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MODEL FOR POPULATION GROWTH WITH MARRIAGE RATE DEPENDENT ON THE MIGRATION

Dr. Umashankar Shukla

Department of Mathematics, L.B.S.S.P.G. College Anandnagar Maharajganj

273155 UP (INDIA)

Email id: dr.umashankarshukla@gmail.com

Abstract

In the present paper, the population growth model with the effect of the migration on marriage rate on studied.

Keywords : Model, Population growth, Marriage, Migration.

1 Introduction

The rapidly increasing world population has a tilling impact on the limited dwelling area of the planet earth and on our finite resources basic needs of human being are food, shelter, clothing, transportation, literacy, medical-aid etc., but the enormous increase of population, especially in India and other developing countries during last three decades, the slow decline in its birth and death rates raid, rapid urbanization and influx due to migration, change in urban and rural population structure varia- tion in sex ratio, rise in the population density.

Rescigno and Richard son (1973) in computing for a common ecological niche has proposed a model for n-species as follows : $\frac{dP_i(t)}{} = (r$

Here the total dementias D may be given by

$$D = \sum_{i=1}^{n} h_i p_i$$

with P_i as the population size of the *ith* species $r_i > 0$ auto increase in population for ith species, h_i, p_i the diminution of the food supply for *ith* species, fi being the co-efficient of utilization of available food and D, the total diminution Srivastava and Ranjan (1977) saved the above equations.

$f D)P$, $i = 1, 2, \ldots n.$

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dt $i \quad i \quad i$

2 Model

It is observed that birth rate and death rate of male and female populations are not same, Mishra (1983) has extended Kendall's 1948 population growth model (1977) to define growth model with marriage rate depending on time in the following way:

$$\frac{dM}{dt} + a_1 M = (a_3 + u_1)Z - K \qquad (1.1)$$

$$\frac{dF}{dt} + a_2 F = (a_4 + u_2)Z - K \qquad (1.2)$$

$$\frac{dZ}{dt} + (a_3 + u_2)Z = K \qquad (1.3)$$

where M, F, Z are the number of unmarried males, unmarried females and mar- ried couples at time t.

a_1, a_2 are the death rates per unmarried male and unmarried female per unit time.

a_3, a_4 are the male and female death rates for married couple per unit time. u_1, u_2 are the male and female birth rates per married couple per unit time. K is the marriage rate.

In a developing country like India, the marriage rate is increasing tremendously every year and effected by various vital events, recently survey words so that is affected by the migration pattern with respect to time and their exist a curlinean relationship behave them.

From the survey records and details information it is so that marriage rate K effected by migration a respect of time as function of the following form:

$$K = A_0 \log A_1$$

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where A_0, A_1 are non-negative constants our proposed population growth model in the form of differential equation is as follows:

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dM

$+ a_1 M = (a_3 + u_1)Z – A_0 \overline{log} A_1$ (2.1)

dt

dF

$+ a_2 F = (a_4 + u_2)Z – A_0 log$ — (2.2)

dt

dZ

$+ \alpha Z = A_0 log A_1$ — (2.3)

dt

where $\frac{1}{2}\alpha$, represent the mean death rate per married males and females in the population.

1 1

– –

Particular case: $_2\alpha = {}_2(a_3 + a_4)$

$_1 = a_2 = a_3 = a_4 = a$

$a_1 = u_2 = u$

$A_1 = 0$

In our population growth model, we get the Kendall's model (1949, 1977).

3 Solution

The equation (2.3) in the above model is linear differential of first order of the form

dy

$= Py = Q$ — (3.1)

dx

where P and Q are the function of x only where solution is

$$ye^{' Pdx} = \int Qe^{' Pdx} dx + constant$$

under the initial conditions at $t = 0$, $Z = Z_0$ initial population of married couple, we get the solution of equation (2.3) is

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I.F. $= e^{\int P dt} = e^{\alpha t}$

