International Journal of Applied Mathematics

Volume 38 No. 3s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

DYNAMICS OF EVOLUTION OF A PREDATOR-PREY SYSTEM SUBJECT TO CERTAIN ENVIRONMENTAL HARVESTING

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Abstract

The development components of a discrete hunter-prey framework of the Lotka-Volterra type are considered with respect to the particular environmental conditions of the prey population and the assumption that the forcing predator dies after one generation. It is believed that the prey harvesting effect is the only factor influencing interactions between two populations, and that the generations involved do not overlap. This study examines three scenarios, zero harvesting, and two harvesting parameter numerical values. The system's fixed points are determined and their stability is thoroughly examined. A variety of periodic windows exhibiting multi-periodic orbits splitting into phenomena of period-doubling and chaos-adding, bi-stability, intermittency character, etc., are observed inside each of the complex bifurcation patterns observed for all the cases. These are in confirmation that the system under investigation is a complex system. Regular and chaotic results are obtained through numerical simulations, attractors, the calculation of attractors' Topological entropies and Lyapunov exponents are indicators of the complexity of the system. By expanding numerical calculations, the correlation dimensions of chaotic attractors were calculated. An intriguing graphic and a tabulated correlation dimension corroborate the findings of this investigation. The research conducted here has implications for both bioscience and the issues surrounding harvesting phenomena in the predator-prey relationship.

Key Words: Chaos, Bifurcation, Lyapunov Exponents, Topological Entropy

1. Introduction

Recent research publications have debated the emergence of chaos during development in a variety of nonlinear dynamical systems in great detail. The intrinsic makeup of many biological and other systems gives rise to great diversity. These systems have multiple structural components inside, which leads to ecological imbalances and environmental

Received: July 26, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

disturbance or discord. This is because individual elements of the multicomponent hardly followed any definite rule during evolution and so possessed specific assets. These systems are referred to as complex systems. A complex system underwent evolution that displays chaos in a certain parameter space along with other phenomena known as complexity. These phenomena are caused by the interplay between several agents in the system and may manifest as bi-stability, intermittency, cascade effects, the display of hysteretic features, the coexistence of several attractors, etc. As a result, systematic nonlinear features of complexity can also be seen. Learning about the dynamics that arise from a group of interdependent pieces is essential to the study of complexity. For any complex system, the investigation of its evolutionary dynamics comprises the study of its regular and chaotic motion as well as the dynamics emerging due to complexity.

A number of investigations on predator-prey evolution models have been published in the literature, with varying assumptions based on the species and their living environments, Beddington, J. R., Free, C. A., & Lawton, J. H. [6], Saha, L. M. Sauresh Das, Til Prasad Sarma, Anunay Choudhary. Das, M. K. [27], Hadeler, K. P., & Freedman, H. I. [17], Neubert, M. G., & Kot, M. [26], Grafton, R. Q., & Silva-Echenique, J. [4], Xiao, Y., Cheng, D., & Tang, S. [37], Abrams, P. A., & Ginzburg, L. R. [1], Liu, X., & Xiao, D. [22], GÜMÜŞ, Ö. A. [16] , Harikrishnan, K. P. [19], Tang, S., & Chen, L. [34], Andrecut, M., & Kauffman, S. A. [3], Saha, L. M., Dixit, P., & Erjaee, G. H. [30]. Evolutionary chaos can best be characterized using Lyapunov exponents. (LCEs), Adler, R. L., Konheim, A. G., & McAndrew, M. H. [2], Saha, L. M. Saureesh Das, Til Prasad Sarma, Anunay Choudhary, Das, M. K. [27], Andrecut, M., & Kauffman, S. A. [3], Saha, L. M., Dixit, P., & Erjaee, G. H. [30], Benettin, G., Galgani, L., Giorgilli, A., & Strelcyn, J. M. [7], Bryant, P., Brown, R., & Abarbanel, H. D. [9], Grassberger, P., & Procaccia, I. [12], Harikrishnan, K. P., & Nandakumaran, V. M. [18], Saha, L. M., Prasad, S., & Erjaee, G. H. [28], Sandri, M. [31], such that at any state of the system, if the measurement LCE > 0the development turns chaotic; in the case where LCE < 0, the evolution turns regular. Improvements in topological entropy reflect the existence of complexity in a system; higher increases in topological entropy indicate a more complex system. Adler, R. L., Konheim, A. G., & McAndrew, M. H. [2], Beddington, J. R., Free, C. A., & Lawton, J. H. [6], Andrecut, M., & Kauffman, S. A. [3], Bowen, R. [8], Gribble, S. [15], Balmforth, N. J., Spiegel, E. A., & Tresser, C. [5], Iwai, K. [21], Baldwin, S. L., & Slaminka, E. E. [4], Nagashima, H. [25], Saha, L. M., Das, M. K., & Bhardwaj, R. [29]. The results of these investigations are highly noteworthy, fascinating, and helpful.

The purpose of the paper is to look into the intricate mechanisms of evolution in a discrete predator-prey system for Lotka-Volterra when forcing predator dies after one generation and harvesting is an issue for the prey population. The interactions between two populations are assumed to be of non-overlapping generations and affected by the prey harvesting effect only. The complex pattern of bifurcations derived in this case is properly analyzed and explained for two cases of harvesting. Numerical calculations are performed, and at various parameter spaces, both regular and chaotic attractors are drawn. Topological entropies, Lyapunov

Received: July 26, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

exponents, and dimensions of correlation between chaotic attractors have been determined through extending numerical simulation.

2. **Description of Model: Fixed points and their stability criteria**

The Lotka-Volterra system's straightforward predator-prey model is

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - \varepsilon NP,$$

$$\frac{dP}{dT} = bNP - \delta P. \tag{1}$$

where N represents the density of prey,

P is predator populations.

The parameter r is the intrinsic rate growth and k is carrying capacity of prey population.

ε is rate of predation,

b is birth rate of predator.

δ is death rate of the predator.

