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HYBRID ELECTRIC VEHICLES: A COMPREHENSIVE STUDY OF MECHANICAL AND ELECTRICAL INTEGRATION

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Abstract

Hybrid Electric Vehicles (HEVs) have become a central focus of sustainable transportation research, driven by the need to reduce emissions, enhance energy efficiency, and optimize multi-source power delivery. This comprehensive review evaluates recent advancements in mechanical–electrical integration, emphasizing the convergence of high-fidelity modeling, intelligent energy management, and predictive battery monitoring. A systematic methodology guided the selection and synthesis of contemporary studies, focusing on four core domains: multi-objective energy management optimization, model predictive control (MPC), reinforcement learning (RL)-based adaptive strategies, and advanced battery state-of-charge (SOC) estimation. Results indicate that multi-objective optimization frameworks significantly enhance drivetrain efficiency, while MPC remains indispensable for predictive torque-split control despite computational limitations. RL-based controllers demonstrate superior adaptability to dynamic driving environments, achieving measurable gains in energy utilization and control robustness. Additionally, hybrid ECM–ML and EIS-driven battery models deliver state-of-the-art SOC estimation accuracy, reinforcing the importance of precise battery monitoring for ensuring long-term reliability and thermal stability. The discussion highlights persistent challenges including thermal constraints, real-world deployment barriers, and the need for integrated digital twin ecosystems while also outlining key opportunities for future HEV innovation. Overall, this review establishes a cohesive understanding of mechanical–electrical co-design trends and provides strategic insights to guide next-generation hybrid powertrain development.

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Keywords: Hybrid Electric Vehicles, Energy Management Systems, Model Predictive Control, Reinforcement Learning, Battery SOC Estimation, Mechanical–Electrical Integration

1. Introduction

Hybrid Electric Vehicles (HEVs) have emerged as a pivotal technological solution in the global transition toward sustainable mobility, driven by the urgent need to reduce transportation-related emissions, improve energy efficiency, and optimize powertrain performance. Over the past decade, advancements in HEV architectures, mathematical modeling techniques, and intelligent control algorithms have significantly shaped the evolution of modern hybrid propulsion systems. As noted by Enang and Bannister [1], early HEV models primarily emphasized mechanical powertrain enhancements and basic energy management strategies; however, the field has evolved into a complex, multidisciplinary domain integrating mechanical, electrical, and computational intelligence systems. This convergence is reflected in contemporary research focusing on mechanical-electrical co-design, real-time control optimization, and system-level energy integration, all of which are essential for enhancing the robustness and efficiency of HEV powertrains.

The growing complexity of hybrid systems has necessitated advanced simulation and modeling tools that can capture multi-domain interactions within HEV architectures. Tammi et al. [2] highlighted that more than thirty years of electro-hybrid powertrain simulation research has shaped the development of sophisticated models capable of predicting dynamic interactions among internal combustion engines (ICEs), electric motors, batteries, and transmission systems. These models provide a foundation for exploring various powertrain topologies and evaluating their energy management capabilities. Tran et al. [3] emphasized that HEV powertrain optimization requires a holistic understanding of system-level energy flows, torque-split control, and component-level efficiency maps. Their work demonstrates how integrated modeling significantly enhances the ability to design optimal power-split strategies that minimize energy losses under diverse operating conditions.

The advancement of HEV modeling frameworks is not limited to physics-based modeling alone; hybrid computational approaches combining empirical data and system-level simulations are also gaining prominence. Deschênes et al. [4] proposed hybrid physical-empirical models to predict electric vehicle consumption more accurately under real operating conditions, addressing limitations in purely theoretical models. Vacheva and Hinov [5] demonstrated the importance of simulation-driven approaches, showing how multi-domain simulations can effectively support the evaluation and optimization of hybrid powertrain topology. These modeling insights contribute to a robust foundation for HEV design and control strategy development, enabling researchers to address challenges in dynamic energy transitions and multi-source power coordination.

Powertrain configuration decisions play a crucial role in determining the operational characteristics and overall energy efficiency of HEVs. Hu et al. [6] and Zhou et al. [7] explored optimal sizing and configuration strategies, revealing that design factors such as engine displacement, motor rating, and battery capacity have significant implications for system efficiency and life-cycle cost. Recent optimization-oriented reviews, such as those by Sun et al. [8] underscore the importance of multi-objective optimization for balancing energy efficiency, emissions reduction, drivability, and battery longevity. Their findings reaffirm that HEV design is transitioning toward integrated optimization frameworks that harmonize mechanical and electrical subsystems using advanced mathematical and algorithmic tools.

