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THE CARTESIAN AND COMPOSITION PRODUCTS OF DISTANCE-HEREDITARY FUZZY GRAPHS

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Abstract

Fuzzy graphs serve as a valuable tool in mathematics, enabling professionals to effectively depict fuzzy relationships among various entities. The inherent property of fuzziness is advantageous in numerous contexts, helping to forecast and address issues. Distance-Conserving Graphs are connected graphs distinguished by the property that all induced paths preserve isometry. Many real-world problems are complex and frequently involve ambiguous information. By integrating Distance-Hereditary graphs with fuzzy sets, we introduce a novel concept. In this paper, we thoroughly examine and introduce operations on Distance-Hereditary Fuzzy Graph, which will explore cartesian and composition products related to Distance-Hereditary Fuzzy Graphs.

Keywords Fuzzy graph, Distance-Hereditary graph, Distance-Hereditary Fuzzy graph, Strong D-HFG, Weak D-HFG.

Introduction

In 1965, L. A. Zadeh introduced fuzzy set theory, representing a pivotal advancement in handling uncertainty and improving decision-making processes[5]. Throughout the 1970s, the idea of distance-hereditary graphs emerged during extensive investigations into graph classes defined by unique structural characteristics[2]. The feature of being distance-hereditary means that for any two vertices, the length of the shortest path is consistent in both the graph and any connected induced subgraph that contains them[4]. This property of preserving distances highlights the significance of distance-hereditary graphs as an intriguing and important subject in the field of graph theory.

In 1977, Howorka[4] offered the initial formal definition of distance-hereditary graphs, highlighting their key characteristics and structural details. This seminal research laid the groundwork for further theoretical and computational inquiries into these graphs. Significant advancements were made during the 1980s and 1990s as researchers examined various algorithmic approaches and classification systems relating to distance-hereditary graphs.

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Notably, Bandelt and Mulder's revelation of an efficient recognition technique sparked heightened research interest in the algorithmic capabilities of these graphs[3]. A major advancement was the creation of a recognition algorithm that operates in linear time, demonstrating the practical usefulness of distance-hereditary graphs in computational contexts. In 2021, Prasanna A and Nishad T M[1] presented weak fuzzy graphs. In 1975, Rosenfeld[6], together with Yeh and Banh, conducted groundbreaking research that investigated the application of fuzzy sets, laying the groundwork for Fuzzy Graphs (FGs).

Sunil M.P. and Suresh Kumar[7] explore the characteristics of HFGs, particularly the behavior of the cartesian product across different types of HFGs, in their study titled On Intuitionistic hesitancy fuzzy graphs. They also highlight the advantages of HFGs for decision-making and problem-solving in the context of mergers.

In this study, we explore the complex characteristics of distance hereditary fuzzy graphs, with a particular emphasis on the cartesian and composition products, accompanied by examples. These products provide a detailed insight into network dynamics, illustrating how uncertainty affects connectivity patterns and structural attributes.

II Preface

Definition 1

Let V_t represent a nonempty set. **The structure of a fuzzy graph is given by $F_G(\varrho, \varpi)$, in which ϱ is a fuzzy subset of a vertex set V_t, and ϖ is a symmetric fuzzy relation defined over ϱ .** Specifically, $\varrho: V_t \to [0,1]$ and $\varpi: V_t \times V_t \to [0,1]$ satisfying the condition that $\varpi(\hat{g}, \varsigma) \leq \varrho(\hat{g}) \wedge \varrho(\hat{h})$ for all elements υ, ς in V_t, where $\wedge$ stands for minimum. The foundational crisp graph related to the fuzzy graph $F_G = (\varrho, \varpi)$ is denoted as $F_G{}^* = (\varrho^*, \varpi)$ where ϱ^* is referred to as the nonempty set V_t of vertices and $\varpi = E_d \subseteq V_t \times V_t$.

Definition 2

The crisp graph (V_t, E_d) is a specific instance of the fuzzy graph F_G with every vertex and edge in (V_t, E_d) possesses a membership degree of 1.

Definition 3

A fuzzy graph $F_g: (\varrho, \varpi)$ is considered a strong fuzzy graph if $\varpi(\hat{g}, \hat{h}) = \varrho(\hat{g}) \wedge \varrho(\hat{h})$ for every $(\hat{g}, \hat{h})$ in ϖ^*.

Definition 4

A fuzzy graph $F_g: (\omega, \varphi)$ is considered a weak fuzzy graph if $\varphi(\hat{g}, \hat{h}) < \omega(\hat{g}) \wedge \omega(\hat{h})$ for every $(\hat{g}, \hat{h})$ in ϖ^*.

Distance-Hereditary Fuzzy Graph

Definition 5

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A Distance-Hereditary Fuzzy Graph is represented as $\boldsymbol{F_G = (V_t, E_d)}$, where $\boldsymbol{v}$ is the set of vertices (i) $\boldsymbol{V_t} = \{\hat{h}_1, \hat{h}_2, \dots, \hat{h}_n\}$ such that $\boldsymbol{\zeta_1, \xi_1, \kappa_1, \lambda_1} \to [\boldsymbol{0}, \boldsymbol{1}]$, indicate the degree of membership, nonmembership, Distance-Hereditary membership and Distance-Hereditary nonmembership for each vertex $\hat{h}_i \in \boldsymbol{V_t}$ respectively. It is required that for any four vertices $\hat{h}_1$, $\hat{h}_2$,$\hat{h}_3$,$\hat{h}_4$ at least two of the following sums hold true:

$$\boldsymbol{d_1} = \boldsymbol{d}_{\hat{h}_1\hat{h}_2} + \boldsymbol{d}_{\hat{h}_3\hat{h}_4}$$

$$\boldsymbol{d_2} = \boldsymbol{d}_{\hat{h}_1\hat{h}_3} + \boldsymbol{d}_{\hat{h}\boldsymbol{v_4}}$$

$$\boldsymbol{d_3} = \boldsymbol{d}_{\hat{h}_1\hat{h}_4} + \boldsymbol{d}_{\hat{h}_2\hat{h}_3}$$

where $\boldsymbol{d}_{\hat{h}_i\hat{h}_j}$ denotes the distance in the graph from vertex $\hat{h}_i$ to $\hat{h}_j$

and it must also be true that $\boldsymbol{0} \le \boldsymbol{\zeta_1}(\hat{h}_i) + \boldsymbol{\xi_1}(\hat{h}_i) + \boldsymbol{\kappa_1}(\hat{h}_i) + \boldsymbol{\lambda_1}(\hat{h}_i) \le \boldsymbol{1}$ for each $\hat{h}_i \in \boldsymbol{V_t}$

