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Implementing Fuzzified Trapezoidal Rule with Triangular Fuzzy Numbers: An Analytical Study

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Abstract

Fuzzy logic was introduced by Lotfi A. Zadeh in 1965 as part of his proposal of fuzzy set theory and it has since been utilized in a wide range of disciplines. This study involves fuzzification of Trapezoidal rule**.** Choice of this topic involves extensive study of large number of literatures for a long period of time**.** Subsequently, this method has been selected for fuzzification. To implement this fuzzified method, a dedicated computer program was created. Scientific analysis is always needed for successful completion of any research work and to draw acceptable conclusion**.** For this purpose, inclusive study has been made in the area of statistics to apply the correct statistical tools to achieve the right goal**.**

Fuzzification of the numerical method was carried out using triangular fuzzy number. The fuzzification method is accompanied by examples to aid in understanding how it works. To compare the fuzzified and classical methods, fifteen examples were considered. A computer program has been developed for the fuzzified methods, and the solutions to these examples were obtained using this newly developed program. Appropriate statistical tools were used to compare the results from the newly developed method with those from the classical methods. It has been observed that the results of the mathematical problems obtained by newly developed fuzzified Trapezoidal rule is more or less same with the classical method.

Keywords: Trapezoidal Rule, Fuzzification, Fuzzy Triangular Number, K-S test, Wilcoxon-signed rank test.

BRIEF INTRODUCTION:

Fuzzy logic was introduced by Lotfi A. Zadeh in 1965 as part of his proposal of fuzzy set theory, and it has since been utilized in a wide range of disciplines[1].

This study involves fuzzification of Trapezoidal rule**.** Choice of this topic involves extensive study of large number of literatures for a long period of time**.** Subsequently, this method has been selected for fuzzification. For this fuzzified method, computer program has been developed. Scientific analysis is always needed for successful completion of any research work and to draw acceptable conclusion**.** For this purpose, inclusive study has been made in the area of statistics to apply the correct statistical tools to achieve the right goal**.**

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Fuzzification of the numerical method was carried out using triangular fuzzy number. The arithmetic operations that have been opted in the calculations are the operation of triangular fuzzy numbers using function principle. The fuzzified method is accompanied by some examples to aid in understanding how it works. Fifteen examples were used to compare the fuzzified methods with the classical ones. These examples have been collected from three different secondary sources. For this study 100 examples are collected and randomly selected 15 out of 100. As noted earlier, the fuzzified method has been implemented through a computer program, accordingly, the solutions for these examples were derived using the computer program designed for the new fuzzified method. Suitable statistical tools were employed to compare the outcomes of the newly developed fuzzified and classical methods. Then the triangular fuzzy numbers have been defuzzified by centroid method. The defuzzified value of the triangular fuzzy number was used for comparison with the corresponding crisp number.

Kaw and Keteltas describe integration as the method for finding the area under a function's graph. Integral calculus has numerous applications in a large range of fields including engineering, statistics, finance, actuarial science, etc. At times, evaluating expressions that involve integrals can be quite complicated. Therefore, to simplify the process of integration, numerous numerical methods have been developed. Numerical integration is employed to approximate values of integrals that cannot be solved analytically. This technique, often known as quadrature, estimates the area under a curve by the area of a square. Various numerical integration methods, including Newton-Cotes, Romberg integration, Gauss Quadrature, and Monte Carlo integration, are employed to evaluate functions that cannot be integrated analytically. Newton-Cotes techniques involve interpolating polynomials. One of the Newton-Cotes methods do not have any restriction on segmentation. However, the number of segments for the Simpson 1/3 rule must be even, while for the Simpson 3/8 rule, it must be a multiple of three. For Boole’s rule, the number of segments must be a multiple of four, while for Weddle’s rule, it must be a multiple of six. Mettle F.O. et al. developed a new Trapezium rule and numerical integration methods that do not impose any restrictions on the number of segments[2].

Integrals play an important role in mathematical analysis. But integrals are effective not just in mathematics but in various other disciplines as well. However, some functions cannot be calculated with analytical mathematical methods. For these reasons, another method called numerical integration was developed to obtain approximate values of the required integral. As needed, one can use numerical methods, which allow calculating results within a specified error margin. There are several fundamental methods of numerical integration, such as the trapezoidal and Simpson’s rules. While each technique provides an approximate integral, the errors differ from method to method [3].

Charles A. Thomson conducted a study on numerical integration techniques for application in the companion circuit method of transient circuit analysis. The numerical integration methods used in circuit transient analysis packages do not always provide the most accurate approximation of a circuit’s actual response. This study focuses on these numerical integration techniques and the degree of their inaccuracy [4].