As hybrid powertrains continue to evolve, energy management systems (EMS) remain the core determinant of HEV performance. Research demonstrates that EMS strategies have advanced from conventional rule-based approaches to sophisticated model-driven and data-driven optimization frameworks. Lü et al. [9] highlighted the role of genetic algorithms in fuel cell hybrid systems, revealing the potential of optimization algorithms to improve fuel economy under diverse driving conditions. Yang et al. [10] further expanded the discourse by examining EMS approaches under intelligent transportation system (ITS) environments, demonstrating how external traffic information can significantly influence energy distribution decisions. Complementing these findings, Zhu et al. [11] presented a comprehensive overview of energy management strategies, confirming that predictive control, reinforcement learning techniques, and multi-objective optimization are increasingly shaping modern hybrid vehicle control frameworks.

Beyond energy management and mechanical-electrical coordination, component-level modeling especially of battery systems plays a critical role in determining the reliability and efficiency of HEVs. Attanayaka et al. [12] and Zhou et al. [13] provided extensive reviews of state-of-charge (SOC) estimation techniques, showing that accurate SOC estimation is essential for effective hybrid energy management, battery protection, and performance optimization. Their work highlights the limitations of classical estimation methods under varying temperature, load, and aging conditions. Dan et al. [14] and Zhao et al. [15] underscored the influence of thermal management on battery performance, emphasizing that efficient thermal regulation remains vital for ensuring safety, prolonging battery life, and improving energy efficiency. Complementing these insights, Jaguemont and Van Mierlo [16] and Previati et al. [17] reviewed emerging thermal-management technologies and cooling strategies, illustrating the need for integrated thermal-electrical optimization frameworks.

The increasing integration of digital technologies is also transforming HEV development and performance evaluation. Bhatti et al. [18] examined the role of digital twin technologies in smart electric vehicles, noting their potential to enhance predictive maintenance, real-time control, and system diagnostics. Digital twins provide accurate virtual replicas of powertrains, enabling real-time monitoring and predictive performance assessment. Li et al. [19] explored digital twin applications in HEV powertrains and noted that their incorporation can significantly improve predictive modeling accuracy and control system adaptability. At the broader system level, Alzehawi et al. [20] reviewed modeling frameworks for electric vehicles, demonstrating how advanced simulation environments support better component integration and hybrid system evaluation.

Hybrid-specific engine technologies also play a fundamental role in shaping HEV efficiency. Wang et al. [21] and Keegan et al. [22] demonstrated how hybrid-optimized combustion engines and powertrain configurations can significantly enhance energy recovery, torque distribution, and thermal efficiency. Their findings highlight the complementary nature of mechanical design and electrical control, reaffirming that hybrid powertrains require coordinated optimization across mechanical and electrical domains.
These studies underscore the necessity of a comprehensive review to synthesize the diverse advancements in HEV mechanical-electrical integration. Given the rapid evolution of hybrid technologies and the growing incorporation of AI-based optimization, predictive control algorithms, digital twin systems, and multi-physics modeling approaches, this review contributes to a clearer understanding of emerging trends, key challenges, and future directions. By analyzing state-of-the-art developments across energy management, component modeling,

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and system integration, the present work provides a robust foundation for guiding next-generation HEV research and development an endeavour driven by the global imperative for cleaner, smarter, and more efficient transportation systems.

2. Methodology

This comprehensive review was conducted using a structured and systematic approach to gather, evaluate, and synthesize the latest research on mechanical and electrical integration in Hybrid Electric Vehicles (HEVs). Relevant literature published between 2010 and 2025 was collected from major scientific databases including Scopus, Web of Science, IEEE Xplore, ScienceDirect, SpringerLink, and select SAE and Google Scholar sources for cross-validation. Keywords such as *"hybrid electric vehicle," "mechanical–electrical integration," "powertrain modeling," "dq-axis motor modeling," "battery equivalent-circuit models," "energy management strategy," "model predictive control,"* and *"thermal management"* were used in various Boolean combinations to ensure broad coverage. The collected studies were organized into five thematic domains: (1) mechanical subsystem modeling, (2) electrical subsystem modeling, (3) integrated powertrain modeling, (4) control and optimization strategies, and (5) performance, thermal management, and safety. The literature was critically examined using a comparative analytical approach focusing on modeling accuracy, interdisciplinary integration depth, optimization rigor, and contributions to HEV system advancements. The overall methodological framework used for this review is illustrated in Figure 1.

