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ON A STRONGLY M-NC RING

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Abstract:

This work aims to investigate and advance the concept of strongly M-nil clean (S-M-NC) rings. In these rings, each element can be expressed as the sum of a nilpotent element and a M-potent element that commute. The study focuses on:

Defining and formalizing the notions of highly M-NC rings and S-M-NC elements.

Proving that if 2 is nilpotent in a strongly S-M-NC ring, then any element

$a \in R$ with the formula $a2$-a is also nilpotent.

Investigating the role of M-units in such rings, particularly when the ring's characteristic

char(R)

demonstrating that, under these conditions, each ideal of R is a M-potent lift module ideal.

This study enhances the understanding of clean rings with specific algebraic and commutativity constraints, providing new insights into the interaction between nilpotent and M-potent elements and their potential applications in algebraic research.

Keywords: M-potent element, strongly μ-nil clean rings, highly M-nil clean rings, nilpotent elements, commutativity**.**

1.1 Introduction

For many years, the foundation of abstract algebra has been the construction of rings and the breakdown of their constituent parts into algebraically important parts. The idea of highly M-nil

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clean rings, according to which each ring element may be expressed as the sum of a nilpotent element and a M-potent element that commute, is one contemporary expansion in this direction. The ideal framework and lifting qualities in commutative as well as noncommutative rings may be better understood thanks to this generalization, which also unites different decomposition theories.

Over the past 20 years, a great deal of work has been done to establish the notion of clean and nil-clean rings. Every component in the ring is a combination of an M-potent with a nilpotent element, according to a formal framework presented by [1]. They also developed structural conclusions for the occurrence of such decomposition and their continued existence under ring homomorphisms. Expanding upon this, [2] investigated the behavior of highly unit nil-clean rings in broader contexts, such as rings that meet modular identification and unit characteristics.

The growth of this field has been greatly aided by the contributions of other writers. After that [3] provided a broader foundation for comprehending cleanness in difficult rings by analyzing rings whose components are sums of commute potent and nilpotent elements. The concept of M-nil cleanliness as a strong algebraic requirement that improves our capacity to categorize rings according to their decomposition behavior is supported by this seminal study. There has also been study on the optimal structure of such rings. Moreover, [4] examined rings with strong lifting capabilities, demonstrating how the ideals these elements produce affect radical conduct. A related work by [5] focused on the importance of the commutativity requirement between the potent as well as nilpotent components when studying highly clean rings over M-potent decompositions. Furthermore, [6] highlighted the modular lifting requirements and the actions of M-units by introducing new circumstances in which a ring may be regarded as M-nil clean. In a more practical setting, [7] showed that nil-clean rings having modular properties provide valuable information for computational algebra and module theory.

The relevance for M-units in strongly nil cleaned rings was further elucidated by recent algebraic frameworks put forth via [8], especially when certain characteristic restrictions like char are assumed $\mathrm{char}(R) = m$. Their results demonstrated that since 2 was nilpotent in R, then the phrase $a^2 - a$ must likewise be nilpotent for any $a \in R$. In order to bridge the conceptual gap among nil cleanness as well as other clean ring variations, [9] conducted a thorough comparison investigation on highly clean-like rings over different lifting and commuting settings.

Lastly, a thorough analysis of M-potent actions by module expansions of rings was provided by [10], who focused on the lifting for M-potents modulo ideals. According to his findings, the highly M-nil clean characteristic has significant ramifications for module theory, particularly in rings with stable features and M-unit attributes.

The current work is constructed on the basis of these joint efforts. This study intends to broaden the theoretical framework and offer new findings that meaningfully generalize earlier findings by investigating the algebraic framework of S-M-NC rings and conditions like the presence of M-units, the nilpotency of 2, and the nature of ideals.

1.2 Research Significance

the significance of this work lies in its ability to expand the frontiers of algebraic research, offering a comprehensive framework for understanding strongly M-nil clean rings, while simultaneously providing tools and insights that may facilitate further discoveries in related areas of mathematics.

