

**SUSTAINABLE POWER GENERATION BY VIBRATIONAL BASED  
PIEZOELECTRIC ENERGY HARVESTER USING RECTANGULAR, CIRCULAR  
AND TRIANGULAR -EDGED SHAPES DESIGN PARAMETER OF CANTILEVER  
FOR CHARGEABLE BATTERIES.**

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**Abstract**

Although batteries are widely used, they have notable limitations, including limited energy capacity and environmental concerns related to their disposal. For low-power devices, piezoelectric energy harvesters (PEHs) present a promising alternative that can reduce the need for frequent battery replacement. PEHs have become increasingly important in various engineering applications, such as sensors and actuators, and are now recognized as a key technology for sustainable energy solutions. This paper focuses on the design and evaluation of PEH devices. Using piezoelectric materials such as lead zirconate titanate (PZT) and polyvinylidene fluoride (PVDF), cantilevers with different edge geometries—rectangular, circular, and triangular—were developed. These cantilevers were fabricated with diverse substrate materials, including rigid materials like structural steel and flexible materials such as polyethylene terephthalate (PET). Finite element modeling was then employed to analyze the output voltages and resonance frequencies of the designed PEHs.

Simulation results showed that the rectangular-edged cantilever, when combined with PZT and a steel substrate, produced the highest output voltage, reaching up to 1.4 V at a resonance frequency of 250 Hz. In comparison, cantilevers with circular and triangular edges generated lower output voltages, demonstrating the significant effect of cantilever geometry on energy harvesting efficiency. The study also revealed that although flexible substrates such as PET are suitable for certain applications, they generally yield lower output voltages than rigid substrates like steel.

Keywords— Piezoelectric energy harvesters, Cantilever beam, Finite element modelling.

**INTRODUCTION:**

Considerable attention has been given to the increasing demand for renewable energy sources such as thermal, vibrational, and solar energy [1]. These technologies provide sustainable

solutions to energy access challenges, particularly in rural and remote areas, by harnessing abundant and inexhaustible resources. Energy harvesting—also known as energy scavenging or power harvesting—refers to the process of capturing, storing, and converting ambient energy into electrical energy, which can be used to power small electronic devices such as sensors and actuators [2]. Depending on the application, energy harvesters may exploit different sources, with mechanical energy being the most extensively studied due to its broad availability. The primary mechanisms for harvesting mechanical energy include electrostatic, piezoelectric, and electromagnetic systems [3]. Among these, piezoelectric energy harvesters (PEHs) are among the most widely adopted energy harvesting technologies. PEHs convert mechanical vibrations or motion into electrical energy, providing a versatile and efficient solution that aligns with the growing demand for renewable energy sources. They are particularly valued for their ability to operate reliably under diverse environmental conditions while consistently supplying power to small-scale electronic devices. Electronic devices enhance energy efficiency and contribute to environmental sustainability [2]. Piezoelectric energy harvesters (PEHs) play a vital role in advancing clean energy generation, directly supporting the objectives of Sustainable Development Goal 7 (Affordable and Clean Energy). By enabling devices to function without reliance on disposable batteries, PEHs help reduce energy costs and minimize environmental impact. Consequently, they contribute to lowering carbon emissions and encouraging the adoption of renewable energy sources, thereby advancing broader sustainability targets [4]. Owing to their adaptability and high sensitivity to mechanical stimuli, PEHs are extensively employed in energy harvesting technologies. By converting vibrational or motion-based energy into electrical power, they provide a reliable and efficient solution to address the increasing demand for renewable energy.

The ability of piezoelectric energy harvesters (PEHs) to operate effectively in diverse environments while supplying power to electronic devices contributes to both energy efficiency and environmental sustainability [2]. By supporting the objectives of Sustainable Development Goal 7 (Affordable and Clean Energy), PEHs play a key role in renewable power generation. Their capacity to enable device operation without disposable batteries helps reduce environmental impact as well as overall energy costs. In doing so, PEHs contribute to broader sustainability goals by lowering carbon emissions and promoting the adoption of renewable energy sources [4]. Extensive research has focused on the design and optimization of PEHs to enhance performance. For example, a bimorph cantilever beam optimized for load resistance and length-to-width ratios achieved a maximum output power of 4.4 mW and a voltage of 10.1 V [5]. In another study, optimization of cantilever geometry revealed that trapezoidal configurations delivered the highest power efficiency [6]. Furthermore, investigations into flexible cantilevers demonstrated that certain designs could generate maximum voltage at specific resonance frequencies [7].

A disk-type sensor was used to examine the effect of applied weight on output voltage in [8]. Similarly, a comparative study demonstrated that E-shaped cantilevers exhibited greater displacement potential than their rectangular counterparts in [9]. These studies highlight the critical influence of material selection and geometric design on the performance of piezoelectric energy harvesters (PEHs). Despite on-going advancements in PEH technologies,

the optimal combination of piezoelectric materials and substrate topologies for maximizing energy output remains insufficiently explored. Much of the existing research focuses primarily on either geometric design or material selection, often neglecting the interaction between these two factors. Therefore, it is essential to investigate a wider range of material combinations—particularly those involving both rigid and flexible substrates—as well as diverse geometric configurations and edge shape in [18].