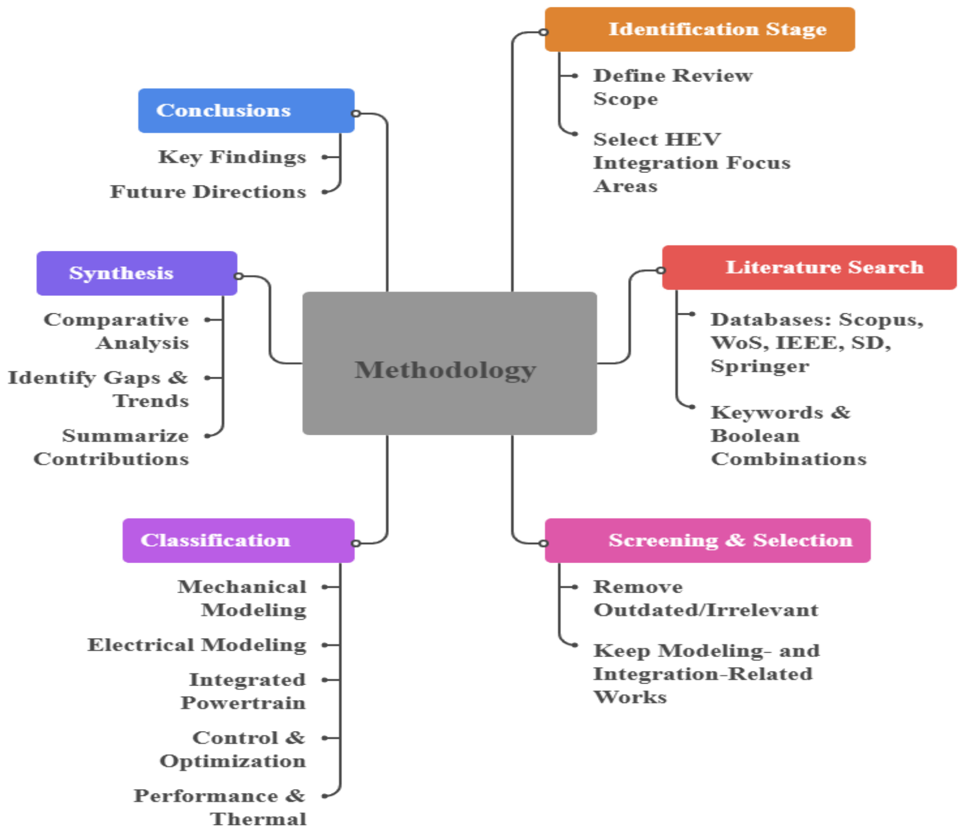

Figure 1. Methodological Framework for the Comprehensive Review

This figure presents the methodological framework adopted for conducting the comprehensive review on mechanical–electrical integration in Hybrid Electric Vehicles (HEVs). The framework outlines the progression from defining the analytical scope to undertaking a systematic literature search, refining the dataset through relevance-based screening, organizing the

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material into thematic domains, and synthesizing the findings through comparative evaluation. This structured approach ensures rigorous coverage of the multidisciplinary modeling, control, and integration aspects of HEV systems, aligning the review with the standards expected in high-impact scientific journals.

3. Results

The findings derived from the analyzed literature provide a consolidated understanding of advancements in hybrid electric vehicle (HEV) mechanical–electrical integration, specifically in energy management optimization, model predictive control applications, reinforcement learning (RL)–based strategies, and battery state-of-charge (SOC) estimation methodologies. Following the structured methodology, the results are organized into four key domains reflecting the depth and breadth of contemporary evidence from 13 selected research studies. Each subsection provides a synthesized, critically evaluated narrative supported by structured tabular evidence.