(ii) $\boldsymbol{E_d} \subseteq \boldsymbol{V_t} \times \boldsymbol{V_t}$ where $\boldsymbol{\zeta_2, \xi_2, \kappa_2, \lambda_2}: \boldsymbol{V_t} \times \boldsymbol{V_t} \to [\boldsymbol{0}, \boldsymbol{1}]$ must satisfy the conditions:

$$\boldsymbol{\zeta_2}(\hat{h}_i, \hat{h}_j) \le \boldsymbol{\zeta_1}(\hat{h}_i) \wedge \boldsymbol{\zeta_1}(\hat{h}_j)$$

$$\boldsymbol{\xi_2}(\hat{h}_i, \hat{h}_j) \le \boldsymbol{\xi_1}(\hat{h}_i) \vee \boldsymbol{\xi_1}(\hat{h}_j)$$

$$\boldsymbol{\kappa_2}(\hat{h}_i, \hat{h}_j) \le \boldsymbol{\kappa_1}(\hat{h}_i) \wedge \boldsymbol{\kappa_1}(\hat{h}_j)$$

$$\boldsymbol{\lambda_2}(\hat{h}_i, \hat{h}_j) \le \boldsymbol{\lambda_1}(\hat{h}_i) \vee \boldsymbol{\lambda_1}(\hat{h}_j)$$

and for every $(\hat{h}_i, \hat{h}_j) \in \boldsymbol{E_d}$ it must hold that $\boldsymbol{0} \le \boldsymbol{\zeta}(\hat{h}_i) + \boldsymbol{\xi_2}(\hat{h}_i) + \boldsymbol{\kappa_2}(\hat{h}_i) + \boldsymbol{\lambda_2}(\hat{h}_i) \le \boldsymbol{1}$.

Definition 6

A Distance-Hereditary Fuzzy Graph, represented as $\boldsymbol{F_G = (V_t, E_d)}$ is referred to as $\boldsymbol{\zeta}$ -strong D-HFG if $\boldsymbol{\zeta_2}(\hat{h}_i, \hat{h}_j) = \boldsymbol{\zeta_1}(\hat{h}_i) \wedge \boldsymbol{\zeta_1}(\hat{h}_j)$ for all $(\hat{h}_i, \hat{h}_j)$.

Definition 7

A Distance-Hereditary Fuzzy Graph, represented as $\boldsymbol{F_g = (V_t, E_d)}$ is referred to as $\boldsymbol{\xi}$-strong D-HFG if $\boldsymbol{\xi_2}(\hat{h}_i, \hat{h}_j) = \boldsymbol{\xi_1}(\hat{h}_i) \vee \boldsymbol{\xi_1}(\hat{h}_j)$ for all $(\hat{h}_i, \hat{h}_j)$.

Definition 8

A Distance-Hereditary Fuzzy Graph, represented as $\boldsymbol{F_G = (V_t, E_d)}$ is referred to as $\boldsymbol{\kappa}$-strong D-HFG if $\boldsymbol{\kappa_2}(\hat{h}_i, \hat{h}_j) = \boldsymbol{\kappa_1}(\hat{h}_i) \vee \boldsymbol{\kappa_1}(\hat{h}_j)$ for all $(\hat{h}_i, \hat{h}_j)$.

Definition 9

A Distance-Hereditary Fuzzy Graph, represented as $\boldsymbol{F_G = (V_t, E_d)}$ is referred to as $\boldsymbol{\lambda}$-strong D-HFG if $\boldsymbol{\lambda}(\hat{h}_i, \hat{h}_j) = \boldsymbol{\lambda_1}(\hat{h}_i) \vee \boldsymbol{\lambda_1}(\hat{h}_j)$ for all $(\hat{h}_i, \hat{h}_j)$.

Definition 10

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A Distance-Hereditary Fuzzy Graph, represented as $F_G = (V_t, E_d)$ is referred to as strong D-HFG if,

$\boldsymbol{\zeta}_2(\hat{h}_i, \hat{h}_j) = \boldsymbol{\zeta}_1(\hat{h}_i) \wedge \boldsymbol{\zeta}_1(\hat{h}_j)$

$\boldsymbol{\xi}_2(\hat{h}_i, \hat{h}_j) = \boldsymbol{\xi}_1(\hat{h}_i) \vee \boldsymbol{\xi}_1(\hat{h}_j)$

$\boldsymbol{\kappa}_2(\hat{h}_i, \hat{h}_j) = \boldsymbol{\kappa}_1(\hat{h}_i) \wedge \boldsymbol{\kappa}_1(\hat{h}_j)$

$\boldsymbol{\lambda}_2(\hat{h}_i, \hat{h}_j) = \boldsymbol{\lambda}_1(\hat{h}_i) \vee \boldsymbol{\lambda}_1(\hat{h}_j)$

Definition 11

A Distance-Hereditary Fuzzy Graph, represented as $\boldsymbol{F_G = (V_t, E_d)}$ is referred to as $\boldsymbol{\zeta}$-weak D-HFG if $\boldsymbol{\zeta_2(\hat{h}_i, \hat{h}_j) < \zeta_1(\hat{h}_i) \wedge \zeta_1(\hat{h}_j)}$ for all $(\boldsymbol{\hat{h}_i, \hat{h}_j})$.

Definition 12

A Distance-Hereditary Fuzzy Graph, represented as $\boldsymbol{F_g = (V_t, E_d)}$ is referred to as $\boldsymbol{\xi}$-weak D-HFG if $\boldsymbol{\xi_2(\hat{h}_i, \hat{h}_j) < \xi_1(\hat{h}_i) \vee \xi_1(\hat{h}_j)}$ for all $(\boldsymbol{\hat{h}_i, \hat{h}_j})$.

Definition 13

A Distance-Hereditary Fuzzy Graph, represented as $\boldsymbol{F_G = (V_t, E_d)}$ is referred to as $\boldsymbol{\kappa}$-weak D-HFG if $\boldsymbol{\kappa_2(\hat{h}_i, \hat{h}_j) < \kappa_1(\hat{h}_i) \vee \kappa_1(\hat{h}_j)}$ for all $(\boldsymbol{\hat{h}_i, \hat{h}_j})$.

Definition 14

A Distance-Hereditary Fuzzy Graph, represented as $\boldsymbol{F_G = (V_t, E_d)}$ is referred to as $\boldsymbol{\lambda}$-weak D-HFG if $\boldsymbol{\lambda(\hat{h}_i, \hat{h}_j) < \lambda_1(\hat{h}_i) \vee \lambda_1(\hat{h}_j)}$ for all $(\boldsymbol{\hat{h}_i, \hat{h}_j})$.