The idea of divided differences can be used to convert this continuous model into a discrete model.

$$\frac{dN}{dt} = \frac{N_{t+h} - N_t}{h},$$

$$\frac{dP}{dt} = \frac{P_{t+h} - P_t}{h}, \tag{2}$$

After that, the appropriate discrete model was produced as

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{k}\right) - \varepsilon N_t P_t$$

$$P_{t+1} = bN_t P_t + (1-d)P_t. \tag{3}$$

We replace the current generation time with a new one and suppose that predators must perish after one generation. Next, following the sealing of equation (3), and by replacing

$$x_t = \frac{N_t}{k}\ ,\ y_t = \frac{\varepsilon P_t}{bk}\ , c = bk$$

Equation (3) is replaced in discrete formulation form as:

$$x_{n+1} = (r+1)x_n - rx_n^2 - cx_n y_n,$$

$$y_{n+1} = c\, x_n\, y_n \tag{4}$$

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In this discrete formulation, x_n and y_n reflect the populations' concentrations of both predators and prey at every generation, n, r and c are positive parameters. Considering specific fixed harvesting of prey population with harvesting rate, H, $(0 < H < 1)$, the system (4) takes the form

$$x_{n+1} = (r+1)x_n - rx_n^2 - cx_n y_n - H\, x_n$$

$$y_{n+1} = c\, x_n\, y_n \tag{5}$$

System (5)'s Jacobian matrix could be discovered as

$$\mathrm{J} = \begin{pmatrix} 1 + r - 2rx - cy - H & -cx \\ cy & cx \end{pmatrix} \tag{6}$$

Fixed points of system (4) be obtained as

$$P_0(0,0),\ P_1\left(\frac{r-H}{r}, 0\right),\ P_2\left(\frac{1}{c}, \frac{c\, r - c\, H - r}{c^2}\right)$$

Then, we have

(i) Eigen values corresponding to fixed point $P_0(0,0)$, obtained as

$\lambda_1 = 1 - H + r, \qquad \lambda_2 = 0$

Since H is very small i.e., $H \ll 1$, and $r > 0$, the fixed point $P_0(0,0)$ is an unstable node.

(ii) Eigen values corresponding to fixed point $P_1\left(\frac{r-H}{r}, 0\right)$ obtained as

$\lambda_1 = 1 + H - r, \qquad \lambda_2 = \frac{c(r-H)}{r}$

As per our further calculations, we assumed, $1 + H < r$ and certainly $\frac{c(r-H)}{r} > 0$, this implies the fixed point $P_1\left(\frac{r-H}{r}, 0\right)$ is an unstable saddle.

(iii) Eigen values corresponding to fixed point $P_2\left(\frac{1}{c}, \frac{c\, r - c\, H - r}{c^2}\right)$, obtained as

(iv) $\lambda_{1,2} = \frac{2c - r \pm \sqrt{4c^2H + 4cr - 4c^2r + r^2}}{2c}$

Since in this case eigen values of the Jacobian matrix (6) depending on c, r and H, stability of fixed point $P_2\left(\frac{1}{c}, \frac{c\, r - c\, H - r}{c^2}\right)$ be established only after assigning numerical values of these constants.

For example, if we assume $r = 2.5$, $c = 2.8$ and $H = 0.02$, fixed points obtained are obtained as

$P_0(0,0)$, $P_1(0.992,\ 0)$ and $P_2(0.357143,\ 0..566837)$

At fixed point $P_2(0.357143,\ 0..566837)$ corresponding eigen values obtained as

Received: July 26, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$\lambda_1 = 1.30165, \qquad \lambda_2 = 1.30165.$$

This suggests that P_2 is a non-stable node.

3. Bifurcation Analysis

For the bifurcation scenario of the system (5), we have fixed $r = 2.5$ and let vary the parameter c.Then, proceed to obtain bifurcation diagrams for cases of $H = 0$, $H = 0.02$ and $H = 0.05$ along prey x_n and predator y_n directions. The respective bifurcation diagrams are shown in Figure 1, Figure 2 and Figure 3. Looking carefully at these figures one can understand the differences. One very common observation is that in each case, within a chaotic region, a number of periodic windows emerge. When magnified, these periodic windows show an amazing scenario of multi periodic orbits, and in some cases, these orbits are again evolving into period-doubling and chaos-adding phenomena. Also, as per the complexity literature, phenomena like bi-stability, intermittency, etc. are seen within these periodic windows. As per some recent literature, Adler, R. L., Konheim, A. G., & McAndrew, M. H. [3], Saha, L. M. Saureesh Das, Til Prasad Sarma, Anunay Choudhary, Das, M. K. [27], Liu, X., & Xiao, D. [22], Harikrishnan, K. P. [19], Saha, L. M., Dixit, P., & Erjaee, G. H. [30]. It is thought that the existence of complexity in the system is what causes these complex patterns to appear within the periodic window. Such complexities may demonstrate a notable rise in the system's topological entropy.

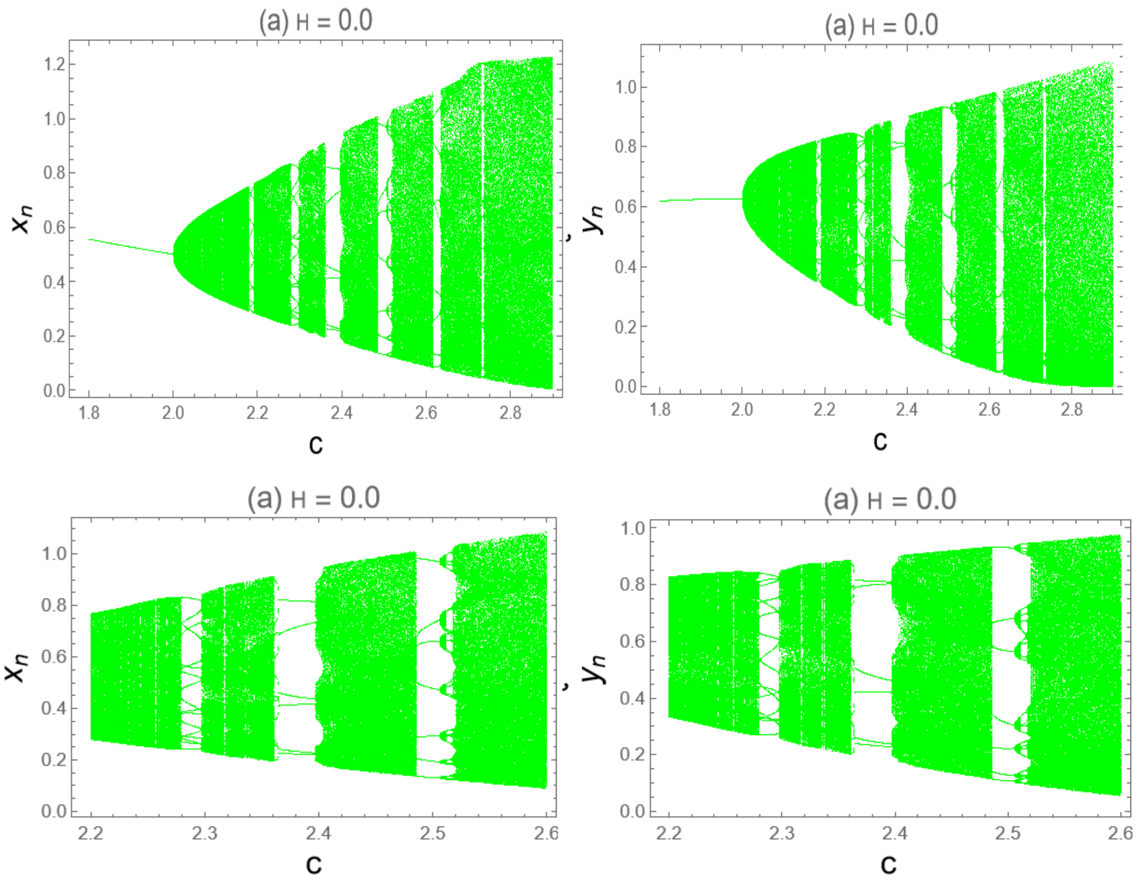

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Figure 1: Bifurcation scenario of predator-prey system (5) along prey and predator axes for $H = 0$ and when $r = 2.5$ and $1.8 \leq c \leq 2.9$. Lower figures are magnifications of corresponding upper figures within range $2.3 \leq c \leq 2.7$.

