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NEUTROSOPHIC BINARY G*S-CLOSED SET IN NEUTROSOPHIC BINARY TOPOLOGICAL SPACE

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Abstract: In this paper, introduce the concepts of neutrosophic binary generalized star semi-closed (G*S-closed) set and neutrosophic binary generalized star semi-open (G*S-open) set in a neutrosophic binary topological space. Additionally, we analyze certain theorems and properties with the help of examples.

Keywords: Neutrosophic Binary, Neutrosophic binary generalized star semi-closed set, Neutrosophic binary generalized star semi- open set.

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1. Introduction

In 1963, N. Levine [8-10] proposed the idea of semi-open set. The exploration of generalized closed set was initiated by C. E. Aull 2] in 1968, where he examined set whose closure is contained within every open superset. The concept of generalized semi-closed set was subsequently introduced by S. P. Arya and T. Nour [1] in 1990. In 1987, P. Bhattacharyya and B. K. Lahiri [4-5] formulated and analysed the notion of semi-generalized closed set through the framework of semi-closed set. In 1994, H. Maki, R. Devi, and K. Balachandran [11] introduced the category of α-generalized closed set. The concept of Neutrosophic set was introduced by Florentin Smarandache based on three segments: T (Truth), I (Indeterminacy), and F (Falsehood). A. A. Salama and S. A. Alblowi [14] later introduced Neutrosophic topological space (NS-T-S). Neutrosophic set have found diverse applications across multiple disciplines, including Electrical and Electronics, Artificial Intelligence, Mechanics, Computer Science, Information Systems, Applied Mathematics, Decision-Making, Medicine, and Management Science. I. Arokiarani, R. Dhavaseelan, and S. Jafari [3] proposed Neutrosophic semi-closed, α-closed, pre-closed, and regular closed set. R. Dhavaseelan and S. Jafari [6] introduced Neutrosophic g-closed set and gα-closed set. The purpose of this paper is to examine

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the contributions of Ishwarya and Bageerathi, [7], on Neutrosophic semi-open set in Neutrosophic Topological Space established in 2016, who proposed Generalized Neutrosophic Closed set. V. K. Shanthi, S. Chandrasekhar, and K. SafinaBegam [15] developed Neutrosophic gs-closed and sg-closed set. Additionally, C. Maheswari et al. introduced Neutrosophic gb-closed set.

In 2011, S. NithyananthaJothi and P. Thangavelu[12-13] put forward the concept of binary topology. Later, in 2022, S. S. Surekha, J. Elekiah, and G. Sindhu[16-18] introduced Neutrosophic binary topological space, consisting of two universal set, each possessing its own truth, indeterminacy, and false membership values. In the same year, S. S. Surekha and G. Sindhu [17] established binary αgs-closed set in binary topological space.

The objective of this paper is to introduce and analyse the concepts of Neutrosophic binary G*S-closed set and Neutrosophic binary G*S-open set in Neutrosophic binary topological space. Furthermore, we investigate the mappings of Neutrosophic binary G*S- closed set in Neutrosophic binary topological space. These findings further encourage researchers to explore the significance of Neutrosophic binary G*S- closed and Open set in Neutrosophic binary topological space.

2. PRELIMINARIES

In this section discuss the basic definition of neutrosophic binary topological space.

Definition 2.1. [16]

A Neutrosophic binary topology from X to Y is a binary structure $M_N \subseteq P(X) \times P(Y)$

That satisfies the following conditions:

1. $(0_x, 0_Y) \in M_N$ and $(1_X, 1_Y) \in M_N$.
2. $(A_1 \cap A_2, B_1 \cap B_2) \in M_N$ whenever $(A_1, B_1) \in M_N$ and $(A_2, B_2) \in M_N$.
3. If $(A\alpha, B\alpha)_{\alpha\in A}$ is a family of members of M_N, then $(\cup_{\alpha\in A}A\alpha, \cup_{\alpha\in A}B\alpha) \in M_N$.

The triplet (X, Y, M_N) is called Neutrosophic Binary Topological space. The members of M_N are called the Neutrosophic binary closed set and the complement of Neutrosophic binary open set are called the Neutrosophic binary closed set in the binary topological space (X, Y, M_N).

