International Journal of Applied Mathematics

Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

STOCHASTIC ASSESSMENT OF COMPLEX REPAIRABLE SYSTEM UNDER MULTI REPAIR APPROACH COPULA DISTRIBUTION

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Abstract:

The best performing and repairable systems are required in almost everywhere in the industry and organizations. This paper analyses a two-subsystem series system's performance. A k-out-of-n:G system, Subsystem 1, is composed of n identical units exhibiting exponentially distributed failure rates and employing both general and copula repair methods. With six identical units, subsystem-2 (the second subsystem) uses a 3-out-of-6: G work policy. A human runs the system, and human error will break it. Stochastic theory facilitates the derivation of differential equations aimed at exposure vital reliability characteristics. These visual aids facilitate comprehensive analysis, allowing engineers and analysts to predict performance trends and identify potential areas of concern or improvement.

Math. Subject Classification: MSC 68M15, MSC 90B25

Keywords: System Availability; MTTF; Supplementary variable; System Reliability; Cost analysis; Gumbel-Hougaard family copula distribution.

1. Introduction:

Maintenance is an indispensable activity that ensures the proper and reliable functioning of repairable systems. In large engineering facilities, the substantial direct and indirect costs of unreliability make this especially important to industrial plant owners. To facilitate cost-economical repairs, a variety of research initiatives are being undertaken worldwide. Researchers have explored different repair and maintenance strategies, validating their effectiveness through the results generated by these policies. A minimum financial return on investment is essential for survival in the profit-driven manufacturing sector. The reputation and customer satisfaction of any manufacturer depends on its financial well-being. System reliability metrics are vital in preserving both reputation and customer loyalty.

Numerous researchers have premeditated various systems, asserting enhanced performance. Nevertheless, the evaluation of such performance improvements can only be confirmed through computational analysis. It is widely recognized that redundancy enhances system performance. A k-out-of-n configuration is typical in almost every industrial system.

Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Historically, numerous researchers have examined the k-out-of-n configuration by employing two-state and three-state working models, which include perfect, degraded, and failed states. Further advancements in k-out-of-n: G models have been explored, wherein the degraded state is subdivided into minor and major degraded states, resulting in a four-state model comprising perfect, inconsequential degraded, main degraded, and whole failed states. Repair processes are critical in the study of system reliability models. Various repair types have been cohesive by researchers to analyze the performance of repairable systems. When a system enters a failed state, prompt repair is essential to restore functionality, achievable through general repair or expedited repair processes such as exponential repair. It is important to note that relying solely on general repair is insufficient for maintenance, while exponential repair can be prohibitively expensive. Additionally, it is a well-established fact that a mixed maintenance approach is necessary for the effective operation of industrial organizations. Several techniques are available to enhance network reliability, including the adjustment of redundant units with central units, and the design of series and parallel configurations, as well as k-out-of-n: G/F configurations. A major factor in system reliability is the application of different repair types.

By creating intricate mathematical models and assessing their effectiveness using a variety of methods and approaches, researchers have made noteworthy advances. Several researchers have used the copula repair function together with the supplementary variable approach to analyze the performance of repairable systems in complex system performance assessment. The study was conducted by V. V. Singh et al. (2009) and V. V. Singh and D. K. Rawal (2015), Jyoti Gulati et al. (2016), V. V. Singh and N. P. Singh (2015). An investigation of the reliability of complex repairable systems with a 2-out-of-3: G subsystem established in both series and parallel configurations was led by Alka Munjal and S. B. Singh (2014). The stochastic analysis of a standby system employing the waiting repair method was investigated by M. Ram et al. (2013). Rawal et al. (2015) used copula linguistics to examine a local area network's reliability metrics. In addition, Maninder et al. (2013) used Kolmogorov differential equations to investigate a complex system in the textile industry under a preventive maintenance scheme. Keeping emphasis on the fault coverage factor, Nupur Goyal et al. (2016) examined the dependability features of a thermal power plant.

There are two subsystems in the current study on the dependability of complex repairable systems, which emphasizes how complicated these systems are and how different approaches are used to evaluate their performance. This topic has benefited from the modeling and availability study of internet data centers conducted by researchers such as Rawal et al. (2014). The impact of various maintenance policies on system performance is examined in this work, which offers important insights into how these policies might improve data centers' efficiency and dependability—two essential elements of contemporary digital infrastructure. Singh et al. (2016) performed an exhaustive performance evaluation of complex systems designed in a series structure in a related study. Using copula approaches, this study looked at a variety of failure and repair disciplines to comprehend component dependencies and how they affect overall system reliability. The study's conclusions are essential for creating systems that can endure breakdowns and continue to function.

Volume 38 No. 12s, 2025

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Expanding on prior series configuration research, Singh, Ram, and Rawal's (2013) study assessed the financial effects of a two-subsystem-series engineering system. Their study draws attention to the cost and reliability trade-offs and offers a framework for decision-makers to maximize system design and maintenance plans. V. V. Singh et al. (2013) included availability, mean time to failure (MTTF), and cost analysis in their study of a controller and two units in series. The significance of comprehending how control methods might affect system efficiency and reliability is emphasized by their work. Another important furtherance was made by V.V. Singh and H. I. Ayagi (2018), who employed a copula to model the interdependencies between system components and examined how well a complex system performed under a preventive resume repair policy. This study provides information on how doing repairs first can improve system availability and save downtime. Monika Gahlot et al. (2018& 2020) introduced a probabilistic method to system dependability by further analyzing a complex system with two subsystems in series pattern under a k-out-of-n: G policy. Authors, Ibrahim Yusuf et al. (2024), studied system performance of serially arranged system comprising subsystems of multi-state through copula repair methods. Jyoti Paliwal et al. (2025), Singh et al. (2025) have analyzed a complex system comprised subsystem in series configuration with concept of multi repair approach. Their research advances our knowledge of how system design and redundancy might reduce the hazards brought on by component failures.

Although numerous researchers worldwide have investigated the reliability of intricate repairable systems, the case of two series subsystems operated by a human remains unexplored. This work aims to bridge this gap through the development of a mathematical model. This analysis uses Laplace transformations and the Gumbel-Hougaard copula family, incorporating supplementary variables. Precise expressions for reliability, availability, MTTF, and cost analysis functions resulted from using MAPLE software. Reliability metrics (reliability, availability, MTTF, and cost) are analyzed through specialized computations that consider failure and repair rates. The copula pair boosts system availability, thus reducing operational costs and increasing net production return, according to the data. Tables and graphs provide a study of results with and without copulas, highlighting the distinctions.

This paper is organized as follows: Section 1 reviews the relevant literature, introducing the model. Section 2 explains the system description, featuring transition diagrams. The assumptions and notations are detailed in Sections 3 and 4. Sections 5 cover the model's formulation and solution, including an analytical study that investigates availability, reliability, and profit analysis. The conclusions of the proposed analysis are found in Section 6.