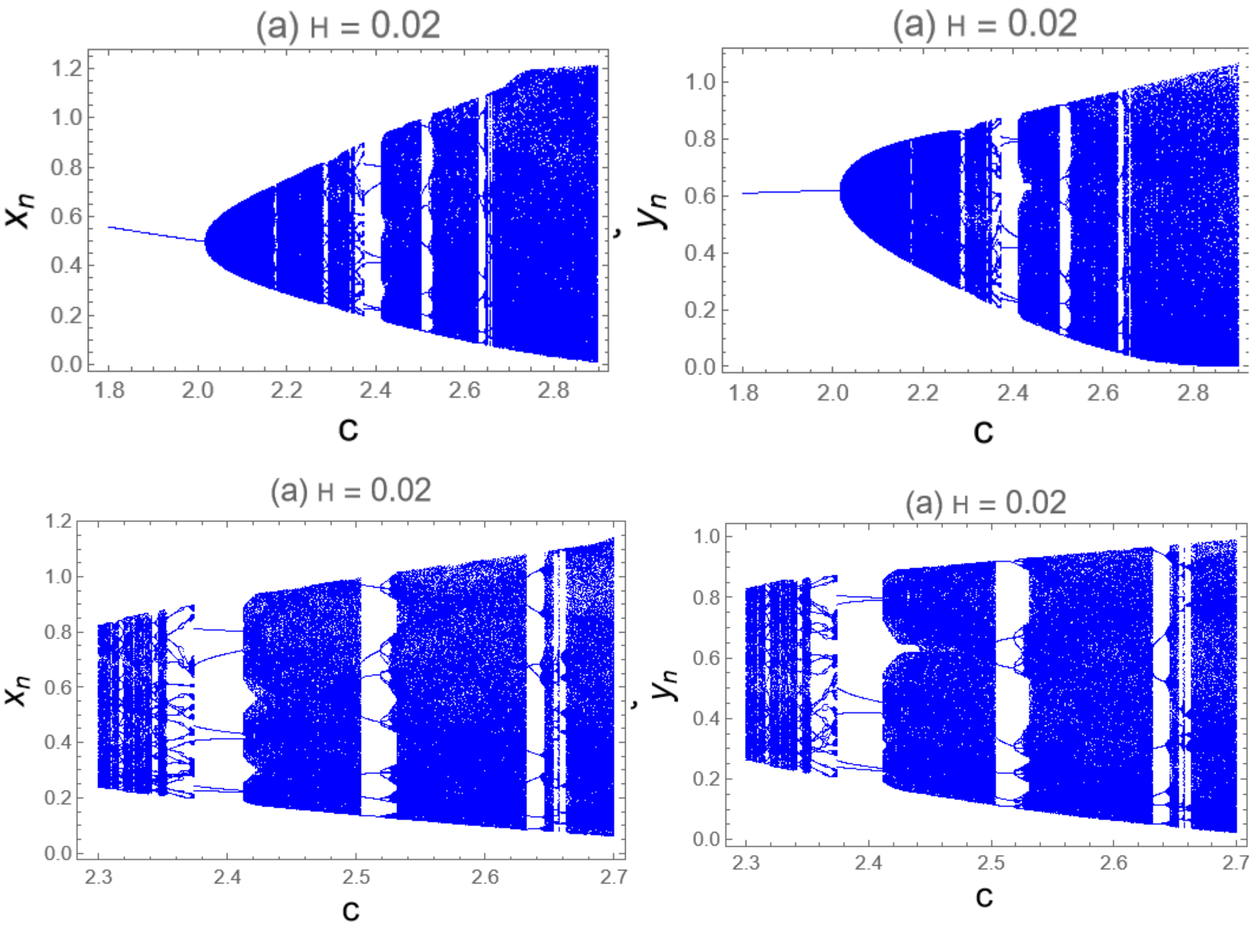
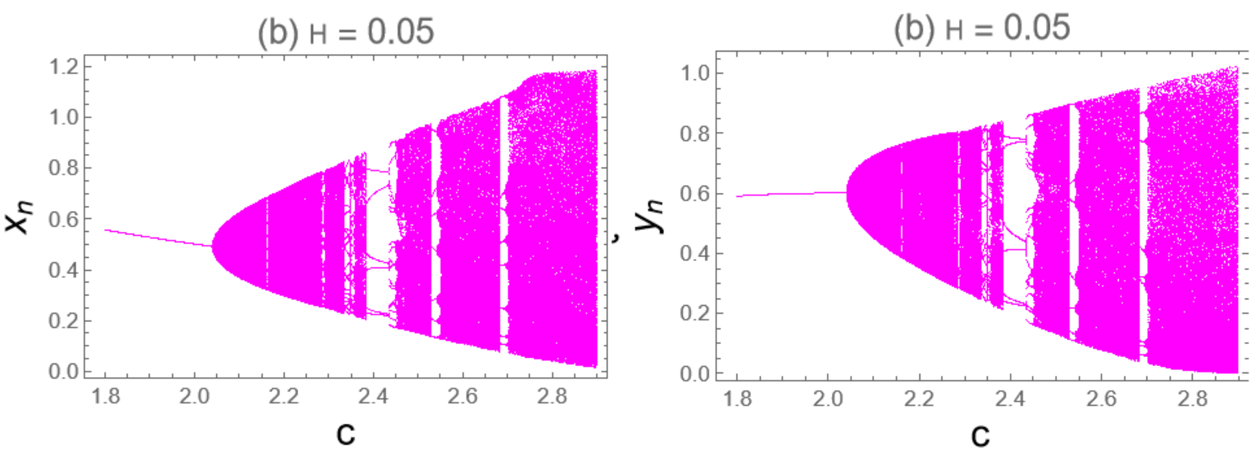


Figure 2: Bifurcation scenario of predator-prey system (5) along prey and predator axes for $H = 0.02$ and when $r = 2.5$ and $1.8 \leq c \leq 2.9$. Lower figures are magnifications of corresponding upper figures within range $2.3 \leq c \leq 2.7$.