Definition 2.2. [16]

Let$(0_X, 0_Y)$ can be defined as

(01) $0_X = \{< x, 0, 0, 1 >: x \in X\}$, $0_Y = \{< y, 0, 0, 1 >: y \in Y\}$

(02) $0_X = \{< x, 0, 1, 1 >: x \in X\}$, $0_Y = \{< y, 0, 1, 1 >: y \in Y\}$

(03) $0_X = \{< x, 0, 1, 0 >: x \in X\}$, $0_Y = \{< y, 0, 1, 0 >: y \in Y\}$

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(04) $0_X = \{< x, 0, 0, 1 >: x \in X\}$, $0_Y = \{< y, 0, 0, 0 >: y \in Y\}$

$(1_X, 1_Y)$ can be defined as

(11) $1_X = \{< x, 1, 0, 0 >: x \in X\}$, $1_Y = \{< y, 1, 0, 0 >: y \in Y\}$

(12) $1_X = \{< x, 1, 0, 1 >: x \in X\}$, $1_Y = \{< y, 1, 0, 1 >: y \in Y\}$

(13) $1_X = \{< x, 1, 1, 0 >: x \in X\}$, $1_Y = \{< y, 1, 1, 0 >: y \in Y\}$

(14) $1_X = \{< x, 1, 1, 1 >: x \in X\}$, $1_Y = \{< y, 1, 1, 1 >: y \in Y\}$

Definition 2.3. [16]

Let $(A,B) = \{<\mu_A, \sigma_A, \gamma_A>,<\mu_B, \sigma_B, \gamma_B>\}$ be a Neutrosophic binary set on (X, Y, M_N), then the complement of the set $C(A,B)$ may be defined as

(C_1) $C(A,B) = \{x,<1 - \mu_{A(x)}, \sigma_{A(x)}, 1 - \gamma_{A(x)}>: x \in X, < y, 1 - \mu_{B(y)}, \sigma_{B(y)}, 1 - \gamma_{B(y)}>: y \in Y\}$

(C_2) $C(A,B) = \{x,< \gamma_{A(x)}, \sigma_{A(x)}, \mu_{A(x)}>: x \in X, < y, \gamma_{B(y)}, \sigma_{B(y)}, \mu_{B(y)}>: y \in Y\}$

(C_3) $C(A,B) = \{x,< \gamma_{A(x)}, 1 - \sigma_{A(x)}, \mu_{A(x)}>: x \in X, < y, \gamma_{B(y)}, 1 - \sigma_{B(y)}, \mu_{B(y)}>: y \in Y\}$

Definition 2.4. [16]

Let (A,B) and (C,D) be two Neutrosophic binary set which is in the form

$(A,B) = \{<\mu_A, \sigma_A, \gamma_A>,<\mu_B, \sigma_B, \gamma_B>\}$ and

$(C,D) = \{<\mu_C, \sigma_C, \gamma_C>,<\mu_D, \sigma_D, \gamma_D>\}$.

Then $(A,B) \subseteq (C,D)$ can be defined as

1. $(A,B) \subseteq (C,D) \Longleftrightarrow \mu_{A(x)} \leq \mu_{C(x)}, \sigma_{A(x)} \leq \sigma_{C(x)}, \gamma_{A(x)} \geq \gamma_{C(x)} \forall x \in X$
$\mu_{B(y)} \leq \mu_{D(y)}, \sigma_{B(y)} \leq \sigma_{D(y)}, \gamma_{B(y)} \geq \gamma_{D(y)} \forall y \in Y$
2. $(A,B) \subseteq (C,D) \Longleftrightarrow \mu_{A(x)} \leq \mu_{C(x)}, \sigma_{A(x)} \geq \sigma_{C(x)}, \gamma_{A(x)} \geq \gamma_{C(x)} \forall x \in X$
$\mu_{B(y)} \leq \mu_{D(y)}, \sigma_{B(y)} \geq \sigma_{D(y)}, \gamma_{B(y)} \geq \gamma_{D(y)} \forall y \in Y$

Definition 2.5. [16]

Let (A,B) and (C,D) be two neutrosophic binary set which is in the form

$(A,B) = \{<\mu_A, \sigma_A, \gamma_A>,<\mu_B, \sigma_B, \gamma_B>\}$ and

$(C,D) = \{<\mu_C, \sigma_C, \gamma_C>,<\mu_D, \sigma_D, \gamma_D>\}$.

(1) $(A,B) \cap (C,D)$ can be defined as

$(A,B) \cap (C,D) = \{ < x, \mu_{A(x)} \wedge \mu_{C(x)}, \sigma_{A(x)} \wedge \sigma_{C(x)}, \gamma_{A(x)} \vee \gamma_{C(x)}>,$
$< y, \mu_{B(y)} \wedge \mu_{D(y)}, \sigma_{B(y)} \wedge \sigma_{D(y)}, \gamma_{B(y)} \vee \gamma_{D(y)} >\}$

$(A,B) \cap (C,D) = \{ < x, \mu_{A(x)} \wedge \mu_{C(x)}, \sigma_{A(x)} \vee \sigma_{C(x)}, \gamma_{A(x)} \vee \gamma_{C(x)}>,$
$< y, \mu_{B(y)} \wedge \mu_{D(y)}, \sigma_{B(y)} \vee \sigma_{D(y)}, \gamma_{B(y)} \vee \gamma_{D(y)}>\}$