2. System Architecture and Description:

The four system states (perfect, minor degraded, major degraded/critical, and completely failed) are represented by a circle, rectangle, diamond, and hexagon, respectively, in the system architecture (Figure 1(a)) and state transition diagram (Figure 1(b)). The state S_0 is a perfect state in which the system work with full efficiency, minor degraded state (S_1, S_4, S_5) are also working states with low efficiency in which system has no danger of failure and a major degraded state/ critical working states (S_2 & S_6) refers to a condition of a system or component

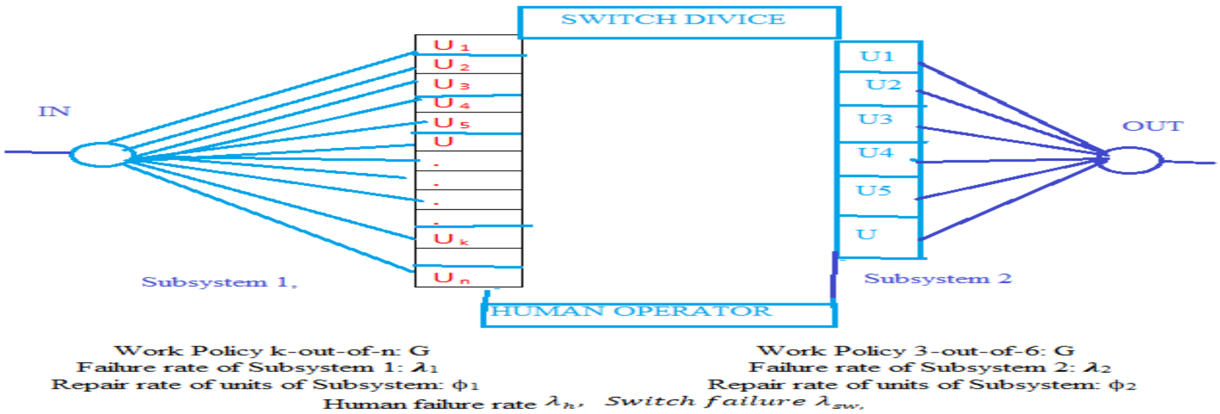
International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

where its performance or functionality has significantly deteriorated but not entirely failed. If all units in any subsystem fail, or due to likely human error, the system (S_3, S_7, S_8) will completely fail. The repair method depends on the level of disintegration: general distribution for minor/major failures, and the Gumbel-Hougaard copula distribution for complete failures.

2.1 State Description:

State	State Description
S_o	The state description for S_0 is one in which the system is perfect, and all subsystems are operational.
S_1	The system is experiencing minor degradation following a component failure within subsystem 1.
S_2	The failure of k subsystem 1 components has resulted in the severely degraded state S_2. A critical state will also result if any single component fails, leading to complete system failure.
S_3	It represents a complete failed state due to failing (k+1) components in subsystem 1.
S_4	It represents a minor partial failure in subsystem 2 due to failing one component in subsystem 2.
S_5	It indicates a slight partial failure in subsystem 2 resulting from a failing second unit.
S_6	State S_6 represents a major degraded state due to failure of third components subsystem 2. It will be a critical state also because if any one component fails it will lead to a complete failed state.
S_7	This state is a completely failed state due to failing the subsystem 2 as per defined working policy 3-out-of-6: G scheme.
S_8	The state where the system is completely failed due to human error

Figure 1(a) (System Block Diagram)



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Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

2.2. State transition diagram of model:

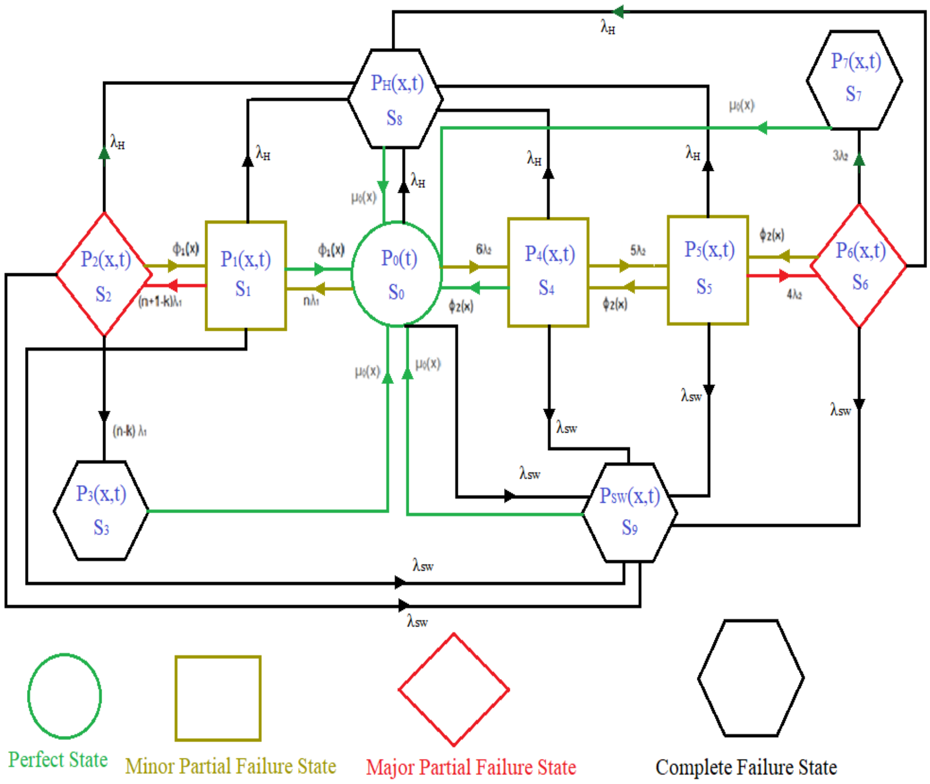

Figure 1b. State Transition diagram of Model

3.Assumptions:

The following assumptions hold for both subsystems. Subsystems 1 and 2 and their components are all working exceptionally well in the initial state (S_0).

i. The subsystem-1 works till k units are operational and fails if more than (n-k) units fail.
ii. The subsystem-2 operates until three units are operationally sound and fails if more than three fail.
iii. If either subsystem 1 or subsystem 2 fails, the system will become inoperable.
iv. When a system unit fails, it can be repaired, whether it is operational or inoperative.
v. All failure rates are constant and assume an exponential distribution.
vi. The repairs are distributed in a general pattern. (i.e. $\emptyset(x)=x$).
vii. It is expected that a repaired system operates as if it were new, and that no damage occurred during the repair process.
viii. As soon as the failed unit is repaired, it will be ready to do the work.

4. Notation used for study of the model:

t/s	Time and Laplace transform variables are in all expressions.