Definition 15

A Distance-Hereditary Fuzzy Graph, represented as $F_G = (V_t, E_d)$ is referred to as strong D-HFG if,

$\boldsymbol{\zeta}_2(\hat{h}_i, \hat{h}_j) < \boldsymbol{\zeta}_1(\hat{h}_i) \wedge \boldsymbol{\zeta}_1(\hat{h}_j)$

$\boldsymbol{\xi}_2(\hat{h}_i, \hat{h}_j) < \boldsymbol{\xi}_1(\hat{h}_i) \vee \boldsymbol{\xi}_1(\hat{h}_j)$

$\boldsymbol{\kappa}_2(\hat{h}_i, \hat{h}_j) < \boldsymbol{\kappa}_1(\hat{h}_i) \wedge \boldsymbol{\kappa}_1(\hat{h}_j)$

$\boldsymbol{\lambda}_2(\hat{h}_i, \hat{h}_j) < \boldsymbol{\lambda}_1(\hat{h}_i) \vee \boldsymbol{\lambda}_1(\hat{h}_j)$

The Cartesian Product of Strong and Weak Distance-Hereditary Fuzzy Graph

In this part, we prove cartesian product theorems of Strong and Weak Distance-Hereditary Fuzzy Graph.

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Definition 16 The cartesian product of two D-HFGs, $F_{G1} = (U, E_{\hat{g}}, \varrho, \varpi)$,$F_{G2} = (V, E_{\hat{h}}, \varrho', \varpi')$ where $\varrho = (\zeta_1, \xi_1, \kappa_1, \lambda_1)$, $\varpi = (\zeta_2, \xi_2, \kappa_2, \lambda_2)$, $\varrho' = (\zeta_1', \xi_1', \kappa_1', \lambda_1')$, $\varpi' = (\zeta_2', \xi_2', \kappa_2', \lambda_2')$ is the D-HFG $F_G = F_{G1} \times F_{G2} = (U \times V, E_{\hat{g}} \times E_{\hat{h}}, \varrho \times \varrho', \varpi \times \varpi')$, $E = E_{\hat{g}} \times E_{\hat{h}}$ such that

$$E_{\hat{g}} = \{\hat{w}: \hat{g}_1 = \hat{g}_2, \hat{w}_2 \in E_{\hat{h}}\}$$

$$E_{\hat{h}} = \{\hat{w}: \hat{h}_1 = \hat{h}_2, \hat{w}_1 \in E_{\hat{g}}\}$$

where, $\hat{w} = ((\hat{g}_1, \hat{h}_1), (\hat{g}_2, \hat{h}_2))$, $w_1 = (\hat{g}_1, \hat{g}_2)$, $w_2 = (\hat{h}_1, \hat{h}_2)$.

$$(\zeta_1 \times \zeta_1')(\hat{j}) = \zeta_1(\hat{g}) \wedge \zeta_1'(\hat{h})$$

$$(\xi_1 \times \xi_1')(\hat{j}) = \xi_1(\hat{g}) \wedge \xi_1'(\hat{h})$$

$$(\kappa_1 \times \kappa_1')(\hat{j}) = \kappa_1(\hat{g}) \wedge \kappa_1'(\hat{h})$$

$$(\lambda_1 \times \lambda_1')(\hat{j}) = \lambda_1(\hat{g}) \wedge \lambda_1'(\hat{h})$$

Where $\hat{j} = (\hat{g}, \hat{h})$

$$(\zeta_2 \times \zeta_2')(\hat{w}) = \begin{cases} \zeta_1(\hat{g}_1) \wedge \zeta_2'(\hat{w}_2), if\ \hat{w} \in E_{\hat{g}} \\ \zeta_1'(\hat{h}_1) \wedge \zeta_2(\hat{w}_1), if\ \hat{w} \in E_{\hat{h}} \end{cases}$$

$$(\xi_2 \times \xi_2')(\hat{w}) = \begin{cases} \xi_1(\hat{g}_1) \vee \xi_2'(\hat{w}_2), if\ \hat{w} \in E_{\hat{g}} \\ \xi_1'(\hat{h}_1) \vee \xi_2(\hat{w}_1), if\ \hat{w} \in E_{\hat{h}} \end{cases}$$

$$(\kappa_2 \times \kappa_2')(\hat{w}) = \begin{cases} \gamma_1(\hat{g}_1) \wedge \gamma_2'(\hat{w}_2), if\ \hat{w} \in E_{\hat{g}} \\ \gamma_1'(\hat{h}_1) \wedge \gamma_2(\hat{w}_1), if\ \hat{w} \in E_{\hat{h}} \end{cases}$$

$$(\lambda_2 \times \lambda_2')(\hat{w}) = \begin{cases} \delta_1(\hat{g}_1) \vee \delta_2'(\hat{w}_2), if\ \hat{w} \in E_{\hat{g}} \\ \delta_1'(\hat{h}_1) \vee \delta_2(\hat{w}_1), if\ \hat{w} \in E_{\hat{h}} \end{cases}$$

Theorem 1 If F_{G1} and F_{G2} are two strong D-HFGs, then their cartesian product $F_{G1} \times F_{G2}$ is also a strong D-HFG.

Proof: Let F_{G1} , F_{G2} be two strong D-HFGs. Then, for $\hat{w}_1 \in E_{\hat{g}}$, $\hat{w}_2 \in E_{\hat{h}}$,

$$\zeta_2(\hat{w}_1) = \zeta_1(\hat{g}_1) \wedge \zeta_1(\hat{g}_2), \zeta_2'(\hat{w}_1) = \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2)$$

$$\xi_2(\hat{w}_1) = \xi_1(\hat{g}_1) \wedge \xi_1(\hat{g}_2), \xi_2'(\hat{w}_1) = \xi_1'(\hat{h}_1) \wedge \xi_1'(\hat{h}_2)$$

$$\kappa_2(\hat{w}_1) = \kappa_1(\hat{g}_1) \wedge \kappa_1(\hat{g}_2), \kappa_2'(\hat{w}_1) = \kappa_1'(\hat{h}_1) \wedge \kappa_1'(\hat{h}_2)$$

$$\lambda_2(\hat{w}_1) = \lambda_1(\hat{g}_1) \wedge \lambda_1(\hat{g}_2), \lambda_2'(\hat{w}_1) = \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2)$$

Case(i) when $\hat{w} \in E_{\hat{g}}$

$$(\zeta_2 \times \zeta_2')(\hat{w}) = \zeta_1(\hat{g}_1) \wedge \zeta_2'(\hat{w}_2)$$

$$= \zeta_1(\hat{g}) \wedge \zeta_1(\hat{g}_2) \wedge \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2), \text{since } \hat{g}_1 = \hat{g}_2$$