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Li J. made a study on composite trapezoidal rule for the estimation of Cauchy principal value integral on circle. This study examined the convergence rate of the trapezoidal rule when the singular point aligns with certain a priori known points. It was concluded that the composite trapezoidal rule exhibits superconvergence at the midpoint of each subinterval [5].

Zhao W. And Zhang Z. Conducted a study entitled as "Derivative Based Trapeziodal Rule for Riemann Stieltjes". The discussion centers on the derivative-based trapezoidal rule for Riemann-Stieltjes integrals and examines the associated error term [6].

Three numerical integration methods—the Trapezoidal rule, Simpson's 1/3 rule, and Simpson's 3/8 rule—were compared by Moheuddin et al. In this study, the accuracy of these methods was compared using their respective error values. Graphical representations were also created to validate the results. Simpson's 1/3 rule has been found to be the most efficient one among these three techniques[7].

In numerical analysis, the trapezoidal rule, also called the trapezoid or trapezium rule, is a technique used to approximate definite integrals.

This paper is mainly dealing with Fuzzification of Trapezoidal rule and its comparison with the classical Trapezoidal rule.

FUZZIFICATION OF TRAPEZOIDAL RULE:

Fuzzification involves converting crisp values into fuzzy values. To fuzzify the classical Trapezoidal Rule, the values in this method are replaced with triangular fuzzy numbers. As a result, the following expression is obtained for the fuzzified trapezoidal rule. Let $Y = F(X)$ be a function that takes the values $Y_0, Y_1, Y_2, Y_3, Y_4, \dots, Y_n$ corresponding to the values $X_0, X_1, X_2, X_3, X_4, \dots X_n$ of X.

$$\int_{X_0}^{X_n} F(X)\, dX = \frac{H}{[2,2,2]}[(sum\ of\ the\ first\ and\ last\ ordinates) + [2,2,2] \times (sum\ of\ the\ remaning\ ordinates)]$$

$$= \frac{H}{[2,2,2]}[(Y_0 + Y_n) + (2,2,2)(Y_1 + Y_2 + Y_3 + Y_4 + \cdots)]$$

where $X_0 = [X_0', X_0'', X_0''']$, $X_n = [X_n', X_n'', X_n''']$ are triangular fuzzy numbers and Y_0, Y_n are the first and the last ordinates and $Y_1, Y_2, Y_3, Y_4, \dots$ are the intermediate ordinates in fuzzy form i.e. $Y_0 = [Y_0', Y_0'', Y_0''']$, $Y_1 = [Y_1', Y_1'', Y_1''']$, … , $Y_n = [Y_n', Y_n'', Y_n''']$

The f.m.f. of $Y_0, Y_1, Y_2, Y_3, Y_4, \dots, Y_n$ are respectively,

$$\mu_{Y_0}(X) = \begin{cases} \dfrac{X - Y_0'}{Y_0'' - Y_0'} & where\ Y_0' \le X \le Y_0'' \\ \dfrac{X - Y_0'''}{Y_0'' - Y_0'''} & where\ Y_0'' \le X \le Y_0''' \\ 0 & otherwise \end{cases}$$

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$$\mu_{Y_1}(X) = \begin{Bmatrix} \frac{X - Y_1'}{Y_1'' - Y_1'} & where\ Y_1' \leq X \leq Y_1'' \\ \frac{X - Y_1'''}{Y_1'' - Y_1'''} & where\ Y_1'' \leq X \leq Y_1''' \\ 0 & otherwise \end{Bmatrix}$$

....

....

$$\mu_{Y_n}(X) = \begin{Bmatrix} \frac{X - Y_n'}{Y_n'' - Y_n'} & where\ Y_n' \leq X \leq Y_n'' \\ \frac{X - Y_n'''}{Y_n'' - Y_n'''} & where\ Y_n'' \leq X \leq Y_n''' \\ 0 & otherwise \end{Bmatrix}$$

Similarly the f.m.f. of $X_0, X_1, \dots, X_n$ are respectively

$$\mu_{X_0}(X) = \begin{Bmatrix} \frac{X - X_0'}{X_0'' - X_0'} & where\ X_0' \leq X \leq X_0'' \\ \frac{X - X_0'''}{X_0'' - X_0'''} & where\ X_0'' \leq X \leq X_0''' \\ 0 & otherwise \end{Bmatrix}$$

$$\mu_{X_1}(X) = \begin{Bmatrix} \frac{X - X_1'}{X_1'' - X_1'} & where\ X_1' \leq X \leq X_1'' \\ \frac{X - X_1'''}{X_1'' - X_1'''} & where\ X_1'' \leq X \leq X_1''' \\ 0 & otherwise \end{Bmatrix}$$

....