(2) $(A,B) \cup (C,D)$ can be defined as

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$(A,B) \cup (C,D) = \{ < x, \mu_{A(x)} \vee \mu_{C(x)}, \sigma_{A(x)} \vee \sigma_{C(x)}, \gamma_{A(x)} \wedge \gamma_{C(x)} >,$

$< y, \mu_{B(y)} \vee \mu_{D(y)}, \sigma_{B(y)} \vee \sigma_{D(y)}, \gamma_{B(y)} \wedge \gamma_{D(y)} > \}$

$(A,B) \cap (C,D) = \{ < x, \mu_{A(x)} \vee \mu_{C(x)}, \sigma_{A(x)} \wedge \sigma_{C(x)}, \gamma_{A(x)} \wedge \gamma_{C(x)} >$

$< y, \mu_{B(y)} \vee \mu_{D(y)}, \sigma_{B(y)} \wedge \sigma_{D(y)}, \gamma_{B(y)} \wedge \gamma_{D(y)} > \}$

Definition 2.6. [16]

Let (X, Y, M_N) be a Neutrosophic Binary Topological Space. Then,

$(A,B)^{1\star N} = \cap\{A\alpha : (A\alpha,B\alpha)$ is Neutrosophic binary closed and $(A,B) \subseteq (A\alpha,B\alpha)\}$

$(A,B)^{2\star N} = \cap\{B\alpha : (A\alpha,B\alpha)$ is Neutrosophic binary closed and $(A,B) \subseteq (A\alpha,B\alpha)\}$.

The ordered pair $((A,B)^{1\star N}, (A,B)^{2\star N})$ is called the Neutrosophic binary closure of (A,B).

Definition 2.7. [16]

Let (X, Y, M_N) be a Neutrosophic Binary Topological Space. Then,

$(A,B)^{1o}N = \cup\{A\alpha : (A\alpha,B\alpha)$ isNneutrosophic binary open and $(A\alpha,B\alpha) \subseteq (A,B)\}$

$(A,B)^{2o}N = \cup\{B\alpha : (A\alpha,B\alpha)$ is Neutrosophic binary open and $(A\alpha,B\alpha) \subseteq (A,B)\}$.

The ordered pair $((A,B)^{1O} N , (A,B)^{2O} N)$ is called the Neutrosophic binary interior of (A,B).

Definition 2.8. [16]

Let (X, Y, M_N) be a Neutrosophic Binary Topological Space. Then (A,B) is called

1. Neutrosophic binary α open if (A,B) ⊆Nbint(Nbcl(Nbint(A,B))).
2. Neutrosophic binary semiopen if (A,B) ⊆Nbcl(Nbint(A,B)).

Definition 2.9. [16]

Let (X, Y, M_N) be a Neutrosophic Binary Topological Space. Then,

$(A,B)^{1\star}N = \cap\{A\alpha : (A\alpha,B\alpha)$ is Neutrosophic binary α closed and $(A,B) \subseteq (A\alpha,B\alpha)\}$

$(A,B)^{2\star}N = \cap\{B\alpha : (A\alpha,B\alpha)$ is Neutrosophic binary α closed and $(A,B) \subseteq (A\alpha,B\alpha)\}$.

The ordered pair $((A,B)^{1\star}N , (A,B)^{2\star}N)$ is called the Neutrosophic binary α closure of (A,B) and is denotedby Nbαcl(A,B).

Definition 2.10. [16]

Let (X, Y, M_N) be a Neutrosophic Binary Topological Space. Let (A,B) ⊆(X, Y).

Then (A,B) is called a Neutrosophic Binary α generalised semiclosed set (shortly Nbαgs-closed set) if

Nbαcl(A,B) ⊆(U, V) whenever (U, V) is Neutrosophic Binary Semiopen.

3. Neutrosophic Binary G*S-closed Set

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In this section, introduce the Neutrosophic Binary Generalized Star Semi-closed (shortly NBG*S-closed) Set their basic definition, NBG*S interior and NBG*S closure their properties and theorem are derived.

Definition 3.1.

Let (P,Q) be a Sub-Set of a NBS (X,Y, τ_{NB}) is called NBG*S-CS if NBSCl(A,B) ⊆(U,V) whenever (P,Q)⊆(U,V)and (U,V) is NBG-OS. The complement of a NBG*S-CS is called the NBG*S-OS.

Example 3.2.

Let X = { a} and Y={b}

(A_1, A_2)= {X:<(0.4, 0.6, 0.5)>,Y:<(0.7, 0.3, 0.6)>}

(B_1, B_2) = {X:<(0.3, 0.7, 0.8)>,Y:<(0.6, 0.4, 0.2)>}

(C_1, C_2)= {X:<(0.4, 0.7, 0.5)>,Y:<(0.7, 0.4, 0.2)>}

(D_1, D_2)= {X:<(0.3, 0.6, 0.8)>,Y:<(0.6, 0.3, 0.6)>} .

