec}(P(W_5)) = 5$.

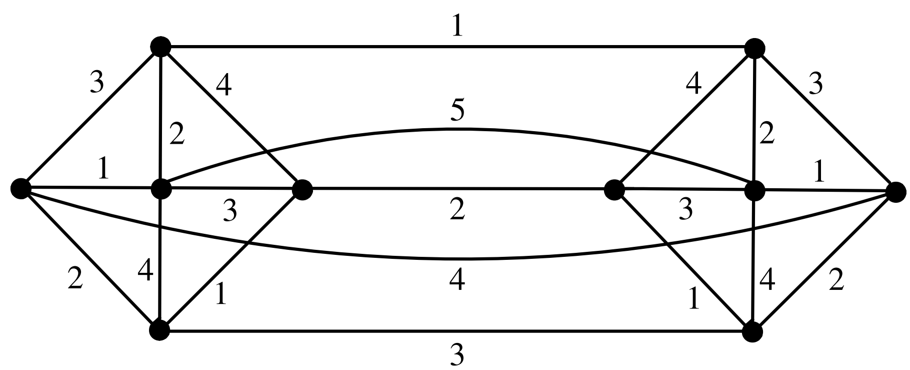

Figure 5: $\chi'_{avrec}(P(W_5)) = 5$

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For $n \geq 6$, $P(W_n)$ has 4-degree vertices except the centre node of both wheel graphs. Moreover, 4-degree vertices are adjacent, so we should make these vertices' color sets the same. As the number of edges increases at the centre node, the required colors also increase, but for all graphs, 4-degree vertices remain stable. Hence, we cannot meet similar color sets for the centre node and 4-degree vertices. □

Theorem 5. *For $n \geq 5$, $\chi'_{avrec}(H) = 7$.*

Proof. Let G be an H-graph; $\{u_i, v_i \mid 1 \leq i \leq n\}$ be the set of vertices and $p = |V| = 2n$ and $q = |E| = 2n - 1$. To determine the chromatic number of the H-graph, we color its edges according to the f-coloring rule. The proof is divided into two cases, and by applying a systematic technique, we establish a tight bound, as outlined in the following algorithm.

Case 1: When n is odd

$$f\{u_{(\frac{n+1}{2})}\} = f\{v_{(\frac{n+1}{2})}\} = \{1, 2, 3\}$$

$$f\{u_1, u_2, u_3, \ldots, u_{(\frac{n+1}{2}-1)}\} = \{2, 4\}$$
$$f\{u_{(\frac{n+1}{2}+1)}, u_{(\frac{n+1}{2}+2)}, u_{(\frac{n+1}{2}+3)}, \ldots, u_n\} = \{3, 5\}$$
$$f\{v_1, v_2, v_3, \ldots, v_{(\frac{n+1}{2}-1)}\} = \{2, 6\}$$
$$f\{v_{(\frac{n+1}{2}+1)}, v_{(\frac{n+1}{2}+2)}, v_{(\frac{n+1}{2}+3)}, \ldots, v_n\} = \{3, 7\}$$

Case 2: When n is even

$$f\{u_{(\frac{n}{2}+1)}\} = f\{v_{(\frac{n}{2})}\} = \{1, 2, 3\}$$

$$f\{u_1, u_2, u_3, \ldots, u_{(\frac{n+1}{2}-1)}\} = \{2, 4\}$$
$$f\{u_{(\frac{n}{2}+2)}, u_{(\frac{n}{2}+3)}, u_{(\frac{n}{2}+4)}, \ldots, u_n\} = \{5, 3\}$$
$$f\{v_1, v_2, v_3, \ldots, v_{(\frac{n}{2}-1)}\} = \{2, 6\}$$
$$f\{v_{(\frac{n}{2}+1)}, v_{(\frac{n}{2}+2)}, v_{(\frac{n}{2}+3)}, \ldots, v_n\} = \{3, 7\}$$

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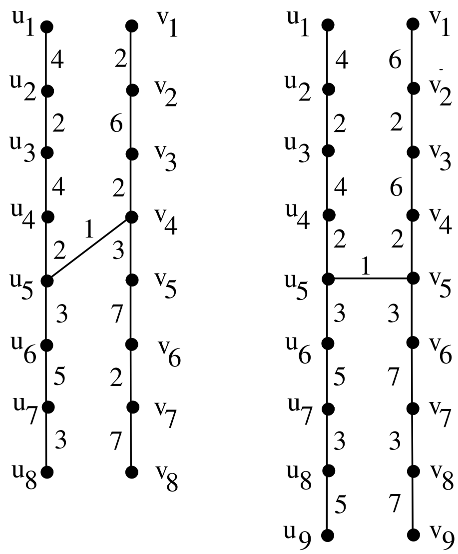