Segment II explores the fundamental principles of piezoelectric energy harvesting. Segment III outlines the design and simulation methodologies employed in the study. Segment IV presents and analyzes the simulation results. Finally, Segment V concludes the paper by summarizing key findings and suggesting directions for future research.

## II. Piezoelectric Energy Scavenging Conviction:

The prefix “*piezo-*” in piezoelectricity is derived from the Greek word meaning “to squeeze” or “to press.” Piezoelectric materials generate an electrical charge when subjected to mechanical stress, deformation, or vibrational energy. This electrical charge can be stored and utilized as a supplementary power source for portable electronic devices. Piezoelectricity is based on two fundamental effects: the direct effect and the converse effect. The direct piezoelectric effect, which is especially valuable in sensor applications, occurs when mechanical stress (such as compression or tension) induces an electrical voltage, as illustrated in Fig. 1 [11][13]. The direct piezoelectric energy harvesting effect, denoted as  $D$ , can be mathematically expressed by the following equation [10].

$$D = dT + \epsilon E \quad (1)$$

In this context,  $E$  represents the electric field,  $T$  denotes mechanical stress,  $\epsilon$  is the permittivity of the material, and  $d$  is the piezoelectric coefficient. The term  $\check{E}$  refers to the energy stored in the material, specifically in the form of the electric field ( $E$ ). Conversely, when an electric field is applied to a piezoelectric material, it induces mechanical deformation—a phenomenon known as the **converse piezoelectric effect**. This effect is widely employed in actuators. The corresponding relationship is given by Equation (2) [11].

$$S = sT + dE \quad (2)$$

In this context,  $T$  represents tension,  $d$  is the piezoelectric coefficient,  $E$  denotes the electric field, and  $s$  refers to elastic compliance—defined as the strain produced in a piezoelectric material per unit of applied stress. For a material to exhibit piezoelectric properties, it must possess a unique structural characteristic known as **non-centrosymmetry**, meaning its molecular structure lacks a center of symmetry. When mechanical pressure or stress is applied to such non-centrosymmetric materials, an electric charge accumulates on their top and bottom surfaces, giving rise to the piezoelectric effect. In contrast, centrosymmetric materials, which have symmetrical molecular structures, cannot exhibit piezoelectric behavior under applied mechanical stress [10].

## A Piezoelectric Design:

The cantilever structure is widely employed in piezoelectric energy harvesters (PEHs), particularly for mechanical energy harvesting. Its effectiveness is largely attributed to its simple design and the substantial mechanical strain generated during vibrations [12]. Cantilever beams offer several advantages, including a high mechanical quality factor, low fabrication cost, and structural simplicity [11]. A notable benefit is that the fundamental flexural modes of a cantilever operate at significantly lower resonance frequencies than other vibrational modes in piezoelectric elements, making them especially suitable for low-frequency energy harvesting. However, under high-impact conditions, cantilever beams often face challenges related to limited mechanical durability. The standard architecture of piezoelectric energy harvesting devices typically employs either a **unimorph** or **bimorph** cantilever design. In a unimorph configuration, a single piezoelectric layer is bonded to a metal substrate, forming a two-layer beam. The metal layer acts as both an electrode and mechanical support, while the piezoelectric layer generates electrical charge when deformed.

When one end of a cantilever is fixed, the structure operates in its flexural mode. In the **unimorph configuration**, only a single piezoelectric layer is active, which gives rise to the term *unimorph* [12]. To enhance power output, a **bimorph configuration** can be employed, in which two piezoelectric layers are bonded to opposite sides of a central metal substrate. This arrangement improves energy conversion efficiency by utilizing both piezoelectric layers simultaneously.

Bimorph configurations are attracting increasing attention in the field of piezoelectric energy harvesting due to their superior power generation capabilities compared to unimorph structures. Piezoelectric cantilevers, in general, are becoming highly significant in energy harvesting applications because of their efficient design and enhanced performance. Their compact architecture reduces the overall device size while simultaneously improving energy output [12]. A bimorph piezoelectric device operating on the direct piezoelectric effect—where mechanical stress induces electrical charge generation—is illustrated in Fig. 1 [11][13].