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Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$\lambda_1/\lambda_2/\lambda_H/\lambda_{sw}$	Failure rates of system (1&2)/human failure/switch failure
$\phi_1(x)$/ $\phi_2(x)$/ $\mu_0(x)$	General repair rates/ copula repair rates for partially failed states/ complete failed states.
$P_0(t)$	The system's probability of being in perfect state S_0.
$\overline{P}(s)$	Laplace transformation of P (t).
$P_i(x, t)$	Probability of a system in state Sj across i=1 to 7.
$E_p(t)$	It denotes the expected profit during the interval [0, t).
K_1, K_2	Revenue and service cost by the time unit.
$\mu_0(x)=$ $C_\theta(u_1(x), u_2(x))$	The Gumbel-Hougaard family copula states that the joint probability from failed state Si to good state S_0 can be expressed as $C_\theta\ (u_1(x), u_2(x)) = \exp\,[x^\theta + \{\log\varphi(x)\}^\theta]^{\frac{1}{\theta}}$ for $1 \leq \theta \leq \infty$. $u_1 = \phi(x)$, and $u_2 = \exp\{x\}$,θ is a parameter.

5. Formulation of a Mathematical Model:

The probability constraint and argument lead to this equation, which is differential, for the model.

If the system is in state (S_0) at time (t), it stays in (S0) during (t, t+Δt); otherwise, after repair, it transitions to (S_0).

$$S_0: \left[\frac{\partial}{\partial t} + n\lambda_1 + 6\lambda_2 + \lambda_H + \lambda_{sw}\right] P_0(t) = \int_0^\infty \phi_1(x) P_1(x,t)dx + \int_0^\infty \phi_2(x) P_4(x,t)dx + \sum_j \int_0^\infty \mu_0(x) P_j(x,t)dx\ ,\ j = 3,7,9,H \quad \ldots(1)$$

$$S_1: \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + (n+1-k)\lambda_1 + \lambda_H + \lambda_{sw} + \phi_1(x)\right] P_1(x,t) = 0 \quad \ldots(2)$$

$$S_2: \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + (n-k)\lambda_1 + \lambda_H + \lambda_{sw} + \phi_1(x)\right] P_2(x,t) = 0 \quad \ldots(3)$$

$$S_3: \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right] P_3(x,t) = 0 \quad \ldots(4)$$

$$S_4: \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 5\lambda_2 + \lambda_H + \lambda_{sw} + \phi_2(x)\right] P_4(x,t) = 0 \quad \ldots(5)$$

$$S_5: \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 4\lambda_2 + \lambda_H + \lambda_{sw} + \phi_2(x)\right] P_5(x,t) = 0 \quad \ldots(6)$$

$$S_6: \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 3\lambda_2 + \lambda_H + \lambda_{sw} + \phi_2(x)\right] P_6(x,t) = 0 \quad \ldots(7)$$

$$S_7: \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right] P_7(x,t) = 0 \quad \ldots(8)$$

$$S_8: \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right] P_H(x,t) = 0 \quad \ldots(9)$$

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$S_9 : \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0 (x)\right] P_{sw}(x, t) = 0$ ...(10)

Boundary Conditions:

The following boundary conditions are accompanied by the present mathematical model. Let us consider that repair is not available; then, the following boundary condition will be associated.

$P_1 (0,t) = n \lambda_1 P_0(t)$ ...(11)

$P_2 (0,t) = (n+1-k)\lambda_1 . n\lambda_1 p_0(t) = n(n+1-k)\lambda_1{}^2 P_0(t)$ …(12)

$P_3 (0,t) = n(n+1-k)(n-k)\lambda_1{}^3 P_0(t)$ ...(13)

$P_4 (0,t) = 6\lambda_2 P_0 (t)$ …(14)

$P_5 (0,t) = 30\lambda_2{}^2 P_0(t)$ ...(15)

$P_6 (0,t) = 120\lambda_2{}^3 P_0(t)$ …(16)

$P_7 (0,t) = 360\lambda_2{}^4 P_0(t)$ …(17)

$P_H(0,t) = \lambda_H (1+n\lambda_1 + n(n+1-k) \lambda_1{}^2 + 6\lambda_2 + 30\lambda_2{}^2 + 120 \lambda_2{}^3) P_0(t)$ ….(18)

$P_9(0,t) = \lambda_{sw} (1+n\lambda_1 + n(n+1-k) \lambda_1{}^2 + 6\lambda_2 + 30\lambda_2{}^2 + 120 \lambda_2{}^3) P_0(t)$ ….(19)

Initial conditions and other concurrent conditions associated with mathematical study are as follows

Initially, $P_0(0)=1$, while all other state transition probabilities at time zero are 0; that is, $P_j(x,0) = 0$ for j = 1, 2, 3..., 7, 8, 9.

Let us define a function

$$\kappa_\phi(x) = \phi(x) \exp\left[-\int_0^x \phi(x)dx\right], \bar{\kappa}_\phi(s) = L\left[\kappa_\phi(x)\right]$$

$\kappa_\phi(x) \& \bar{\kappa}_\phi(s)$ *reperesents general repair function with repair rate* $\phi(x)$ *and Laplace transform of* $\kappa_\phi(x)$ *for solving mathematical equations.*

The solution of above differential equations can be obtained by Laplace applications and stated boundary conditions. The Laplace transform of various state transition probability can be presented as.

$$\bar{P}_0(s) = \frac{1}{D(s)} = 1 \quad …(20)$$

$$\bar{P}_1(s) = \frac{n\lambda_1}{D(s)}\left\{\frac{1-\bar{\kappa}_{\phi_1}(S+(n+1-k)\lambda_1+\lambda_H+\lambda_{sw})}{s+(n+1-k)\lambda_1+\lambda_H+\lambda_{sw}}\right\} \quad …(21)$$

$$\bar{P}_2(s) = \frac{n(n+1-k)\lambda_1{}^2}{D(s)}\left\{\frac{1-\bar{\kappa}_{\phi_1}(S+(n-k)\lambda_1+\lambda_H+\lambda_{sw})}{s+(n-k)\lambda_1+\lambda_H+\lambda_{sw}}\right\} \quad …(22)$$

$$\bar{P}_3(s) = \frac{n(n+1-k)(n-k)\lambda_1{}^3}{D(s)}\left\{\frac{1-\bar{\kappa}_{\mu o}(s)}{s}\right\} \quad …(23)$$

Received: August 15, 2025

Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$\bar{P}_4(s) = \frac{6\lambda_2}{D(s)}\left\{\frac{1-\bar{\kappa}_{\phi_1}(S+5\lambda_2+\lambda_H+\lambda_{sw})}{s+5\lambda_2+\lambda_H+\lambda_{sw}}\right\}$ …(24)

$\bar{P}_5(s) = \frac{30\lambda_2{}^2}{D(s)}\left\{\frac{1-\bar{\kappa}_{\phi_1}(S+4\lambda_2+\lambda_H+\lambda_{sw})}{s+4\lambda_2+\lambda_H+\lambda_{sw}}\right\}$ …(25)

$\bar{P}_6(s) = \frac{120\lambda_2{}^3}{D(s)}\left\{\frac{1-\bar{\kappa}_{\phi_1}(S+3\lambda_2+\lambda_H+\lambda_{sw})}{s+3\lambda_2+\lambda_H+\lambda_{sw}}\right\}$ …(26)

$\bar{P}_7$ (s) $=\frac{360\lambda_2{}^4}{D(s)}\left\{\frac{1-\bar{\kappa}_{\mu o}(s)}{s}\right\}$ …(27)