Figure 6: $\chi'_{avrec}(H) = 6, n = 8, \quad \chi'_{avrec}(H) = 7, n = 9$

For all $n \geq 5$, the H-graph contains two vertices of maximum degree, each having degree 3 and being adjacent to one another. These 3-degree vertices are assigned the color set $\{1, 2, 3\}$. In addition to these, the graph also contains vertices of degree 2 and degree 1. The 2-degree vertices are adjacent only at the discontinuities $f\{u_{(\frac{n+1}{2})}\}$ and $f\{v_{(\frac{n+1}{2})}\}$ for odd and $f\{v_{\frac{n}{2}+1}\}$ to $f\{u_{\frac{n}{2}}\}$ for even, which leads to four distinct color sets. This maximizes the chromatic number. Therefore, $\chi'_{avrec}(H) = 7$. □

Result 1: For $n = 3$, $\chi'_{avrec}(H) = 3$.
Result 2: For $n = 4$, $\chi'_{avrec}(H) = 5$.

Theorem 6. *For $n \geq 3$, the antiprism graph A_n admits $\chi'_{avrec}(A_n) = 4$.*

Proof. Let the vertex set of the antiprism graph A_n be $u_i,\ 1 \leq i \leq n$, and $v_j,\ 1 \leq j \leq n$. Color the edges of A_n based on the

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f-coloring rule to get the chromatic number by proposing an algorithm. Here, the proof is divided into two cases using a systematic technique to attain a tight bound, as shown below:

Case 1: For even n

$$f\{u_iu_{i+1}\} = \begin{cases} 1, & i = 1, 3, 5, \ldots \\ 4, & i = 2, 4, 6, \ldots, n-2 \end{cases}$$

$$f\{u_nu_1\} = 4, \quad f\{v_nv_1\} = 4$$

$$f\{u_iv_{j+1}\} = 3, \quad i = 1, 2, 3 \ldots n-1, j = 1, 2, 3, \ldots$$

$$f\{u_iv_j\} = 2, \quad i = j = 1, 2, 3, \ldots$$

$$f\{v_jv_{j+1}\} = \begin{cases} 1, & j = 1, 3, 5, \ldots \\ 4, & j = 2, 4, 6, \ldots, n-2 \end{cases}$$

$$f\{u_nv_1\} = 3$$

Case 2: For odd n

$$f\{u_iu_{i+1}\} = \begin{cases} 1, & i = 1, 3, 5, \ldots, n-2 \\ 4, & i = 2, 4, 6, \ldots \end{cases}$$

$$f\{u_nu_1\} = 3, \quad f\{u_nv_1\} = 2$$

$$f\{u_iv_{j+1}\} = 2, \quad i = 1, 2, 3, \ldots, n-1, j = 1, 2, 3, \ldots$$

$$f\{u_iv_j\} = 3, \quad i = j = 2, 3, 4, \ldots, n-1$$

$$f\{v_jv_{j+1}\} = \begin{cases} 1, & i = 1, 3, 5, \ldots, n-2 \\ 4, & i = 2, 4, 6, \ldots \end{cases}$$

$$f\{u_1v_1\} = 4$$

$$f\{u_nv_n\} = 1$$

$$f\{u_nv_1\} = 3$$

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Since every vertex in the antiprism graph has degree 4, it is a 4-regular graph. Each vertex has cyclic and cross-linked connections with neighboring vertices. According to the f-coloring rule, adjacent vertices of the same degree must have the same color sets. Thus, the antiprism graph requires four distinct colors to achieve the necessary color sets without conflict. Hence, $\chi'_{avrec}(A_n) = 4$.

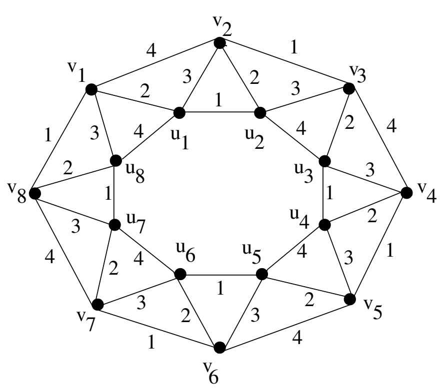

Figure 7: Figure: $\chi'_{avrec}(A_8) = 4$

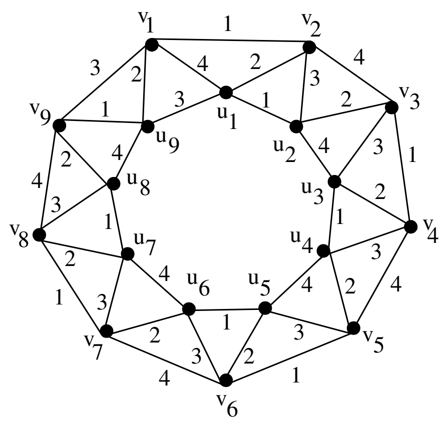

Figure 8: $\chi'_{avrec}(A_9) = 4$

□

Theorem 7. *If n is a positive even integer, the alternate triangular cycle $A(C_{2n})$ satisfies $\chi'_{avrec}(A(C_{2n})) = 3$.*

Proof. The graph $A(C_{2n})$ consists of $2n$ vertices that are connected in an alternating triangular structure, where each vertex is close to one another. The maximum degree of the graph is $\Delta = 3$.

