

**MHD FLOWING OF CONVECTIVE HEAT FLOW ROTATING OF
RECTANGULAR HOLLOW WITH ACHIEVE OF NON-ISOTHERMAL
AND ISOTHERMAL HEDGE**

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Abstract

The study investigates the magneto hydrodynamic (MHD) behaviour of convective heat flow in a rotating rectangular hollow enclosure subjected to both non-isothermal and isothermal boundary conditions. The interaction of thermal buoyancy, rotational forces, and externally applied magnetic fields produces a complex flow structure, significantly influencing heat transfer characteristics within the cavity. A two-dimensional incompressible electrically conducting fluid is considered, and the governing equations are formulated using the Bossiness approximation, incorporating Coriolis acceleration and Lorentz force effects. The rectangular hollow is examined under mixed boundary conditions, wherein selected walls are maintained at uniform isothermal temperatures, while others follow a non-isothermal spatial distribution. Numerical simulations are employed to evaluate the influence of key controlling parameters, including Hartmann number, Taylor number, Prandtl number, and Rayleigh number. The results reveal the critical role of magnetic damping in suppressing convective currents, while rotation enhances secondary circulatory structures depending on angular velocity. The interplay between isothermal and non-isothermal hedges yields distinct thermal stratifications and modifies heat transfer rates along the cavity walls. Overall, the analysis provides insights into optimizing thermal regulation in rotating MHD systems with relevance to advanced cooling technologies, geophysical flows, and energy-conversion devices.. In this study we assume the left vertical wall of the cavity is at temperature $T_h (T_h + By^\lambda)$ and right

vertical wall temperature T_c ($T_h > T_c$) the top and bottom horizontal surfaces of the cavity are adiabatic. Darcy law is to be obeyed to the flow inside the porous medium the properties of the fluid and porous medium are homogeneous, isotropic and constant expect variation of fluid density with temperature. The fluid and porous medium are in thermal equilibrium and flux of heat radiation in y - direction is negligible in comparison to that in x -direction. Galerkin Finite Element Method of three noded triangular elements has been used to convert the partial differential equations into the matrix form equations. Results are presented in terms of stream functions and isotherms for various values of Aspect ratios, Radiation parameters, Rayleigh numbers, Nusselt numbers and Power law exponents(λ).

1. INTRODUCTION

Magneto hydrodynamic (MHD) convective heat flow in rotating enclosures continues to be a significant area of research due to its broad applicability in engineering, geophysics, metallurgy, and energy systems. Among various geometrical configurations, the **rectangular hollow cavity** provides a versatile domain for examining the combined influence of thermal gradients, magnetic fields, and rotational effects on fluid motion. When a fluid with finite electrical conductivity is subjected to an external magnetic field, Lorentz forces arise, modifying the flow structure, suppressing instabilities, and altering the heat transport characteristics. These interactions become even more complex under rotation, where Coriolis forces introduce additional coupling between momentum and thermal fields.

A key aspect of such systems concerns the **boundary thermal conditions**. The distinction between **isothermal** and **non-isothermal hedges (walls)** plays a crucial role in determining the nature of buoyancy-driven convection. Isothermal boundaries maintain uniform temperatures, facilitating classical natural convection patterns, whereas non-isothermal boundaries introduce spatially varying thermal distributions that generate complex, multi-cellular flow structures and enhanced heat transfer rates. The interplay between these boundary conditions and the imposed magnetic and rotational environments leads to rich physical phenomena, including flow stabilization, boundary-layer deformation, heatline reorientation, and modification of Nusselt number behavior.

Studying MHD convective flow in a rotating rectangular hollow with mixed thermal boundaries not only provides fundamental insights into fluid-thermal-electromagnetic coupling but also supports technological advancements in **cooling of rotating machinery, crystal growth, electromagnetic casting, energy storage devices, and geo-astrophysical modelling**. The formulation and analysis of this configuration allow researchers to explore parameter influences such as Hartmann number, Taylor number, Rayleigh number, Prandtl number, and thermal boundary non-uniformity, thereby offering a complete framework for understanding stability, transport characteristics, and flow morphology.

Darcy's law is usually written as:

$$q = -K/\mu \, dp/dx$$

where: q = Darcy velocity (specific discharge), K = permeability of the porous medium
 μ = dynamic viscosity of the fluid ,

dp/dx = pressure gradient

The negative sign indicates that flow occurs from high pressure to low pressure.

The literature, natural convection heat Transfer in enclosures by Ostrach [1]. Unsteady natural convection in a rectangular cavity by Patterson [2], Applied Finite element analysis by Segerland [3] is inspired in research. An experimental investigation of mixed cavity natural convection in the high Rayleigh number regime by Kirkpatrick [4], Natural convection in rectangular enclosures from below and cooled along one side explained detailed by November et.al.[5], Transient natural convection in a rectangular enclosure with one heated side wall by Hall and Bejan [6] Heat transfer in square cavities with partially active vertically walls numerically explained by Valencia and Frederick [7], Numerical solutions of transient natural convection in a square cavity with different side wall temperature and Natural convection in a differentially heated square cavity with internal heat generation are given by Hyun et.al..[8], Natural convection in rectangular enclosures heated from below and symmetrically cooled from the sides numerically explained by Milanez [9], Cheung et.al..[10] Problems in Isotropic Seepage by Finite Elements are solved numerically. Natural convection in rectangular enclosures heated from one side and cooled from the ceiling investigated by Ayhan [11], Free Convection in a Thermally Stratified Non-Darcy Porous Medium Saturated with a Non-Newtonian Fluid, P.V.S.N. Murthy [12], Forced Convection Flow of Power-Law Fluids Over a Flat Plate Embedded in a Darcy-Brinkman Porous Medium Asterios and Eugun [13], A numerical study on natural convection in porous media-filled an inclined triangular enclosure with heat sources using nano-fluid in the presence of heat generation effect by Mansour, S. E. Ahmed[14], The flow of a rarefied gas in a rectangular enclosure due to the non-isothermal walls with no synergetic contributions from external force fields is investigated by Stefanov[15], The radiant efflux from a cylindrical cavity having non-isothermal bounding surfaces is determined by solving a problem of combined radiation and conduction by Heinisch[16] ,Combined effects of thermal radiation and internal heat source/sink, formulation for natural-convection flow in a square cavity filled with porous medium using isothermal vertical walls and adiabatic horizontal ones, has been studied numerically by using finite-difference technique in staggered grid distribution by Sabyasachi et. al..[17].

2. MATHEMATICAL FORMULATION

Consider a two dimensional cavity of height and width (L) filled with saturated porous medium, the left vertical surface of the cavity is maintained at temperature T_h greater than right vertical surface temperature T_c , the top and bottom horizontal surfaces of the cavity are adiabatic. The flow inside the porous medium is assumed to obey Darcy law and there is no phase changes of the fluid the properties of the fluid and porous medium are homogeneous, isotropic and constant except variation of fluid density with temperature. The fluid and porous medium are in thermal equilibrium and radioactive heat flux in y direction is negligible in comparison to that in the x - direction.

The governing equations in Cartesian Coordinates can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{gK\beta}{v} \frac{\partial T}{\partial x} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{1}{\rho c_p} \frac{4n^2 \sigma}{3\beta_R} \frac{\partial^2 T^4}{\partial x^2} \quad (3)$$

with the boundary conditions are

At hot wall	$x = 0$	$u = 0, v = 0$	$T = T_\infty + B y^\lambda \quad (4)$
At cold wall	$x = L$	$u = 0, v = 0$	$T = T_c$
At hot wall	$y = 0$	$u = 0, v = 0$	$\frac{\partial T}{\partial y} = 0$
At cold wall	$y = L$	$u = 0, v = 0$	$\frac{\partial T}{\partial y} = 0$

where x and y are the distances measured along the horizontal and vertical directions respectively and u and v are the velocity components in the x - and y - directions respectively.