P_H (s) = $\frac{\lambda_H . A}{D(s)}\left\{\frac{1-\bar{\kappa}_{\mu o}(s)}{s}\right\}$ …(28)

P_{sw} (s) = $\frac{\lambda_{sw} . B}{D(s)}\left\{\frac{1-\bar{\kappa}_{\mu o}(s)}{s}\right\}$ …(29)

Here, A=$\left(1 + n\lambda_1 + n(n+1-k)\lambda_1{}^2 + 6\lambda_2 + 30\lambda_2{}^2 + 120\lambda_2{}^3\right)$

B=$\left(1 + n\lambda_1 + n(n+1-k)\lambda_1{}^2 + 6\lambda_2 + 30\lambda_2{}^2 + 120\lambda_2{}^3\right)$

$$D(s) = s + n\lambda_1 + 6\lambda_2 + \lambda_H - \frac{n\lambda_1\phi_1}{S+(n+1-k)\lambda_1+\lambda_H+\phi_1} - \frac{n\,(n+1-k)(n-k)\lambda_1{}^3\mu_0}{s+\mu_0} + \frac{\lambda_H\left(1+n\lambda_1+n(n+1-k)\lambda_1{}^2+6\,\lambda_2+30\,\lambda_2{}^2+120\,\lambda_2{}^3\right)\mu_0}{s+\mu_0} - \left[\frac{\lambda_{sw}\left(1+n\lambda_1+n(n+1-k)\lambda_1{}^2+6\,\lambda_2+30\,\lambda_2{}^2+120\,\lambda_2{}^3\right)\mu_0}{s+\mu_0} - \frac{6\lambda_2\phi_2}{S+5\lambda_2+\lambda_H+\phi_2} - \frac{360\lambda_2{}^3\mu_0}{s+\mu_0}\right] \quad \ldots(30)$$

The sum of Laplace transforms the state transition probabilities where the system is in operational mode is:

$P_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_4(s) + \bar{P}_5(s) + \bar{P}_6(s)$. …(31)

$$P_{up}(s) = \frac{1}{D(s)}\left(1 + \frac{n\lambda_1}{s+(n+1-k)\lambda_1+\lambda_H+\phi_1} + \frac{n(n+1-k)\lambda_1{}^2}{s+(n-k)\lambda_1+\lambda_H+\phi_1} + \frac{6\lambda_2}{s+5\lambda_2+\lambda_H+\phi_2} + \frac{30\lambda_2{}^2}{s+4\lambda_2+\lambda_H+\phi_2} + \frac{120\lambda_2{}^3}{s+3\lambda_2+\lambda_H+\phi_2}\right) \quad \ldots(32$$

4. Analysis & discussion of results

4.1 Availability Analysis:

If a repair follows general or exponential distributions,

Setting

$\bar{S}_{\mu_0}(s) = \frac{\mu_0}{s+\mu_0}$, $\bar{S}_{\varphi_s}(s) = \frac{\varphi_s}{s+\varphi_s}$ *and using parametric values of failure and repair as*; $\lambda_1 = 0.02$, $\lambda_2 = 0.015$, $\lambda_H = 0.02$, $\lambda_{sw} = 0.02$, $\varphi_1 = \varphi_2 = 1$ *and* $\mu_0 = \mu_0 = 2.7183$, in $P_{up}(s)$ and taking inverse Laplace Transform we get the following expression for the system performance metric as system availability for copula repair. Here we have evaluated the performance of four types

International Journal of Applied Mathematics

Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

of complex systems (10-out-of-50: G, 20-out-of-50: G, 30-out-of-50: G & 40-out-of-50: G) as the cases in equation (32) and graphically represented in figure:

For $k = 10$, $n = 50$:

$$\begin{aligned}\mathrm{AV}_I(t) = {} & -0.0036007\, e^{-2.9124t} - 0.15254\, e^{-2.4088t} + 0.20701 \times 10^{-1} e^{-1.1689t} \\ & + 1.1368\, e^{-0.25808t} - 0.39331 \times 10^{-3} e^{-1.8400t} - 0.90439 \times 10^{-3} e^{-1.1000t} \\ & - 0.91583 \times 10^{-4} e^{-1.0850t}.\end{aligned}$$

For $k = 20$, $n = 50$:

$$\begin{aligned}\mathrm{AV}_{II}(t) = {} & -0.0036007\, e^{-2.9124t} - 0.15254\, e^{-2.4088t} + 0.20701 \times 10^{-1} e^{-1.1689t} \\ & + 1.1368\, e^{-0.25808t} - 0.39331 \times 10^{-3} e^{-1.8400t} - 0.90439 \times 10^{-3} e^{-1.1000t} \\ & - 0.91583 \times 10^{-4} e^{-1.0850t}.\end{aligned}$$

For $k = 30$, $n = 50$:

$$\begin{aligned}\mathrm{AV}_{III}(t) = {} & -0.0036007\, e^{-2.9124t} - 0.15254\, e^{-2.4088t} + 0.20701 \times 10^{-1} e^{-1.1689t} \\ & + 1.1368\, e^{-0.25808t} - 0.39331 \times 10^{-3} e^{-1.8400t} - 0.90439 \times 10^{-3} e^{-1.1000t} \\ & - 0.91583 \times 10^{-4} e^{-1.0850t}.\end{aligned}$$

For $k = 40$, $n = 50$:

$$\mathrm{AV}_{IV}(t) = -0.0036007\, e^{-2.9124t} - 0.15254\, e^{-2.4088t} + 0.20701 \times 10^{-1} e^{-1.1689t} + 1.1368\, e^{-0.25808t} - 0.39331 \times 10^{-3} e^{-1.8400t} - 0.90439 \times 10^{-3} e^{-1.1000t} - 0.91583 \times 10^{-4} e^{-1.0850t} \quad \ldots(33)$$

Table 2: Variation of Availability with respect to time (Copula Repair)

Time(t)	Availability Copula Repair			
	10-out-of-50: G	20-out-of-50: G	30-out-of-50: G	40-out-of-50: G
0	1.000	1.000	1.000	1.000
1	0.870	0.907	0.948	0.995
2	0.679	0.749	0.834	0.939
3	0.525	0.611	0.726	0.879
4	0.405	0.500	0.631	0.822
5	0.313	0.407	0.548	0.768
6	0.242	0.332	0.476	0.718
7	0.187	0.271	0.414	0.671
8	0.144	0.221	0.360	0.627
9	0.112	0.180	0.313	0.586

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Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