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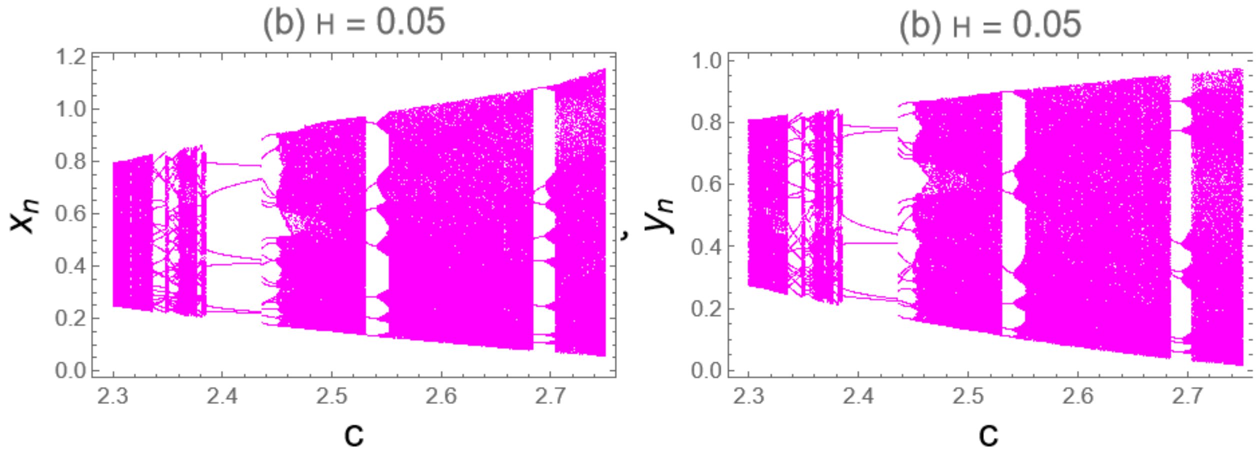

Figure 3: Bifurcation scenario of predator-prey system (5) along prey and predator axes for $H = 0.05$ and when $r = 2.5$ and$1.8 \leq c \leq 2.9$. Lower figures are magnifications of corresponding upper figures within range $2.3 \leq c \leq 2.75$.

4. Results of Numerical Simulations

The parameter $r = 2.5$ fixed, two types of attractor are drawn: regular and chaotic for different values of parameter c for all three cases $H = 0.0$, $H = 0.02$ and $H = 0.05$. These are presented in Figure 4, Figure 5 and Figure 6.

Case I: When $H = 0.0$

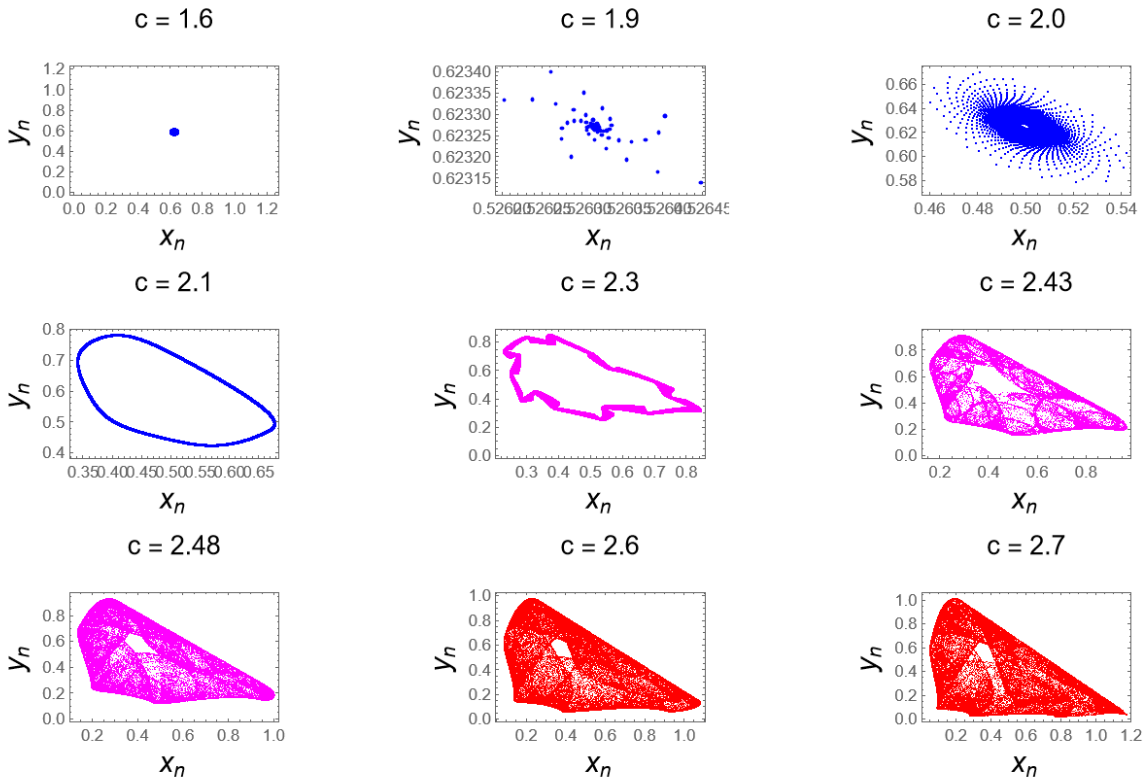

Figure 4: Plots of attractors that are regular and chaotic for varying values of parameter c when $H = 0.0$ and $r = 2.5$.

Received: July 26, 2025

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Case II: When $H = 0.02$.