Using the following non dimensional variables,

$$\text{Width, } X = \frac{x}{L} \quad \text{Height, } Y = \frac{y}{L} \quad \text{and Stream function, } \bar{\psi} = \frac{\psi}{\alpha}$$

The governing equations (1) to (3) reduced to non-dimensional form and introducing stream function ' ψ '

$$\frac{\partial^2 \bar{\psi}}{\partial X^2} + \frac{\partial^2 \bar{\psi}}{\partial Y^2} = -Ra \frac{\partial \theta}{\partial X} \quad (5)$$

$$\left[\frac{\partial \bar{\psi}}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \bar{\psi}}{\partial X} \frac{\partial \theta}{\partial Y} \right] = \left(\left(1 + \frac{4R_d}{3} \right) \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (6)$$

Where T denotes the temperature ϑ and α are kinematic viscosity and thermal diffusivity respectively, K is the medium permeability, T_h and T_c are the temperatures at hot bottom wall and cold vertical walls respectively L is the side of the square cavity.

$$\text{Temperature} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\text{Radiation Parameter} \quad R_d = \frac{4\sigma n^2 T_\infty^3}{\beta_R K_S}$$

$$\text{Rayleigh number} \quad Ra = \frac{g\beta_T \Delta T K L}{v\alpha}$$

With the non-dimensional boundary conditions are

$$\text{at } X = 0 \quad \bar{\psi} = 0 \quad \theta = 1$$

$$\begin{array}{lll}
 \text{at } X = 1 & \bar{\psi} = 0 & \theta = 1 \\
 \text{at } Y = 0 & \bar{\psi} = 0 & \frac{\partial \theta}{\partial y} = 0 \\
 \text{at } Y = 1 & \bar{\psi} = 0 & \frac{\partial \theta}{\partial y} = 0
 \end{array}$$

3. SOLUTION OF THE PROBLEM

Thus far we have derived the partial differential equations, which describe the heat and fluid flow behavior in the vicinity of porous medium. The development of governing equations is one part but the second and important part is to solve these equations in order to predict the various parameters of interest in the porous medium. There are various numerical methods available to achieve the solution of these equations, but the most popular numerical methods are Finite difference method, Finite volume method and the Finite element method. The selection of these numerical methods is an important decision, which is influenced by variety of factors amongst which the geometry of domain plays a vital role. Other factors include the ease with which these partial differential equations can be transformed into simple forms, the computational time required and the flexibility in development of computer code to solve these equations. In the present study, we have predominantly used Finite Element Method (FEM). The following sections enlighten the Finite element method and present its application to solve the governing equations.

The Finite Element Method is a deservingly popular method amongst scientific community. This method was originally developed to study the mechanical stresses in a complex air frame structure popularized by Zienkiewicz and Cheung by applying it to continuum mechanics. Since the application of Finite Element Method has been exploited to solve the number of problems in various engineering disciplines. The great thing about finite element method is its ease with which it can be generalized to myriad engineering problems comprised of different materials.

Another admirable feature of the Finite Element Method (FEM) is that it can be applied wide range of geometries having irregular boundaries, which is highly difficult to achieve with other contemporary methods. FEM can be said to have comprised of roughly 5 steps to solve any particular problem.

The steps can be summarized as

Descritizing the domain: This step involves the division of whole physical domain into smaller segments known as elements, and then identifying the nodes, coordinates of each node and ensuring proper connectivity between the nodes.

• **Specifying the equation:** In this step, the governing equation is specified and an equation is written in terms of nodal values.

- **Development of Global matrix:** The equations are arranged in a global matrix which takes into account the whole domain.
- **Solution:** The equations are solved to get the desired variable at each table in the domain.
- **Evaluate the quantities of interest:** After solving the equations a set of values is obtained for each node, which can be further processed to get the quantities of interest.

There are varieties of elements available in FEM, which are distinguished by the presence of number of nodes. The present study is carried out by using a simple 3-noded triangular element.

Good insight into the FEM Galerkin method is employed to convert the partial differential equations into matrix form for an element. The steps invented are as given below. Please note that the nodal terms i, j & k are replaced by 1, 2 & 3 respectively in subsequent discussions for simplicity.

The momentum and energy balance equations are solved using the Galerkin finite element method. Continuity equation will be used as a constraint due to mass conservation and this constraint may be used to obtain the pressure distribution. In order to solve equations, we use the finite element method where the pressure P is eliminated by a penalty parameter γ and the incompressibility criteria given by equation (1) which results in

$$P = -\gamma \left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right)$$

The continuity equation (1) is automatically satisfied for large values of γ .

Application of Galerkin method to equation (5) yields

$$\{R^e\} = - \int_A N^T \left(\frac{\partial^2 \bar{\psi}}{\partial x^2} + \frac{\partial^2 \bar{\psi}}{\partial y^2} + Ra \frac{\partial \bar{T}}{\partial x} \right) dA \quad (7)$$

where R^e is the residue. Considering individual terms of equation (7), the differentiation of following terms results into

$$\frac{\partial}{\partial X} \left([N]^T \frac{\partial \bar{\psi}}{\partial X} \right) = [N]^T \frac{\partial^2 \bar{\psi}}{\partial X^2} + \frac{\partial [N]^T}{\partial X} \frac{\partial \bar{\psi}}{\partial X} \quad (8)$$

Thus

$$\int_A N^T \frac{\partial^2 \bar{\psi}}{\partial X^2} dA = \int_A \frac{\partial}{\partial X} \left([N]^T \frac{\partial^2 \bar{\psi}}{\partial X^2} \right) dA - \int_A \frac{\partial [N]^T}{\partial X} \frac{\partial \bar{\psi}}{\partial X} dA \quad (9)$$

The first term on right hand side of equation (9) can be transformed into surface integral by the application of Greens theorem and leads to inter-element requirement at boundaries of an element. The boundary conditions are incorporated in the force vector.

Making use of $T = N_i T_i + N_j T_j + N_k T_k$

$$\int_A N^T \frac{\partial^2 \bar{\psi}}{\partial X^2} dA = - \int_A \frac{\partial N^T}{dX} \frac{\partial N}{\partial X} \begin{Bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{Bmatrix} dA \quad (10)$$

Substitution of $N_m = \frac{a_m + b_m x + c_m y}{2a}$ $m = i, j, k$ into equation (10)

$$\begin{aligned} &= - \frac{1}{(2A)^2} \int_A \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} [b_1 b_2 b_3] \begin{Bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{Bmatrix} dA \\ &= - \frac{1}{4A} \begin{bmatrix} b^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^2 \end{bmatrix} \begin{Bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{Bmatrix} \end{aligned} \quad (11)$$

$$\text{Similarly } \int_A N^T \frac{\partial^2 \bar{\psi}}{\partial X^2} dA = - \frac{1}{4A} \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_3 & c_2 c_3 & c_3^2 \end{bmatrix} \begin{Bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{Bmatrix} \quad (12)$$

The third term of the equation (7) is

$$\int_A N^T R a \frac{\partial \theta}{\partial X} dA = R a \int_A N^T \frac{\partial \theta}{\partial X} dA \quad (13)$$

in order to get the matrix equation of (13) the triangular elements can be subdivided into three triangles with a point in the center of original triangles, replace shape functions and using the area integration like first problem.

$$\begin{aligned} \text{We get } &= Ra \frac{A}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{1}{2A} b_1 + b_2 + b_3 \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \\ &= \frac{Ra}{6} \begin{Bmatrix} b_1 \theta_1 + b_2 \theta_2 + b_3 \theta_3 \\ b_1 \theta_1 + b_2 \theta_2 + b_3 \theta_3 \\ b_1 \theta_1 + b_2 \theta_2 + b_3 \theta_3 \end{Bmatrix} \end{aligned} \quad (14)$$