1.3 Research Problem

the primary research problem of this study is to rigorously determine the necessary and sufficient conditions for a ring to be strongly M-nil clean, with particular attention to the structural roles of M-units, the nilpotency of 2, and the influence of ideals. By addressing this problem, the research aims to bridge the existing gaps in the theoretical framework, clarify the relationships between strongly M-nil clean rings and other related classes, and provide a solid foundation for further studies in this emerging area of algebra.

1.4 Research Objectives

The primary objectives of this study are as follows:

1. To define and formalize strongly M-nil clean rings (S-M-NC rings): Establish precise algebraic definitions and properties that distinguish S-M-NC rings from other classes, including clean, nil-clean, and highly unit nil-clean rings.

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2. To investigate decomposition properties of elements: Determine the necessary and sufficient conditions under which every element in a strongly M-nil clean ring can be expressed as the sum of a nilpotent element and a commuting M-potent element.
3. To explore the role of M-units and nilpotency conditions: Analyze how M-units and specific nilpotency conditions, such as the nilpotency of 2, influence the structural behavior and decomposition properties of S-M-NC rings.
4. To examine ideal structures and lifting properties: Study how ideals and their lifting characteristics interact with the strongly M-nil clean property, providing insights into the broader algebraic framework and module-theoretic implications.
5. To establish connections with related ring classes: Compare S-M-NC rings with other clean-like rings to understand their similarities, differences, and potential generalizations, thereby situating the findings within the larger context of ring theory.
6. To contribute to theoretical and practical algebraic knowledge: Provide results that not only expand the theoretical understanding of ring structures but also offer potential applications in module theory, computational algebra, and related areas of mathematics.

2. Preliminaries

We represent the set of Jacobson radicals, the set of all nilpotent elements, the set of units, the right annihilator, and the M-potent, by $J(R)$, $N(R)$, $U(R)$, $r(a)$, and $m(R)$ respectively.

2- M-NC rings

This section aims to define M-NC rings with some elementary properties.

Definition 2.1.[11] Assume that $m \geq 2$ is a positive number. If x is a component of a ring R can be expressed as $x = f + b$, where $b \in N(R)$ and $f \in m(R)$, subsequently R is considered to contain M-nil clean rings (M-NC), if all of R's elements are M-NC.

Example. 2.2

1. A ring Z_8, Z_9, Z_{16} are M-NC rings.
2. The Boolean ring is M-NC ring.

Proposition 2.3. A ring R's component x is M-NC iff $1 - x$ is a M-NC element.

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Proof: Suppose that x is an M- NC element, then there exists $f \in m(R\ )$ and $b \in N(R\ )$such that $x = f + b$. Then $1 - x$=$(1 - f)$–b. $(1 - f)$is M-potent element and b is nilpotent element. Therefore $(1 - x)$ is an M- NC element.

Conversely, suppose that $(1 - x)$ is an M-NC element, that is $1 - x = f + b, f \in m(R\ )$ and $b \in N(R\ )$. Hence $-x = f - 1 + b$, thus $x = (1 - f) + (-b)$. Now $(1 - f)$ is M-potent and $(-b) \in N(R\ )$. Therefore x is an M-NC element.

Proposition 2.4. The M-NC ring is the homomorphic image of the M-NC ring.

Proof: If $h: R_1 \to R_2$ is a ring homomorphism, and 〖R_1 is an M-NC ring, thus $a \in h(R_1)$, and for any $r \in R_1$, $a = h(r)$. Because R_1 represents a M-NC ring. We are able to formulate $r = f + b$, where f is a M-potent component for R_1 and$b \in N(R)$.

Currently, $h(r) = h(f + b) = h(f) + h(b)$ and $h(f) = h(f^m) = (h(f))^m$, since h is homomorphic. Thus $h(f)$ represents an element with M-potency of $h(R_1)$ and $(h(b))^n = h(b^n) = h(0) = \acute{0}$, Consequently,$h(b)$ is a nilpotent element in $h(R_1)$. Therefore $h(R_1)$ is M-NC ring.

Now we present the desired property for the quotient ring.

Proposition 2.5. Let I represent a nil ideal of a ring R . Thus, R is M-NC ring iff, R/I is M-NC ring.