$Ze^{\alpha t} = A_0 \log A_1 \int te^{\alpha t}dt$

$$Z = t\frac{A0}{\alpha} \log A_1 - \frac{A0}{\alpha^2} \log A_1 + Ce^{-\alpha t} \quad (3.2)$$

$$Z_0 = -\frac{A0}{\alpha^2} \log A_1 + C \quad (3.3)$$

$$C = Z_0 + \frac{A0}{\alpha^2} \log A_1 \quad (3.4)$$

Now, substituting the value of Z in equation (2.2), we get

$$\frac{dm}{dt} + a_1 M = [a_3 + u_1] \left[\frac{A0}{\alpha} \log A^t_1 - \frac{A0}{\alpha^2} \log A_1 + Ce^{-\alpha t} - \log A^t_1\right] \quad (3.5)$$

$$\frac{df}{dt} + a_2 F = [a_4 + u_2] \left[\frac{A0}{\alpha} \log A^t_1 - \frac{A0}{\alpha^2} \log A_1 + Ce^{-\alpha t} - \log A^t_1\right] \quad (3.6)$$

which are the form of (3).

Let $\beta = (a_3 + u_1)$ = the natural increases of married males.

$\gamma = (a_4 + u_2)$ = the natural increases of married females.

Substituting β, γ in equation (3.5) and (3.6)

$$\frac{dm}{dt} + a_1 M = \beta \left[\frac{A0}{\alpha} \log A^t_1 - \frac{A0}{\alpha^2} \log A_1 + Ce^{-\alpha t} - A\right]$$

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$\log A^t$ 0 1

(3.7)

$$\frac{df}{dt} + a\, F = \gamma\, \frac{A0}{\alpha} \log A^t - \frac{A0}{\alpha^2} \log A_1 + Ce^{-e^{\alpha t}} - A$$

$$Me^{\alpha_1 t} = \frac{A_0}{\alpha} \log A_1^t - \frac{A_0}{\alpha^2} \log A_1 + Ce^{-e^{\alpha t}} - A_0 \log A_1^t e^{\alpha_1 t}$$

$$M = \beta\ \frac{-}{\alpha} \log A_1 \frac{t}{\alpha_1} - \frac{1}{\alpha_1^2} - \frac{A_0}{\alpha^2} \log A_1 + \frac{Ce^{-\alpha t}}{\alpha_{1-\alpha}} - \frac{A}{\alpha_1} \log A_1 \frac{t}{\alpha_1} - \frac{1}{\alpha_1^2} + \eta e^{-\alpha_1 t} \quad (3.9)$$

$$M_0 = \beta\ - \frac{A_0}{\alpha_1^2\alpha} \log A_1 - fracA_0\alpha^2\alpha_1 \log A_1 + \frac{C}{\alpha_{1-\alpha}} + \frac{A_0}{\alpha_1} \log A_1 + \eta \quad (3.10)$$

$$\eta = M_0 - A_0 \log A_1\ \frac{\beta}{\alpha_1^2\alpha} \frac{\beta}{\alpha_2\alpha_1} \frac{1}{\alpha_1^3} \frac{BC}{\alpha_{1-\alpha}} \quad (3.11)$$

and solving (3.8), we get

$$F = \gamma\ \frac{-}{\alpha} \log A_1 \frac{t}{\alpha_2} - \frac{1}{\alpha_2^2} - \frac{A_0}{\alpha^2\alpha_2} \log A_1 + \frac{Ce^{-\alpha t}}{\alpha_{2-\alpha}} - A_0 \log A_1 \frac{t}{\alpha_2} - \frac{1}{\alpha_2^2} + \epsilon e^{-\alpha_2 t} \quad (3.12)$$

$$F_0 = \gamma\ - \frac{A_0}{\alpha\alpha_2^2} \log A_1 - \frac{A_0}{\alpha^2\alpha_2} \log A_1 + \frac{C}{\alpha_{2-\alpha}} + \frac{A_0 \log A_1}{\alpha_2^2} + \epsilon \quad (3.13)$$

$$\epsilon = F_0 - A_0 \log A_1\ - \frac{\gamma}{\alpha\alpha_2^2} + \frac{1}{\alpha^2\alpha_2} \log A_1\ \frac{1}{\alpha\alpha_2^2} - \frac{\gamma C}{\alpha_{2-\alpha}} \quad (3.14)$$