The equation (7) can be written in the matrix form as:

$$= \frac{1}{4A} \left\{ \begin{bmatrix} b^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^2 \end{bmatrix} + \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_2 & c_2 c_3 & c_3^2 \end{bmatrix} \right\} \begin{Bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \\ \bar{\psi}_3 \end{Bmatrix} = \frac{Ra}{6} \begin{Bmatrix} b_1 \theta_1 + b_2 \theta_2 + b_3 \theta_3 \\ b_1 \theta_1 + b_2 \theta_2 + b_3 \theta_3 \\ b_1 \theta_1 + b_2 \theta_2 + b_3 \theta_3 \end{Bmatrix} \quad (15)$$

The Finite Element Method of the Energy Equation

$$\{R^e\} = - \int_A [N]^T \left[\left(\frac{\partial \bar{\psi}}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \bar{\psi}}{\partial X} \frac{\partial \theta}{\partial Y} \right) - \left(\left(1 + \frac{4R_d}{3} \right) \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \right] dA \quad (16)$$

$$\int_A [N]^T \frac{\partial \bar{\psi}}{\partial Y} \frac{\partial \theta}{\partial X} dA = \int_A \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} \frac{\partial [N]}{\partial Y} \{\bar{\psi}\} \frac{\partial [N]}{\partial X} \{\theta\} dA \quad (17)$$

$$= \int_A \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} dA \frac{1}{4A^2} [c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3] [b_1, b_2, b_3] \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \quad (18)$$

$$= \frac{1}{12A} \begin{Bmatrix} c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \\ c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \\ c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \end{Bmatrix} [b_1, b_2, b_3] \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \quad (19)$$

$$\text{Similarly } \int_A N^T \frac{\partial \bar{\psi}}{\partial X} \frac{\partial \theta}{\partial Y} dA = \frac{1}{12A} \begin{Bmatrix} b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \\ b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \\ b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \end{Bmatrix} [c_1, c_2, c_3] \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

(20)

The remaining two terms of energy equation are evaluated as

$$\int_A N^T \left[1 + \frac{4}{3} R_d \right] \frac{\partial^2 \theta}{\partial X^2} dA = -\frac{1}{4A} \left(1 + \frac{4R_d}{3} \right) \begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \quad (21)$$

$$\int_A N^T \frac{\partial^2 \theta}{\partial Y^2} dA = -\frac{1}{4A} \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_3 & c_2 c_3 & c_3^2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \quad (22)$$

Thus the stiffness matrix is given by

$$\begin{aligned} & \frac{1}{12A} \begin{bmatrix} c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \\ c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \\ c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \end{bmatrix} [b_1, b_2, b_3] - \frac{1}{12A} \begin{bmatrix} b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \\ b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \\ b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \end{bmatrix} [c_1, c_2, c_3] \\ & + \frac{1}{4A} \left(\begin{bmatrix} 1 + \frac{4}{3} R_d \end{bmatrix} \begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^2 \end{bmatrix} + \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_3 & c_2 c_3 & c_3^2 \end{bmatrix} \right) \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = 0 \quad (23) \end{aligned}$$

4. NUSSELT NUMBER

The dimensionless average Nusslet number ($\overline{Nu}$) can be calculated using the formula

$$\overline{Nu} = \int_0^1 Nu_y dY \quad (24)$$

where $Nu_y = - \left[\left(1 + \frac{4R_d}{3} \right) \frac{\partial \theta}{\partial X} \right]_{X=0,1}$

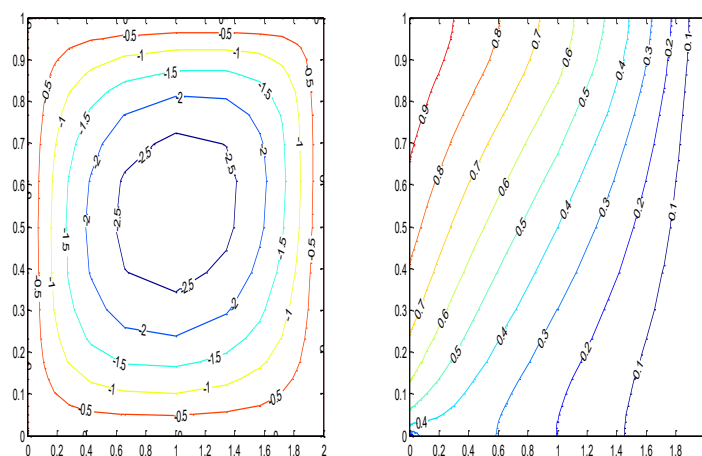
5. RESULTS AND DISCUSSION

The effect of λ on Streamlines and Isotherms of the fluid is illustrated in fig (1) and Fig. (2), which corresponds to the values $Ra = 50$, $A_r = 1$, and $R_d = 1$ and $Ra = 100$, $A_r = 1$ and $R_d = 1$ respectively. In both figures the magnitude of the streamlines decreases when λ increases from 0 to 1 and streamlines tend to move towards upper horizontal wall. In isotherms thickness of the thermal boundary layer becomes small with increase in λ .

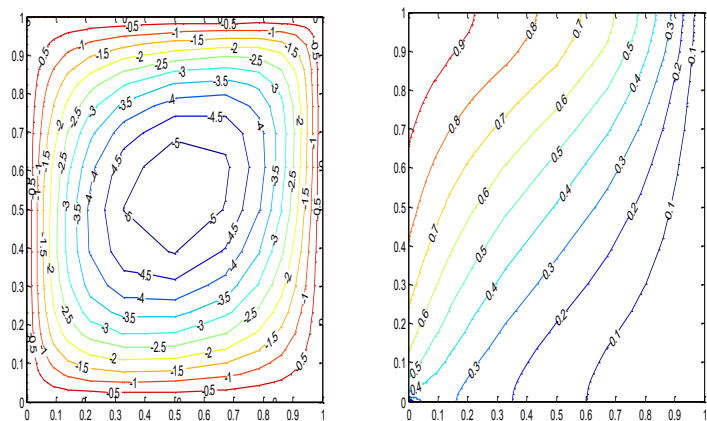
The effect of Ra on Stream lines and Isotherms of the fluid is illustrated in fig (3) and Fig. (4), which corresponds to the values, $\lambda = 0.25$, $A_r = 1$, and $R_d = 1$ and $\lambda = 1$, $A_r = 1$ and $R_d = 1$ respectively. In both figures the increased Raleigh numbers is associated with higher convection heat transfer thus the streamlines crowded and covers almost whole domain. In both cases the area occupied by the temperature lines decreases when Ra is maximum.

The effect of A_r on Stream lines and Isotherms of the fluid is illustrated in Fig. (5) and Fig. (6), which corresponds to the values, $Ra = 100$, $\lambda = 0.25$ and $R_d = 3$ and $\lambda = 1$, $Ra = 100$ and $R_d = 3$ respectively. In both cases with increase in A_r the streamlines are distorted and crowded near upper horizontal wall of the cavity, indicating the increasing fluid velocity at that position. Particularly near $A_r = 2$ we observe convective effect is observed at lower left, upper right corners of the cavity, in Fig.(6), but it shift into upper portion of the hot wall in figure (6).