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$$= (\zeta_1 \times \zeta_1')(\hat{g}_1, \hat{h}_1) \wedge (\zeta_1 \times \zeta_1')(\hat{g}_2, \hat{h}_2)$$

$$(\xi_2 \times \xi_2')(\hat{w}) = \xi_1(\hat{g}_1) \wedge \xi_2'(\hat{w}_2)$$

$$= \xi_1(\hat{g}_1) \wedge \xi_1(\hat{g}_2) \wedge \xi_1'(v_1) \wedge \xi_1'(v_2), \text{ since } \hat{g}_1 = \hat{g}_2$$

$$= (\xi_1 \times \xi_1')(\hat{g}_1, \hat{h}_1) \wedge (\xi_1 \times \xi_1')(\hat{g}_2, \hat{h}_2)$$

$$(\kappa_2 \times \kappa_2')(\hat{w}) = \kappa_1(\hat{g}_1) \wedge \kappa_2'(\hat{w}_2)$$

$$= \kappa_1(\hat{g}_1) \wedge \kappa_1(\hat{g}_2) \wedge \kappa_1'(v_1) \wedge \kappa_1'(v_2), \text{ since } \hat{g}_1 = \hat{g}_2$$

$$= (\kappa_1 \times \kappa_1')(\hat{g}_1, \hat{h}_1) \wedge (\kappa_1 \times \kappa_1')(\hat{g}_2, \hat{h}_2)$$

$$(\lambda_2 \times \lambda_2')(\hat{w}) = \lambda_1(\hat{g}_1) \wedge \lambda_2'(\hat{w}_2)$$

$$= \lambda_1(\hat{g}_1) \wedge \lambda_1(\hat{g}_2) \wedge \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2), \text{ since } \hat{g}_1 = \hat{g}_2$$

$$= (\lambda_1 \times \lambda_1')(\hat{g}_1, \hat{h}_1) \wedge (\lambda_1 \times \lambda_1')(\hat{g}_2, \hat{h}_2)$$

Case(ii) When $\hat{w} \in E_{\hat{h}}$

$$(\zeta_2 \times \zeta_2')(\hat{w}) = \zeta_1'(\hat{h}) \wedge \zeta_2(\hat{w}_1)$$

$$= \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2) \wedge \zeta_1(\hat{g}_1) \wedge \zeta_1(\hat{g}_2), \text{ since } \hat{h}_1 = \hat{h}_2$$

$$= (\zeta_1 \times \zeta_1')(\hat{g}_1, \hat{h}_1) \wedge (\zeta_1 \times \zeta_1')(\hat{g}_2, \hat{h}_2)$$

$$(\xi_2 \times \xi_2')(\hat{w}) = \xi_1'(\hat{h}_1) \wedge \xi_2(\hat{w}_1)$$

$$= \xi_1'(\hat{h}_1) \wedge \xi_1'(\hat{h}_2) \wedge \xi_1(\hat{g}_1) \wedge \xi_1(\hat{g}_2), \text{ since } \hat{h}_1 = \hat{h}_2$$

$$= (\xi_1 \times \xi_1')(\hat{g}_1, \hat{h}_1) \wedge (\xi_1 \times \xi_1')(\hat{g}_2, \hat{h}_2)$$

$$(\kappa_2 \times \kappa_2')(\hat{w}) = \kappa_1'(\hat{h}_1) \wedge \kappa_2(\hat{w}_1)$$

$$= \kappa_1'(\hat{h}_1) \wedge \kappa_1'(\hat{h}_2) \wedge \kappa_1(\hat{g}_1) \wedge \kappa_1(\hat{g}_2), \text{ since } \hat{h}_1 = \hat{h}_2$$

$$= (\kappa_1 \times \kappa_1')(\hat{g}_1, v_1) \wedge (\kappa_1 \times \kappa_1')(\hat{g}_2, \hat{h}_2)$$

$$(\lambda_2 \times \lambda_2')(\hat{w}) = \lambda_1'(\hat{h}_1) \wedge \lambda_2(\hat{w}_1)$$

$$= \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2) \wedge \lambda_1(\hat{g}_1) \wedge \lambda_1(\hat{g}_2), \text{ since } \hat{h}_1 = \hat{h}_2$$

$$= (\lambda_1 \times \lambda_1')(\hat{g}_1, \hat{h}_1) \wedge (\lambda_1 \times \lambda_1')(\hat{g}_2, \hat{h}_2)$$

Thus, $F_G = F_{G1} \times F_{G2}$ is a strong D-HFG.

Theorem 2 If F_{G1}, F_{G2} are two D-HFGs such that $F_{G1} \times F_{G2}$ is strong, then at least one of F_{G1}or F_{G2} will be strong.

Proof: Assume that the two D-HFGs F_{G1}, F_{G2} are weak. Then there exists at least one $\hat{w}_1 \in E_{\hat{g}}$, $\hat{w}_2 \in E_{\hat{h}}$, with

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$\zeta_2(\hat{w}) < \zeta_1(\hat{g}_1) \wedge \zeta_1(\hat{g}_2), \zeta_2'(\hat{w}_1) < \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2)$,

$\xi_2(\hat{w}_1) < \xi_1(\hat{g}_1) \wedge \xi_1(\hat{g}_2), \xi_2'(\hat{w}_1) < \xi_1'(\hat{h}_1) \wedge \xi_1'(\hat{h}_2)$,

$\kappa_2(\hat{w}_1) < \kappa_1(\hat{g}_1) \wedge \kappa_1(\hat{g}_2), \kappa_2'(\hat{w}_1) < \kappa_1'(\hat{h}_1) \wedge \kappa_1'(\hat{h}_2)$,

$\lambda_2(\hat{w}_1) < \lambda_1(\hat{g}_1) \wedge \lambda_1(\hat{g}_2), \lambda_2'(\hat{w}_1) < \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2)$.

Let $\hat{w} \in E_1$. Then,

$$\begin{aligned}(\zeta_2 \times \zeta_2')(\hat{w}) &= \zeta_1(\hat{g}_1) \wedge \zeta_2'(\hat{w}_2) \\ &< \zeta_1(\hat{g}_1) \wedge \zeta_1(\hat{g}_2) \wedge \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2), \text{since } \hat{g}_1 = \hat{g}_2 \\ &= (\zeta_1 \times \zeta_1')(\hat{g}_1, \hat{h}_1) \wedge (\zeta_1 \times \zeta_1')(\hat{g}_2, \hat{h}_2)\end{aligned}$$

i.e., $(\zeta_2 \times \zeta_2')(\hat{w}) < (\zeta_1 \times \zeta_1')(\hat{g}_1, \hat{h}_1) \wedge (\zeta_1 \times \zeta_1')(\hat{g}_2, \hat{h}_2)$.

$$\begin{aligned}(\xi_2 \times \xi_2')(\hat{w}) &= \xi_1(\hat{g}_1) \wedge \xi_2'(\hat{w}_2) \\ &< \xi_1(\hat{g}_1) \wedge \xi_1(\hat{g}_2) \wedge \xi_1'(\hat{h}_1) \wedge \xi_1'(\hat{h}_2), \text{ since } \hat{g}_1 = \hat{g}_2 \\ &= (\xi_1 \times \xi_1')(\hat{g}_1, \hat{h}_1) \wedge (\xi_1 \times \xi_1')(\hat{g}_2, \hat{h}_2)\end{aligned}$$

i.e., $(\xi_2 \times \xi_2')(\hat{w}) < (\xi_1 \times \xi_1')(\hat{g}_1, \hat{h}_1) \wedge (\xi_1 \times \xi_1')(\hat{g}_2, \hat{h}_2)$.