....

$$\mu_{X_n}(X) = \begin{Bmatrix} \frac{X - X_n'}{X_n'' - X_n'} & where\ X_n' \leq X \leq X_n'' \\ \frac{X - X_n'''}{X_n'' - X_n'''} & where\ X_n'' \leq X \leq X_n''' \\ 0 & otherwise \end{Bmatrix}$$

and α cut is $[X_n]^\alpha = [X_n' + (X_n'' - X_n')\alpha, X_n''' - (X_n''' - X_n'')\alpha]$

Example Let us evaluate $\int_{[3.99,4,4.1]}^{[5.19,5.2,5.21]} \log X \, dX$ using fuzzified trapezoidal rule.

Solution: Here F(x)= $\log X \, dX$ and H=[0.19, 0.2, 0.21]

f.m.f. of H is

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$$\mu_H(X) = \begin{cases} \dfrac{X - 0.19}{0.2 - 0.19} & where\ 0.19 \leq X \leq 0.2 \\ \dfrac{X - 0.21}{0.2 - 0.21} & where\ 0.2 \leq X \leq 0.21 \\ 0 & otherwise \end{cases}$$

Let us form a table for the X and Y values

X	Y=F(X)
X_0 = 3.99 4 4.01	Y_0 =1.38379 1.38629 1.38879
X_1 =4.18 4.2 4.22	Y_1 = 1.43031 1.43508 1.43984
X_2 =4.37 4.4 4.43	Y_2 =1.47476 1.4816 1.4884
X_3 =4.56 4.6 4.64	Y_3 =1.51732 1.52606 1.53471
X_4 =4.75 4.8 4.85	Y_4 =1.55814 1.56862 1.57898
X_5 =4.94 5 5.06	Y_5 =1.59737 1.60944 1.62137
X_6 =5.13 5.2 5.27	Y_6 =1.63511 1.64866 1.66203

By fuzzified trapezoidal rule,

$$\int_{[3.99,4,4.1]}^{[5.19,5.2,5.21]} \log X\, dX = \frac{H}{[2,2,2]}[(sum\ of\ the\ first\ and\ last\ ordinates) + [2,2,2] \times (sum\ of\ the\ remaning\ ordinates)]$$

$$= \frac{H}{[2,2,2]}[(Y_0 + Y_6) + (2,2,2)(Y_1 + Y_2 + Y_3 + Y_4 + Y_5)]$$

Say Y=[1.7266, 1.82651, 1.92963]

with f.m.f

$$\mu_Y(X) = \begin{cases} \dfrac{X - 1.7266}{1.82651 - 1.7266} & where\ 1.7266 \leq X \leq 1.82651 \\ \dfrac{X - 1.92963}{1.82651 - 1.92963} & where\ 1.82651 \leq X \leq 1.92963 \\ 0 & otherwise \end{cases}$$

and α cut is

$$[Y]^{\alpha} = [1.7266 + (1.82651 - 1.7266)\alpha, 1.92963 - (1.92963 - 1.82651)\alpha]$$

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Example Let us evaluate $\int_{[-.01,0,.01]}^{[0.99,1,1.01]} \frac{dX}{[1,1,1]+X^3}$

Here F(X)= $\int_{[-.01,0,.01]}^{[0.99,1,1.01]} \frac{dX}{[1,1,1]+X^3}$ and H=[0.24,0.25,0.26]

The f.m.f of H is

$$\mu_H(X) = \begin{cases} \frac{X-0.24}{0.25-0.24} & where\ 0.24 \le X \le 0.25 \\ \frac{X-0.26}{0.25-0.26} & where\ 0.25 \le X \le 0.26 \\ 0 & otherwise \end{cases}$$

Let us form the table for X and Y

X	**Y=F(X)**
X_0= -0.01 0 0.01	Y_0= 0.999999 1 1
X_1= 0.23 0.25 0.27	Y_1= 0.980697 0.984615 0.987979
X_2= 0.47 0.5 0.53	Y_2= 0.870415 0.888889 0.905942
X_3= 0.71 0.75 0.79	Y_3= 0.669775 0.703297 0.736425
X_4= 0.95 1 1.05	Y_4= 0.463473 0.5 0.538394

By fuzzified trapezoidal rule

$$\int_{[3.99,4,4.1]}^{[5.19,5.2,5.21]} \log X\, dX = \frac{H}{[2,2,2]}[(sum\ of\ the\ first\ and\ last\ ordinates) + [2,2,2] \times (sum\ of\ the\ remaning\ ordinates)]$$

$$= \frac{H}{[2,2,2]}[(Y_0 + Y_4) + (2,2,2)(Y_1 + Y_2 + Y_3)]$$

Say Y=[0.78063, 0.83151,0.883882]

The f.m.f. of Y is

$$\mu_Y(X) = \begin{cases} \frac{X-0.78063}{0.83151-0.78063} & where\ 0.78063 \le X \le 0.83151 \\ \frac{X-0.883882}{0.83151-0.883882} & where\ 0.83151 \le X \le 0.883882 \\ 0 & otherwise \end{cases}$$

and α cut is $[Y]^{\alpha} = [0.78063 + (0.83151 - 0.78063)\alpha, 0.883882 - (0.883882 - 0.83151)\alpha]$

FUZZIFIED TRAPEZOIDAL RULE VS CLASSICAL TRAPEZOIDAL RULE:

In this section fifteen examples have been considered for comparison of Fuzzified Trapezoidal Rule and Classical Trapezoidal Rule. The number of sub interval has been taken even in case of these examples. C++ programs have been developed for Fuzzified

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Trapezoidal Rule and Classical Trapezoidal Rule. The solutions of these examples were obtained using these programs. The solutions have been recorded in the Table 1.