The effect of R_d on Stream lines and Isotherms of the fluid is illustrated in fig (7) and Fig. (8), which corresponds to the values, $Ra = 100$, $A_r = 1$, $\lambda = 0.25$, and $\lambda = 1$, $Ra = 100$ and $A_r = 1$ respectively. In both cases we observe from the streamlines that the flow consists of a single cell and it is clear from the streamlines that the velocity of the fluid increases with increase in R_d from isotherms we conclude that the thermal boundary layer become thin.



b)



c)

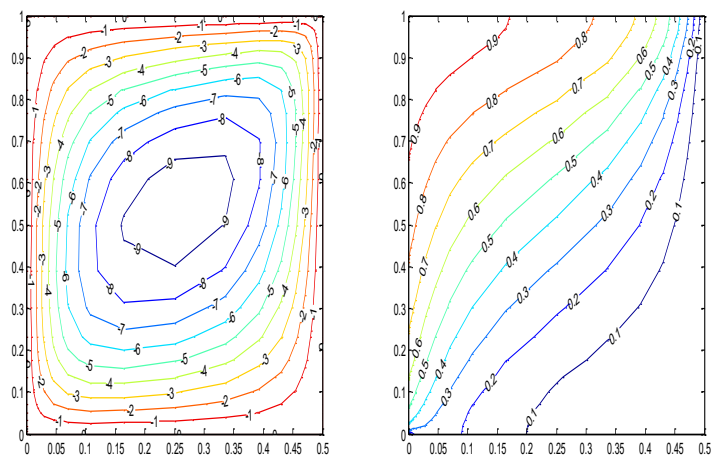
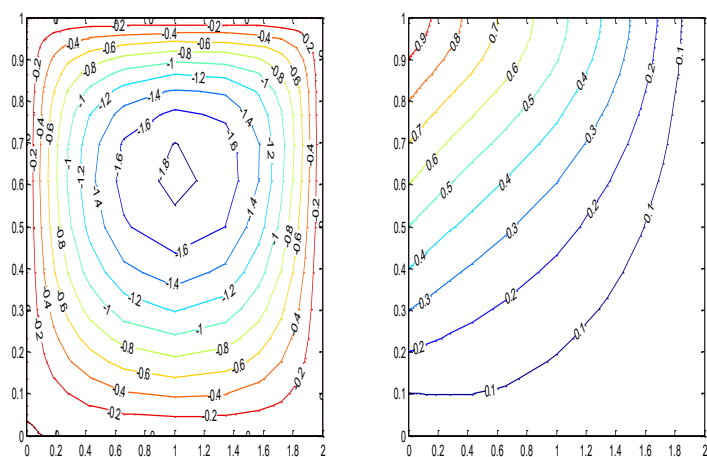


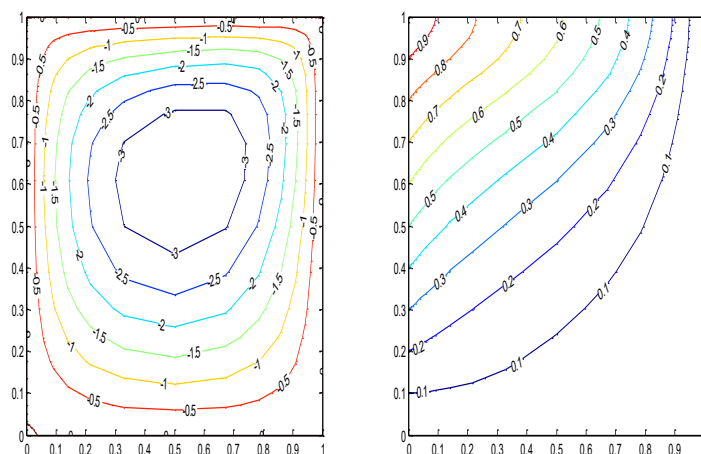
Fig: (1): Streamlines (Left) and Isotherms (Right) for $Ra=100$, $R_d=3$, $\lambda=0.25$

a) $A_r=0.5$ b) $A_r=1$ c) $A_r=2$

d)



e)



f)

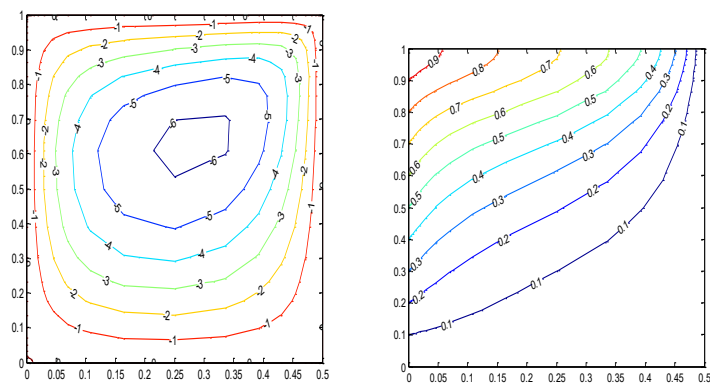
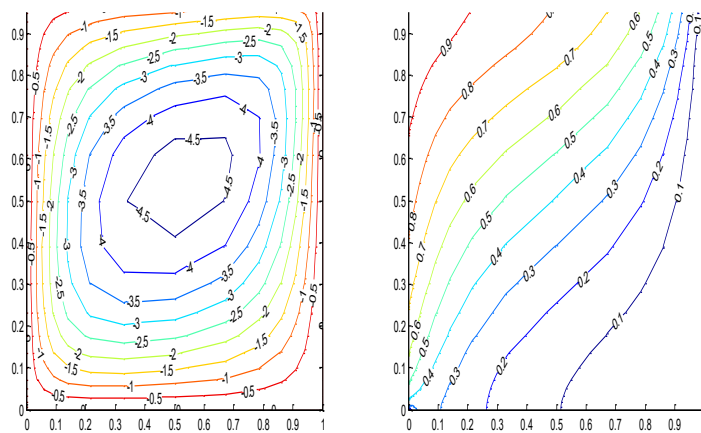
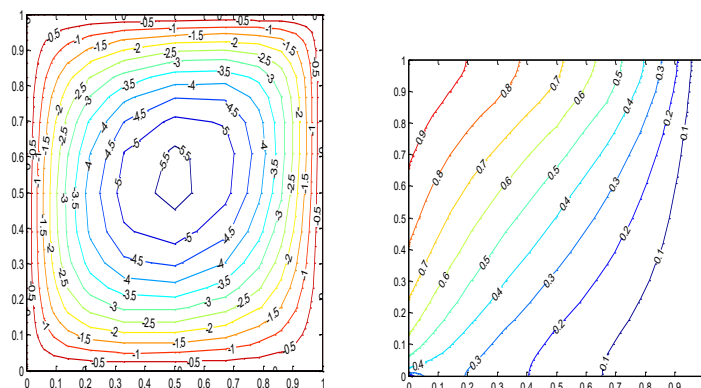


Fig: (2): Streamlines (Left) and Isotherms (Right) for $Ra=100$, $R_d=3$, $\lambda=1$
a) $A_r=0.5$ b) $A_r=1$ c) $A_r=2$



h)



i)