Then τ_{NB}={ 0_{NB}, A_1, A_2, A_3, A_4, 1_{NB} } is NBTS on (X,Y).

Now, (P,Q) ={X:<(0.5, 0.7, 0.5)>,Y:<(0.9, 0.4, 0.5)>} is NBG*S-CS.

Definition 3.3.

Let(X, Y, τ_{NB}) be a NBTS. Then for a NB Sub-Set (P,Q) of (X,Y). The NB semi-closure of (P,Q) is the intersection of all NBG*S-CS of (X,Y) contained in (P,Q). That is,

NBG*S-Cl(P,Q)=∩{ (K,L) :(K,L) is a NBG*S-CS in (X,Y) and (K,L)⊇ (P,Q)}.

Definition 3.4.

Let(X,Y, τ_{NB}) be a NBTS , for a NB Sub-Set (P,Q) of (X,Y) the NB semi-interior of (P,Q) is the union of all **N** semi-open set of (X,Y) contained in (P,Q)..i.e. NBG*S-Int(P,Q)=∪{ (G,H): (G,H) is a NB g*s-open set in (X,Y) and (G,H)⊆(P,Q)}.

Proposition 3.5.

NB Sub-Set (P,Q) and (R,S) of a NBTS

(X,Y,τ_{NB}) we have

(i) NBG*S-Int(P,Q)⊆(P,Q)

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(ii) (P,Q) is NBG*S-OS in (X,Y)⟺NBG*S-Int(P,Q)=(P,Q)

(iii) NBG*S-Int(NBG*S-Int(P,Q))=NBG*S-Int(P,Q)

(iv) If (P,Q)⊆(R,S) then NBG*S-Int(P,Q)⊆NBG*S-Int(R,S)

Proof:

(i) Follows from Definition 3.3.

Let (P,Q) be NBG*S-OS in (X,Y). Then (P,Q)⊆NBG*S-Int(P,Q). By using (i) we get (P,Q)=NBG*S-Int(P,Q). Conversely assume that (P,Q)=NBG*S-Int(P,Q).

(ii) By using Definition 3.3, (P,Q) is NBG*S-OS in (X,Y). Thus (ii)is proved.

(iii) By using (ii), NBG*S-Int(NB-G*S-Int(P,Q)) = NBG*S-Int(P,Q). This proves (iii).

(iv) Since (P,Q)⊆ (R,S), by using(i),NBG*S-Int(P,Q)⊆(P,Q)⊆(R,S). That is NBG*S-Int(P,Q)⊆(R,S). By(iii),NBG*S-Int(NBG*S-Int(P,Q))⊆NBG*S-Int(R,S).ThusNBG*S-Int(P,Q)⊆NBG*S-Int(R,S). This proves (iv).

Theorem 3.6.

Let(X, Y, τ_{NB}) be a NBTS. Then for any NB Sub-Set (P,Q) and (R,S) of a NBTS , we have

(i) NBG*S-Int((P,Q)∩(R,S))=NBG*S-Int(P,Q)∩ NB-G*S-Int(R,S)

(ii) NBG*S-Int((P,Q)∪(R,S))⊇NBG*S-Int(P,Q)∪NB-G*S-Int(R,S).

Proof : Since (P,Q)∩(R,S)⊆ (P,Q) and (P,Q)∩(R,S)⊆ (R,S) by using Proposition 3.5(iv), NBG*S-Int((P,Q)∩(R,S)) ⊆NB-G*S-Int(P,Q) and NBG*S-Int((P,Q)∩(R,S))⊆NBG*S-Int(R,S). This implies that NBG*S-Int((P,Q)∩(R,S))⊆NBG*S-Int(P,Q)∩NBG*S-Int(R,S) ---(1).Byusing Proposition 3.5(i), NBG*S-Int(P,Q)⊆(P,Q) and NBG*S-Int(R,S)⊆ (R,S).This implies that NBG*S-Int(P,Q) ∩ NBG*S-Int(R,S)⊆(P,Q)∩(R,S).

Now applying proposition3.5(iv),NBG*S-Int((NBG*S-Int(P,Q)∩NBG*S-Int(R,S))⊆NBG*S-Int((P,Q)∩(R,S)).By(1),NBG*S-Int(NBG*S-Int(P,Q))∩NBG*S-Int(NBG*S-Int(R,S))⊆NBG*S-Int((P,Q)∩(R,S)).Byproposition3.5(iii),NBG*S-Int(P,Q)∩NBG*S-Int(R,S)⊆NBG*SInt((P,Q)∩(R,S)) -----(2).

