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AN ANALYTICAL APPROACH FOR SOLVING FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS

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Abstract:

A local fractional variational homotopy perturbation method which is based on local fractional variational iteration method and local fractional homotopy perturbation method is applied to solve the approximate solution of the local fractional Poisson equations in two independent variables that appear in mathematical physics involving the local fractional derivatives are investigated in this paper. Illustrative examples are included to demonstrate the high accuracy and fast convergence of this new algorithm.

Keywords: Local rractional Poisson equation; local fractional variational iteration method; local fractional homotopy perturbation method.

1. Introduction

Fractional calculus has attracted much attention for its potential applications in various scientific fields such as fluid mechanics, biology, viscoelasticity, engineering, and other areas of science [1-4]. So it becomes important to find some efficient methods for solving fractional differential equations.

As it is known the Poisson equation plays an important role in mathematical physics [5,6]; that is, it describes the electrodynamics and intersecting interface (see, e.g., [7,8] and the cited references therein).The solution of this equation was discussed by using different methods [9-13]. We notice that recently fractional Poisson equations based on fractional derivatives were analyzed in [14] and the existence and approximations of its solutions can be found in [15]. The Legendre wavelet method was used to find the fractional Poisson equation with Dirichlet boundary conditions [16]. In [17], the Dirichlet problem for the fractional Poisson’s equation with Caputo derivatives was reported. Furthermore, the fractional Poisson equation based on the shifted Gunwale estimate was obtained in [18]. A variety of analytical and estimate approaches for solving fractional partial differential equations were invented lately [19-81].

Local fractional Variational homotopy perturbation method has a very simple solution procedure and absorbs all of the positive features of local fractional variational iteration and homotopy perturbation methods and is highly compatible with the diversity of the physical problems. In this work, we will use local fractional variational homotopy perturbation

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method to solve local fractional Poisson equations with initial and boundary conditions. The proposed algorithm provides the solution in a rapid convergent series which may lead to the solution in a closed form. The local fractional Poisson equation in two independent variables namely,

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} = f(x,y),\ (x,y) \in [0,+\infty] \times [0,1], \tag{1.1}$$

where the nondifferentiable functions $u(x,y)$ and $f(x,y)$ are adopted the local fractional differential operators and α denotes the fractal dimension, subject to the initial and boundary conditions

$$u(x,0) = 0, \quad u(x,l) = 0, \quad u(0,y) = \varphi(y), \quad \frac{\partial^{\alpha}}{\partial x^{\alpha}} u(0,y) = \phi(y) \tag{1.2}$$

The paper has the following organization. In Section 2 the concepts of local fractional complex derivatives and integrals are briefly reviewed. In Section 3 the local fractional variational homotopy perturbation method is recalled. In Section 4 the nondifferentiable solutions for local fractional Poisson equations are presented. Finally, Section 5 outlines the main conclusions.

2. A Brief Review of the Local Fractional Calculus

Definition 1 (see [49,50]). Suppose that there is the relation

$$|f(x) - f(x_0)| < \varepsilon^{\alpha} \ ,0 < \alpha \le 1 \tag{2.1}$$

with $|x - x_0| < \delta$, for $\varepsilon, \delta > 0$ and $\varepsilon, \delta \in R$, then the function $f(x)$ is called local fractional continuous at $x = x_0$ and it is denoted by $\lim\limits_{x \to x_0} f(x) = f(x_0)$.

Definition 2 (see [49,50]). Suppose that the function $f(x)$ satisfies condition (2.1), for $x \in (a,b)$; it is so called local fractional continuous on the interval (a,b), denoted by $f(x) \in C_{\alpha}(a,b)$.

Definition 3 (see [49,50]). In fractal space, let $f(x) \in C_{\alpha}(a,b)$, local fractional derivative of $f(x)$ of order α at $x = x_0$ is given by

$$D_x^{\alpha} f(x_0) = \frac{d^{\alpha}}{dx^{\alpha}} f(x)\Big|_{x=x_0} = f^{(\alpha)}(x_0) = \lim_{x \to x_0} \frac{\Delta^{\alpha}(f(x) - f(x_0))}{(x - x_0)^{\alpha}} \tag{2.2}$$

where $\Delta^{\alpha}(f(x) - f(x_0)) \cong \Gamma(\alpha + 1)(f(x) - f(x_0))$.