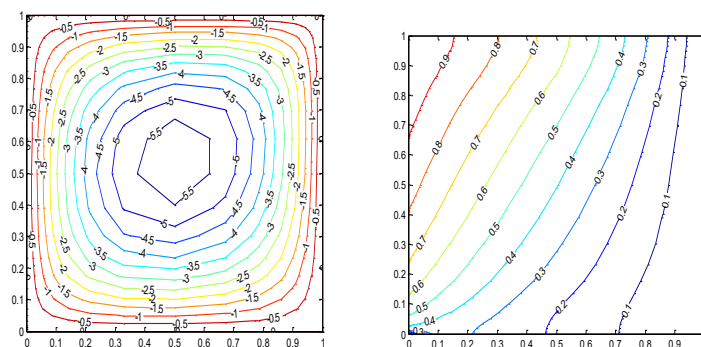
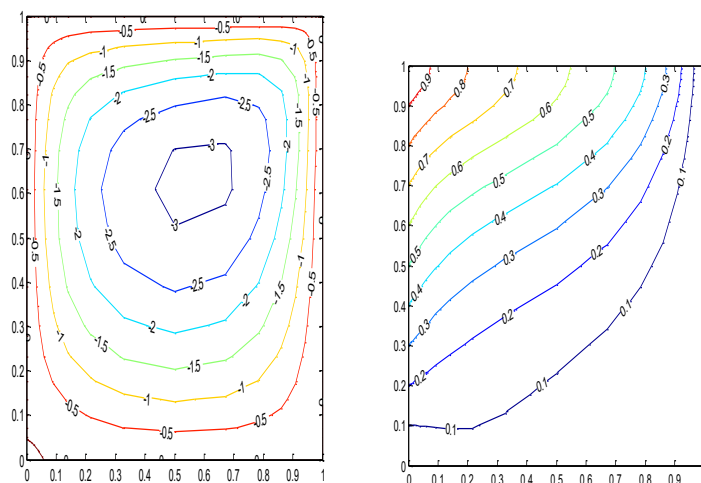
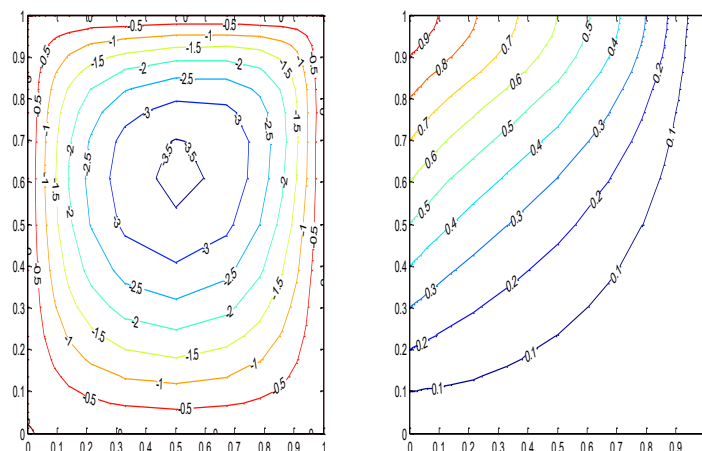


Fig: (3): Streamlines (Left) and Isotherms (Right) for $Ra=100$, $Ar=1$, $\lambda=0.25$

a) $R_d=1$ b) $R_d=5$ c) $R_d=10$ j)



k)



l)

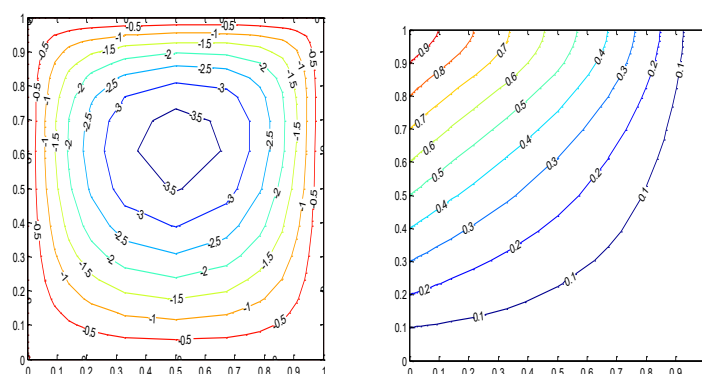
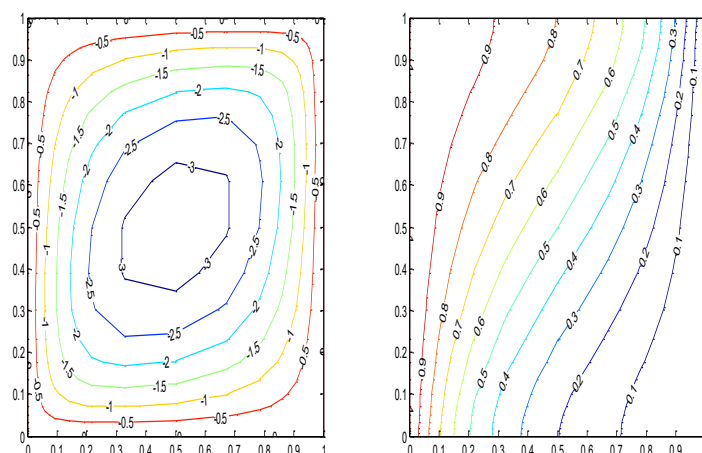
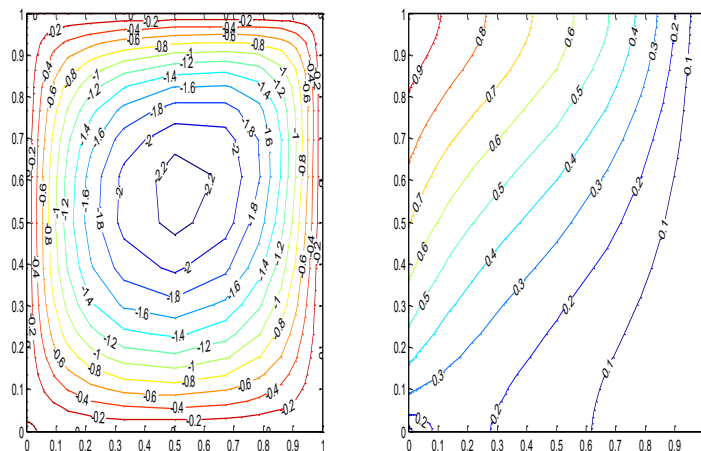


Fig: (4): Streamlines (Left) and Isotherms(Right) for $Ra=100$, $Ar =1$, $\lambda=1$

a) $R_d=1$ b) $R_d=5$ c) $R_d=10m$



n)



o)

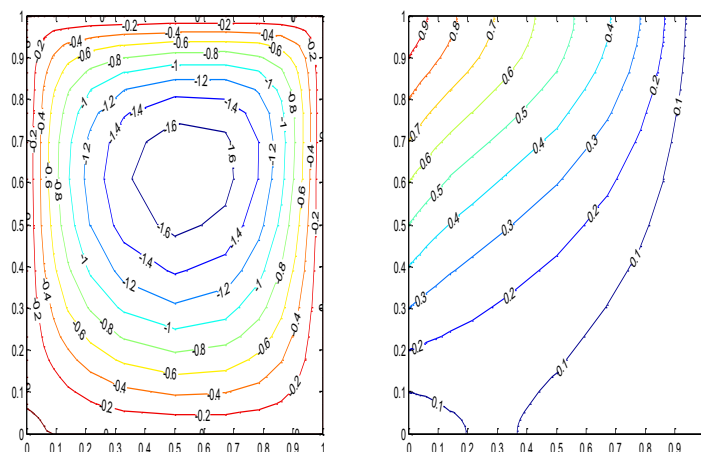
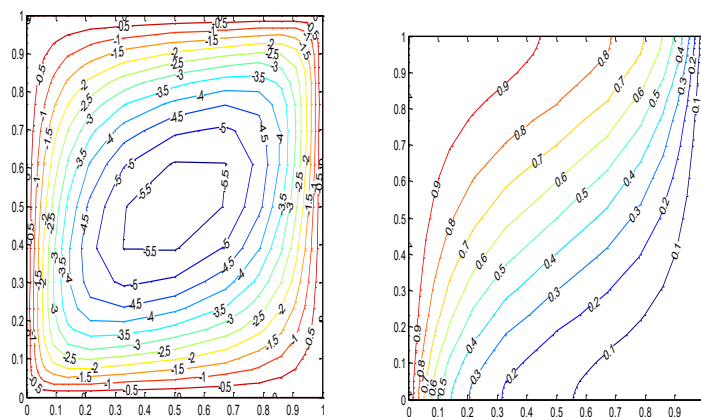