If the total population is P then

$$P = M + F + 2Z$$

With the help of the equation (3.2), (3.9), (3.12), we get

$$P = \beta\ \frac{A_0}{\alpha} \log A_1 \frac{t}{\alpha_1} - \frac{1}{\alpha_1^2} \frac{A_0}{\alpha^2\alpha_1} \log A_1 + \frac{Ce^{-\alpha t}}{\alpha_{1-\alpha}} \frac{A_0}{\alpha_1} \log A_1 \frac{t}{\alpha_1} - \frac{1}{\alpha_1^2} + \eta e^{-\alpha_1 t}$$
$$+ \gamma\ \frac{-}{\alpha} \log A_1 \frac{t}{\alpha_2} - \frac{1}{\alpha_2^2} - \frac{A_0}{\alpha^2\alpha_2} \log A_1 + \frac{Ce^{-\alpha t}}{\alpha_{2-\alpha}} - A_0 \log A_1 \frac{t}{\alpha_2} - \frac{1}{\alpha_2^2} + \epsilon e^{-\alpha_2 t}$$
$$+ 2\frac{A_0}{\alpha} \log A_1^t - \frac{A_0}{\alpha^2} \log A_1 + Ce^{-e^{\alpha t}} \quad (3.15)$$

(3.8)

$\log A^t$ 0 1

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Solving (3.7) and (3.8) under the initial candidates at $t = 0$, $M = M_0$ and $F = F_0$ being initial population of unmarried males and unmarried females respectively we get

Solution is

I.F. $= e^{\int a_1 dt} = e^{a_1 t}$

$-A \log A \quad \frac{1}{1} - \frac{1}{} \quad + \eta e^{-a_1 t} - \epsilon e^{-a_2 t}$ (3.17)

0 $\quad a_1 \quad a_2$

Putting

$a_2 \quad 2$

$M = RZ \; F = SZ$

where R and S are called parameters of population we may obtained the value of R an with the help of the equal (3.2), (3.9) and (3.12), we get

$ZdR \; dt \quad = \frac{dm}{dt} - \frac{dZ}{dt}$

$\frac{ZdR}{dt} = A \log A \quad \frac{\beta}{0} \frac{1}{1} - \frac{R}{\alpha a_1} - C\alpha e^{-\alpha t} \frac{1}{a_1 - \alpha} - \frac{R}{} = \eta a_1 e^{-a_1 t}$

$Zds \quad 2 \quad a_1 \quad a$

$\frac{Zds}{dt} = \frac{df}{dt} - \frac{dz}{dt}$

$\frac{Zds}{dt} = A \log A \quad \frac{\gamma^2}{0} \frac{1^2}{1} - \frac{S}{\alpha a} - C\alpha e^{-\alpha t} \frac{\gamma}{a_2 - \alpha} - \frac{S}{} = \epsilon a_2 e^{-a_2 t}$

$a \quad a$

Let us define

$D = M - F =$ Difference of unmarried males and unmarried females D may be positive, negative or zero, according to the sex ratio of unmarried population is less than unity greater than unity or equal to one or according to the sex ratio, for un- married population.

Then,

Sex Ratio $< 1, > 1 or = 1$ Then,

D is given by $\quad A_0$

$t \quad A_0$

$A_0 \quad 1 \quad t \quad 1$

$Ce^{-\alpha t}$

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$-a\ t$

D $\log A_1$ $-$ $-$ $\log A_1 +$ $-$ $\log A_1 -$ 2 $+ \eta e$ 1
$= \beta$ α 2 a a $\alpha^2 a_1$ 1 a a_1 a

1 1

a_1- 1

α

$\underline{A_0}$ $\underline{t}$ $\underline{1}$ $\underline{A_0}$ $Ce^{-\alpha t}$ $\underline{t}$ $\underline{1}$

$-a\ t$

$+ \gamma$ α $\log A_1$ a $-$ a $-$ $\log A_1 + a_2 -$ $- A_0 \log A_1$ $-$ $+ \epsilon e$ 2

2 $\alpha^2 a_2$ α 2

2 2 2 2 a a

β $\underline{t}$ (3.16)