From(1)and(2),NBG*S-Int((P,Q)∩(R,S))=NBG*S-Int(P,Q)∩NBG*S-Int(R,S). This implies(i).Since (P,Q)⊆(P,Q)∪(R,S) and (R,S)⊆ (P,Q) ∪ (R,S) , by using Proposition 3.5(iv), NBG*S-Int(P,Q)⊆NBG*S-Int((P,Q)∪(R,S)) and NBG*S-Int(R,S)⊆NBG*S-

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Int((P,Q)∪(R,S)). This implies that NBG*S-Int(P,Q)∪NBG*S-Int(R,S)⊆NBG*S-Int((P,Q)∪(R,S)). Therefore satisfied (ii).The following example shows that the equality need not be hold in Theorem 3.6(ii).

Example 3.7.

Let X = $\{a_1, b_1, c_1\}$ and Y=$\{a_2, b_2, c_2\}$ then

τ_{NB}= $\{0_{NB}$, (A_1, A_2), (B_1,B_2), $1_{NB}\}$ where

(A_1, A_2) ={X:⟨ (0.41, 0.71, 0.11),(0.51, 0.61, 0.21),(0.91, 0.71, 0.31)⟩,

Y:⟨(0.41, 0.61, 0.11),(0.71, 0.71, 0.21),(0.91, 0.51, 0.11)⟩}

(B_1,B_2) = {X:⟨(0.41, 0.71, 0.11), (0.71, 0.71, 0.21),(0.91, 0.71, 0.11)⟩,

Y:⟨ (0.41, 0.61, 0.11),(0.51, 0.61, 0.21),(0.91, 0.51, 0.31)⟩}

Then(X, Y, τ_{NB}) is NBTS.

Consider the NBS are

(E,F) = {X;⟨(0.71, 0.61, 0.11),(0.71, 0.61, 0.11),(0.91, 0.51, 0.01)⟩ ,

Y:⟨(0.41, 0.61, 0.11),(0.51, 0.71, 0.21),(2.1, 0.71, 0.11)⟩}.

Then NBG*S-Int(E,F) =(A_1, A_2) and NBG*S-Int(E,F) = (A_1, A_2) This implies that NBG*S-Int(E,F)∪NB-G*S-Int(F,F)= (A_1, A_2).

4 Properties Between Neutrosophic Binary G*S-closed Set and Neutrosophic Binary G*S-open Set

In this section, introduce the concept of Neutrosophic Binary generalized star semi closure operators in a Neutrosophic Binary topological space.

Proposition 4.1

Let(X, Y, τ_{NB}) be a NBTS . Then for any NB Sub-Set (P,Q) of (X,Y)

(i) C(NBG*S-Int(P,Q))=NBG*S-Cl(C(P,Q)),

(ii) C(NBG*S-Cl(P,Q))=NBG*S-Int(C(P,Q)).

Proof :

By using Definition 3.4,

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NBG*S-Int(P,Q) = ∪{(G,H):(G,H) is a NBG*S-OS in (X,Y) and (G,H)⊆(P,Q)}. Taking complement on both sides, C(NBG*S-Int(P,Q)) = C(∪{G :G is a NBG*S-OS in (X,Y) and (G,H)⊆(P,Q)})=∩{C(G,H):C(G,H) is a NBG*S-CS in (X,Y) and C(P,Q)⊆ C(G,H) }. Replacing C(G,H) by (K,L) , C(NBG*S-Int(P,Q))=∩{ (K,L):(K,L) is a NBG*S-CS in (X,Y) and (K,L)⊇ C(P,Q)}.

By Definition 3.3, C(NBG*S-Int(P,Q))=NBG*S-Cl(C(P,Q)). This proves(i). By using(i), C(NBG*S-Int(C(P,Q)))=NBG*S-Cl(C(C(P,Q)))=NBG*S-Cl(P,Q). Taking complement on both sides, NBG*S-Int(C(P,Q))=C(NBG*S-Cl(P,Q)). Hence proved(ii)

Proposition 4.2

Let (X, Y, τ_{NB}) be a NBTS. Then for any NB Sub-Set (P,Q) and (R,S) of a NBTS (X,Y) we have

(i) (P,Q)⊆NBG*S-Cl(P,Q)

(ii) (P,Q) is NBG*S-CS in X ⟺NBG*S-Cl(P,Q)=(P,Q)

(iii) NBG*S-Cl(NB-G*S-Cl(P,Q))=NBG*S-Cl(P,Q)

(iv) If (P,Q)⊆(R,S) then NBG*S-Cl(P,Q)⊆NBG*S-Cl(R,S)

Proof :

(i) Follows from Definition 3.3.

Let (P,Q) be NBG*S-CS in (X,Y). By using Proposition 4.3,C(P,Q) is NBG*S-OS in (X,Y).By Proposition 4.1(ii), NBG*S-Int(C(P,Q))=C(P,Q)⟺C(NB-G*S-Cl(P,Q))=C(P,Q)⟺NBG*S-Cl(P,Q)=(P,Q). Thus proved (ii)

Therefore (ii), NBG*S-Cl(NBG*S-Cl(P,Q))=NBG*S-Cl(P,Q). This proves (iii), since A⊆B,C(R,S)⊆C(P,Q). By using Proposition 3.5(iv),NBG*S-Int(C(R,S))⊆NBG*S-Int(C(P,Q)). Taking complement on both sides, C(NBG*S-Int(C(R,S)))⊇C(NBG*S-Int(C(P,Q))). By Proposition 4.2(ii),NBG*S-Cl(P,Q)⊆NBG*S-Cl(R,S).This proves(iv).