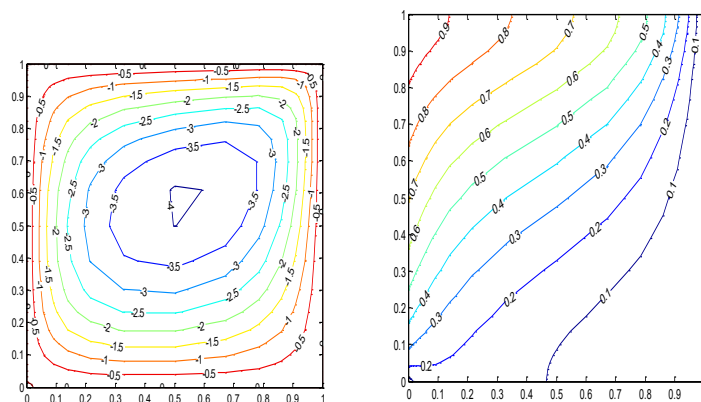
Fig: (5) : Streamlines (Left) and Isotherms(Right) for $Ra=50$, $Ar=1$, $Rd=1$

a) $\lambda=0$ b) $\lambda=0.5$ c) $\lambda=1$

p)



q)



r)

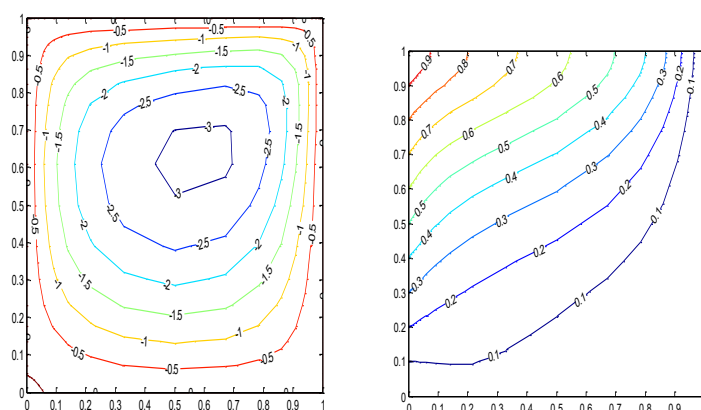
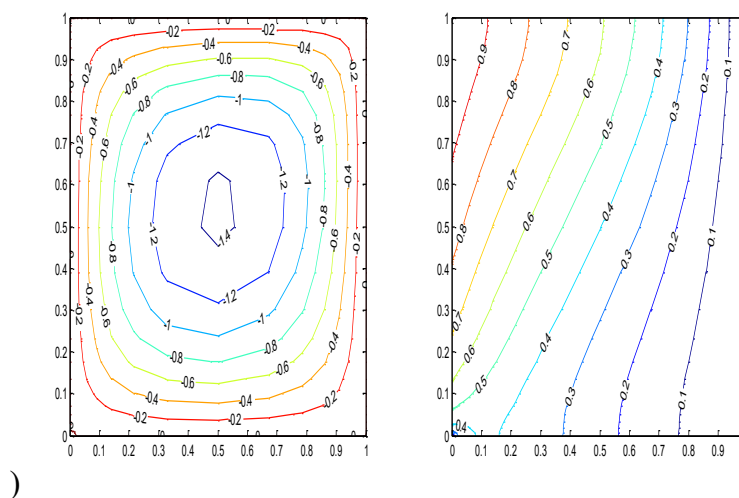
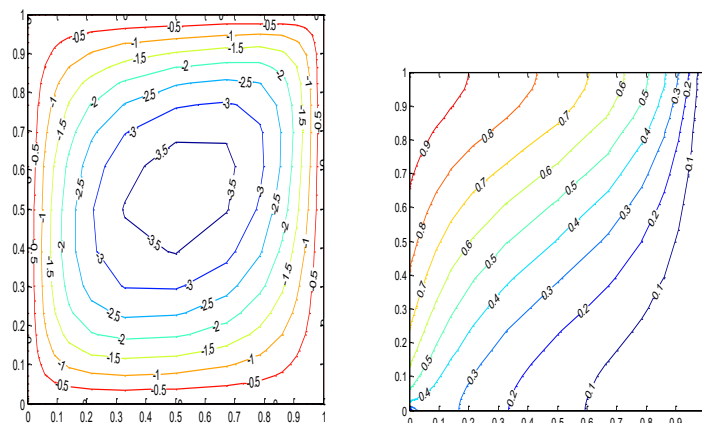


Fig: (6) : Streamlines (Left) and Isotherms (Right) for $Ra=100$, $Ar = 1$, $R_d = 1$

a) $\lambda=0$ b) $\lambda=0.5$ c) $\lambda=1$



t)



u)

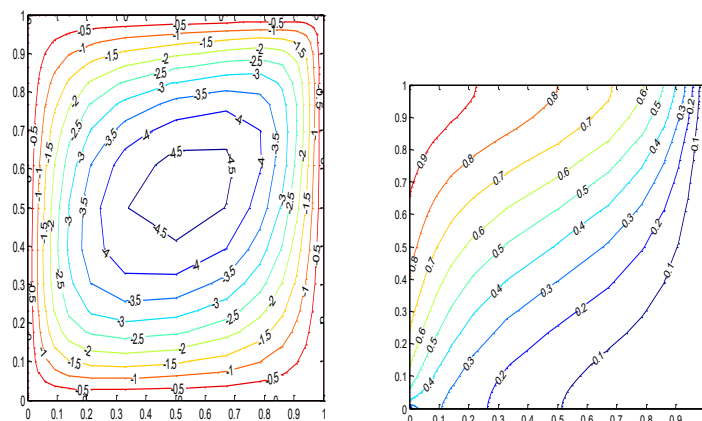
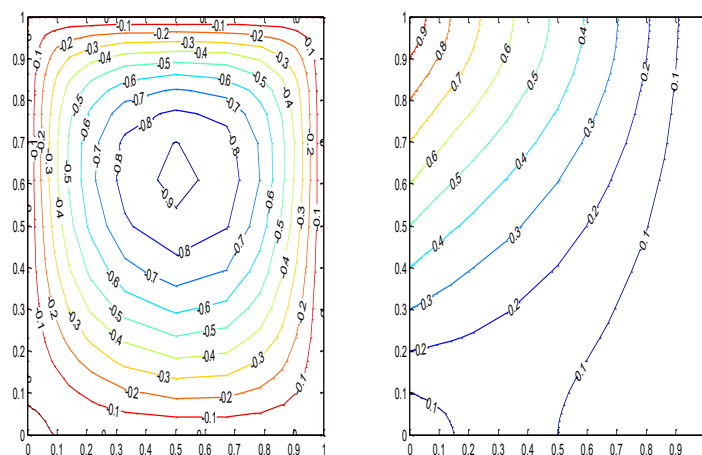