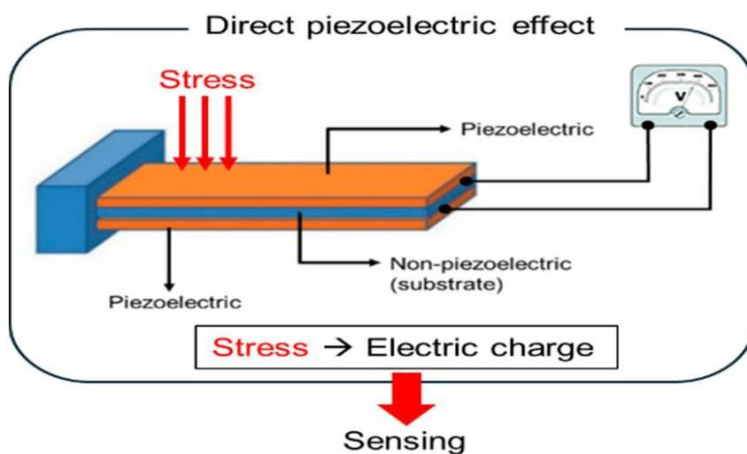


Fig. 1. Bimorph piezoelectric design with a direct piezoelectric effect.

**B. Mode of Operation & Working with PZT & PVDF:**

With respect to the poling direction, the piezoelectric coefficient exhibits two primary modes: the  $d_{31}$  mode, where the applied mechanical force is parallel to the generated electric field, and the  $d_{33}$  mode, where the force is applied perpendicular to the electric field [12]. Among these, the  $d_{33}$  mode is particularly effective for harvesting energy from small vibrations, as it typically generates a higher output voltage in response to minimal cantilever displacement. Studies have shown that vibrations in the  $d_{33}$  mode have a more substantial impact on voltage generation compared to the  $d_{31}$  mode [14]. Consequently, the  $d_{33}$  mode is often employed in thin piezoelectric elements to achieve enhanced voltage output [15], and it is widely recommended for use in piezoelectric energy harvesters [3]. Figure 4 illustrates the configuration of the  $d_{33}$  mode [11], [16].

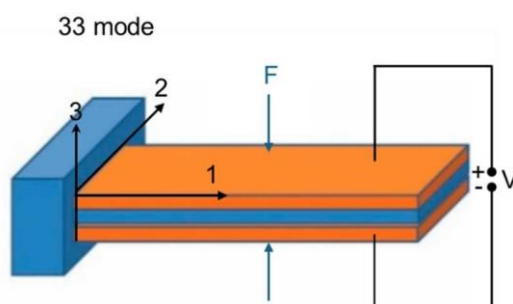


Fig. 2. The  $d_{33}$  mode in bimorph piezoelectric design.

Piezoelectric materials are generally classified into four main categories: single crystals, ceramics, polymers, and polymer composites/nanocomposites. Among these, piezoelectric ceramics—particularly lead zirconate titanate (PZT)—are widely used due to their excellent piezoelectric properties, thermal stability, and mechanical durability. The performance of single crystals is strongly influenced by factors such as raw material purity, growth conditions, and crystal orientation. PZT has been extensively investigated for its efficient mechanical-to-electrical energy conversion and its capability to operate at elevated temperatures, with a Curie temperature of approximately 350 °C. However, its inherent brittleness and limited flexibility restrict its use in applications that demand mechanical adaptability. The high density and Young's modulus of PZT contribute to this brittleness, leading to a high resonance frequency and reduced sensitivity to low-frequency ambient vibrations [3]. In contrast, polymeric materials such as polyvinylidene fluoride (PVDF) exhibit greater strain tolerance, lower stiffness, and enhanced mechanical flexibility, making them particularly suitable for wearable devices and flexible sensor applications. Compared to ceramics, polymers offer several advantages—including sufficient voltage and power output, lower manufacturing costs, and faster processing—despite their relatively low piezoelectric constant [17]. This study focuses on the use of PVDF, selected for its mechanical flexibility and durability, alongside PZT, chosen for its high piezoelectric efficiency, across a range of design configurations and substrate materials.

### III. Piezoelectric Scavenger Model:

Wind vibrations form the primary component of the electromechanical model diagram for a piezoelectric-based energy harvester, which also consists of several other subsystems. The functional schematic of a piezoelectric wind vibration energy harvester is shown in Figure 1. In our model, particular emphasis is placed on the cantilever design. The rectangular cantilever configuration produces an output of 1.4 volts. To enhance the voltage, two rectangular cantilevers will be connected in series, and a voltage doubler circuit will be incorporated, enabling the charging of a 3.7 V battery.

Since mechanical stress can also be induced through wind, this design approach has been chosen for the proposed model. Piezoelectric energy harvester will be developed on a minimum resonance frequency of 250Hz. This energy harvester is also known as energy scavenger also. As generated voltage are below 12voltage sothat these voltage can be used in to charging as well as storing the power in portable devices to use in remote areas. As generated power is alternating current by nature and it can converted in direct current using converter circuits. Converter circuit will be alternating current to direct current converter and dc to dc converter after converting alternating current to direct current. Maximum power point tracking will be used to track the total generated power through electromechanical piezoelectric energy harvester and this is applicable in charging and storing the total power. Voltage can be enhanced using voltage doubler circuit and using connection and by arrangement of the cantilevers.