Proof: Suppose that $R\ /I$ is M-NC ring and let $a \in R$. Then $a + I \in R\ /I$ and there $f \in m(R\ )$ and $b \in N(R\ )$. Such that $a + I = (f + I) + (b + I)$. Consequently, $a + I = (f + b) + I$, that is $a - (f + b) \in I$. Now $f = f^m$ it implies to $(f^m)^2 = (f^m)(f^m) = f.f = f^2 = f$ for $m \geq 2$, This means that f becomes idempotent. By [12. Proposition 3.15], the idempotent is lift modulo any nil ideal. Also b is a nilpotent element, $(a - f)$ is nilpotent modulo I, that is $a - (f + b) = b_1$, where b_1 is a nilpotent element, thus $a = f + b + b_1$, hence $a = f + b_2$. Therefore R is a M-NC ring.

The converse is clear from the fact that M-NC rings are closed under homomorphic images.

Finally, in this section we give the relation between M-NC rings and M-clean rings.

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Proposition 2.6. Each M-NC ring represents M-clean ring.

Proof: Suppose that R is a M-NC ring, and let a be an element of R. Then from (Proposition 2.3) $a-1=f+b$, where $f^m=f,\ m\geq 2$ and $b\in N(R)$. Implies $a=f+(b+1)$ is M-clean where f is M-potent and $b+1$ is unit element.

3. Some Properties of Strongly M-NC rings

This part presents the concept of strongly M-NC rings, referred to strongly M-nil clean rings, and investigates a characterization of M-NC rings.

Definition 3.1. If a can be expressed as $a=f+b$, where f denotes an element with M-potency with $m\geq 2$ as a positive integer and $b\in N(R)$ with $fb=bf$, then a component for a ring R is assumed being S-M-NC. When all of the elements in a ring R are strongly M-NC, they say the ring has highly M-NC.

Whenever $a$component an of a ring R can be expressed via $a=f+b$, where f is a M-potent component with $m\geq 2$ which is positive as well as $b\in N(R)$ with $fb=bf$, then the element is considered to be strongly M-NC. If all of the elements of a ring R are highly M-NC, then it is stated that the ring is strongly M-NC (S-M-NC).

Proposition 3.2. Assume R be a S-M-NC ring. Thus, for every element $a\in R$, $a^2-a\in N(R)$, if $2\in N(R)$.

Proof: Let $a\in R$. Then there exists a M-potent element $f^m=f,\ m\geq 2,\ m\in z^+$ and an element $b\in N(R\ )$ such that $a=f+b$. Now; $a^2=(f+b)^2=f^2+2fb+b^2$, Hence $a^2-a=(f^2+2fb+b^2)-(f+b)=f^2-f+2fb+b^2-b=(f-f)+(2fb+b^2-b)=(2fb+b^2-b)\in N(R\ )$. Therefore a^2-a is a nilpotent element.

Proposition 3.3: Please allow R be a S-M-NC ring. Consequently, every single one $a\in R$, $a-a^3$ is a nilpotent element if $3\in N(R)$.

Proof: Let $a\in R$. Then there exists a M-potent element $f^m=f,\ m\geq 2,\ m\in z^+$ and an element $b\in N(R)$ such that $a=f+b$. Now; $a^3=(f+b)^3=(f+b)(f+b)^2=f+3fb+$

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$3fb^2 + b^3$. Hence $a - a^3 = (f + b) - (f + 3fb + 3fb^2 + b^3) = (f - f) - (3fb + 3fb^2 + b^3) + b = b - b^3 - 3f(b - b^2 - b^3) \in N(R)$. Therefore $a - a^3$ is a nilpotent element.

Proposition 3.4. Let R represent a ring. Then for all element $a \in R$, $a - a^3 \in N(R)$ if $a^2 \in R$ is a S-M-NC element and $2 \in N(R)$.

Proof: Let $a \in R$. If $a^2 \in R$ is a S-M-NC element, then $a^4 - a^2$is a nilpotent component in R by proposition 3.1; hence $a^2 - a^4$ is a nilpotent element in R. Thus $a(a - a^3)$ is a nilpotent element in R. Now $(a - a^3)^2 = a(a - a^3)(1 - a^2)$ is nilpotent in R, and this shows that $a - a^3$ is a nilpotent element.