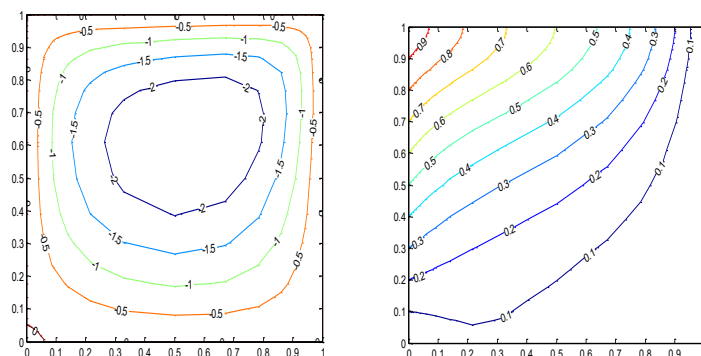
Fig: (7): Streamlines (Left) and Isotherms(Right) for $A_r=1, R_d=1, \lambda=0.25$

a) $Ra=25$ b) $Ra=75$ c) $Ra=100$

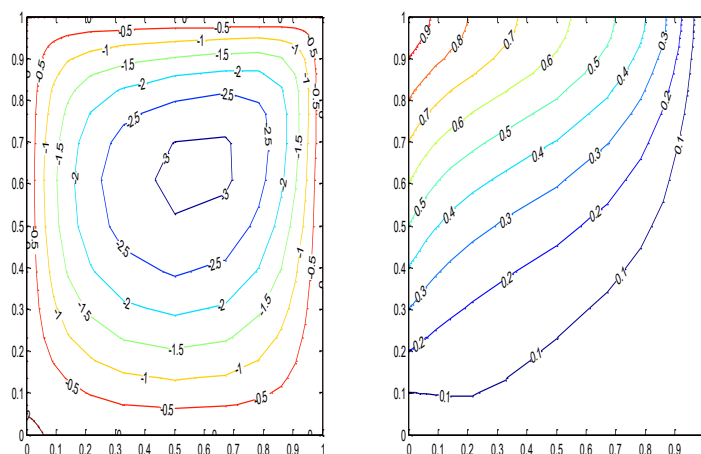
v)



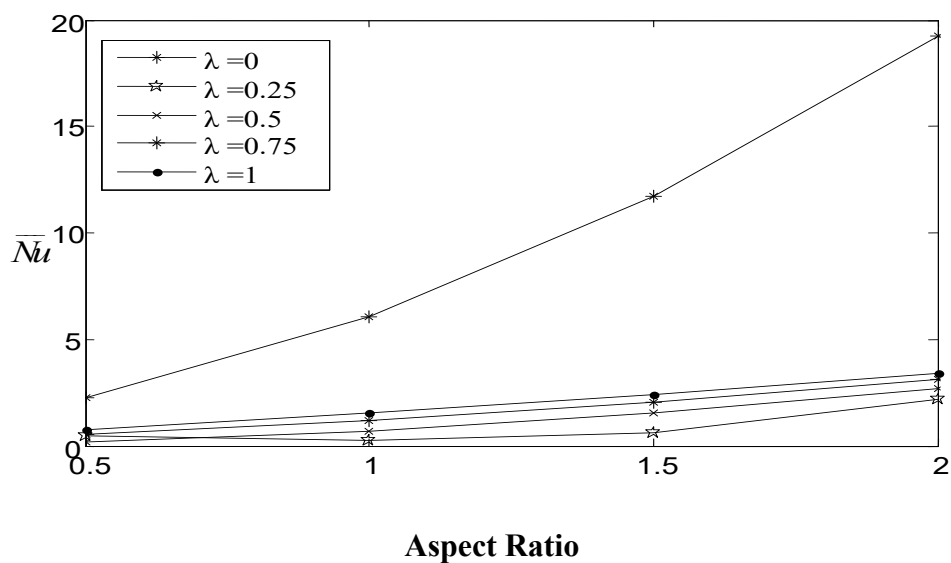
w)



x)

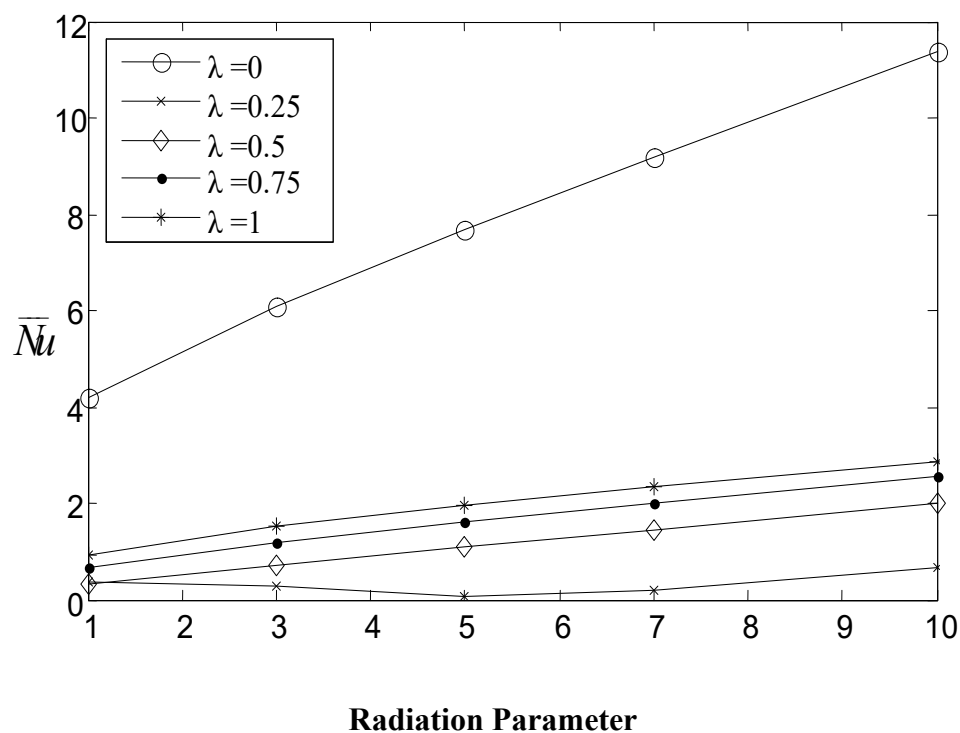


**Fig. (8): Streamlines (Left) and Isotherms (Right) for $Ar=1, Ra=1, \lambda=1$
a) $Ra=25$ b) $Ra=75$ c) $Ra=100$**



**Fig: (9): $\bar{Nu}$ Variations with A_r at hot surface for different values of λ
at $R_d=3, Ra=100$**

Fig (9) shows the variation of $\bar{Nu}$ with respect to A_r of the cavity for various values of power law exponent at $R_d = 3, Ra = 100$. The $\bar{Nu}$ is higher for the case of isothermal wall temperature. For a given value of A_r the lines corresponds to $\lambda > 0$ come close in between A_r , 0.5 to 1 but it changes with increase in A_r .



**Fig: (10): $\bar{Nu}$ Variations with R_d at hot surface for different values of λ
at $A_r=1, Ra=100$**

Fig (10) shows the variation of $\bar{Nu}$ with respect to R_d for variation values of λ at $A_r = 1, Ra = 100$. $\bar{Nu}$ increases with increase in R_d except at $\lambda = 1$. But the line corresponds to $\lambda = 1$ first gradually decreases next it increases with increase in R_d . It is clear from the figure that heat content of the wall is more at $\lambda = 0$, compared with other values of λ .