10	0.086	0.147	0.272	0.548

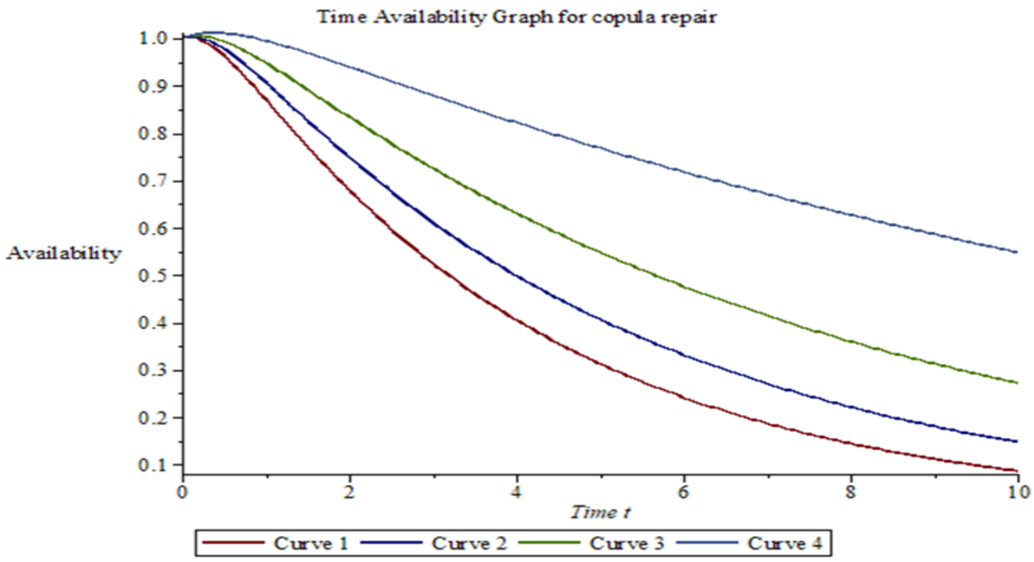

Figure 2: Variation of Availability with respect to time (Copula Repair)

Setting: $\lambda_1 = 0.02,\ \lambda_2 = 0.015,\ \lambda_H = 0.02,\ \lambda_{sw} = 0.02,\ \varphi_1 = \varphi_2 = 1\ and\ \mu_0 = 1,$ in $P_{up}(s)$ and taking inverse Laplace Transform we get the following expression for the system performance metric as system availability for copula repair.

(AV: I), $k = 10, n = 50$

$$\begin{aligned}\mathrm{AV}_I = &-0.10002\times10^{-2}e^{-1.1000t} - 0.16541\,e^{-2.5681t} + 0.49394\times10^{-1}e^{-1.1852t}\\ &+ 0.35820\times10^{-1}e^{-1.0278t} + 1.0817\,e^{-0.24889t} - 0.39535\times10^{-3}e^{-1.8400t}\\ &- 0.11250\times10^{-3}e^{-1.0850t}\end{aligned}$$

(AV: II), $k = 20, n = 50$

$$\begin{aligned}\mathrm{AV}_{II} = &-0.10002\times10^{-2}e^{-1.1000t} - 0.16541\,e^{-2.5681t} + 0.49394\times10^{-1}e^{-1.1852t}\\ &+ 0.35820\times10^{-1}e^{-1.0278t} + 1.0817\,e^{-0.24889t} - 0.39535\times10^{-3}e^{-1.8400t}\\ &- 0.11250\times10^{-3}e^{-1.0850t}\end{aligned}$$

(AV: III), $k = 30, n = 50$

$$\begin{aligned}\mathrm{AV}_{III} = &-0.10002\times10^{-2}e^{-1.1000t} - 0.16541\,e^{-2.5681t} + 0.49394\times10^{-1}e^{-1.1852t}\\ &+ 0.35820\times10^{-1}e^{-1.0278t} + 1.0817\,e^{-0.24889t} - 0.39535\times10^{-3}e^{-1.8400t}\\ &- 0.11250\times10^{-3}e^{-1.0850t}\end{aligned}$$

(AV: IV), $k = 40, n = 50$

$$\begin{aligned}\mathrm{AV}_{IV} = &-0.10002\times10^{-2}e^{-1.1000t} - 0.16541\,e^{-2.5681t} + 0.49394\times10^{-1}e^{-1.1852t}\\ &+ 0.35820\times10^{-1}e^{-1.0278t} + 1.0817\,e^{-0.24889t} - 0.39535\times10^{-3}e^{-1.8400t}\\ &- 0.11250\times10^{-3}e^{-1.0850t}\end{aligned}$$

…(34)

By using the inverse Laplace transform, we were able to estimate availability. Pup (t) may vary for different values of the time variable t=0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 units of time.

Received: August 15, 2025

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Table 3**:** Variation of Availability with respect to time (General repair)

Time(t)	Availability General Repair			
	10-out-of-50: G	20-out-of-50: G	30-out-of-50: G	40-out-of-50: G
0	1.000	1.000	1.000	1.000
1	0.858	0.895	0.936	0.982
2	0.666	0.734	0.818	0.922
3	0.516	0.600	0.712	0.862
4	0.401	0.492	0.621	0.807
5	0.312	0.404	0.541	0.755
6	0.243	0.331	0.472	0.707
7	0.190	0.272	0.412	0.662
8	0.148	0.223	0.359	0.619
9	0.115	0.183	0.313	0.580
10	0.090	0.151	0.273	0.543

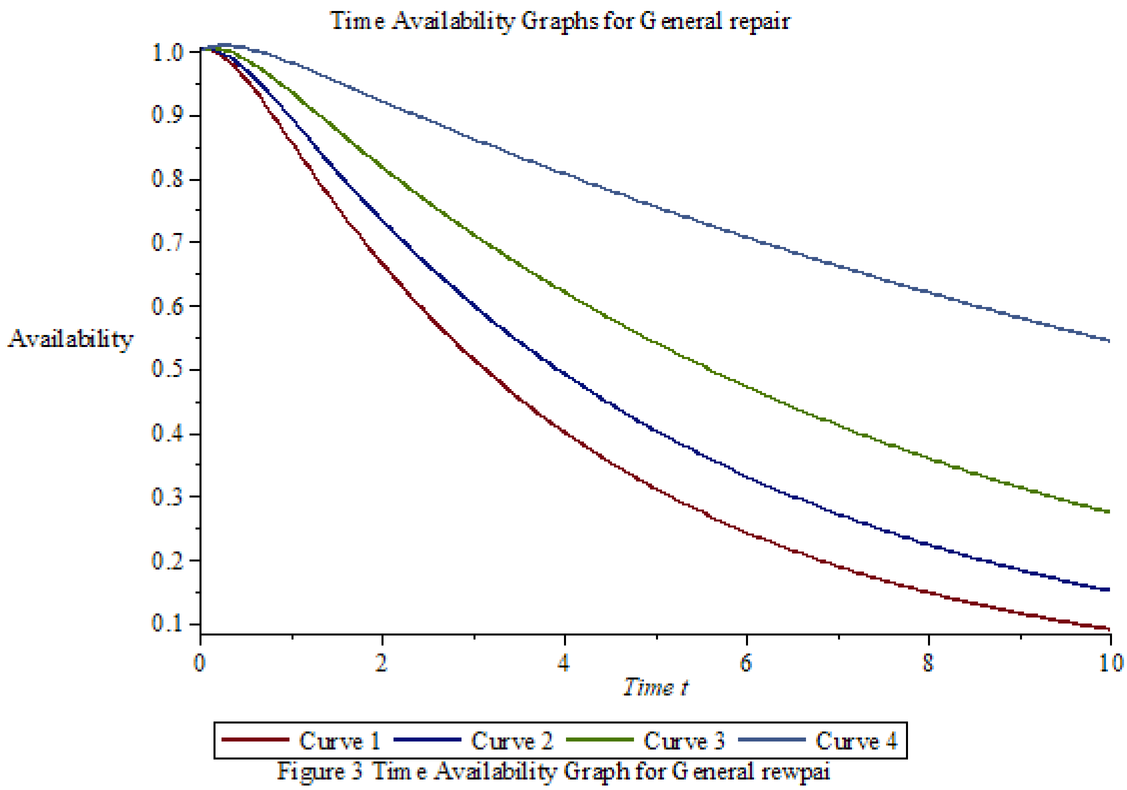

Figure 3: Variation of Availability with respect to time (General repair)