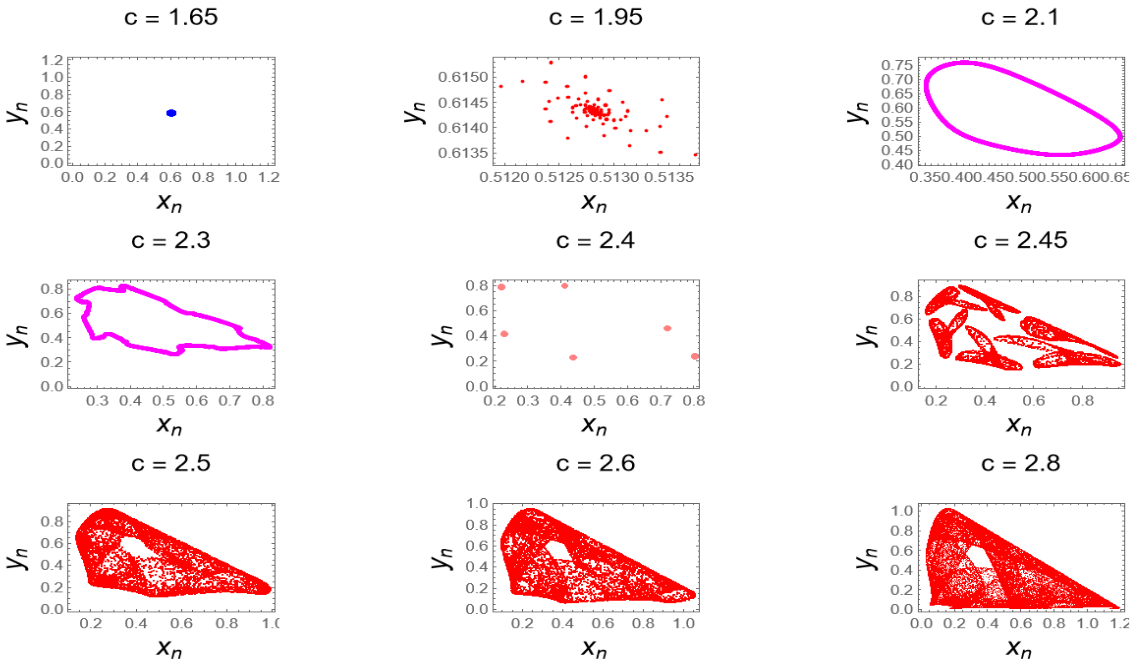

Figure 5: Plots of attractors that are regular and chaotic for varying values of parameter c when $H = 0.2$ and $r = 2.5$.

Case II: When $H = 0.05$.

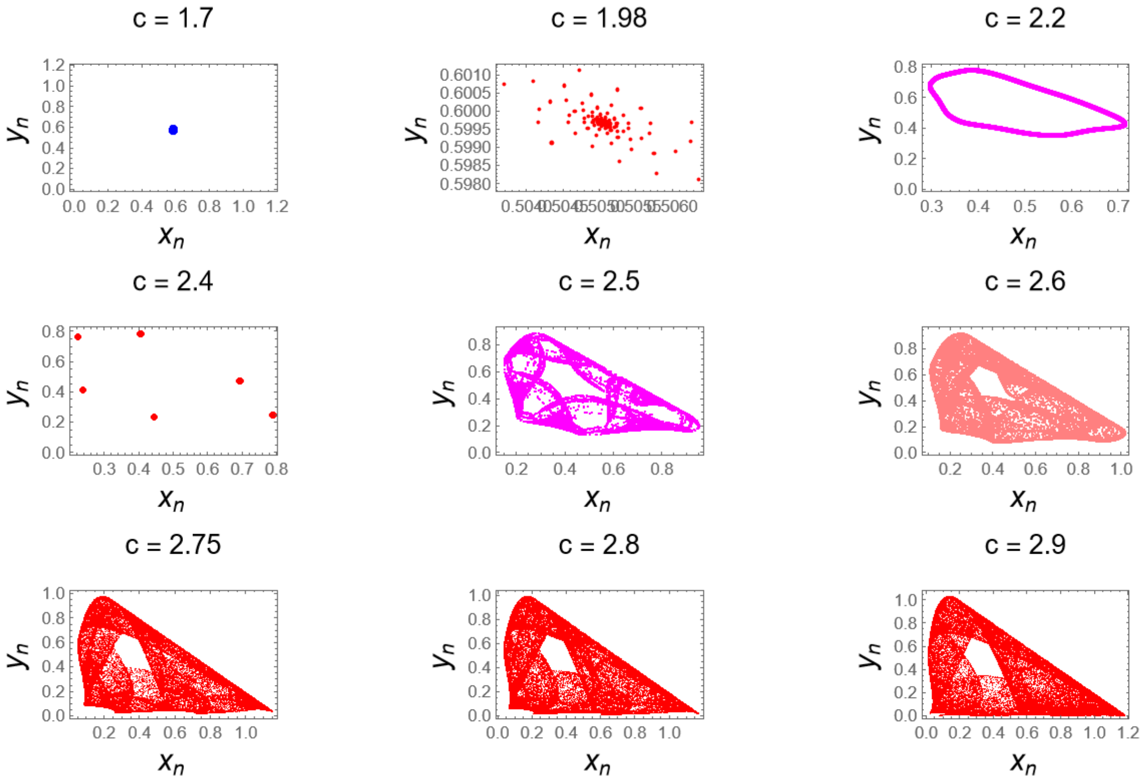

Figure 6: Plots of attractors that are regular and chaotic for varying values of parameter c when $H = 0.05$ and $r = 2.5$.

Received: July 26, 2025

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Lyapunov Exponents (LCEs): When chaos arises, the system demonstrates sensitivity to initial conditions, meaning that during long-term development, the two trajectories that were started very near to one another exhibit divergent behavior.A measure of the divergence of paths beginning closely is given by Lyapunov exponents, and if the system evolves chaotically, then LCEs > 0 and for LCEs < 0 the system evolves regularly. In Figure 4, plots of Lyapunov exponents are shown against cases of chaotic and regular attractors. To calculate Lyapunov exponents and plot them, we have used the Mathematica codes of Martelli, M. [24].