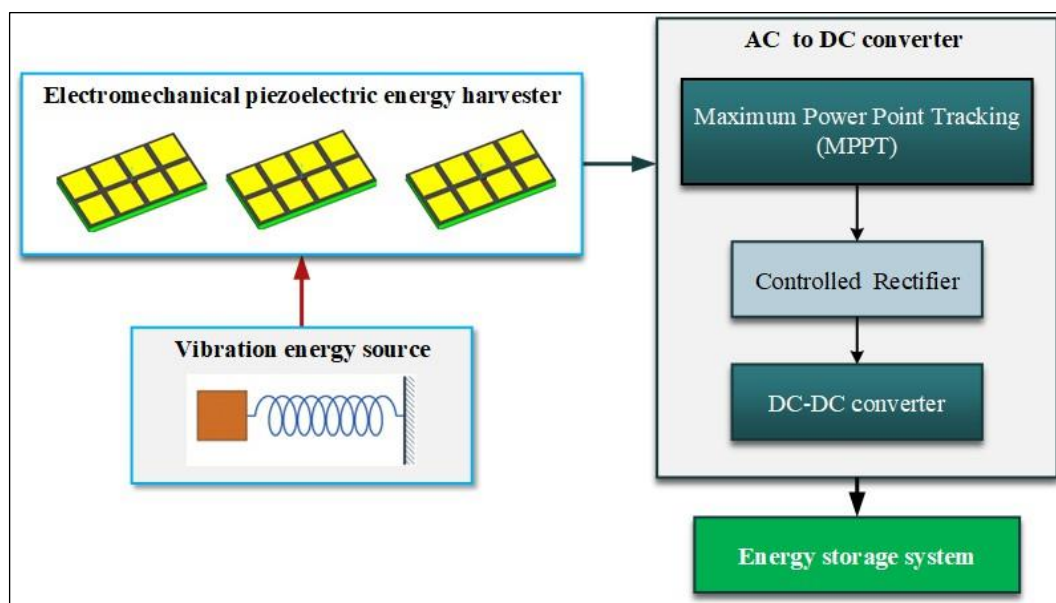


Figure 3. Diagram of the piezoelectric energy harvester for stress-generated vibrations.

1. Source of vibration: A structure may vibrate due to wind or stress can be given to Cantilever beams or any other resonant structure designed to oscillate as a result of vibrations caused by the wind can be used for this purpose.

2. Piezoelectric materials: Mechanical strain is transformed into a train of electrical charges via the direct piezoelectric action.

3. Electrical output: An electrical charge is produced by the piezoelectric material, which it instantly absorbs, stores, or converts into electrical power.

A collection of capacitors or batteries is separated into rectifiers, voltage regulators, a DC-to-DC converter, and potentially an energy storage device via this circuit.

### **A.Designing and Simulation of Piezoelectric Energy Scavenger &Geometry:**

Finite element modeling was employed for the design, analysis, and simulation process. Based on specific parameters—including geometry type, number of layers, dimensions, and layer arrangement—a 3D cantilever model was developed. Appropriate piezoelectric and substrate materials were selected, and the corresponding physical properties were assigned. Mesh generation, boundary condition definitions, and study parameters were then established. To ensure a fair comparison of performance across different designs, all cantilever models were normalized to the same volume, allowing evaluation solely on the basis of material properties and geometry.

Three different cantilever designs—rectangular, triangular, and circular—were developed to evaluate the effect of edge geometry on resonance frequency and output voltage. To compare and validate the results, these designs were further analyzed to understand their stress distribution characteristics. Table I summarizes the specifications of all cantilever beams, including their dimensions, materials, and edge geometries. The configurations of the rectangular, triangular, and circular-edged beams are illustrated in Fig. 5. Comsol Multiphasic will be the tool on which every function will be carried out. The table mention below is for substrate material and piezoelectric layers.

<b>Components</b>	<b>Materials</b>	<b>Dimensions</b>
Substrate layer	Steel	W=10 mm L=23.5 mm
	PET	T= 0.1mm
Piezoelectric layers	PZT	W=10 mm L= 23.5 mm T=0.1mm

TABLE I. THE PARAMETERS OF THE CANTILEVER BEAMS

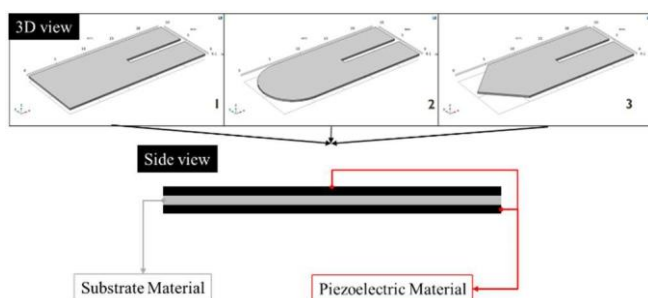


Figure 4. Rectangular, circular and triangular -edged shapes design parameter

**B. Choosing Suitable Material:**

Four distinct materials were selected for the components of the cantilever beam. Polyethylene terephthalate (PET) was chosen for its flexibility, while structural steel was used as the stiff substrate material. For the piezoelectric layers, polyvinylidene fluoride (PVDF) and lead zirconate titanate (PZT-5A) were employed. The substrate material was positioned between the two piezoelectric layers. The specific properties of the substrate materials are presented in Table II, and those of the piezoelectric materials are listed in Table III.