Proposition 3.5. Assume R form a ring, also a considered be a S-M-NC component in R. Then $a(a - 1)(a - 2)$ is a nilpotent element in R if $3 \in N(R)$.

Proof: Let $a \in R$, since a is S-M-NC element then, there exists M-potent element $f = f^m$, $m \geq 2$, $m \in z^+$ and nilpotent element b such that $a = f + b$, Now, $a(a - 1)(a - 2) = (f + b)(f + b - 1)(f + b - 2)$ since $fb = bf$ and $f^m = f$, consequently, $a(a - 1)(a - 2) = 3bf + 3b^2f + b^3 - 3b^2 + 2b = 3fb(b - 1) + b(b - 1)(b - 2) \in N(R)$. Therefore $a(a - 1)(a - 2)$ is a nilpotent element in R.

Now, we introduce the fundamental theorem as follows:

Theorem 3.6. Each subring for a ring that is highly M-NC is also S-M-NC, if $2, 3 \in N(R)$.

Proof: Let l be a subring of R and a be any element in l. Since R is S-M-NC ring, by proposition 3.2, we have $a - a^3 \in N(R)$. Then $(a - a^3)^n = 0$ for some $n \in N$, that is $a - a^3 \in N(l)$. Now $6 = 2^3 - 2 \in N(l)$ and $(a^2 - a) - (a^2 - a)^3, a^3 - a \in N(l)$ and so $3a^2 - 3a \in N(l)$, it follows that ; $(-2a^2)^2 - (-2a^2) = 4a^4 + 2a^2 = (6a^4 - 2a^4) + 2a^2 = 6a^4 + 2a(a - a^3) \in N(l)$. Moreover $(a + 2a^2)^2 - (a + 2a^2) = a^2 + 4a^3 + 4a^4 - a - 2a^2 = (3a^2 - 3a) + 4(a^3 - a) + 6a + 4a(a^3 - a) = 3(a^2 - a) + 4(a^3 - a) + 6a + 4a(a^3 - a) \in N(l)$. Now by Lemma 3.5 in [12], there is a monic polynomial $f(t) \in z[t]$, s.t $f(a)^2 = f(a)$ and $a - f(a) \in N(l)$. Now, $(((-2a^2) - f(a))^n = 0$.Consequently,$af(a) = f(a)a$ and therefore l is strongly M-NC ring.

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Definition 3.7. [13] Let I represent a ring R ideal. M-potent I denotes the existence of R, M-potent e that produces $e - x \in I$, provided for $x \in R,\ x - x^m \in I$.

Definition 3.8. [13] Let I represent the ideal for a ring R. As we claim, R has "Module commutative property" if both of the components $\xi + I$ with $\nu + I$ of $R/I, (\xi + I)(\nu + I) = (\nu + I)(\xi + I)$ satisfies $\xi\nu = \nu\xi\ \ in\, R$.

Moreover, we introduce following property of a S-M-NC ring R.

Proposition 3.9. Assume R be a S-M-NC also " M-unit property " with $char(R) = m$ **.** Thus, M-potent lift each notion for R modul **.**

Proof: Assume R has the "M-unit property" and is a highly M-NC ring and $char(R) = m.$ Now let I be any ideal in R and $a \in R$ with $a - a^3 \in I$. Because R is S-M-NC, we possess $a = b + f$, where $b \in N(R\ \ )$ where f is a component with M-potency, $m \geq 2, m \in z^+$ and $bf = fb$. Now, $a - a^3 \in I$ implies that $(b + f) - (b + f)^m \in I$ it follows that $(b + f) - (b^m + mb^{m-1}f + \frac{m(m-1)}{2!}b^{m-2}f^2 + \cdots + mbf^{m-1} + f^m)$, since $bf = fb$ then, $a - a^3 = b - b^3 = b(1 - b^{m-1}) = b(1 - b)(1 + b + b^2 + \cdots + b^{m-2}) \in I$ it follows that: $a - a^3 = b(1 - b)(1 + b + 1 + b^2 - 1 + b^3 + \cdots + 1 + b^{m-3} - 1 + b^{m-2}) \in I.$ Now; assume that $e = 1 + f$, we discover that $e^m = (1 + f)^m = 1 + mf + \frac{m(m-1)}{2!}f^2 + \cdots + mf^{m-1} + f^m = 1 + f = e$, (since char($R$) =$m$ and $f^m = f$), e consequently constitutes an M-potent component.. Hence, $e - a = 1 + f - (b + f) = 1 - b$ and also; $b - b^m \in I$ implies that $b(1 - b^{m-1}) \in I$ it follows that $(1 - b^{m-1}) \in I$ (I represent an ideal) implies that $(1 - b)(1 + b + b^2 + \cdots + b^{m-2}) \in I.$ Because, R has property of M-unit, then $(1 - b) \in I.$ Hence $e - a \in I$ therefore, we possess M-potent raise each of R's ideals modulo.