The formulas of local fractional derivatives of special functions [24] used in the paper are as follows:

$$D_x^{(\alpha)} ag(x) = aD_x^{(\alpha)} g(x), \tag{2.3a}$$

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$$\frac{d^{\alpha}}{dx^{\alpha}}\left(\frac{x^{n\alpha}}{\Gamma(1+n\alpha)}\right)=\frac{x^{(n-1)^{\alpha}}}{\Gamma(1+(n-1)\alpha)}\ ,\ n\in N \tag{2.3b}$$

Definition 4 (see [49,50]). A partition of the interval $[a,b]$ is denoted as (t_j, t_{j+1}), $j=0,...,N-1$, $t_0=a$ and $t_N=b$ with $\Delta t_j=t_{j+1}-t_j$ and $\Delta t=\max\{\Delta t_0,\Delta t_1,.....\}$. Local fractional integral of $f(x)$ in the interval $[a,b]$ is given by

$${}_aI_b^{(\alpha)}\, f(x)=\frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^{\alpha}=\frac{1}{\Gamma(1+\alpha)}\lim_{\Delta t\to 0}\sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^{\alpha}. \tag{2.4}$$

The formulas of local fractional integrals of special functions [24] used in the paper are as follows:

$${}_0I_x^{(\alpha)}ag(x)=a\ {}_0I_x^{(\alpha)}g(x), \tag{2.5a}$$

$${}_0I_t^{(\alpha)}\left(\frac{x^{n\alpha}}{\Gamma(1+n\alpha)}\right)=\frac{x^{(n+1)^{\alpha}}}{\Gamma(1+(n+1)\alpha)}\ ,\ n\in N \tag{2.5b}$$

3. Analysis of the method

In this section, we introduce the idea of the local fractional variational homotopy perturbation method. Let us consider the local fractional operator equation in the form

$$L_{\alpha}u(x,t)+N_{\alpha}u(x,t)=g(x,t) \tag{3.1}$$

where L_α and N_α are linear and nonlinear local fractional operators, respectively, and $g(x,t)$ is the source term within the nondifferentiable function.

Local fractional variational iteration algorithm reads as

$$u_{n+1}(x,t)=u_n(x,t)+{}_0I_t^{(\alpha)}\left\{\frac{\lambda(\xi)^{\alpha}}{\Gamma(1+\alpha)}\left(L_{\alpha}u_n(x,\xi)+N_{\alpha}u_n(x,\xi)-g(x,\xi)\right)\right\}, \tag{3.2}$$

where $\frac{\lambda(\zeta)^{\alpha}}{\Gamma(1+\alpha)}$ is a general fractal Lagrange's multiplier and $L_{\alpha}=\frac{\partial^{m\alpha}}{\partial t^{m\alpha}}$.

Therefore, a local fractional correction functional was structured as follows:

$$u_{n+1}(x,t)=u_n(x,t)+{}_0I_t^{(\alpha)}\left\{\frac{\lambda(\xi)^{\alpha}}{\Gamma(1+\alpha)}\left(L_{\alpha}u_n(x,\xi)+N_{\alpha}\tilde{u}_n(x,\xi)-g(x,\xi)\right)\right\}, \tag{3.3}$$

where $\tilde{u}_n$ is considered as a restricted local fractional variation. That is, $\delta^{\alpha}\,\tilde{u}_n=0$ [27, 30].

After the fractal Lagrangian multiplier is determined, for $n\geq 0$, the successive approximations u_{n+1} of the solution $u(x,t)$ can be readily given by using any selective local fractional function u_0. Consequently, we obtain the solution $u(x,t)=\lim_{n\to\infty}u_n(x,t)$. In the