<b>Structural steel</b>	
Density [kg/m <sup>3</sup> ]	7850
Young's modulus [Pa]	200 × 10 <sup>9</sup>
Poisson's ratio	0.30
<b>PET</b>	
Density [kg/m <sup>3</sup> ]	1.32
Young's modulus [Pa]	2 × 10 <sup>9</sup>
Poisson's ratio	0.40

TABLE II.SUBSTRATE MATERIALS PROPERTIES

<b>PZT-5A</b>						
Density [kg/m <sup>3</sup> ]	7750					
Elasticity matrix [Pa]	1.20346 × 10 <sup>11</sup>					
Coupling matrix [C/m <sup>2</sup> ]	0	0	0	0	12.290	
	(	0	0	0	12.29 <sup>5</sup>	)

	5 0 -5.35 -5.351 0 0 0 1 15.784
Relative permittivity	919.1
<b>PVDF</b>	
Density [ $kg/m^3$ ]	1780
Elasticity matrix [ $Pa$ ]	$3.8 \times 10^9$
Coupling matrix [ $C/m^2$ ]	0 0 0 00 0 ( 0 0 0 00 0) 0.024 0.001 0 0 -0.027 0
Relative permittivity	7.4

TABLE III. PIEZOELECTRIC MATERIALS PROPERTIES

To examine the impact of the piezoelectric energy harvester, two physics subjects—solid mechanics and electrostatics—were chosen from Matlab. The fixed constraint and the all-domain body load are the boundaries used in solid mechanics. Ground and terminal limits were chosen for electrostatics. On the top and bottom layers of the piezoelectric materials, respectively, are the terminal boundaries, which serve as electrodes. On the other hand, the ground boundaries are located on the top and bottom layers of the piezoelectric materials, respectively. A 1 kΩ resistor is also connected to the output in parallel. The creation of mesh tetrahedrons for every cantilever beam is shown in Fig. 6.

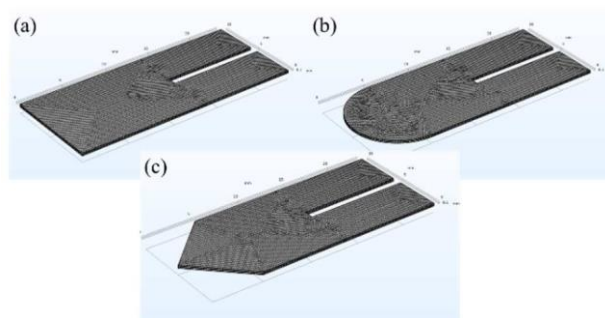


Figure 5. Tetrahedrons Mesh Generation for (a) Rectangular-Edged Shape,

(b) Circular-Edged Shape And (c) Triangular-Edged Shape

#### IV. Simulation Results and Analysis:

Figure 6 presents the maximum voltage output and resonant frequency for three cantilever edge geometries using PZT/steel combinations. At a resonance frequency of 250 Hz, the rectangular edge design produces the highest voltage output (1.3914 V), followed by the circular edge design (1.38 V) at 273 Hz and the triangular edge design (1.09 V) at 309 Hz. The simulation results also illustrate the distribution of electric potential across the surface of the piezoelectric cantilevers, represented by a color scale. Red indicates regions of high electric potential, while blue represents regions of low potential. The concentration of red at the central region reflects areas of maximum potential, as the terminal borders (electrodes) are located in these zones.

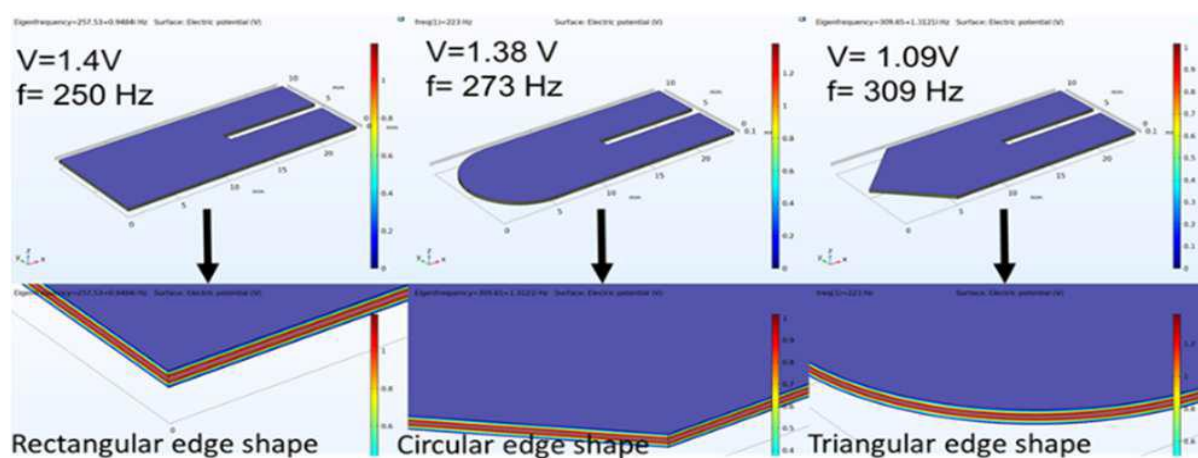


Figure 6. Highest output voltage in each edge shapes when combined with PZT/steel.