$$\begin{aligned}(\kappa_2 \times \kappa_2')(\hat{w}) &= \kappa_1(\hat{g}_1) \wedge \kappa_2'(\hat{w}_2) \\ &< \kappa_1(\hat{g}_1) \wedge \kappa_1(\hat{g}_2) \wedge \kappa_1'(\hat{h}_1) \wedge \kappa_1'(\hat{h}_2), \text{ since } \hat{g}_1 = \hat{g}_2 \\ &= (\kappa_1 \times \kappa_1')(\hat{g}_1, \hat{h}_1) \wedge (\kappa_1 \times \kappa_1')(\hat{g}_2, \hat{h}_2)\end{aligned}$$

i.e., $(\kappa_2 \times \kappa_2')(\hat{w}) < (\kappa_1 \times \kappa_1')(\hat{g}_1, \hat{h}_1) \wedge (\kappa_1 \times \kappa_1')(\hat{g}_2, \hat{h}_2)$.

$$\begin{aligned}(\lambda_2 \times \lambda_2')(\hat{w}) &= \lambda_1(\hat{g}_1) \wedge \lambda_2'(\hat{w}_2) \\ &< \lambda_1(\hat{g}_1) \wedge \lambda_1(\hat{g}_2) \wedge \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2), \text{ since } \hat{g}_1 = \hat{g}_2 \\ &= (\lambda_1 \times \lambda_1')(\hat{g}_1, \hat{h}_1) \wedge (\lambda_1 \times \lambda_1')(\hat{g}_2, \hat{h}_2)\end{aligned}$$

i.e., $(\lambda_2 \times \lambda_2')(\hat{w}) < (\lambda_1 \times \lambda_1')(\hat{g}_1, \hat{h}_1) \wedge (\lambda_1 \times \lambda_1')(\hat{g}_2, \hat{h}_2)$.

Hence $F_{G1} \times F_{G2}$ is not strong, a contradiction. So at least one of F_{G1}or F_{G2} will be strong.

The Composition of Strong and Weak Distance-Hereditary Fuzzy Graph

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Definition 17 The composition of two D-HFGs, $F_{G1} = (U, E_{\hat{g}}, \varrho, \varpi), F_{G2} = (V, E_{\hat{h}}, \varrho', \varpi')$ where $\varrho = (\zeta_1, \xi_1, \kappa_1, \lambda_1)$, $\varpi = (\zeta_2, \xi_2, \kappa_2, \lambda_2)$, $\varrho' = (\zeta_1', \xi_1', \kappa_1', \lambda_1')$, $\varpi' = (\zeta_2', \xi_2', \kappa_2', \lambda_2')$ is the D-HFG $F_G = F_{G1} \circ F_{G2} = (U \times V, E, \varrho \circ \varrho', \varpi \circ \varpi')$, $E = E_1 \cup E_2 \cup E_3$ such that

$$E_1 = \{\hat{w}: \hat{g}_1 = \hat{g}_2, \hat{w}_2 \in E_{\hat{g}}\}$$

$$E_2 = \{\hat{w}: \hat{h}_1 = \hat{h}_2, \hat{w}_1 \in E_{\hat{g}}\}$$

$$E_3 = \{\hat{w}: \hat{h}_1 \neq \hat{h}_2, \hat{w}_1 \in E_{\hat{g}}\}$$

where, $\hat{w} = ((\hat{g}_1, \hat{h}_1), (\hat{g}_2, \hat{h}_2))$, $\hat{w}_1 = (\hat{g}_1, \hat{g}_2)$, $\hat{w}_2 = (\hat{h}_1, \hat{h}_2)$.

$$(\zeta_1 \circ \zeta_1')(\hat{j}) = \zeta_1(\hat{g}) \wedge \zeta_1'(\hat{h})$$

$$(\xi_1 \circ \xi_1')(\hat{j}) = \xi_1(\hat{g}) \wedge \xi_1'(\hat{h})$$

$$(\kappa_1 \circ \kappa_1')(\hat{j}) = \kappa_1(\hat{g}) \wedge \gamma_1'(\hat{h})$$

$$(\lambda_1 \circ \lambda_1')(\hat{j}) = \lambda_1(\hat{g}) \wedge \lambda_1'(\hat{h})$$

Where $\hat{j} = (\hat{g}, \hat{h})$

$$(\zeta_2 \circ \zeta_2')(\hat{w}) = \begin{cases} \zeta_1(\hat{g}_1) \wedge \zeta_2'(\hat{w}_2), \text{if } \hat{w} \in \mathrm{E}_1 \\ \zeta_1'(\hat{h}_1) \wedge \zeta_2(\hat{w}_1), \text{if } \hat{w} \in \mathrm{E}_2 \\ \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2) \wedge \zeta_2(\hat{w}_1), \text{if } \hat{w} \in \mathrm{E}_3 \end{cases}$$

$$(\xi_2 \circ \xi_2')(\hat{w}) = \begin{cases} \xi_1(\hat{g}_1) \wedge \xi_2'(\hat{w}_2), \text{if } \hat{w} \in \mathrm{E}_1 \\ \xi_1'(\hat{h}_1) \wedge \xi_2(\hat{w}_1), \text{if } \hat{w} \in \mathrm{E}_2 \\ \xi_1'(\hat{h}_1) \wedge \xi_1'(\hat{h}_2) \wedge \xi_2(\hat{w}_1), \text{if } \hat{w} \in \mathrm{E}_3 \end{cases}$$

$$(\kappa_2 \circ \kappa_2')(\hat{w}) = \begin{cases} \kappa_1(\hat{g}_1) \wedge \kappa_2'(\hat{w}_2), \text{if } \hat{w} \in \mathrm{E}_1 \\ \kappa_1'(\hat{h}_1) \wedge \kappa_2(\hat{w}_1), \text{if } \hat{w} \in \mathrm{E}_2 \\ \kappa_1'(\hat{h}_1) \wedge \kappa_1'(\hat{h}_2) \wedge \kappa_2(\hat{w}_1), \text{if } \hat{w} \in \mathrm{E}_3 \end{cases}$$

$$(\lambda_2 \circ \lambda_2')(\hat{w}) = \begin{cases} \lambda_1(\hat{g}_1) \wedge \lambda_2'(\hat{w}_2), \text{if } \hat{w} \in \mathrm{E}_1 \\ \lambda_1'(\hat{h}_1) \wedge \lambda_2(\hat{w}_1), \text{if } \hat{w} \in \mathrm{E}_2 \\ \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2) \wedge \lambda_2(\hat{w}_1), \text{if } \hat{w} \in \mathrm{E}_3 \end{cases}$$

Theorem 3 If F_{G1} and F_{G2} are two strong D-HFGs, then their composition $F_{G1} \circ F_{G2}$ is also a strong D-HFG.