By using the color set $\{1, 2, 3\}$, all 3-degree vertices can be assigned the same color sets according to the f-coloring rule. In this structure, the 2-degree vertices are non-adjacent and can be assigned colors without conflict. Therefore, three colors are sufficient to satisfy the f-coloring condition.

Hence, $\chi'_{avrec}(A(C_{2n})) = 3$.

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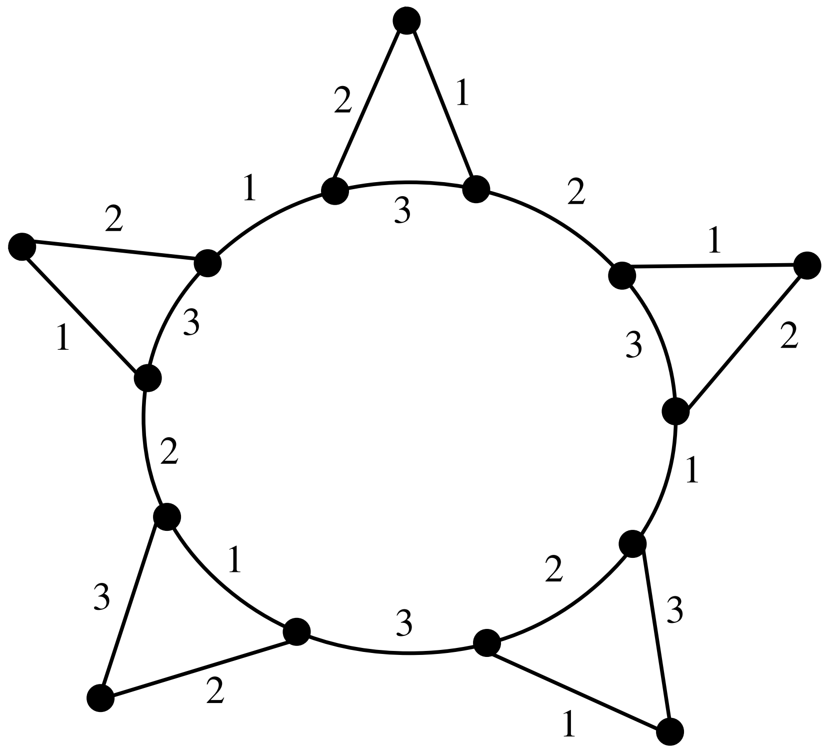

Figure 9: $\chi'_{avrec}(A(C_{10})) = 3$

□

Theorem 8. *For $n \geq 3$, the Mirror graph $M(P_n)$ satisfies $\chi'_{avrec}(M(P_n)) = 5$.*

Proof. Let the vertices of the Mirror graph of G be $v_1, v_2, v_3, \ldots, v_n$ and those of G' be $v'_1, v'_2, v'_3, \ldots, v'_n$. Color the edges of the Mirror graph according to the f-coloring rule as follows:

$$f\{v_i v_{i+1}\} = \begin{cases} 1, & i = 1, 3, 5, \ldots \\ 3, & i = 2, 4, 6, \ldots \end{cases}$$

$$f\{v'_i v'_{i+1}\} = \begin{cases} 1, & i = 1, 3, 5, \ldots \\ 3, & i = 2, 4, 6, \ldots \end{cases}$$

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$$f\{v_i v_i'\} = 2, \quad i = 2, 3, 4, \ldots, n-1$$
$$f\{v_1 v_1'\} = 4$$
$$f\{v_n v_n'\} = 5$$

The vertex set of the mirror graph $M(P_n)$ divides into two degree classes. Vertices in the first class have the highest degree $\Delta = 3$, which arises because each vertex of the original path is connected to its mirror counterpart. Since these degree-3 vertices are adjacent and form the same color sets, three distinct colors are sufficient for an admissible assignment. Vertices of degree 2, which belong to the second class, require two additional colors to maintain distinct coloring for adjacent vertices. By combining the color requirements for both degree classes, the chromatic number of the mirror path graph is obtained as: $\chi'_{avrec}(M(P_n)) = 5$.