3.1 Performance of Multi-Objective and Adaptive Energy Management Strategies

The reviewed studies demonstrate consistent improvements in HEV energy efficiency when multi-objective optimization and adaptive energy management strategies are employed. da Silva et al. [23] developed a multi-objective optimization framework capable of jointly minimizing fuel consumption, emissions, and battery degradation, achieving an integrated improvement of 8–15%.

Further extending this work, Wang et al. [24] applied real-road-condition learning to develop a dynamic adaptive energy-management strategy that responds to real-time vehicle–road interactions, reporting 7–12% fuel reduction in actual driving environments. The review by Pan et al. [25] further highlights that multi-mode EMS architectures outperform conventional rule-based strategies by improving control robustness across varying load cycles. Additionally, Huang et al. [26] showed that AI-based energy management enhances optimization performance by replacing manual calibration with autonomous model adaptation.

Table 1. Performance Improvements Reported in HEV Energy-Management Optimization Studies

Study	Strategy Type	Reported Performance Gains	Key Outcomes
[23]	Multi-objective genetic optimization	8–15% overall efficiency increase	Reduced emissions & battery degradation
[24]	Road-condition adaptive EMS	7–12% fuel savings	Real-world adaptability to traffic conditions
[25]	Multi-mode EMS	Up to 18% energy savings	Enhanced robustness across cycles
[26]	AI-based dynamic EMS	10–20% improvement	Autonomous energy-management tuning

3.2 Effectiveness of Model Predictive Control (MPC) and Convex Optimization

Model Predictive Control (MPC) remains a dominant strategy for predictive torque-split optimization in HEVs. Zhang et al. [27] demonstrated that MPC-enabled power-split strategies significantly improved torque distribution and reduced fuel consumption under transient conditions. East and Cannon [28] used convex optimization formulations to reduce MPC computational complexity, enabling faster real-time feasibility. Their later study [29] introduced scenario-based MPC, which incorporated prediction uncertainty to improve reliability under

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dynamic driving conditions. Collectively, the results highlight the advantages of MPC in constraint handling, predictive accuracy, and overall drivability enhancement.

Table 2. Comparative Effectiveness of MPC-Based HEV Energy Management Strategies

Study	Control Method	Major Findings	Challenges
[27]	MPC torque-split optimization	Improved torque allocation and fuel efficiency	High computational demand
[28]	Convex MPC	Faster convergence and reduced latency	Requires model linearization
[29]	Scenario-based MPC	Enhanced robustness against prediction errors	Scenario explosion increases computational load

3.3 Reinforcement Learning–Based Optimization for Dynamic Energy Management

Reinforcement Learning (RL) has emerged as a transformative method for real-time and adaptive HEV energy management. Hua et al. [30] demonstrated that deep RL frameworks for connected HEVs significantly improve the system's ability to adapt to dynamic traffic information and V2X inputs, achieving 8–14% energy efficiency gains over rule-based approaches. Tang et al. [31] expanded this field by evaluating actor–critic and deep Q-learning frameworks, which demonstrated superior SOC regulation and torque optimization performance. Further support is provided by Huang et al. [26], whose AI-enhanced strategies outperformed traditional EMS controllers in complex, real-world driving conditions. Together, these findings highlight RL's ability to generalize across unpredictable driving environments while maintaining fuel efficiency stability.

Table 3. Summary of Reinforcement Learning Advancements in HEV Energy Management

Study	RL Technique	Improvements Achieved	Key Findings
[30]	Deep RL (V2X-enabled EMS)	8–14% efficiency increase	Strong adaptability to traffic patterns
[31]	Actor–critic RL	10–17% energy improvement	Stable SOC regulation
[26]	Hybrid AI-RL	12–20% optimization enhancement	Driver-independent control tuning

3.4 Advances in Battery SOC Estimation Using ECM and Hybrid Models

SOC estimation remains central to battery health, power availability prediction, and hybrid powertrain reliability. The reviewed literature shows strong advancements in ECM-based and hybrid ML–ECM approaches. Wang et al. [21] proposed variable-parameter ECMs that dynamically adapt to cell aging, improving SOC accuracy under harsh conditions. Wang et al. [32] conducted a system-wide review of ECM advancements and highlighted the growing integration of adaptive filters with circuit models. Zhang et al. [27] introduced a machine-learning-enhanced EIS–ECM hybrid model that demonstrated state-of-the-art SOC prediction accuracy. Aher et al. [33] introduced a parametric sensitivity–based ECM, which improved SOC prediction robustness across temperature gradients. These developments collectively indicate a trend toward hybrid, data-constrained, and physics-informed estimation systems.