Proof: Let F_{G1} , F_{G2} be two strong D-HFGs. Then, for $\hat{w}_1 \in E_{\hat{g}}$, $\hat{w}_2 \in E_{\hat{h}}$,

$$\zeta_2(\hat{w}_1) = \zeta_1(\hat{g}_1) \wedge \zeta_1(\hat{g}_2), \zeta_2'(\hat{w}_1) = \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2)$$

$$\xi_2(\hat{w}_1) = \xi_1(\hat{g}_1) \wedge \xi_1(\hat{g}_2), \xi_2'(\hat{w}_1) = \xi_1'(\hat{h}_1) \wedge \xi_1'(\hat{h}_2)$$

$$\kappa_2(\hat{w}_1) = \kappa_1(\hat{g}_1) \wedge \kappa_1(\hat{g}_2), \kappa_2'(\hat{w}_1) = \kappa_1'(\hat{h}_1) \wedge \kappa_1'(\hat{h}_2)$$

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$$\lambda_2(\hat{w}_1) = \lambda_1(\hat{g}_1) \wedge \lambda_1(\hat{g}_2), \lambda_2'(\hat{w}_1) = \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2)$$

Case(i) when $\hat{w} \in E_1$

$$(\zeta_2 \circ \zeta_2')(\hat{w}) = \zeta_1(\hat{g}_1) \wedge \zeta_2'(\hat{w}_2)$$
$$= \zeta_1(\hat{g}_1) \wedge \zeta_1(\hat{g}_2) \wedge \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2), \text{ since } \hat{g}_1 = \hat{g}_2$$
$$= (\zeta_1 \times \zeta_1')(\hat{g}_1, \hat{h}_1) \wedge (\zeta_1 \times \zeta_1')(\hat{g}_2, \hat{h}_2)$$

$$(\xi_2 \circ \xi_2')(\hat{w}) = \xi_1(\hat{g}_1) \wedge \xi_2'(\hat{w}_2)$$
$$= \xi_1(\hat{g}_1) \wedge \xi_1(\hat{g}_2) \wedge \xi_1'(\hat{h}_1) \wedge \xi_1'(\hat{h}_2), \text{ since } \hat{g}_1 = \hat{g}_2$$
$$= (\xi_1 \times \xi_1')(\hat{g}_1, \hat{h}_1) \wedge (\xi_1 \times \xi_1')(\hat{g}_2, \hat{h}_2)$$

$$(\kappa_2 \circ \kappa_2')(\hat{w}) = \kappa_1(\hat{g}_1) \wedge \kappa_2'(\hat{w}_2)$$
$$= \kappa_1(\hat{g}_1) \wedge \kappa_1(\hat{g}_2) \wedge \kappa_1'(\hat{h}_1) \wedge \kappa_1'(\hat{h}_2), \text{ since } \hat{g}_1 = \hat{g}_2$$
$$= (\kappa_1 \times \kappa_1')(\hat{g}_1, \hat{h}_1) \wedge (\kappa_1 \times \kappa_1')(\hat{g}_2, \hat{h}_2)$$

$$(\lambda_2 \circ \lambda_2')(\hat{w}) = \lambda_1(\hat{g}_1) \wedge \lambda_2'(\hat{w}_2)$$
$$= \lambda_1(\hat{g}_1) \wedge \lambda_1(\hat{g}_2) \wedge \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2), \text{ since } \hat{g}_1 = \hat{g}_2$$
$$= (\lambda_1 \times \lambda_1')(\hat{g}_1, \hat{h}_1) \wedge (\lambda_1 \times \lambda_1')(\hat{g}_2, \hat{h}_2)$$

Case(ii) When $\hat{w} \in E_2$

$$(\zeta_2 \circ \zeta_2')(\hat{w}) = \zeta_1'(\hat{h}_1) \wedge \zeta_2(\hat{w}_1)$$
$$= \zeta_1'(\hat{h}_1) \wedge \zeta_1'(\hat{h}_2) \wedge \zeta_1(\hat{g}_1) \wedge \zeta_1(\hat{g}_2), \text{ since } \hat{h}_1 = \hat{h}_2$$
$$= (\zeta \times \zeta_1')(\hat{g}_1, \hat{h}_1) \wedge (\zeta_1 \times \zeta_1')(\hat{g}_2, \hat{h}_2)$$

$$(\xi_2 \circ \xi_2')(\hat{w}) = \xi_1'(\hat{h}_1) \wedge \xi_2(\hat{w}_1)$$
$$= \xi_1'(\hat{h}) \wedge \xi_1'(\hat{h}_2) \wedge \xi_1(\hat{g}_1) \wedge \xi_1(\hat{g}_2), \text{ since } \hat{h}_1 = \hat{h}_2$$
$$= (\xi_1 \times \xi_1')(\hat{g}_1, \hat{h}_1) \wedge (\xi_1 \times \xi_1')(\hat{g}_2, \hat{h}_2)$$

$$(\kappa_2 \circ \kappa_2')(\hat{w}) = \kappa_1'(\hat{h}_1) \wedge \kappa_2(\hat{w}_1)$$
$$= \kappa_1'(\hat{h}_1) \wedge \kappa_1'(\hat{h}_2) \wedge \kappa_1(\hat{g}_1) \wedge \kappa_1(\hat{g}_2), \text{ since } \hat{h}_1 = \hat{h}_2$$
$$= (\kappa_1 \times \kappa_1')(\hat{g}_1, \hat{h}_1) \wedge (\kappa_1 \times \kappa_1')(\hat{g}_2, \hat{h}_2)$$

$$(\lambda_2 \circ \lambda_2')(\hat{w}) = \lambda_1'(\hat{h}_1) \wedge \lambda_2(\hat{w}_1)$$
$$= \lambda_1'(\hat{h}_1) \wedge \lambda_1'(\hat{h}_2) \wedge \lambda_1(\hat{g}_1) \wedge \lambda_1(\hat{g}_2), \text{ since } \hat{h}_1 = \hat{h}_2$$
$$= (\lambda_1 \times \lambda_1')(\hat{g}_1, \hat{h}_1) \wedge (\lambda_1 \times \lambda_1')(\hat{g}_2, \hat{h}_2)$$

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Thus, $F_{G1} = F_{G1} \circ F_{G2}$ is a strong D-HFG.

Theorem 4 If F_{G1}, F_{G2} are two D-HFGs such that $F_{G1} \circ F_{G2}$ is strong, then at least one of F_{G1}or F_{G2} will be strong.