#### A. Outcome of PZT and PVDF on Output Voltage:

**Figure 7** compares PZT and PVDF materials in terms of output voltage generation. A clear difference in voltage efficiency is observed when combined with steel. PZT consistently outperforms PVDF, producing output voltages of 1.3914 V, 1.3816 V, and 1.0899 V for rectangular, circular, and triangular shapes, respectively. In contrast, PVDF generates significantly lower voltages of 0.0069 V, 0.0258 V, and 0.01815 V for the same shapes. This trend remains consistent when the substrate material is switched to PET, where PZT achieves higher output voltages (1.0971 V, 0.9634 V, and 0.8565 V) compared to PVDF (0.0056 V, 0.0127 V, and 0.0481 V). The superior performance of PZT is attributed to its higher relative permittivity, as shown in Table III, which indicates a greater capacity to store electrical energy. Furthermore, PZT has higher piezoelectric coupling coefficients, underscoring its superior efficiency in converting mechanical energy into electrical energy compared to PVDF.

Therefore, PZT is more suitable for high-energy conversion applications. These findings align with the conclusions of study [18], which also confirmed the superior conversion efficiency of PZT over PVDF.

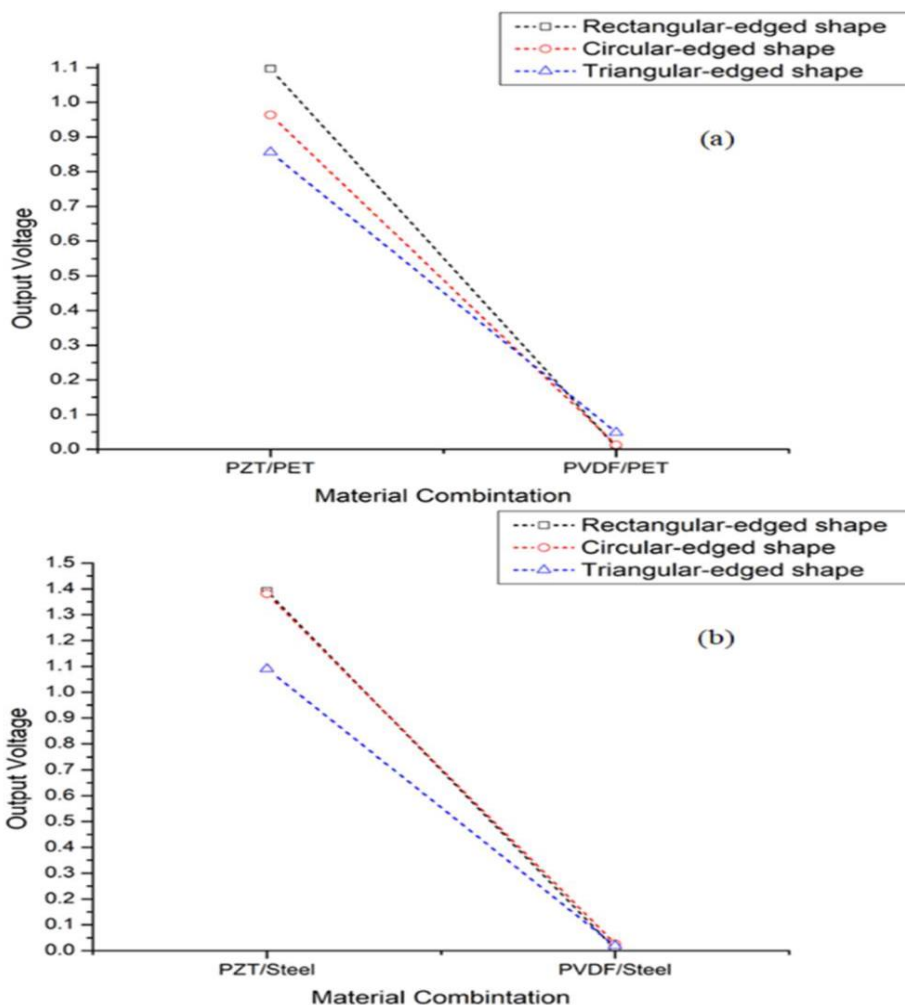


Figure 7. Output Voltage of PZT and PVDF paired with (a) PET and (b) Steel.