Next, we set up properties for a S-M-NC ring for the ring of quotients, before that we prove below desired characterization proposition:

Proposition 3.10. The homomorphic form of S-M-NC ring is S-M-NC ring.

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Proof: Similar to proof of proposition 2.4.

Following, A ring R is said to be NJ-ring if $N(R) \subseteq J(R)$.

Proposition 3.11: Assume R be **a** NJ-ring fulfilling has "property of M-unit" with "modulo commutative property " with $char(R) = m$ and let I be an ideal of R. Then R is S-M-NC if and only if R/I is S-M-NC ring.

Proof: Assume R be S-M-NC ring. Thus, via proposition 3.10, R/I S-M-NC ring and proposition 3.9, It obeys modulo I for M-potent lift.

Conversely: Assume that R/I is S-M-NC ring and M-potent lift modulo I. Let $a \in R$. Thus, $a + I \in R/I$. Because R is S-M-NC ring, we are able to write $a + I = (f + I) + (b + I)$, with $f + I$ represent an M-potent component in R/I. Where $f = f^m,\ m \geq 2,\ m \in z^+$, and $b + I$ is a nilpotent element in $R\ \ /I$. Thus, R is NJ-ring that is b is a unit element in R. Next, $f + I$ was M-potent, suggesting that $f - f^m \in I$. Also R satisfies M-potent lift modulo I, An M-potent $l \in R$ exists s.t, $\ell - f \in I$, and so $\ell + I = f + I$. Hence $a + I = (b + I) + (\ell + I)$, that gives us $b + I = (a - \ell) + I$, consequently, $(a - \ell) + I$ is a nilpotent element in R/I also thus $(a - \ell)$ is a nilpotent element in R. Therefore, $a = (a - \ell) + \ell$, where $a - \ell$ is a nilpotent element of R., and ℓ represents a component of R that has M potency. Also, we possess $(f + I) + (b + I) = (b + I)(f + I)$ that is $\big((a - \ell) + I\big) + (\ \ell + I\ ) = (\ell + I)\big((a - \ell) + I\big)$. Given that the module commutative condition is satisfied by R, we possess $(a - \ell)\,\ell = \ell(a - \ell)$, therefore, R is a strongly M-NC ring.

4. Conclusions:

The structure and conduct of highly M-NC rings—rings who's each element may be expressed as the sum of a nilpotent element and a commuting M-potent component were investigated in this study. Our results expand various fundamental theorems on nil-clean ring to an additional generic context specified by M-potency and prove that such rings display rich algebraic characteristics.

For any a that is a part of R, the element $a^2 - a$ must be nilpotent if R represents a strongly M-NC ring and element 2 in R is nilpotent. This is one of the main results reached. This trait emphasizes the compatibility with the fundamental nilpotent behavior caused by the ring's characteristics and the structural subdivision of elements.

Furthermore, we showed that any ideal in R is a M-potent lift module in a strongly M-NC ring with a fixed characteristic $char(R) = m$ and a M-unit condition. This finding greatly expands our knowledge of ideal behavior in these rings by indicating that the M-NC characteristic encourages advantageous lifting actions, which are crucial for module theory and ring extensions.

In summary, by describing new kinds of rings from the perspective of M-potency with commutativity criteria, this work adds to a thorough expansion of clean ring theory. Future research in related fields, including ring homomorphisms, expansions to grade along with module rings, and their utilization in fuzzy with neuromorphic algebraic systems, is made possible by these results.

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