Proof: Assume that the two D-HFGs F_{G1}, F_{G1} are weak. Then there exists at least one $\hat{\mathrm{w}}_1 \in E_{\hat{\mathrm{g}}}$, $\hat{\mathrm{w}}_2 \in E_{\hat{\mathrm{h}}}$, with

$\zeta_2(\hat{\mathrm{w}}_1) < \zeta_1(\hat{\mathrm{g}}_1)\wedge\zeta_1(\hat{\mathrm{g}}_2), \zeta_2'(\hat{\mathrm{w}}_1) < \zeta_1'(\hat{\mathrm{h}}_1)\wedge\zeta_1'(\hat{\mathrm{h}}_2)$,

$\xi_2(\hat{\mathrm{w}}_1) < \xi_1(\hat{\mathrm{g}}_1)\wedge\xi_1(\hat{\mathrm{g}}_2), \xi_2'(\hat{\mathrm{w}}_1) < \xi_1'(\hat{\mathrm{h}}_1)\wedge\xi_1'(\hat{\mathrm{h}}_2)$,

$\kappa_2(\hat{\mathrm{w}}_1) < \kappa_1(\hat{\mathrm{g}}_1)\wedge\kappa_1(\hat{\mathrm{g}}_2), \kappa_2'(\hat{\mathrm{w}}_1) < \kappa_1'(\hat{\mathrm{h}}_1)\wedge\kappa_1'(\hat{\mathrm{h}}_2)$,

$\lambda_2(\hat{\mathrm{w}}_1) < \lambda_1(\hat{\mathrm{g}}_1)\wedge\lambda_1(\hat{\mathrm{g}}_2), \lambda_2'(\hat{\mathrm{w}}_1) < \lambda_1'(\hat{\mathrm{h}}_1)\wedge\lambda_1'(\hat{\mathrm{h}}_2)$.

Let $\hat{\mathrm{w}} \in E_1$. Then,

$$\begin{aligned}(\zeta_2 \circ \zeta_2')(\hat{\mathrm{w}}) &= \zeta_1(\hat{\mathrm{g}}_1)\wedge\zeta_2'(\hat{\mathrm{w}}_2)\\ &< \zeta_1(\hat{\mathrm{g}}_1)\wedge\zeta_1(\hat{\mathrm{g}}_2)\wedge\zeta_1'(\hat{\mathrm{h}}_1)\wedge\zeta_1'(\hat{\mathrm{h}}_2), \text{ since } \hat{\mathrm{g}}_1 = \hat{\mathrm{g}}_2\\ &= (\zeta_1 \circ \zeta_1')(\hat{\mathrm{g}}_1, \hat{\mathrm{h}}_1)\wedge(\zeta_1 \circ \zeta_1')(u_2, \hat{\mathrm{h}}_2)\end{aligned}$$

i.e., $(\zeta_2 \circ \zeta_2')(\hat{\mathrm{w}}) < (\zeta_1 \circ \zeta_1')(\hat{\mathrm{g}}_1, \hat{\mathrm{h}}_1)\wedge(\zeta_1 \circ \zeta_1')(\hat{\mathrm{g}}_2, \hat{\mathrm{h}}_2)$.

$$\begin{aligned}(\xi_2 \circ \xi_2')(\hat{\mathrm{w}}) &= \xi_1(\hat{\mathrm{g}}_1)\wedge\xi_2'(\hat{\mathrm{w}}_2)\\ &< \xi_1(\hat{\mathrm{g}}_1)\wedge\xi_1(\hat{\mathrm{g}}_2)\wedge\xi_1'(\hat{\mathrm{h}}_1)\wedge\xi_1'(\hat{\mathrm{h}}_2), \text{ since } \hat{\mathrm{g}}_1 = \hat{\mathrm{g}}_2\\ &= (\xi_1 \circ \xi_1')(\hat{\mathrm{g}}_1, \hat{\mathrm{h}}_1)\wedge(\xi_1 \circ \xi_1')(\hat{\mathrm{g}}_2, \hat{\mathrm{h}}_2)\end{aligned}$$

i.e., $(\xi_2 \circ \xi_2')(\hat{\mathrm{w}}) < (\xi_1 \circ \xi_1')(\hat{\mathrm{g}}_1, \hat{\mathrm{h}}_1)\wedge(\xi_1 \circ \xi_1')(\hat{\mathrm{g}}_2, \hat{\mathrm{h}}_2)$.

$$\begin{aligned}(\kappa_2 \circ \kappa_2')(\hat{\mathrm{w}}) &= \kappa_1(\hat{\mathrm{g}}_1)\wedge\kappa_2'(\hat{\mathrm{w}}_2)\\ &< \kappa_1(\hat{\mathrm{g}}_1)\wedge\kappa_1(\hat{\mathrm{g}}_2)\wedge\kappa_1'(\hat{\mathrm{h}}_1)\wedge\kappa_1'(\hat{\mathrm{h}}_2), \text{ since } \hat{\mathrm{g}}_1 = \hat{\mathrm{g}}_2\\ &= (\kappa_1 \circ \kappa_1')(\hat{\mathrm{g}}_1, \hat{\mathrm{h}}_1)\wedge(\kappa_1 \circ \kappa_1')(\hat{\mathrm{g}}_2, \hat{\mathrm{h}}_2)\end{aligned}$$

i.e., $(\kappa_2 \circ \kappa_2')(\hat{\mathrm{w}}) < (\kappa_1 \circ \kappa_1')(\hat{\mathrm{g}}_1, \hat{\mathrm{h}}_1)\wedge(\kappa_1 \circ \kappa_1')(\hat{\mathrm{g}}_2, \hat{\mathrm{h}}_2)$.

$$\begin{aligned}(\lambda_2 \circ \lambda_2')(\hat{\mathrm{w}}) &= \lambda_1(\hat{\mathrm{g}}_1)\wedge\lambda_2'(\hat{\mathrm{w}}_2)\\ &< \lambda_1(\hat{\mathrm{g}}_1)\wedge\lambda_1(\hat{\mathrm{g}}_2)\wedge\lambda_1'(\hat{\mathrm{h}}_1)\wedge\lambda_1'(\hat{\mathrm{h}}_2), \text{ since } \hat{\mathrm{g}}_1 = \hat{\mathrm{g}}_2\\ &= (\lambda_1 \circ \lambda_1')(\hat{\mathrm{g}}_1, \hat{\mathrm{h}}_1)\wedge(\lambda_1 \circ \lambda_1')(\hat{\mathrm{g}}_2, \hat{\mathrm{h}}_2)\end{aligned}$$

i.e., $(\lambda_2 \circ \lambda_2')(\hat{\mathrm{w}}) < (\lambda_1 \circ \lambda_1')(\hat{\mathrm{g}}_1, \hat{\mathrm{h}}_1)\wedge(\lambda_1 \circ \lambda_1')(\hat{\mathrm{g}}_2, \hat{\mathrm{h}}_2)$.

Hence $F_{G1} \circ\ F_{G2}$ is not strong, a contradiction. So at least one of F_{G1}or F_{G2} will be strong.