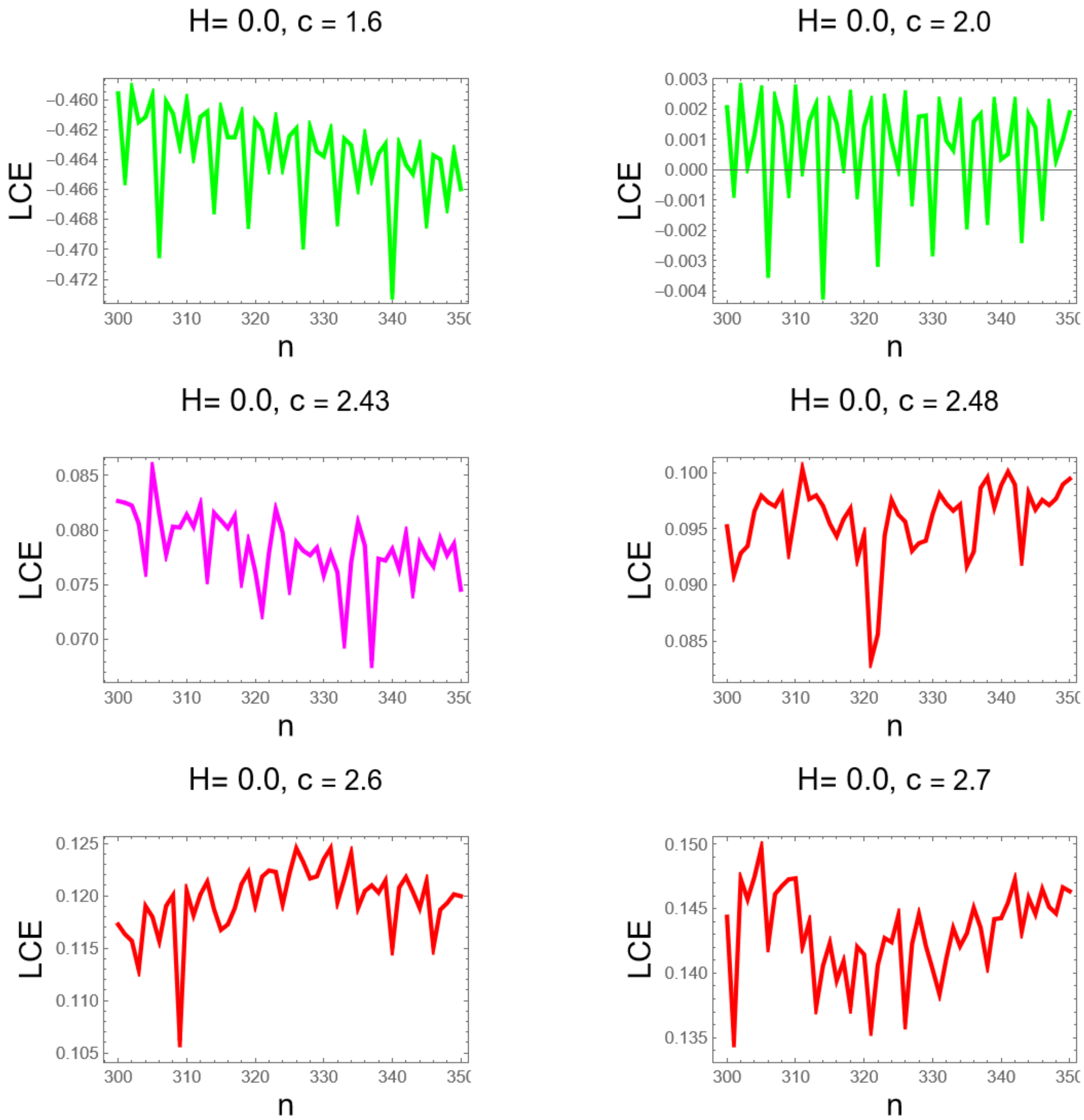

Figure 7: Plots of Lyapunov exponents (LCEs) that are chaotic and regular states of system (5) at $r = 2.5$, $H = 0.0$ and varying values for parameter c.

Received: July 26, 2025

International Journal of Applied Mathematics

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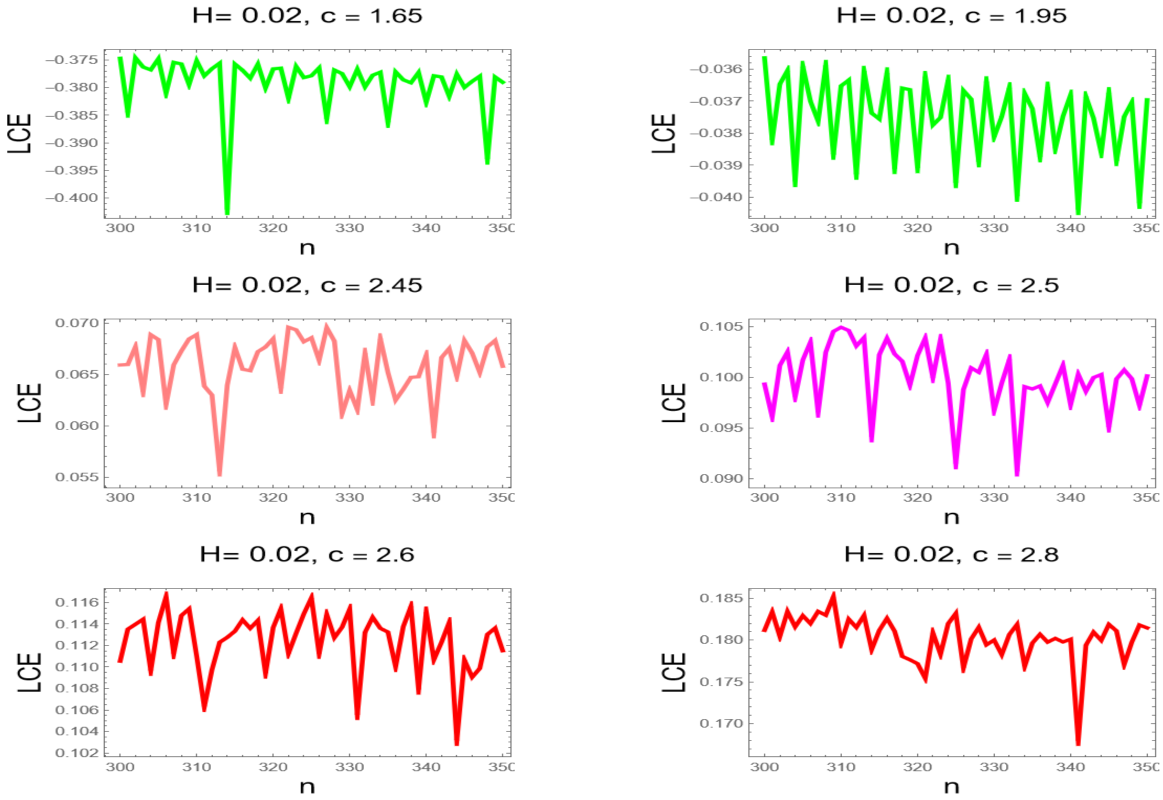

Figure 8: Plots of Lyapunov exponents (LCEs) that are chaotic and regular states of system (5) at $r = 2.5$, $H = 0.02$ and varying values for parameter c.

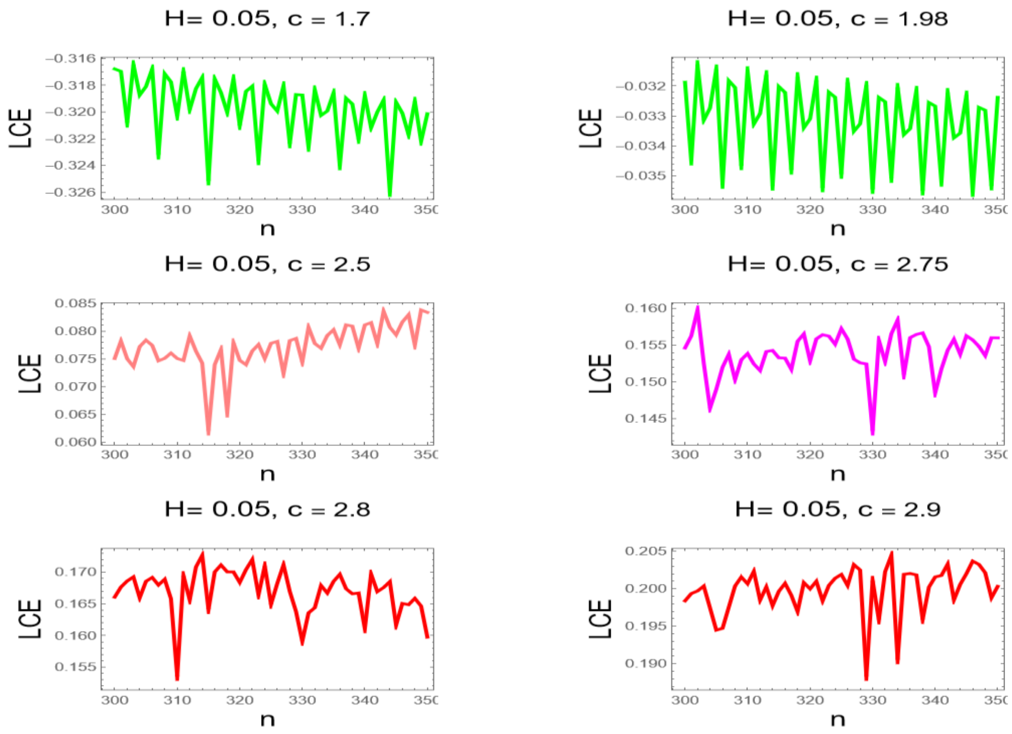

Figure 9: Plots of Lyapunov exponents (LCEs) that are chaotic and regular states of system (5) at $H = 0.05,\ r = 2.5$, and varying values for parameter c.