Received: August 15, 2025

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4.2 Reliability Analysis:

Setting all repairs to zero in (32) and applying the inverse Laplace transform yields the system reliability expression, usable with specified failure rates, $\lambda_1 = 0.02$, $\lambda_2 = 0.015$, $\lambda_H = 0.02$& $\lambda_{sw} = 0.02$, in (32), one can get an expression of the reliability of the system as;

1. Reliability I: 10-out-of-50: G

$$\text{Reliability I}(t) = -0.10002 \times 10^{-2}\, e^{-1.1000t} - 0.16541\, e^{-2.5681t} + 0.49394 \times 10^{-1}\, e^{-1.1852t} + 0.35820 \times 10^{-1}\, e^{-1.0278t} + 1.0817\, e^{-0.24889t} - 0.39535 \times 10^{-3}\, e^{-1.8400t} - 0.11250 \times 10^{-3}\, e^{-1.0850t}.$$

2. Reliability II: 20-out-of-50: G

$$\text{Reliability II}(t) = -0.10002 \times 10^{-2}\, e^{-1.1000t} - 0.16541\, e^{-2.5681t} + 0.49394 \times 10^{-1}\, e^{-1.1852t} + 0.35820 \times 10^{-1}\, e^{-1.0278t} + 1.0817\, e^{-0.24889t} - 0.39535 \times 10^{-3}\, e^{-1.8400t} - 0.11250 \times 10^{-3}\, e^{-1.0850t}.$$

3. Reliability III: 30-out-of-50: G

$$\text{Reliability III}(t) = -0.10002 \times 10^{-2}\, e^{-1.1000t} - 0.16541\, e^{-2.5681t} + 0.49394 \times 10^{-1}\, e^{-1.1852t} + 0.35820 \times 10^{-1}\, e^{-1.0278t} + 1.0817\, e^{-0.24889t} - 0.39535 \times 10^{-3}\, e^{-1.8400t} - 0.11250 \times 10^{-3}\, e^{-1.0850t}.$$

4. Reliability IV:40-out-of-50: G

$$\text{Reliability IV}(t) = -0.10002 \times 10^{-2}\, e^{-1.1000t} - 0.16541\, e^{-2.5681t} + 0.49394 \times 10^{-1}\, e^{-1.1852t} + 0.35820 \times 10^{-1}\, e^{-1.0278t} + 1.0817\, e^{-0.24889t} - 0.39535 \times 10^{-3}\, e^{-1.8400t} - 0.11250 \times 10^{-3}\, e^{-1.0850t} + 0.095745\, e^{-0.095000t} + 0.0070681\, e^{-0.080000t}. \quad \ldots(35)$$

Table 4. Variation of Reliability with respect to time

Time (t)	Reliability Variation of different configurations			
	10-out-of-50: G	20-out-of-50: G	30-out-of-50: G	40-out-of-50: G
0	1.000	1.000	1.000	1.000
1	0.797	0.840	0.890	0.947
2	0.494	0.568	0.667	0.799
3	0.287	0.359	0.470	0.646
4	0.170	0.225	0.326	0.514
5	0.107	0.145	0.226	0.406
6	0.075	0.098	0.159	0.322
7	0.057	0.071	0.115	0.255
8	0.046	0.054	0.085	0.202

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9	0.039	0.044	0.064	0.161
10	0.034	0.037	0.050	0.129

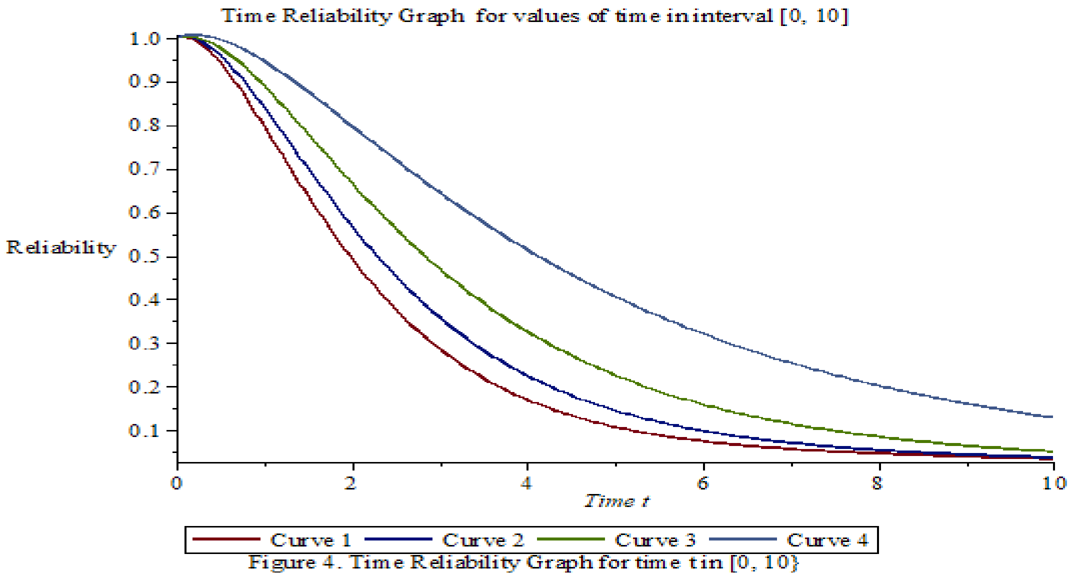

Figure 4. Variation of Reliability with respect to time

4.3 Profit Analysis/ Cost Analysis:

The expected profit of the system over the interval $[0\ to)$ t can be expressed as follows: Assume system failure rates $of\ \lambda_1 = 0.02,\ \lambda_2 = 0.015,\ \lambda_H = 0.02,\ \lambda_{sw} = 0.02\ with\ n = 50, k = 40$, and a constantly available repair facility. The revenue generation K_1 and service cost K_2 per unit time in the interval [0, t) then the expected profit can be calculated by the formula.

$$E_p(t) = K_1 \int_0^t P_{up}(t)\ \ dt - K_2 t \quad \dots \quad (36)$$

$$E_p(t) = \quad K_1[0.31329 \times 10^{-1} e^{-1.1150t} - 0.52961 \times 10^{-3} e^{-1.1000t} + 0.25117 \times 10^{-4} e^{-1.0850t} + 0.32454 \times 10^{-3} e^{-1.2400t} - 0.10964 \times 10^{-1} e^{-2.5125t} + 0.51125 \times 10^{-1} e^{-2.0010t} - 0.33212 \times 10^{-1} e^{-1.1083t} - 16.122\, e^{-0.066537t} + 16.084] - K_2 t \quad \dots (37)$$