$= \frac{A_0}{\alpha} \log A$ $-\frac{1}{a_1}$ $- \gamma$ $\frac{t}{a_1^2}$ $-\frac{\beta}{a_2}$ $- \frac{A_0}{2\ \alpha^2} \log A$ $\frac{\beta}{1\ a}$ $-\frac{\gamma}{a^2}$ $+ Ce^{-\alpha t}$ $\frac{\beta}{a_1-\alpha}$ $-\frac{\gamma}{a_2-\alpha}$

1

$-A \log A$ $\frac{t}{1}$ $-\frac{1}{a_1}$ $-$ $\frac{t}{1}$ $-\frac{1}{a_2}$ $+ \eta e^{-a_1 t} - \epsilon e^{-a_2 t}$ (3.17)

0

Putting

a_2 2

$M = RZ\ F = SZ$

where R and S are called parameters of population we may obtained the value of R an
with the help of the equal (3.2), (3.9) and (3.12), we get

$ZdR\ dt$ $= \frac{dm}{dt}$ $-$ $dZ\ dt$

$\frac{ZdR}{dt\ Zds}$ $= A$ $\log A_0$ $\frac{\beta}{1}$ $\frac{1}{\alpha a_1}$ $- \frac{R}{a_1^2}$ $- Ca e^{-\alpha t}$ $\frac{1}{a_1-\alpha}$ $- \frac{R}{a}$ $- \eta a\ e^{-a_1 t}$

$= \frac{df}{dt}$ $- \frac{dz}{dt}$

dt

$\frac{Zds}{dt}$ $= A$ $\log A_0$ $\frac{\gamma^2}{1\ \alpha a}$ 1^2 $- \frac{S}{a}$ $- Ca e^{-\alpha t}$ $\frac{\gamma}{a_2-\alpha}$ $- \frac{S}{a}$ $- \epsilon a\ e^{-a_2 t}$

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4 Conclusion

We are not interested in an infinite population therefore. We want to investigate the condition when the marriage rate is constant. Marriage rate has to be kept nearly constant. If the population explosion is to be checked.

If Z, M and F are independent of time

Then

dz

$= 0$ $\overline{dt}$

dm

$= 0$

dt

dF

dt $= 0$

which implies that the marriage rate is not a function of time so that

dK

$=0$ (4.2)

dt

$K = A_0$

which is along with.

Further (2.1), (2.2) and (2.3) gives

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$Z = \frac{A0}{}$

0
α

$M = \frac{(u_1 - a_4)}{}Z$ 0 0

a_1

$F = \frac{(u_3 - a_3)}{}Z$ 0 0

a_2

Conversely if condition (4.2) and (4.3) holds then

$K = A_0$ *and* $A_1 = A_2 = 0$

which is the value of K at $t = 0$ when$Z = Z_0$

dZ

$= \frac{A_0}{}$

α

0 —

dt

The Z is independent of time; similarly it can be shown that M and F are also independent of time hence we get theorem.

Theorem 4.1 : The necessary and sufficient condition that the population of mar- ried couples and the sex ratio of unmarried population be independent of time are:

1. The marriage rate is constant and
2. The initial population of married couples, unmarried males and unmarried females are:

(i) $Z_0 = \frac{A0}{\alpha}$

(ii) $M_0 = \beta - 1 \; \frac{A0}{\alpha} = \frac{u1-a4}{a_1} \; Z_0$ a_1

(iii) $F_0 = \gamma - 1 \; \frac{A0}{\alpha} = \frac{u2-a3}{a_2} \; Z_0$ a_2

The balance of unmarried sexes of the population in the marriageable ages is an important factor to explain the short term fluctuation in marriage rate K, in general the number of marriage is greater if the sexes at the marriage age or equal than if

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either sex markedly outnumber the other. Therefore we may be tempted to determine the time when the pre-dominance of one sex over the other vanishes in that situation:

$\beta = \gamma A_1 = a_2$

Therefore for (2.1), (2.2) and (2.3) we have the following theorem.

Theorem 4.2: The time at which the pre-dominance of one sex over the other vanishes is given by:

1

$-\alpha D = 0$

2

Now, $\alpha \neq 0$ Therefore $D = 0$ $M = F$ Or

(4.3)

F

$= 1$

M

Hence, the following theorem.

Theorem (4.3): T he time at which the pre-dominance of one sex over the other vanishes in the population if the sex ratio of unmarried population becomes unity.

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