Table – 1 Showing the Output of the C++ Program Developed for the Fuzzified Trapezoidal Rule and Classical Trapezoidal Rule

Sl No	Crisp root	Fuzzy Triangular Number	Defuzzified value
1	6.5215	6.15898, 6.5216, 6.9037	6.52161
2	0.8320	0.78063, 0.8317, 5883882	.8317
3	0.3885	0.356421, 0.38923, 0 .42508	.389346
4	0.7428	0.692763, 0.74287, 0.794832	.742984
5	57.982	55.0399, 57.981, 61.1012	57.992
6	1.82761	1.7266,1.82651, 1.92963	1.82766
7	0.79411	0.726453, 0.794119, 0.864693	0.794119
8	1.3077	1.2867, 1.3076, 1.32908	1.30776
9	0.69711	0.657266, 0.6971, 0.738279	0.697024
10	0.4054	0.391958, 0.4055, 0.41945	0.40551
11	0.2172	0.179214, 0.2171, 0.261052	0.217206
12	0.14567	0.117367, 0.14581, 0.17793	0.145784
13	0.7760	0.760266, 0.77601, 0.929214	0.77613
14	0.5212	0.4512, 0.5150, 0.594	0.5202
15	0.50028	0.474094, 0.50121, 0.525353	0.500398

Comparison of the Solutions obtained from Fuzzified Trapezoidal Rule and Classical Trapezoidal Rule:

To determine whether the results obtained using the fuzzified Trapezoidal rule differ significantly from those of the classical method on the same randomly selected set of problems, an appropriate statistical test is applied.

Box plots and the K-S test are used to check the normality of the results from both methods on the same randomly selected problems, which helps in choosing the proper statistical test. The box-plot of the results is presented in Figure 1,

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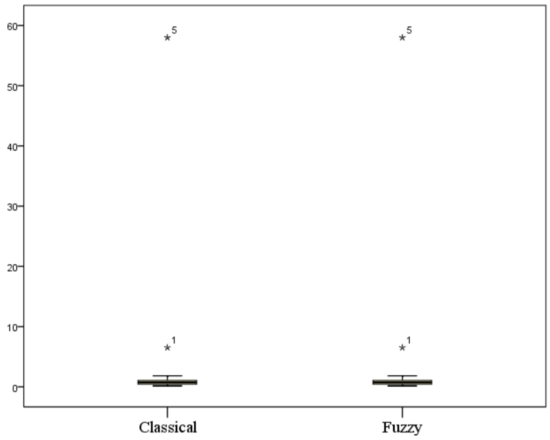

Figure 1 : Box-plot of Trapezoidal rule

By examining the median position and the presence of extreme values in the box plot (Figure 1) for the results of both the classical and fuzzy methods, it can be concluded that the results are unlikely to follow a normal distribution, which is further confirmed by the K-S test. Table 2 shows the results of the K-S test.

Table - 2 Results of K-S test

	Kolmogorov-Smirnov		
	Statistic	d.f.	Sig.
Classical	.449	15	0.00000000432
Fuzzy	.449	15	0.00000000422

From the K-S test, it is evident that the results obtained by both the classical and fuzzy methods are statistically significant and deviate from a normal distribution (p-value<0.01)

As the data are not normally distributed, the Wilcoxon signed-rank test, a non-parametric method, is employed to compare the statistical significance of results obtained by both methods. The outcomes are presented in Table 3

Table -3 Showing the Results of Descriptive Statistics and Wilcoxon-signed rank test

	Classical	Fuzzy	Z-value	p-value
Mean	4.91129	4.91130		
Median	0.74298	0.74298		
Std. Deviation	14.76653	14.76654		
Minimum	0.14578	0.14578		
Maximum	57.99195	57.99200	-	0.109

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			1.604	

Table 3 presents the descriptive statistics of the results obtained using the classical and fuzzified Trapezoidal rules. The Wilcoxon signed-rank test results indicate that the outcomes from both methods are not statistically significant (p-value>0.05). Thus, it can be concluded that the results of the mathematical problems obtained using the newly developed fuzzified Trapezoidal rule are largely similar to those from the classical method.

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