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homotopy perturbation method, the basic assumption is that the solution can be written as a power series in p:

$$u(x,t)=\sum_{n=0}^{\infty}p^n u_n(x,t)=p^0u_0+p^1u_1+p^2u_2+\cdots, \tag{3.4}$$

and the nonlinear term can be decomposed as

$$N_\alpha u(x,t)=\sum p^n H_n(u), \tag{3.5}$$

where $p\in[0,1]$ is an embedding parameter. $H_n(u)$ is He‹s polynomials [26] that can be generated by

$$H_n(u_0,u_1,\ldots,u_n)=\frac{1}{n!}\frac{\partial^n}{\partial p^n}\left\{N_\alpha\left(\sum_{i=0}^{n}p^i u_i\right)\right\}_{p=0},n=0,1,2,\ldots \tag{3.6}$$

The variational homotopy perturbation method is obtained by the elegant coupling of correction function (3.3) of variational iteration method with He‹s polynomials and is given by

$$\begin{aligned}\sum_{n=0}^{\infty}p^n u_n(x,t)=u_0(x,t)+{}_0I_t^{(\alpha)}\left\{\frac{(\xi-t)^\alpha}{\Gamma(1+\alpha)}g(x,\xi)\right\}+\\ p\;{}_0I_t^{(\alpha)}\left\{\frac{\lambda(\xi)^\alpha}{\Gamma(1+\alpha)}\left(\sum_{n=0}^{\infty}p^n\frac{\partial^{m\alpha}}{\partial\xi^{m\alpha}}u_n(x,\xi)+\sum_{n=0}^{\infty}p^n H_n(u)\right)\right\}\end{aligned} \tag{3.7}$$

Comparisons of like powers of p give solutions of various orders. If $L_\alpha=\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}$, then we have the local fractional variational homotopy perturbation formula as follows:

$$\begin{aligned}\sum_{n=0}^{\infty}p^n u_n(x,t)=u_0(x,t)+{}_0I_t^{(\alpha)}\left\{\frac{(\xi-t)^\alpha}{\Gamma(1+\alpha)}g(x,\xi)\right\}+\\ p\;{}_0I_t^{(\alpha)}\left\{\frac{(\xi-t)^\alpha}{\Gamma(1+\alpha)}\left(\sum_{n=0}^{\infty}p^n\frac{\partial^{2\alpha}}{\partial\xi^{2\alpha}}u_n(x,\xi)+\sum_{n=0}^{\infty}p^n H_n(u)\right)\right\}\end{aligned} \tag{3.8}$$

By equating the terms (3.8) with identical powers of p, and taking the limit as p tends to 1, we get

$$u(x,t)=\lim_{p\to 1}\sum_{n=0}^{\infty}p^n u_n(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+\cdots$$

The formula (3.8) plays an important role in dealing with the 2α-order local fractional differential equation with either linearity or nonlinearity.

4. The Nondifferentiable Solutions for Local Fractional Poisson Equations

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In this section we investigate the nondifferentiable solutions for the local fractional Poisson equations in two independent variables with different initial-boundary conditions.

Example 1. We analyze the local fractional Poisson equation in the following form:

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}=\frac{y^{\alpha}}{\Gamma(1+\alpha)}, \tag{4.1}$$

subject to the initial and boundary conditions

$$\begin{aligned} &u(x,0)=0,\ u(x,l)=0\\ &u(0,y)=\frac{y^{3\alpha}}{\Gamma(1+3\alpha)},\\ &\frac{\partial^{\alpha}}{\partial x^{\alpha}}u(0,y)=\sin_{\alpha}(y^{\alpha}). \end{aligned} \tag{4.2}$$

Using (3.2) we have the correct functional is given by

$$u_{n+1}(x,y)=u_n(x,y)+{}_0I_x^{(\alpha)}\left\{\frac{(\xi-x)^{\alpha}}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}u_n(\xi,y)}{\partial\xi^{2\alpha}}+\frac{\partial^{2\alpha}u_n(\xi,y)}{\partial y^{2\alpha}}-\frac{y^{\alpha}}{\Gamma(1+\alpha)}\right)\right\} \tag{4.3}$$

where the initial value given by

$$u_0(x,y)=\frac{y^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{\alpha}}{\Gamma(1+\alpha)}\sin_{\alpha}(y^{\alpha}) \tag{4.4}$$