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Applications

Distance-hereditary fuzzy graphs model evolutionary trees well, where distances between nodes represent evolutionary time or genetic dissimilarity. These graphs can be used when constructing evolutionary trees that preserve exact distances. Their distance-preserving property is useful in designing routing and communication protocols where distance accuracy is critical. Due to their structure, they allow for fast algorithms to compute distances and shortest paths. Their structural properties help in analyzing fault-tolerance and redundancy. Certain forms of RNA structures, which can be modeled using graphs, show distance-hereditary properties.

Conclusion

This article presents the idea of a distance-hereditary fuzzy graph, a new type of fuzzy graph, along with visual examples that illustrate its membership degrees. Distance-Hereditary Fuzzy Graphs can be utilized in humanoid robots, which frequently navigate dynamic and unpredictable environments. These models enable the robots to effectively represent spatial relationships and uncertainties in their surroundings. To ensure natural interactions, humanoid robots need to recognize and convey human-like emotions. Fuzzy Controllers can evaluate the level of certainty about object characteristics, allowing humanoid robots to modify their grip in real-time. This flexibility is crucial for a range of applications, from industrial automation to assistive robotics. Additionally, we examined various attributes of D-HFG, such as ζ-strong, ξ-strong, κ-strong, λ-strong, strong, ζ -weak, ξ -weak, κ -weak, λ -weak, weak, cartesian and composition. Looking ahead, we intend to elaborate on further concepts related to D-HFG.

Conflicts of interest

The author declare that there is no conflict of interest regarding the publication of this paper.

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References

1. A. Prasanna and T. M. Nishad. Some properties of weak fuzzy graphs, J. Math. Comput. Sci. 2021, 11(3), 3594-3601.
2. E. Howorka. A characterization of distance-hereditary graphs, Qunrt. J. M&h. Oxford Ser.,1977, 2 28, 417420.
3. H.J. Bandelt, H.M. Mulder. Distance-hereditary graphs, J. Combin. Theory Ser. B, 1986, 41 (2), 182–208
4. Howorka. E. A characterization of ptolemaic graphs; survey of results, in "Proceedings, 8th SE Conf. Combinatorics, Graph Theory, and Computing," 1977, pp. 355-361.
5. L.A.Zadeh. Fuzzy sets, Information and Control.1965,8:338–53.

Received: July 22, 2025

Volume 38 No. 3s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

6. Rosenfeld. A. Fuzzy graphs. In Fuzzy Sets and their Applications to Cognitive and Decision Processes. Zadeh LA, Fu KS, Shimura M, editors.1975.New York: Academic press.
7. Sunil, M. P., & Kumar, S. On intuitionistic hesitancy fuzzy graphs. Mathematics and Statistics, 2023.12(1), 48–54.

Illustractions:

Example 1

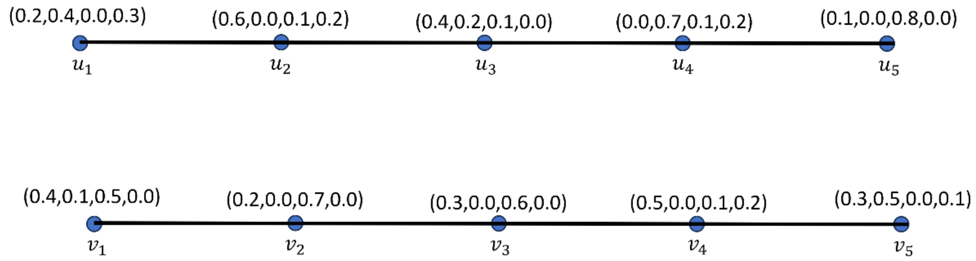

Fig 1 Strong D-HFGs of $\boldsymbol{F_{G1}}$ and $\boldsymbol{F_{G2}}$

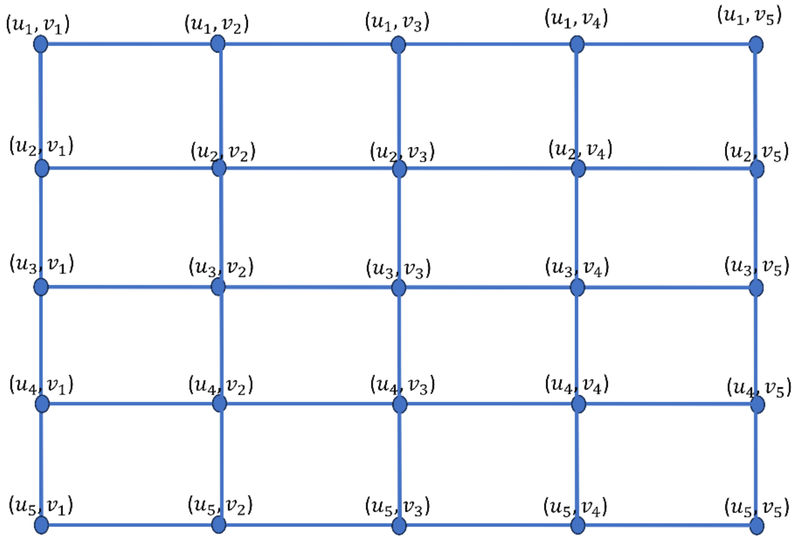

Fig. 2 Strong Distance-Hereditary Fuzzy Graph of $\boldsymbol{F_{G1} \times F_{G2}}$

Example 2

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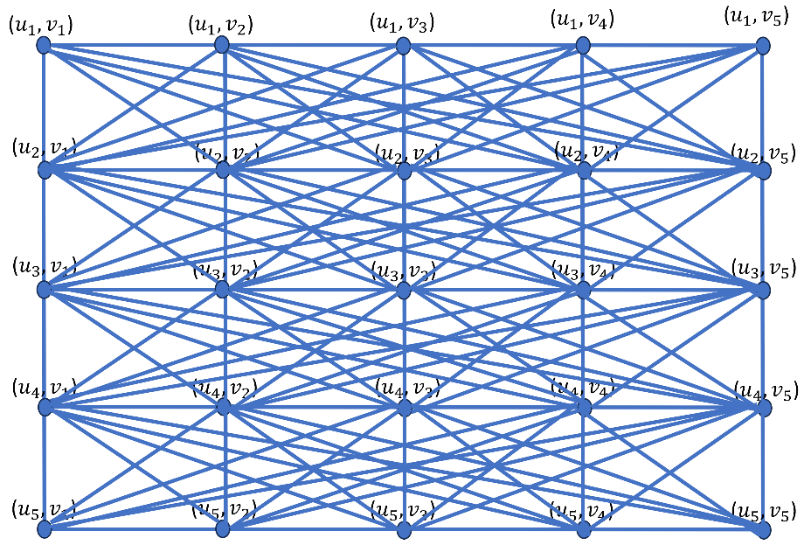

Fig. 3 **Strong Distance-Hereditary Fuzzy Graph of $F_{G1} \circ F_{G2}$**

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