By setting K_1 to 1 and K_2 to 0.6, 0.5, 0.4, and 0.3 sequentially, and varying t from 0 to 10, we obtain table 4(a). Figure 5(a) illustrates the profit's variation with respect to time (t). The Local maxima for cases 1: [1.82714987360595416, [t = 8.73241909129779]], case 2: [2.83323866980762418, [t = 11.4725782893063], case3: [4.14194992288932617, [t = 14.8262693491564]], case 4: [5.83040881257896348, [t = 19.149996594279]; was found at different values of cost per unit and revenue of time presented in cases 1,2, 3 & 4

Table 5(a): Expected profit in [0, t) t = 0,1,2, 3….10 for copula repair

Time t	Expected Profit $E_P(t)$ for values of K_1 & K_2

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	$E_P(t)$, K_1 =1, K_2=0.60	$E_P(t)$, K_1 =1, K_2=0.50	$E_P(t)$, K_1 =1, K_2=0.40	$E_P(t)$, K_1 =1, K_2=0.30
0	0	0	0	0
1	0.405	0.505	0.605	0.705
2	0.771	0.971	1.171	1.371
3	1.079	1.379	1.679	1.979
4	1.329	1.729	2.129	2.529
5	1.525	2.025	2.525	3.025
6	1.669	2.269	2.869	3.469
7	1.765	2.465	3.165	3.865
8	1.816	2.616	3.416	4.216
9	1.826	2.726	3.626	4.526
10	1.796	2.796	3.796	4.796

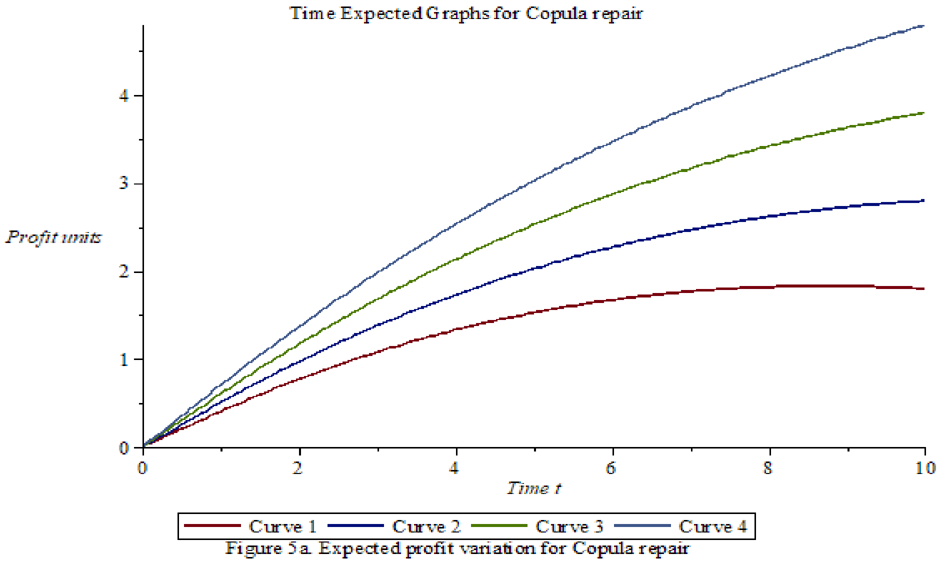

Figure 5(a): Expected profit analysis for Copula repair

The anticipated general repair profit is displayed below, assuming similar copula repair failure rates. Profit expectations for the periods [0, t) where t = 0, 1, 2, 3...10... This is tracked as seen in Table 5a, and Figure 5a.

Expected Profit

$$\begin{aligned} E_p(t) = \quad & K_1[0.30037\times 10^{-1}e^{-2.2231t} - 0.45775\times 10^{-1}e^{-1.1096t} - 0.13310\times 10^{-1}e^{-1.0121t} \\ & -16.097\times e^{-0.65178t} + 0.41948\times 10^{-1}e^{-1.1150t} + 0.332710\times 10^{-3}e^{-1.2400t} \\ & +0.26616\times 10^{-4}e^{-1.10850t} - 0.441948x10^{-3}\, e^{-1.1000t} + 16.084] - K_2 t \quad \ldots (37) \end{aligned}$$

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By setting K_1=1 and letting K_2 range from 0.6 to 0.3, while changing t from 0 to 10, we obtain table 5(b). Figure 5(b) illustrates the change in profit as a function of time (t). The expected profit values are lower than the copula repair case values, which is important to remember.

Table 5(b): Expected profit in [0, t) t = 0,1,2,3,….10 for General repair

Time t	Expected Profit $E_P(t)$ for values of K_1 & K_2			
	$E_P(t)$, K_1=1, K_2=0.60	$E_P(t)$, K_1=1, K_2=0.50	$E_P(t)$, K_1=1, K_2=0.40	$E_P(t)$, K_1=1, K_2 =0.30
0	0	0	0	0
1	0.400	0.500	0.600	0.700
2	0.753	0.953	1.153	1.353
3	1.045	1.345	1.645	1.945
4	1.281	1.681	2.081	2.481
5	1.464	1.964	2.464	2.964
6	1.597	2.197	2.797	3.397
7	1.684	2.384	3.084	3.784
8	1.728	2.528	3.328	4.128
9	1.731	2.631	3.531	4.431
10	1.696	2.696	3.696	4.696

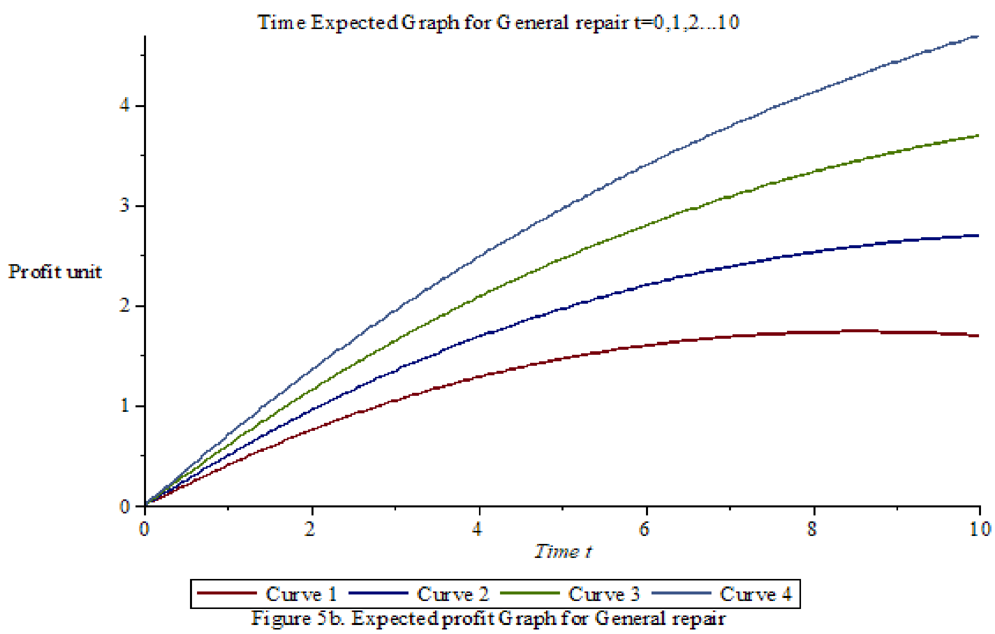

Figure 5(b): Expected profit in [0, t) t = 0,1,2, 3,.20 for General repair

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5. MTTF Analysis

Let's assume that ϕ_1, ϕ_2, and μ_0 are zero in equation (54). By approaching zero, the MTTF can be derived by taking the limit as s tends to zero.