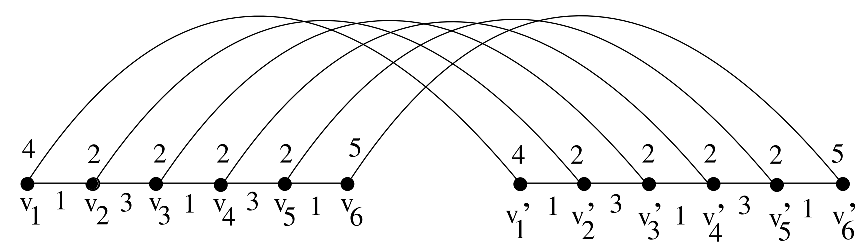

Figure 10: $\chi'_{avrec}(M(P_6)) = 5$

□

Theorem 9. *For $n \geq 3$, the crown or sun graph CW_n satisfies $\chi'_{avrec}(CW_n) = 3$.*

Proof. A corona of form $C_n \circ K_1$, Where $n \geq 3$ is the crown graph. In the sun graph of C_n , let $u_i,\ 1 \leq i \leq n$, and $v_j,\ 1 \leq j \leq n$ be the vertex sets. Using an algorithm, colour the edges of CW_n according to the f colouring rule to determine the chromatic number. Here, we divided the evidence into two cases using the methodical

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approach to achieve a tight constraint, as seen in the methodology that follows.

Case 1: For even n

$$f\{u_i v_j\} = 1, \quad i = j = 1, 2, 3, \ldots$$

$$f\{u_i u_{i+1}\} = \begin{cases} 2, & i = 1, 3, 5, \ldots \\ 3, & i = 2, 4, 6, \ldots, n-2 \end{cases}$$

$$f\{u_n u_1\} = 3$$

Case 2: For odd n

$$f\{u_i v_j\} = 1, \quad i = j = 1, 2, 3, \ldots n-2$$

$$f\{u_i u_{i+1}\} = \begin{cases} 2, & i = 1, 3, 5, \ldots \\ 3, & i = 2, 4, 6, \ldots n-3 \end{cases}$$

$$f\{u_{(n-1)} v_{(n-1)}\} = 3$$

$$f\{u_{(n-1)} u_n\} = 1$$

$$f\{u_n v_n\} = 2$$

$$f\{u_n u_1\} = 3$$

In the sun graph, the maximum degree is $\Delta = 3$, and these vertices are adjacent. To obtain the required color sets for such vertices, it is sufficient to use the three colors $\{1, 2, 3\}$ In addition, the graph contains pendant vertices of degree one, which are not adjacent to each other and therefore do not impose additional coloring constraints. Hence, $\chi'_{avrec}(CW_n) = 3$.

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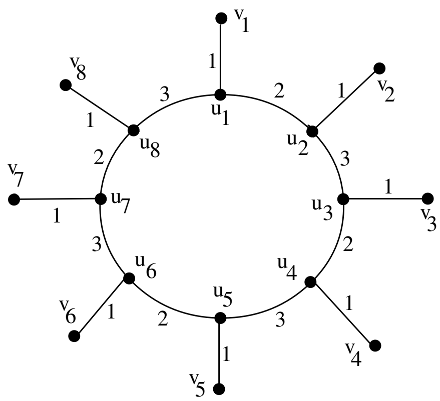

Figure 11: $\chi'_{avrec}(CW_8) = 3$

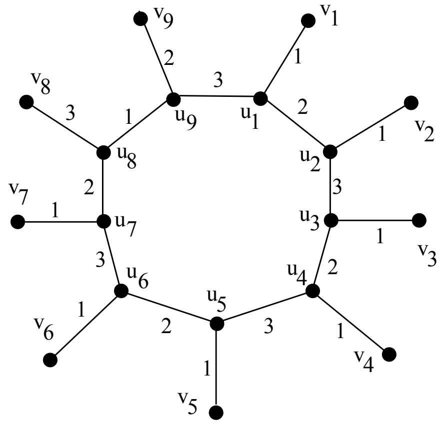

Figure 12: $\chi'_{avrec}(CW_9) = 3$

□

4 Conclusion

In this study, we explored the concept of adjacent vertex reducible edge coloring and determined the corresponding chromatic number for several well-structured graph families. By examining the behavior of edge color assignments under the condition that adjacent vertices of equal degree receive identical incident-edge color sets, we identified the maximum value of K required for each graph class. Our analysis provides explicit chromatic numbers for mirror graphs, alternate triangular cycles, prism wheel graphs, double wheel graphs, antiprism graphs, crown (sun) graphs, degree-splitting graphs, and H-graphs. These findings contribute to a deeper understanding of constrained edge colorings and highlight how structural properties influence color requirements. The results not only extend existing knowledge on specialized graph colorings but also open avenues for further investigation into broader families of graphs and potential algorithmic approaches for determining adjacent vertex reducible edge chromatic numbers in complex networks.

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