Proposition 4.3

Let (P,Q) be a NBS in a NBTS

(X,Y,τ_{NB}). Then NB-Int(P,Q)⊆NB-G*S-Int(P,Q)⊆(P,Q)⊆NB-G*S-Cl(P,Q)⊆NB-Cl(P,Q).

Proof :

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It follows from the definitions of corresponding operators.

Proposition 4.4

Let (X, Y, τ_{NB}) be a NBTS . Then for a NB Sub-Set (P,Q) and (R,S) of a NBTS (X,Y), the following statements are,

(i) NBG*S-Cl((P,Q)∪(R,S)) =NBG*S-Cl(P,Q)∪NBG*S-Cl(R,S) and

(ii) NBG*S-Cl((P,Q)∩(R,S))⊆NBG*S-Cl(P,Q)∩NBG*S-Cl(R,S).

Proof :

Since NBG*S-Cl((P,Q)∪(R,S))=NBG*S-Cl(C(C((P,Q)∪(R,S)))),

ByusingProposition4.2(i),NB-G*S-Cl((P,Q)∪(R,S))=C(NBG*S Int(C((P,Q)∪(R,S))))=C(NBG*S-Int(C(P,Q)∩C(R,S))).Again using Proposition3.5(i),NBG*S-Cl((P,Q)∪(R,S))=C(NBG*S-Int(C(P,Q))∩NB-G*S-Int(C(R,S)))=C(NBG*S-Int(C(P,Q)))∪C(NBG*S-Int(C(R,S))). By using proposition 4.2(i),NBG*S-Cl((P,Q)∪(R,S))=NBG*S-Cl(C(C(P,Q)))∪NBG*S-Cl(C(C(R,S)))= NBG*SCl(P,Q)∪NB-G*S-Cl(R,S).Thus proved(i). Since (P,Q)∩(R,S)⊆(P,Q) and (P,Q)∩(R,S)⊆ (R,S), by using Proposition 4.3(iv),NBG*S-Cl((P,Q)∩(R,S))⊆NBG*S-Cl(P,Q) and NBG*S-Cl((P,Q)∩(R,S)) ⊆NBG*S-Cl(R,S). This implies that NBG*S-Cl((P,Q)∩(R,S))⊆NBG*S-Cl(P,Q)∩NBG*S-Cl(R,S).This proves(ii). The following example shows that the equality need not be hold in Proposition 4.5(ii).

Example 4.5

Let X = { a_1 , b_1 ,c_1 } Y={a_2, b_2, c_2}with τ_{NB}={ 0_{NB}, (A_1, A_2), (B_1, B_2), 1_{NB} } and
C(τ_{NB})={ 1_{NB}, (C_1, C_2), (D_1, D_2), 0_{NB} } where
(A_1, A_2)= {X:⟨(0.5, 0.6, 0.1),(0.6, 0.7, 0.1),(0.9, 0.5, 0.2)⟩,
Y,:⟨(0.4, 0.5, 0.2),(0.8, 0.6, 0.3),(0.9, 0.7, 0.3)⟩}
(B_1, B_2)= {X:⟨(0.4, 0.5, 0.2),(0.6, 0.6, 0.3),(0.9, 0.5, 0.3)⟩,
Y:⟨(0.5, 0.6, 0.1),(0.8, 0.7, 0.1),(0.9, 0.7, 0.2)⟩}
(C_1, C_2) = {X:⟨(0.1, 0.4, 0.5),(0.1, 0.3, 0.6),(0.2, 0.5, 0.9)⟩,
Y:⟨(0.2, 0.5, 0.4),(0.3, 0.4, 0.8),(0.3, 0.3, 0.9)⟩}
(D_1, D_2)= {X:⟨(0.2, 0.5, 0.4),(0.3, 0.4, 0.6),(0.3, 0.5, 0.9)⟩,
Y:⟨(0.1, 0.4, 0.5),(0.1, 0.3, 0.8),(0.2, 0.3, 0.9)⟩}.

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Then(X,Y, τ_{NB}) is a NBTS.

Consider the NBS are

(E_1, E_2)= {X:⟨(0.1, 0.2, 0.5),(0.2, 0.3, 0.7),(0.3, 0.3, 1)⟩,

Y:⟨(0.2, 0.4, 0.8),(0.1, 0.2, 0.8),(0.2, 0.5, 0.9)⟩}.

Then NBG*S-Cl(E_1, E_2)=(D_1, D_2) and NBG*S-Cl(C_1, C_2)=(D_1, D_2).