$$\text{MTTF} = \frac{1}{n\lambda_1+6\lambda_2+\lambda_H+\lambda_{sw}}\left(\begin{array}{c}1+\frac{n\lambda_1}{(n+1-k)\lambda_1+\lambda_H+\lambda_{sw}}+\frac{n(n+1-k){\lambda_1}^2}{(n-k)\lambda_1+\lambda_H+\lambda_{sw}}+\frac{6\lambda_2}{5\lambda_2+\lambda_H+\lambda_{sw}}\\+\frac{30{\lambda_2}^2}{4\lambda_2+\lambda_H+\lambda_{sw}}+\frac{120{\lambda_2}^3}{3\lambda_2+\lambda_H+\lambda_{sw}}\end{array}\right) \quad \ldots(38)$$

Setting, λ_2=0.015, λ_H = 0.02, λ_{sw} = 0.02 and varying λ_1 in the range [0.01, 0.09] with an incremental value 0.01 in each entry, one can get the values of the MTTF corresponding to the failure rate λ_1. Column 2 of table 6 shows the variation of MTTF regarding the failure λ_1.

We can find MTTF values by changing λ_2, within the [0.01, 0.09] range with an incremental value of 0.01, when λ_1 = 0.015, λ_H = 0.02, and λ_{sw} = 0.02. The third column of Table 6 illustrates how MTTF changes with failure λ_2.

Setting, λ_2=0.015, λ_1 = 0.02, and varying λ_H as; 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 We can get the values of the MTTF corresponding to the failure rate λ_H. Column 4 of table 6 shows the variation of MTTF regarding the failure λ_H.

Setting, λ_2=0.015, λ_1 = 0.02, λ_H = 0.02 and varying λ_{sw} as; 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 We can get the values of the MTTF corresponding to the failure rate λ_{sw}. Column 4 of table 6 shows the variation of MTTF regarding the failure λ_{sw}.

Table 6 MTTF Variation with respect to failure rates

Failure Rate	MTTF λ_1	MTTF λ_2	MTTF λ_H	MTTF λsw
0.01	2.930	2.892	3.084	3.094
0.02	2.034	2.936	2.930	2.950
0.03	1.560	2.913	2.802	2.842
0.04	1.267	2.879	2.692	2.712
0.05	1.068	2.845	2.595	2.700
0.06	0.924	2.815	2.510	2.600
0.07	0.815	2.791	2.433	2.533
0.08	0.729	2.772	2.340	2.442
0.09	0. 626	2.760	2.301	2.352

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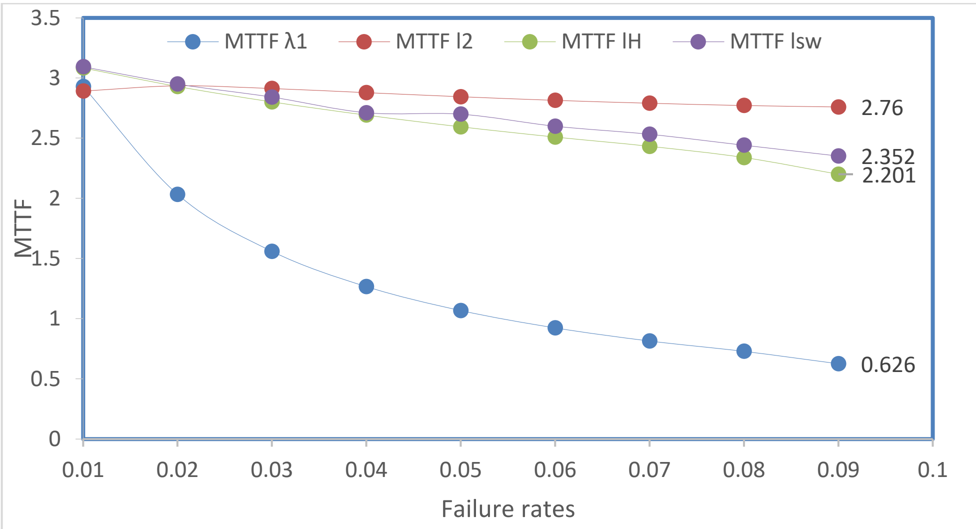

Figure 6. MTTF Variation respect to failure rates

6. Results Analysis and Conclusions:

In this model the various reliability metrics have been studied. The system structure have designed on a combination of different working nature k-out-of-n: G & 3-out-of-6: G. Tables and figures present the analytical findings. When failure rates are fixed at different values, Figure 2 illustrates the relationship between the availability of the repair system and time. The system's availability decreases in response to lower failure rate settings, such as λ_1=0.02, λ_2=0.015, and λ_H=0.02, λ_{sw}=0.02 and eventually stabilizes at zero after an appropriate period. Consequently, it is evident from the model's graphical analysis that it is possible to predict with confidence how a complex system will behave in the future for any given set of parametric values at any point in time. Up to time t=9, the availability of systems that overlook human failure declines, but it then starts to decline once more. The fluctuation in a non-repairable system's reliability is shown in Figure 3. The four cases of system Availability i.e., 10-out-of-50: G, 20-out-of-50: G, 30-out-of-50: G, 40 -out-of-50: G, have computed for analytic study for copula repair as well as for general repair. From table 1& concern figure for copula repair predicts that system performance is found better when k values increase. The four cases of system Reliability i.e., 10-out-of-50: G, 20-out-of-50: G, 30-out-of-50: G, 40 -out-of-50: G, have calculated for analytical study for non-repairable system and same pattern was evidently observed as for availability. Tables 5(a) and 5(b), and Figures 5(a) and 6(b) display how profit changes over time (t = 0, 1...10) for copula and general repairs, with a fixed revenue cost (K1=1) and varying service costs (K2=0.6, 0.5, 0.4, 0.3). Finally, rising service expenses clearly correlate with falling profits. Profits are predicted to be higher with low service costs compared to high service costs.

Figure 6 presents the system's mean time to failure (MTTF). Considering fixed parameters, we're studying the variations in λ_1, λ_2, and λ_H. The differences in MTTF linked to λ_1, λ_2, λ_H, λ_{sw} are generally high; however, they are very close for λ2.

Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Conflict of interest statement: Authors state that there is no conflict-of-interest associated with this manuscript.

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