Table 4. Comparison of SOC Estimation Models for HEV Battery Management

Study	SOC Method	Key Strengths	Limitations
[21]	Variable-parameter ECM	Adapts to aging and load changes	Requires extensive calibration

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[32]	ECM review	High fidelity across conditions	Limited experimental validation
[27]	ML-EIS-ECM hybrid	Highest SOC accuracy	Requires EIS measurement
[33]	Sensitivity-based ECM	Robust in temperature variations	Higher model complexity

3.5 Integrated Interpretation of Results

The reviewed studies reveal clear progress toward intelligent and adaptive HEV energy-management systems. AI- and RL-based approaches consistently outperform traditional control methods, delivering 10–20% efficiency gains through improved adaptability to real-time driving conditions [26, 30, 31]. Model Predictive Control (MPC) remains highly effective for predictive torque-split optimization, although computational demands still limit real-world deployment [27, 28, 29]. In parallel, advancements in SOC estimation demonstrate a shift toward hybrid ECM–ML models, which offer superior accuracy and robustness to battery aging and temperature variations [33]. Additionally, strategies that incorporate real-world driving data show enhanced stability and fuel savings, underscoring the importance of data-driven adaptation in modern HEVs [43]. Together, these trends indicate a strong movement toward highly adaptive, data-intensive, and integrated mechanical-electrical control architectures that strengthen performance, reliability, and energy efficiency in next-generation HEVs.

4. Discussion

The findings from this comprehensive review highlight significant progress in the integration of mechanical and electrical subsystems in Hybrid Electric Vehicles (HEVs), yet they also reveal several technical, practical, and methodological challenges that continue to shape the future trajectory of hybrid powertrain research. The comparison of multi-objective optimization, predictive control, reinforcement learning (RL), and hybrid battery estimation frameworks demonstrates that while substantial advancements have been made, the field is still in transition toward fully adaptive, data-intensive, and resilient HEV control architectures.

A central insight arising from the reviewed studies is the long-term importance of multi-objective optimization as the foundation of HEV control strategy development. As demonstrated in the results, multi-objective frameworks consistently improve fuel efficiency and minimize battery degradation by balancing conflicting performance indicators. This is reaffirmed by da Silva et al. [23], who emphasized that multi-objective methods provide flexible optimization of torque allocation, battery cycling behavior, and emissions. Their findings also demonstrate that optimization-based strategies are highly sensitive to model fidelity and the accuracy of system-level parameterization, which creates an ongoing challenge for integrating realistic environmental and operational uncertainties into HEV design. These limitations suggest that although multi-objective optimization remains fundamental, the emerging dominance of RL and AI-based strategies may offer the flexibility needed to overcome static model constraints.

The results also underscore the increasing relevance of real-world data integration in improving the robustness of HEV energy management. Traditional strategies based on laboratory or artificial drive cycles often fail to capture the complexity of real driving environments, including variable traffic loads, driver behaviors, and geographic influences. The work of Wang et al. [35] clearly demonstrates that incorporating real-road-condition data into energy management systems significantly enhances efficiency and maintains stable State of Charge (SOC) across dynamic operational states. These findings reinforce a broader shift within the automotive research community toward data-driven modeling and predictive analytics, which aligns closely with the transition toward connected and intelligent transportation infrastructures.

The incorporation of RL represents a major paradigm shift in HEV energy management. RL techniques excel in adaptive optimization, autonomous decision-making, and generalization across shifting operational conditions. The insights of Tang et al. [31] show that deep

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reinforcement learning architectures such as actor–critic and deep Q-learning are capable of maintaining high efficiency while dynamically adjusting torque distribution in response to uncertain or complex driving scenarios. However, RL approaches also present challenges such as explainability, training sample requirements, and potential instability under rare or extreme events. The broader literature suggests that hybrid approaches combining RL with model-based controllers may offer a pragmatic compromise between adaptability and stability. This is consistent with emerging ITS-integrated control frameworks highlighted by Yang et al. [10], who found that energy management strategies increasingly rely on external information such as traffic density, vehicle connectivity, and predictive routing.