### B. Outcome of substrate Steel material on output voltage:

Figure 8 illustrates that steel generally performs better than PET as a substrate for piezoelectric materials. When paired with PZT, steel consistently yields higher output voltages: 1.3914 V for rectangular shapes, 1.3816 V for circular shapes, and 1.0899 V for triangular shapes, compared to PET, which produces 1.0971 V, 0.9634 V, and 0.8565 V for the same geometries. These results indicate that the high density, high Young's modulus, and low Poisson's ratio of steel enhance the piezoelectric effect when combined with PZT. In contrast, PVDF shows a different performance trend. Notably, for the triangular-edge shape, PET outperforms steel, generating a higher output voltage of 0.0481 V. This suggests that PET's material properties may offer advantages in certain configurations, particularly in enhancing PVDF's piezoelectric response for specific geometries.

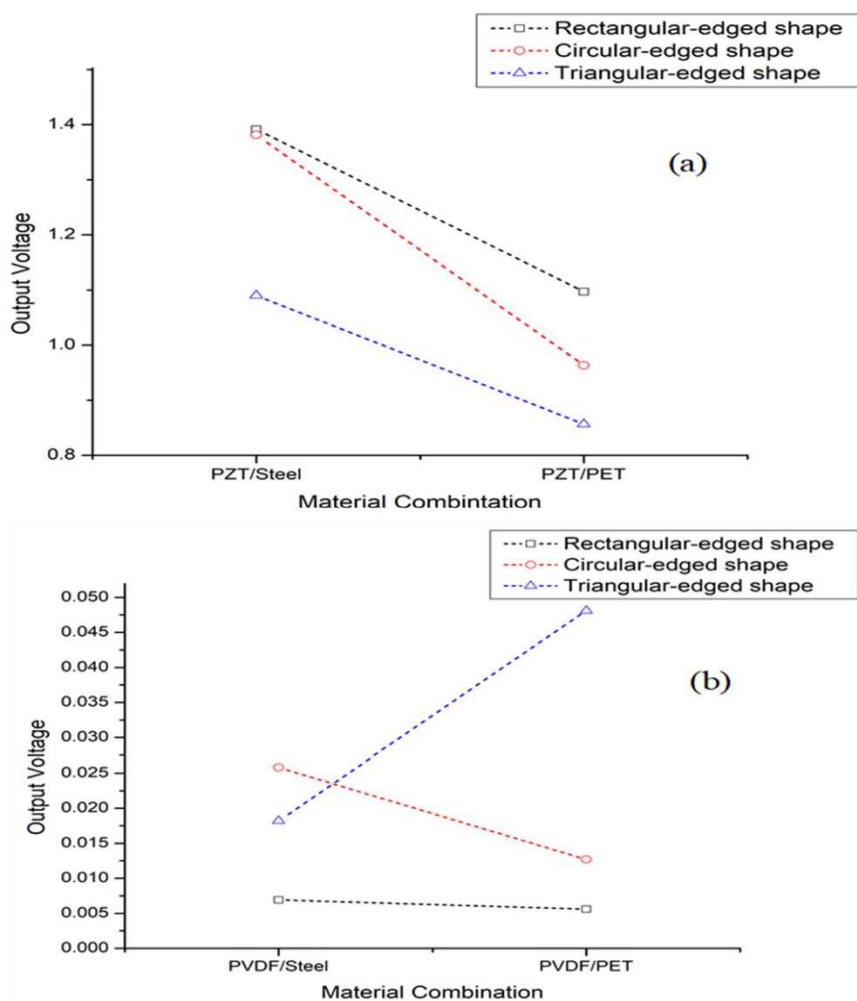


Figure 8. Output Voltage of Steel and PET paired with (a) PZT and (b) PVDF.

C. Outcome of all cantilevers shape on output voltage:

Figure 9 shows that the PZT/Steel and PZT/PET rectangular configurations yield the highest output voltages, 1.3914 V and 1.0971 V, respectively. This indicates that, in terms of energy conversion, the rectangular geometry may be the most effective for certain material pairings. For PVDF/Steel, however, the circular geometry performs better, producing 0.0258 V—higher than its rectangular counterpart—although it does not surpass the output of PZT-based combinations. This suggests that PVDF materials may benefit more from circular shapes. In contrast, triangular geometries generally produce the lowest output voltages in PZT combinations, yielding 1.0899 V for PZT/Steel and 0.8565 V for PZT/PET. Interestingly, PVDF/PET stands out as an exception: its triangular geometry achieves the highest output voltage of 0.0481 V. This indicates that the triangular form enhances the energy conversion efficiency of PVDF/PET, likely due to improved uniform stress distribution, which facilitates greater voltage generation [3].