Received: July 26, 2025

International Journal of Applied Mathematics

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Topological Entropies

As mentioned, complex systems are internally composed of structurally multiple components, and these individual elements exhibit mixed nonlinearity features, chaos, and complexity as they develop independently of a dynamical system's mixing rate. The law of probability is thus involved in the measure of complexity and can be used to explain the rate at which evolutions mix. The idea of topological entropy was presented, with the idea that it represents a measurement of a dynamical system's rate of mixing, Adler, R. L., Konheim, A. G., & McAndrew, M. H. [3], Saha, L. M. Saureesh Das, Til Prasad Sarma, Anunay Choudhary, Das, M. K. [27], Bowen, R. [8], Balmforth, N. J., Spiegel, E. A., & Tresser, C. [5], Gribble, S. [15], Iwai, K. [21], Baldwin, S. L., & Slaminka, E. E. [4], Nagashima, H. [25]. Topological entropy measures how quickly the number of recognizable orbits increases exponentially over time. Topological entropy is also called Kolmogorov-Sinai entropy Grassberger, P., & Procaccia, I. [13]. The steps involved in computing topological entropy are as follows:

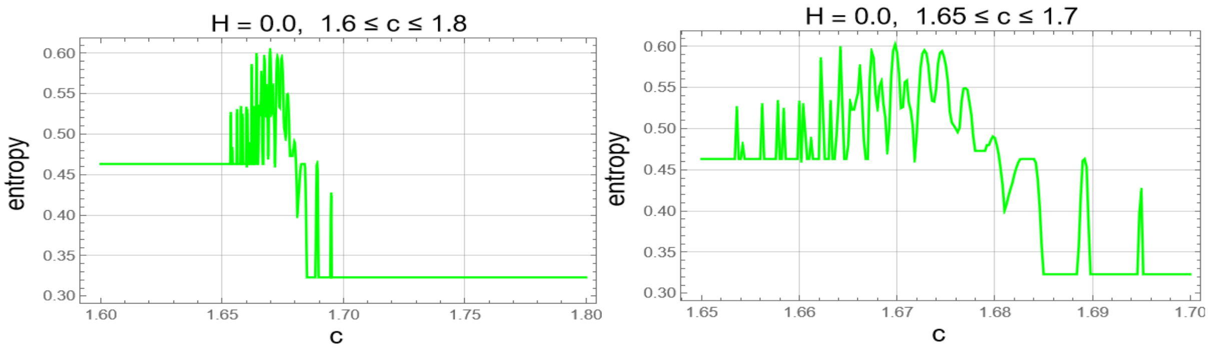
Suppose we have a finite partition $\mathrm{P} = \{ \mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3, \ldots, \mathrm{A}_\mathrm{d}\}$ of a state space X. Hence, the probability of a particular reading is defined as follows for a measure μ on X with total measure μ(X) = 1.

$$\mathrm{p}_\mathrm{i} = \mu(\mathrm{A}_\mathrm{i}), \mathrm{i} = 1, 2, .., \mathrm{d}. \tag{7}$$

The entropy of the partition be given by

$$\mathrm{H(P)} = -\sum_{\mathrm{i}=0}^{\mathrm{N}} \mathrm{p}_\mathrm{i} \,\mathrm{Log}\, \mathrm{p}_\mathrm{i} \tag{8}$$

Fix $r = 2.5$, Topological entropies calculated for cases $H = 0$, $H = 0.02$ and $H = 0.05$ for various ranges of values of c as shown in Figure 8.



Received: July 26, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

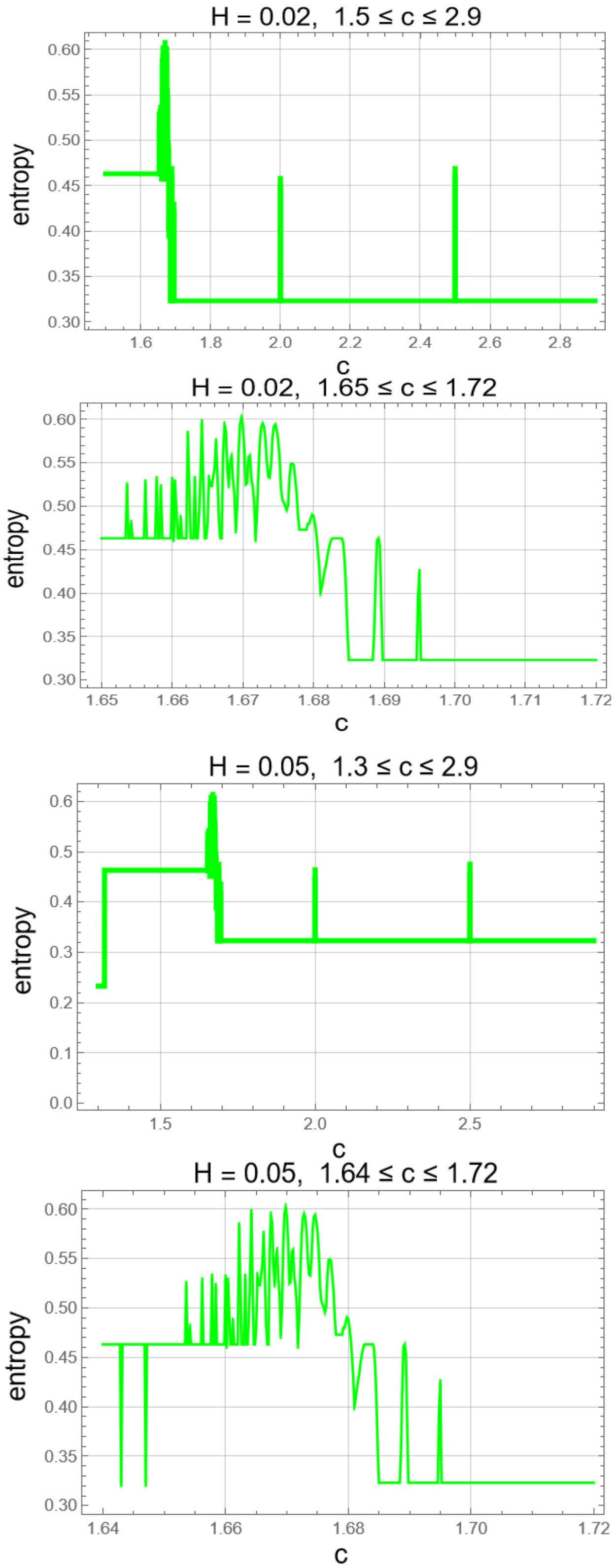


Figure 10: Plots of topological entropies for predator-prey system, eq$^{n.}$ (5) for $r = 2.5$ and in different intervals of values of parameter c for three cases, $= 0.0$, $H = 0.02$ and $H = 0.05$.