This implies that NBG*S-Cl(E_1, E_2)∩NBG*S-Cl(C_1, C_2) = A7.

Now, (E_1, E_2) ∩ (C_1, C_2)= {X:⟨(0.2, 0.5, 0.4),(0.3, 0.4, 0.6),(0.3, 0.5, 0.9)⟩,

Y: ⟨(0.1, 0.4, 0.5),(0.1, 0.3, 0.8),(0.2, 0.3, 0.9)⟩}. It follows that NBG*S-Cl((E_1, E_2) ∩ (C_1, C_2))= . (D_1, D_2)

Then NBG*S-Cl(E_1, E_2) ∩NBG*S-Cl(C_1, C_2) ⊈ NBG*S- ((E_1, E_2)∩ (C_1, C_2)).

Theorem 4.6

Let(X,Y,τ_{NB}) be a NBTS. Then NB Sub-Set (P,Q) and (R,S) of (X,Y) we have

(i) NBG*S-Cl(P,Q)⊒(P,Q) ∪NBG*S-Cl(NBG*S-Int(P,Q)),

(ii) NBG*S-Int(P,Q)⊑(P,Q)∩NBG*S-Int(NBG*S-Cl(P,Q)),

(iii) NBG*S-Int(NB-G*S-Cl(P,Q)) ⊆NBG*S-Int(NBG*S-Cl(P,Q)),

(iv) NBG*S-Int(NBG*S-Cl(P,Q)) ⊇NBG*S-Int(NBG*S-Cl(NBG*S-Int(P,Q))).

Proof : By Proposition 4.3(i), (P,Q)⊑NBG*S-Cl(P,Q)---(1). Again using Proposition 3.4(i), NBG*S-Int(P,Q)⊑(P,Q).Then NBG*S-Cl(NBG*S-Int(P,Q))⊑NBG*S-Cl(P,Q),(2).By(1) &(2) we have(P,Q)∪NBG*S-Cl(NBG*S-Int(P,Q))⊑NBG*S-Cl(P,Q). This proves (i).By Proposition 3.4(i), NBG*S-Int(P,Q)⊑(P,Q) -----(3).Again using proposition 4.3(i), (P,Q)⊑NBG*S-Cl(P,Q).Then NBG*S-Int(P,Q)⊑NBG*S-Int(NBG*S-Cl(P,Q))--(4).From(3) &(4), we have NBG*S-Int(P,Q)⊑ (P,Q) ∩NBG*S-Int(NBG*S-Cl(P,Q)). This proves(ii).By Proposition 4.4, NBG*S-Cl(P,Q)⊑NBG*S-Cl(P,Q).We get NBG*S-Int(NBG*S-Cl(P,Q))⊑NBG*S-Int(NB-Cl(P,Q)). Hence(iii).By(i), NBG*S-Cl(P,Q)⊒ (P,Q)∪NBG*S-Cl(NBG*S-Int(P,Q)). We have NBG*S-Int(NBG*S-Cl(P,Q)⊒NBG*S-Int(A∪NBG*S-Cl(NBG*S-Int(P,Q))).Since NB-Int((P,Q)∪(R,S))⊒NB-Int(P,Q)∪NB-Int(R,S),NB-Int(NBG*S-Cl(P,Q)⊒NBG*S-Int(P,Q)∪NBG*S-Int(NBG*S-Int(NBG*S-Int(P,Q)))⊒NBG*S-Int(NBG*S-Cl(NBG*S-Int(P,Q))).Hence(iv) proved.

Theorem 4.7

Let (X, Y,τ_{NB}) be a NBTS. Then NB Sub-Set (P,Q) and (R,S) of (X,Y) are satisfies,

(i) NBG*S-Cl(P,Q)⊒(P,Q) ∪ NBG*S-Cl(NBG*S-Int(P,Q)),

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(ii) NBG*S-Int(P,Q)⊆(P,Q) ∩ NBG*S-Int(NBG*S-Cl(P,Q)),

(iii) NBG*S-Int(NBG*S-Cl(P,Q))⊆NB-Int(NB-Cl(P,Q)),

(iv)NBG*S-Int(NBG*S-Cl(P,Q))⊇NB-Int(NBG*S-Cl(NBG*S-Int(P,Q))).

Proof:

By Proposition 4.2(i), (P,Q)⊆NBG*S-Cl(P,Q) -----(1).