Applying the local fractional variational homotopy perturbation method, one has

$$\begin{aligned}\sum_{n=0}^{\infty}p^n u_n(x,t)=&\frac{y^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{\alpha}}{\Gamma(1+\alpha)}\sin_{\alpha}(y^{\alpha})-{}_0I_x^{(\alpha)}\left\{\frac{(\xi-x)^{\alpha}}{\Gamma(1+\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)}\right\}+\\ &p\ {}_0I_t^{(\alpha)}\left\{\frac{(\xi-t)^{\alpha}}{\Gamma(1+\alpha)}\left(\sum_{n=0}^{\infty}p^n\frac{\partial^{2\alpha}}{\partial\xi^{2\alpha}}u_n(\xi,y)+\sum_{n=0}^{\infty}p^n\frac{\partial^{2\alpha}}{\partial y^{2\alpha}}u_n(\xi,y)\right)\right\},\end{aligned} \tag{4.5}$$

or

$$\begin{aligned}u_0+pu_1+p^2u_2+\cdots=&\frac{y^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{\alpha}}{\Gamma(1+\alpha)}\sin_{\alpha}(y^{\alpha})-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)}+p\ {}_0I_x^{(\alpha)}\times\\ &\left\{\frac{(\xi-x)^{\alpha}}{\Gamma(1+\alpha)}\left(\left[\frac{\partial^{2\alpha}u_0}{\partial\xi^{2\alpha}}+p\frac{\partial^{2\alpha}u_1}{\partial\xi^{2\alpha}}+p^2\frac{\partial^{2\alpha}u_2}{\partial\xi^{2\alpha}}+\cdots\right]+\left[\frac{\partial^{2\alpha}u_0}{\partial y^{2\alpha}}+p\frac{\partial^{2\alpha}u_1}{\partial y^{2\alpha}}+p^2\frac{\partial^{2\alpha}u_2}{\partial y^{2\alpha}}+\cdots\right]\right)\right\}\end{aligned} \tag{4.6}$$

Comparing the coefficient of like powers of p, we have

$$p^0: u_0(x,y)=\frac{y^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{\alpha}}{\Gamma(1+\alpha)}\sin_{\alpha}(y^{\alpha})-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)} \tag{4.7}$$

$$p^1: u_1(x,y)={}_0I_x^{(\alpha)}\left\{\frac{(\xi-x)^{\alpha}}{\Gamma(1+\alpha)}\left(\left[\frac{\partial^{2\alpha}u_0}{\partial\xi^{2\alpha}}\right]+\left[\frac{\partial^{2\alpha}u_0}{\partial y^{2\alpha}}\right]\right)\right\}=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)}+\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\sin_{\alpha}(y^{\alpha})$$

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(4.8)

$$p^2 : u_2(x,y) = {}_0I_x^{(\alpha)}\left\{\frac{(\xi-x)^\alpha}{\Gamma(1+\alpha)}\left(\left[\frac{\partial^{2\alpha}u_1}{\partial\xi^{2\alpha}}\right]+\left[\frac{\partial^{2\alpha}u_1}{\partial y^{2\alpha}}\right]\right)\right\} = \frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\sin_\alpha(y^\alpha) \tag{4.9}$$

and so on.

The solution of the LFVHPM is obtained

$$u(x,t) = \lim_{p\to 1}\sum_{n=0}^{\infty} p^n u_n(x,t)$$

$$= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^{\alpha)}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{x^{3\alpha}}{\Gamma(1+3\alpha}+\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\cdots\right) \tag{4.10}$$

So the solution in a closed form is:

$$= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^{\alpha)}\sinh_\alpha(x^\alpha) \tag{4.11}$$

Example 2. We analyze the local fractional Poisson equation in the following form:

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}} = E_\alpha(y^\alpha), \tag{4.1}$$

subject to the initial and boundary conditions given as follows:

$$\begin{aligned} &u(x,0)=0,\ u(x,l)=0 \\ &u(0,y)=E_\alpha(y^\alpha), \\ &\frac{\partial^\alpha}{\partial x^\alpha}u(0,y)=\cos_\alpha(y^\alpha). \end{aligned} \tag{4.2}$$