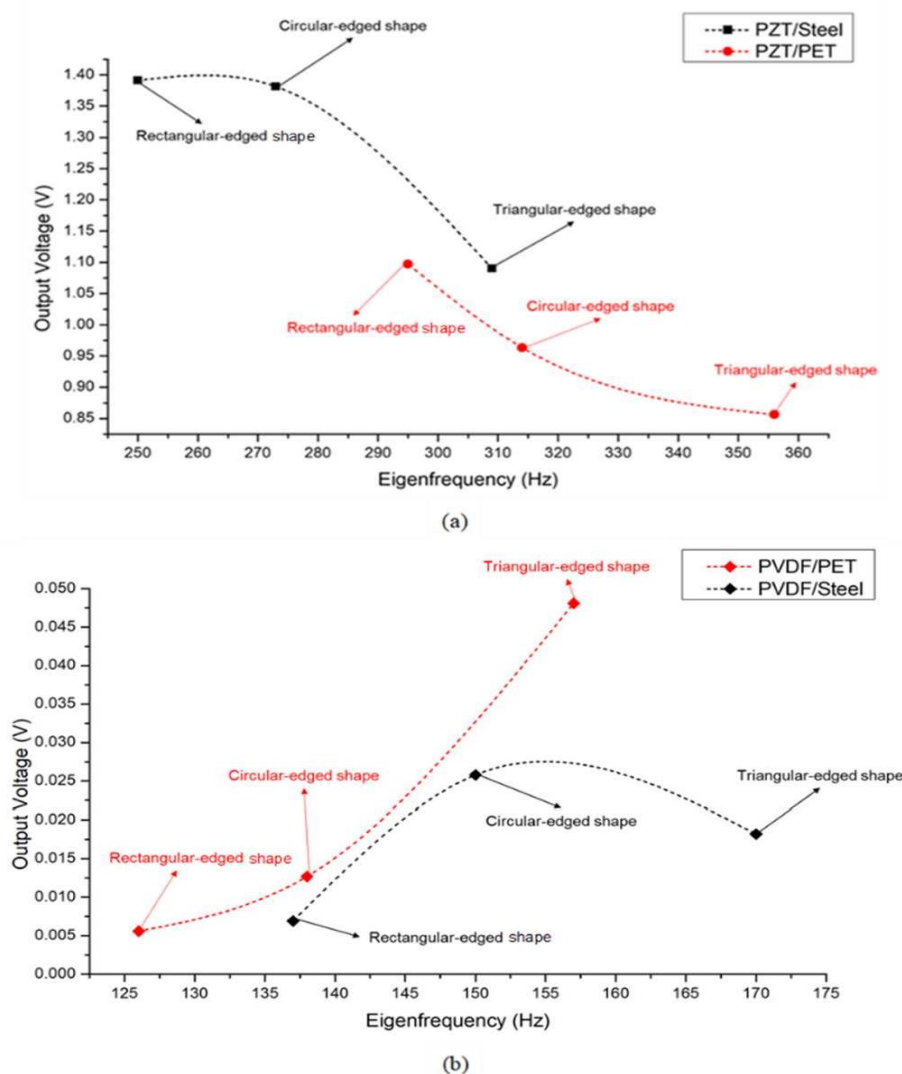


Figure 9. Output voltage across different edge shapes between (a) PZT and (b) PVDF paired with different substrates

Extensive analysis of different piezoelectric materials and substrate pairings demonstrates that the **PZT/Steel combination is the most efficient** for maximizing output voltage and piezoelectric performance. Across all geometries, this pairing consistently achieves the highest output voltages. Specifically, PZT/Steel produces 1.3914 V in rectangular shapes, 1.3816 V in circular shapes, and 1.0899 V in triangular shapes. This superior performance can be attributed to the complementary material properties of PZT and steel, making this combination highly suitable for applications that demand high energy conversion efficiency. Rectangular geometries, in particular, exhibit excellent performance when paired with PZT. The PZT/Steel rectangular configuration achieves the highest voltage output of 1.3914 V, while the PZT/PET rectangular pairing also delivers a comparatively high value of 1.0971 V. These findings confirm that the rectangular design is especially advantageous for optimizing piezoelectric action in PZT-based systems.

**V. Conclusion:**

By exploring different combinations of piezoelectric materials, substrate materials, and cantilever geometries, this study aimed to design and analyze piezoelectric energy harvesters (PEHs). The findings highlight the effectiveness of specific material–geometry pairings in optimizing energy conversion. The highest output voltages were consistently achieved with the PZT/Steel combination, where the rectangular cantilever produced the maximum recorded output of 1.3914 V at a resonance frequency of 250 Hz. This superior performance is attributed to the synergy between steel’s high density, high Young’s modulus, and low Poisson’s ratio, and PZT’s high piezoelectric constant and electromechanical coupling coefficient. Rectangular geometries proved to be the most effective design for energy harvesting, as they delivered the highest voltage outputs for both PZT/Steel and PZT/PET combinations. These results underscore the importance of selecting optimal material–geometry pairings for maximizing efficiency. Future work should focus on investigating a broader range of substrate materials and evaluating their performance with both PVDF and PZT. Moreover, the proposed concepts will be advanced into prototypes and validated through experimental testing and practical applications.

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**Author’s contribution:** Vikas Kumar, Dr. Rohtash Dhiman, Dr. Jitendra Singh

**Other Ethics Statements:** Research is in proper Ethics.

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