Received: July 26, 2025

Volume 38 No. 3s, 2025

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Correlation Dimensions

One form of fractal dimension is the correlation dimension as it gives a measure of the chaotic attractor's dimensionality. All chaotic attractors, (also sometimes called strange attractors), possess properties such as self-similarity and non-integer dimensions. Any chaotic attractor's correlation dimension can be statistically determined by using the Heaviside function. To obtain this, first one has to calculate the correlation integral data C(r), for a certain r ≪ 1. Then, plot the correlation graph by plotting $\frac{\log C(r)}{\log r}$ against r, as presented in Figure 11. Next, we found the equation for the straight line that fitted the data points in order to apply the linear fit criterion (linear regression) to the data of correlation. This line's y-intercept yields the correlation dimension D_C .

Thus the correlation dimension for the chaotic attractor obtained for $r = 2.5$, $c = 2.48$ and $H = 0.0$.

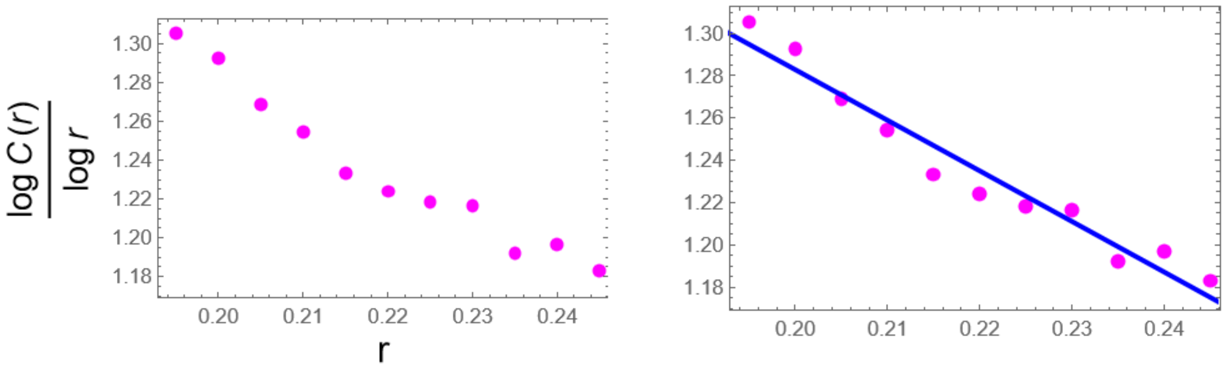

Figure 11: The left plot represents the correlation curve, and the right plot represents the linear fitting of the data points of the correlation curve.

The straight line's equation fitting the correlation data point curve's obtained as

$$y = 1.76126 - 2.39216\, x \tag{9}$$

Thus, the obtained chaotic attractor's correlation dimension for $r = 2.5$, $H = 0.0$ and $c = 2.8$ is obtained as $D_c = 1.76126 \approx 1.761$.

Following the steps of calculations, we can calculate correlation dimensions of many more chaotic attractors for cases of $H = 0.02$ and $H = 0.05$ as displayed in the Table 1.

Table 1: Calculations of Correlation Dimensions D_c of Chaotic Attractors:

r	H	c	D_c
	$H = 0.0$	2.43	1.598
		2.48	1.761

Received: July 26, 2025

$r = 2.5$		2.6	1.51
		2.7	1.795
	$H = 0.02$	2.45	1.402
		2.5	1.736
		2.6	1.479
		2.7	1.618
		2.8	1.389
	$H = 0.05$	2.5	1.487
		2.6	1.637
		2.75	1.549
		2.8	1.578
		2.9	1.613

5. **Concluding Remarks**

The phenomenon of evolution of a Lotka-Volterra discrete type predator-prey system is at the forcing predator dies after one generation and the prey population is subject to harvesting are investigated in equation (5). This system's fixed points are determined, and their stability is discussed in detail. Bifurcation plots, Figure 1, Figure 2, and Figure 3, of this system are drawn for cases of zero harvesting, ($H = 0$), $H = 0.02$ and $H = 0.05$. Bifurcation patterns in all cases appeared to be very complex, with a number of periodic windows. When magnified, lower figures in bifurcation diagrams, shows the amazing scenario of multi periodic orbits, and in some cases, these orbits are again evolving into period-doubling and chaos adding phenomena. The phenomena like bi-stability, intermittency, etc. seen within these periodic windows which are in confirmation that the system under observation is a complex system.

Three sets of regular and chaotic attractors, corresponding to the cases $H = 0$, $H = 0.02$, $H = 0.05$, are drawn and shown through the Figure 4 – Figure 6. These show a transition from regular motion to chaos due to changes in parameter spaces. For a few regular and chaotic attractors, corresponding plots of Lyapunov exponents (LCEs) were obtained, as demonstrated through Figure 7 – Figure 9. Regular evolution indicates a balanced, orderly situation, while chaotic evolution indicates a disorderly situation. In cases of chaos, there is a possibility of the extinction of one or both populations.

As measures of complexity, plots of topological entropies are obtained for all three cases of harvesting, and for certain closed regions of parameter$c$, these plots are magnified, as shown in Figure 10. We observe here in all cases significant increase and fluctuations in topological confirming the presence of complexity in the system (5).

Received: July 26, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Finally, we compute the correlation dimension for a chaotic attractor, which is a kind of fractal dimension. After describing the methodology of calculation, correlation dimensions for chaotic attractors obtained for different cases, $H = 0$, $H = 0.02$, $H = 0.05$, calculated and presented in tabular form, Table.

The results obtained through this study are significant, and we hope that these results will contribute a lot to mathematical biological problems relating to predator - prey problems with harvesting. For numerical simulation works in this study, we have used Mathematica codes described in the books by Martelli, M. [24], and Lynch, S. [23].

Conflict of Interest

The authors have no conflict of interest to declare for publication of this paper.

Acknowledgments

The authors acknowledge the supports provided by the North Cap University, Gurgaon, during preparation of this paper. Authors are also thankful to the Editor-in-Chief and the reviewers for their valuable suggestions.

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Received: July 26, 2025