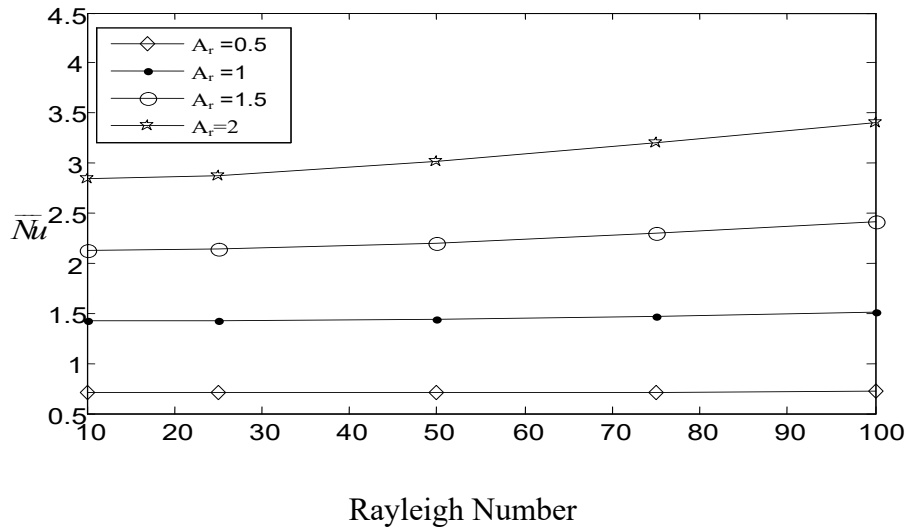


Fig: (11): $\bar{Nu}$ Variations with Ra at hot surface for different values of A_r at $\lambda = 1$, $R_d = 3$

Fig (11) shows the variation of $\bar{Nu}$ with respect to Ra for various values of A_r at $\lambda = 1$, $R_d = 3$. $\bar{Nu}$ increases with increase in A_r . By comparing the lines corresponds to $A_r = 0.5$, $A_r = 2$ the effective change is observed.

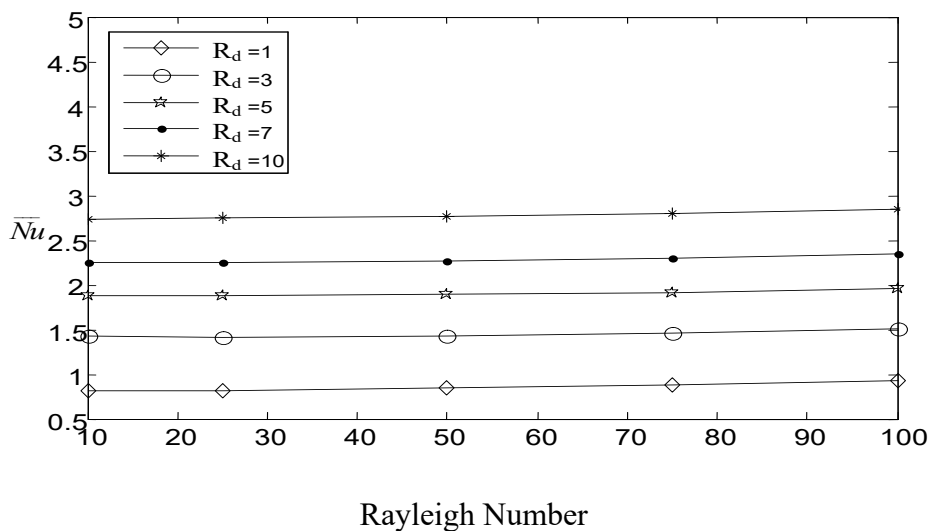


Fig: (12): $\bar{Nu}$ Variations with Ra at hot surface for different values of R_d at $\lambda = 1$, $A_r = 1$

Fig (12) shows the variation of $\bar{Nu}$ with respect to Ra for various values of R_d at $\lambda = 1$, $A_r = 1$. It is clear from the figure that increase in $\bar{Nu}$ is almost linear with respect to Ra .

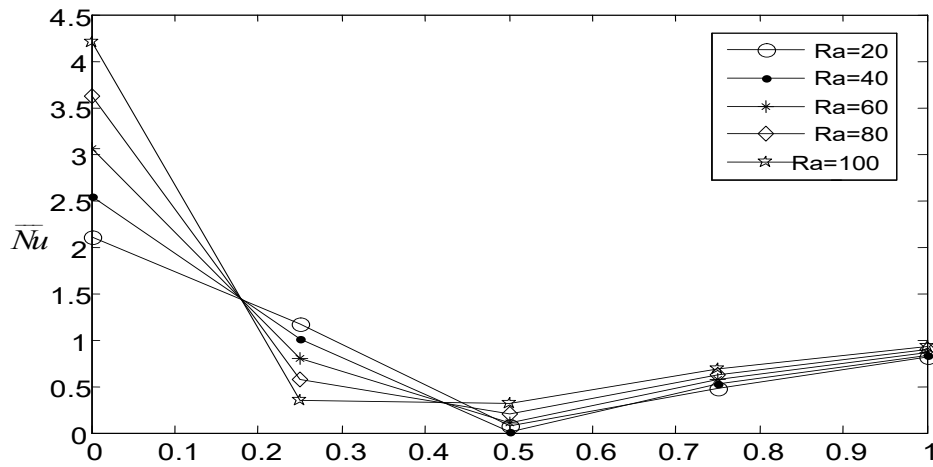


Fig: (13): $\bar{Nu}$ Variations with λ at hot surface for different values of Ra at $Ar=1, Rd=1$

Fig (13) shows the variation of $\bar{Nu}$ with respect to λ for hot surface at various values of Ra at $Ar=1, Rd=1$. It is obvious from the figure that the line corresponds to different Ra decreases with increase in λ , but they merge two times in between $\lambda = 0.1$ to 0.5 and next they increases with increase in λ . But all the lines come close at high value of λ .

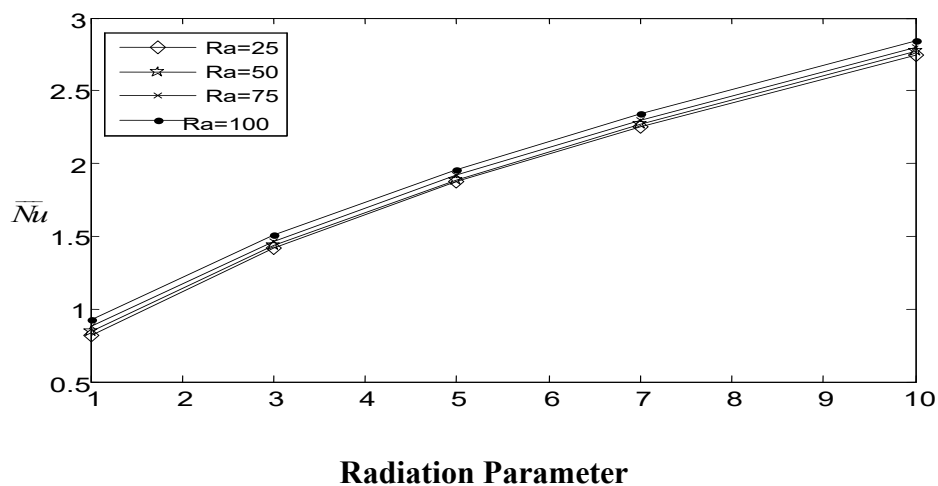


Fig: (14): $\bar{Nu}$ Variations with R_d at hot surface for different values of Ra at $Ar=1, \lambda=1$

Fig (14) shows the variation of $\bar{Nu}$ with respect to R_d for hot surface at different values of Ra at $Ar=1, \lambda=1$. $\bar{Nu}$ increases with increase in R_d , the line is corresponds to different Ra come close there is no change even the R_d is maximum.

6. CONCLUSIONS

The magnitude of the streamlines decreases when λ as increase from 0 to 1 is and streamlines tend to move towards upper horizontal wall. The isotherms thickness of the thermal boundary layer becomes small with increase in λ . The increased Raleigh numbers is associated with higher convection heat transfer thus the streamlines crowded and covers almost whole domain. The area occupied by the temperature lines decreases when Ra is maximum.

Increase in A_r the streamlines are distorted and crowded near upper horizontal wall of the cavity, indicating the increasing fluid velocity at that position. Particularly near $A_r = 2$ we observe convective effect is observed at lower left, upper right corners of the cavity.

The flow consists of a single cell and it is clear from the streamlines that the velocity of the fluid increases with increase in R_d form isotherms. We conclude that the thermal boundary layer become thin.

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