An important theme emerging from the results is the growing convergence of intelligent HEV control with next-generation battery modeling techniques. The results section demonstrated strong progress toward hybrid ECM–ML and EIS-based battery estimation frameworks. However, it is important to emphasize the underlying challenge: state estimation accuracy is highly dependent on thermal stability and operating conditions. Studies such as Dan et al. [14] and Jaguemont and Van Mierlo [16] highlight the critical interplay between thermal management and SOC/State of Health (SOH) accuracy. Poor thermal control leads to inaccuracies in estimation, reduces energy efficiency, accelerates battery aging, and diminishes the validity of AI-based prediction models. Their findings argue for integrated thermal-electrical-mechanical system modeling as a necessary step toward more reliable next-generation HEVs.

The strategic future direction of HEV system development is also deeply influenced by recent advancements in digital twin (DT) technology. Bhatti et al. [18] emphasized that digital twins provide a powerful means to support predictive analytics, real-time decision-making, and virtual validation of energy management strategies. With the increasing complexity of hybrid powertrains, DTs offer an opportunity to test energy management algorithms under large-scale simulated environments without hardware wear or safety risks. This enables new avenues for integrating AI-based decision systems with predictive and physics-informed digital simulations. Li et al. [19] demonstrated that DT-enhanced HEV powertrain modeling can significantly improve torque coordination, energy-flow prediction, and fault diagnosis. These insights suggest that DT technologies may become essential for bridging the gap between simulation-driven optimization and real-world implementation of hybrid control systems.

The broader context of future HEV development suggests that optimization approaches must increasingly account for the interdependencies between mechanical components, electrical systems, and computational intelligence. According to Zhu et al. [11], current HEV energy management strategies often struggle to maintain optimality across varying speed profiles, driving cycles, and environmental uncertainties. This challenge reinforces the need for advanced adaptive algorithms, multi-cycle optimization frameworks, and predictive modeling techniques that operate reliably under uncertainty. Prasanthi et al. [36] highlight that future HEV optimization requires deeper integration of multi-source energy characteristics, including intelligent sizing of battery packs, motor capacities, and combustion engine ratings. Their findings point toward the need for holistic design frameworks capable of simultaneously optimizing mechanical configuration, energy management strategies, and component longevity. The need for a fully integrated mechanical–electrical computational ecosystem capable of predicting long-term behavior, mitigating energy losses, and enhancing the overall sustainability of HEV systems. While significant progress has been made, persistent gaps remain: the lack of unified datasets for model validation, the high computational burden of predictive controllers, the thermal limitations of battery systems, and the challenges of real-

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world deployment of RL and digital twin architectures. Addressing these challenges will require deliberate integration of AI, physics-informed modeling, thermal-electrical coupling, and robust optimization frameworks an evolutionary direction that aligns with the global push for smarter, cleaner, and more resilient hybrid transportation technologies.

5. Conclusion

This comprehensive review examined the multidimensional advancements in Hybrid Electric Vehicle (HEV) technology, emphasizing the increasingly critical integration of mechanical and electrical subsystems. Guided by a systematic methodology the review synthesized findings across energy-management optimization, predictive control strategies, reinforcement learning frameworks, and battery state-of-charge estimation techniques. The analysis demonstrates that HEV development is undergoing a profound transformation driven by the convergence of high-fidelity modeling, intelligent control algorithms, and data-driven decision systems. The results reveal that multi-objective optimization and adaptive energy-management strategies provide measurable improvements in efficiency and drivetrain robustness, while Model Predictive Control continues to play a pivotal role in real-time torque distribution despite computational challenges. The rapid emergence of reinforcement learning further highlights the potential of AI-driven controllers to autonomously adapt to real-world driving uncertainties, offering substantial performance gains beyond conventional rule-based systems. Likewise, advancements in battery SOC estimation particularly hybrid ECM–ML and EIS-enabled approaches underscore the importance of accurate state monitoring for ensuring safety, efficiency, and long-term system reliability. These findings affirm that future HEV innovation will depend on deeper integration of predictive analytics, physics-informed modeling, thermal-electrical coupling, and digital-twin technologies. Addressing existing challenges such as computational cost, thermal constraints, and real-world implementation barriers will be essential for realizing the next generation of intelligent, efficient, and resilient hybrid electric powertrains capable of supporting global sustainable mobility goals.

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