Using (3.2) we have the correct functional is given by

$$u_{n+1}(x,y) = u_n(x,y) + {}_0I_x^{(\alpha)}\left\{\frac{(\xi-x)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}u_n(\xi,y)}{\partial\xi^{2\alpha}}+\frac{\partial^{2\alpha}u_n(\xi,y)}{\partial y^{2\alpha}} - E_\alpha(y^\alpha)\right)\right\} \tag{4.3}$$

where the initial value given by

$$u_0(x,y) = E_\alpha(y^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)}\cos_\alpha(y^\alpha) \tag{4.4}$$

Applying the local fractional variational homotopy perturbation method, one has

$$\begin{aligned}\sum_{n=0}^{\infty} p^n u_n(x,t) = E_\alpha(y^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)}\cos_\alpha(y^\alpha) + {}_0I_x^{(\alpha)}\left\{\frac{(\xi-x)^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha)\right\} + \\ p\ {}_0I_t^{(\alpha)}\left\{\frac{(\xi-t)^\alpha}{\Gamma(1+\alpha)}\left(\sum_{n=0}^{\infty} p^n\frac{\partial^{2\alpha}}{\partial\xi^{2\alpha}}u_n(\xi,y)+\sum_{n=0}^{\infty} p^n\frac{\partial^{2\alpha}}{\partial y^{2\alpha}}u_n(\xi,y)\right)\right\},\end{aligned} \tag{4.5}$$

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or

$$u_0 + pu_1 + p^2u_2 + \cdots = \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\sin_\alpha(y^{\alpha)} + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)} + p\ {}_0I_x^{(\alpha)} \times \left\{\frac{(\xi-x)^{\alpha}}{\Gamma(1+\alpha)}\left(\left[\frac{\partial^{2\alpha}u_0}{\partial\xi^{2\alpha}} + p\frac{\partial^{2\alpha}u_1}{\partial\xi^{2\alpha}} + p^2\frac{\partial^{2\alpha}u_2}{\partial\xi^{2\alpha}} + \cdots\right] + \left[\frac{\partial^{2\alpha}u_0}{\partial y^{2\alpha}} + p\frac{\partial^{2\alpha}u_1}{\partial y^{2\alpha}} + p^2\frac{\partial^{2\alpha}u_2}{\partial y^{2\alpha}} + \cdots\right]\right)\right\} \tag{4.6}$$

Comparing the coefficient of like powers of p, we have

$$p^0 : u_0(x,y) = \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\sin_\alpha(y^{\alpha)} + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)} \tag{4.7}$$

$$p^1 : u_1(x,y) = {}_0I_x^{(\alpha)}\left\{\frac{(\xi-x)^{\alpha}}{\Gamma(1+\alpha)}\left(\left[\frac{\partial^{2\alpha}u_0}{\partial\xi^{2\alpha}}\right] + \left[\frac{\partial^{2\alpha}u_0}{\partial y^{2\alpha}}\right]\right)\right\} = -\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)} + \frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\sin_\alpha(y^{\alpha})$$

(4.8)

$$p^2 : u_2(x,y) = {}_0I_x^{(\alpha)}\left\{\frac{(\xi-x)^{\alpha}}{\Gamma(1+\alpha)}\left(\left[\frac{\partial^{2\alpha}u_1}{\partial\xi^{2\alpha}}\right] + \left[\frac{\partial^{2\alpha}u_1}{\partial y^{2\alpha}}\right]\right)\right\} = \frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\sin_\alpha(y^{\alpha}) \tag{4.9}$$

and so on.

The solution of the LFVHPM is obtained

$$u(x,t) = \lim_{p\to 1}\sum_{n=0}^{\infty} p^n u_n(x,t)$$

$$= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^{\alpha)}\left(\frac{x^{\alpha}}{\Gamma(1+\alpha)} + \frac{x^{3\alpha}}{\Gamma(1+3\alpha} + \frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\cdots\right) \tag{4.10}$$

So the solution in a closed form is:

$$= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^{\alpha)}\sinh_\alpha(x^{\alpha}) \tag{4.11}$$

5. Conclusions

The local fractional operators started to be deeply investigated during the last few years. One of the major problems is to find new methods and techniques to solve some given important local fractional partial differential equations on Cantor set. In this line of thought we consider that three local fractional Poisson equations with differential initial and boundary values were solved by using the local fractional variational homotopy perturbation method.

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