Again using Proposition 3.5(i), NBG*S-Int(P,Q)⊆ (P,Q). Then NBG*S-Cl(NB-G*S-Int(P,Q))⊆NBG*S-Cl(P,Q)--(2)By(1)&(2) and (P,Q)∪NBG*S-Cl(NBG*S-Int(P,Q))

⊆NBG*S-Cl(P,Q). This proves(i).By Proposition 3.4(i), NBG*S-Int(P,Q)⊆(P,Q) -----(1). Again using proposition 4.4(i), (P,Q)⊆NBG*S-Cl(P,Q). Then NBG*S-Int(P,Q)⊆NBG*S-Int(NBG*S-Cl(P,Q)) -----(2).From(1) &(2), we have NBG*S-Int(P,Q)⊆(P,Q)∩NBG*S-Int(NBG*S-Cl(P,Q)).This proves(ii).By Proposition 4.3, NBG*S-Cl(P,Q)⊆NBCl(P,Q).We get NBG*S-Int(NBG*S-Cl(P,Q))⊆NBG*S-Int(NB-Cl(P,Q)).Hence(iii).By(i),NBG*S-Cl(P,Q)⊇A ∪ NBG*S-(NBG*S-Int(P,Q)).We have NBG*S-Int(NBG*S-Cl(P,Q)⊇NBG*S-Int((P,Q)∪NBG*S-Cl(NB-G*S-Int(P,Q))).SinceNBG*S-Int((P,Q)∪(R,S))⊇NBG*S-Int(P,Q)∪NBG*S-Int(R,S),NBG*S-Int(NBG*S-Cl(P,Q)⊇NBG*S-Int(P,Q)∪NBG*S-Int(NBG*S-Cl(NBG*S-Int(P,Q)))⊇NBG*S-Int(NBG*S-Cl(NBG*S-Int(P,Q))).Hence(iv) proved.

Theorem 4.8
A Sub-Set (P,Q) in a NBTS (X,Y, τ_{NB}) is a NBG*S-OS if and only if (P,Q)⊆NBG*S-Cl(NBG*S-Int(P,Q)).

Proof:
Sufficiency: Let (P,Q)⊆NBG*S-Cl(NBG*S-Int(P,Q)). Then for NBG*S-O=NB G*S--Int(P,Q), and NBG*S-O⊆(P,Q)⊆NBG*S-Cl(NBG*S-Int).

Necessity: Let (P,Q) be a NBG*S-OS in (X,Y). Then NBGSO⊆(P,Q)⊆NBG*S-Cl(NB G*S--O)NO for some NB-OS also known as NBG*S-OS. But NBG*SO⊆NBG*S-Int(P,Q) and thus NBG*S-Cl(NBG*SO)⊆NBG*S-Cl(NBG*S-Int(P,Q)). Hence (P,Q)⊆NBG*S-Cl(NBG*S-O)⊆NBG*S-Cl(NBG*S-Int(P,Q))

Theorem 4.9
Let (X,Y, τ_{NB}) be a NBTS. Then the union of two NBG*S-OS is NBG*S-OS in the (X,Y, τ_{NB}).

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Proof:
Let (P,Q) and (R,S) be NBG*S-O sets in (X,Y, τ_{NB}). Then (P,Q)⊆NBG*S-Cl(NBG*S-Int(P,Q)) and (R,S)⊆NBG*S-Cl(NBG*S-Int(R,S)). Therefore,

(P,Q)∪(R,S)⊆NBG*S-Cl(NBG*S-Int(P,Q))∪NBG*S-Cl(NBG*S-Int(R,S))=NBG*S-Cl(NBG*S-Int(P,Q)∪NBG*S-Int(R,S))⊆NBG*S-Cl(NBG*S-Int((P,Q)∪(R,S))).Hence, (P,Q) ∪ (R,S) is NBG*S-open set in (X,Y, τ_{NB}).

Theorem 4.10
Let (X,Y, τ_{NB}) be a NBTS. If $\{(P,Q)\}_{\alpha\in\Delta}$ is a collection of NBG*S-OS in (X,Y, τ_{NB}), then $\bigcup_{\alpha\in\Delta}$(P,Q) is a NBG*S-O.

Proof:
For each α∈Δ, there exists a NBG*S-O such that NBG*S-O⊆(P,Q)⊆NBG*S-Cl(NBG*S-O)
Then,$\bigcup_{\alpha\in\Delta}$ NBG*S- O ⊆ $\bigcup_{\alpha\in\Delta}$ (P,Q) ⊆ $\bigcup_{\alpha\in\Delta}$ NBG*S-Cl(NBG*S-O)⊆NBG*S-Cl($\bigcup_{\alpha\in\Delta}$NBG*S-O. Hence, let NBG*S-O=$\bigcup_{\alpha\in\Delta}$NBG*S-O

5 Conclusion

In this article, Neutrosophic Binary Generalized Star Semi-Closed set (NBG*S-CS) and Neutrosophic Binary Generalized Star Semi-Open (NBG*S-OS) set are introduced. Their relationships with other set in Neutrosophic Binary Topological Space(NBTS) are analysed. The characteristics of these set are thoroughly examined and illustrated with examples. In the future, Neutrosophic Binary Generalized Star Semi-Closed problems using NBG*S-closure and NBG*S-Interior in Neutrosophic Binary set (NBS) are explored.

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