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SOLUTION OF INTEGRAL EQUATIONS AND MAYAN TRANSFORM

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Abstract

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Due to the importance of integral equations of all types and branches in applied, life and engineering sciences, in this paper exact solutions were found using the Mayan integral technique. The convolution theorem for this technique was presented and proven and applied to the first and second types of Volterra linear integral equations as well as Abel's integral equations with various examples.

Math. Subject Classification: 34A12, 44A35, 45AXX, 45A05, 45DXX, 45D05, 45E10, 47B35.

Key Words and Phrases: Mayan transform, Volterra integral equation, Convolution theorem, Abel's integral equations.

1 Introduction

Many researchers have in their research the finding of exact solutions to integral equations using integral transforms change the integral equation into an easy algebraic equation in terms of algebraic calculations, such as using Laplace, Tarig, SEE, complex SEE ,Sumudu, and many others in finding exact solutions to integral equations [2, 3, 4, 5, 6, 7, 8, 9, 10].

In this paper, a new integral technique "Mayan transform" [27], is employed in calculating the solutions of certain integral equations. Some illustrative analysis are given. It is noted that the solution of integral equations obtained via applying Mayan transform can be defined on distribution spaces; this article contains the analysis of the same. Mayan transform was presented to facilitate the process of finding the solution of ordinary and partial differential operator equations in the time domain. The transform applied for Mayan technique is based on the classical Fourier integral. Mayan transform is known for functions of exponential order, is proclaimed. We consider functions in the set B defined as:

$$B = \{\mathrm{g}\left(\tau\right) : \ \exists \quad H, l_1, l_2 > 0, \qquad \left|g\left(t\right)\right| \leq He^{-u\tau}\}.$$

Where H is any constant, real finite number and l_1 and l_2 may be

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finite or infinite. Mayan transform, denoted as the operator $[\cdot]$, is defined as equations

$$M\left[g\left(\tau\right)\right]=\eta\left(u\right)=\frac{1}{u^{i\alpha}}\int_{0}^{\infty}g\left(\tau\right)e^{-u^{i\beta}\tau}d\tau,\qquad \tau\geq 0,\qquad l_1\leq u\leq l_2\ ,$$

$$Im\left(u^{i\beta}\right)<0\ ,\ i\epsilon C.[27] \tag{1}$$

The parameter u in this technique is applied to factor the variable τ in the argument of $g\left(\tau\right)$. This technique has connection with Laplace and Elzaki transform. Some propositions are given as:

1. Mayan technique of derivative and n^{th} order derivative of $g(t)$, respectively, are given as [27]:

$$M\left[g'\left(\tau\right)\right]=u^{i\beta}\eta\left(u\right)-\frac{g\left(0\right)}{u^{i\alpha}} \tag{2}$$

$$M\left[g''\left(\tau\right)\right]=u^{2i\beta}\eta\left(u\right)-\frac{g'\left(0\right)}{u^{i\alpha}}-\frac{u^{i\beta}}{u^{i\alpha}}g\left(0\right) \tag{3}$$

$$M\left[g'''\left(\tau\right)\right]=u^{3i\beta}\eta\left(u\right)-\frac{u^{i\beta}}{u^{i\alpha}}g'\left(0\right)-\frac{g''\left(0\right)}{u^{i\alpha}}-\frac{u^{2i\beta}}{u^{i\alpha}}g\left(0\right) \tag{4}$$

$$M\left[g^{(m)}\left(\tau\right)\right]=u^{mi\beta}\eta\left(u\right)-\sum_{r=0}^{m-1}\frac{g^{(m-1)-r}\left(0\right)u^{ir\beta}}{u^{i\alpha}},m=1,2,3\,,\ldots.$$

2. When $g\left(t\right)=\tau^{m}$, $m=1,2,3\ ,\ldots.$

$$M\left[\tau^{m}\right]=\frac{m!}{u^{i(m+1)\beta+\alpha}} \tag{5}$$

If a,b are any constants and $r(t)$ and $n(t)$ are functions, then the linear property for Mayan transform is defined as

$$M\left[a\ r\left(t\right)\underline{+}b\ n\left(t\right)\right]=a\ R\left(u\right)\underline{+}b\ N\left(u\right). \tag{6}$$

3. The relation between the Laplace transform $K(s)$ and Mayan transform is $\eta\left(u\right)$ defined by

$$\eta\left(u\right)=\frac{K\left(u^{i\beta}\right)}{u^{i\alpha}} \tag{7}$$

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The sufficient conditions for the existence of Mayan transform are that $g(\tau)$ for $\tau \geq 0$ be piecewise continuous and should be of exponential order, Mayan transform exist. We prove some properties for Mayan transform:

Theorem 1. *If $M[g(\tau)] = K(u)$, so Mayan transform is expressed as:*

$$M\left[\int_0^\infty g\left(u\right)du\right] = \frac{M\left(u\right)}{u^{i\beta}}$$

Proof. Let $K\left(\tau\right) = \int_0^\infty f\left(v\right)dv$. Then $K'\left(\tau\right) = g\left(\tau\right)$, $K\left(0\right) = 0$.

Taking Mayan transform both side, we get:

$$u^{i\beta}M\left[K\left(\tau\right)\right] - \frac{K\left(0\right)}{u^{i\alpha}} = M\left(u\right)$$

Then

$$M\left[K\left(\tau\right)\right] = \frac{M\left(u\right)}{u^{i\beta}}.$$

And

$$M\left[\int_0^\infty g\left(u\right)du\right] = \frac{M\left(u\right)}{u^{i\beta}}.$$

□

Theorem 2. *Let $n(\tau)$ and $r(\tau)$ being Laplace transform of $N(s)$ and $R(s)$, respectively and Mayan transform $P(v)$ and $K(v)$, respectively, then*

$$M[(n{*}r)(\tau)] = u^{i\alpha}P(v)K(v).$$

Proof. The convolution of Laplace transform is written as:

$$L\left[\left(n * r\right)\left(\tau\right)\right] = N\left(s\right)R\left(s\right)$$

Applying the duality relation as in (7), we get:

$$M\left[\left(n * r\right)\left(\tau\right)\right] = \frac{1}{u^{i\alpha}}L\left[n * r\right].$$

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Since $P(v) = \frac{N(v)}{u^{i\alpha}}, K(v) = \frac{R(v)}{u^{i\alpha}}$, the above equation will be

$$M[(n*r)(\tau)] = \frac{1}{u^{i\alpha}}[N(v).R(v)],$$

$$M[(n*r)(\tau)] = \frac{1}{u^{i\alpha}}\left[u^{i\alpha}P(v).u^{i\alpha}K(v)\right],$$

$$M[(n*r)(\tau)] = u^{i\alpha}P(v)K(v). \tag{8}$$

This proves the theorem. □

2 Solution of Integral Equations via Mayan Technique

Solution of different kinds of integral equations are given by applying different kinds of techniques [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. In this part, we apply Mayan transform to get the exact solution of certain integral equation.

1. Consider the Volterra integral equation of first kind with a convolution type kernel

$$u(x) = \int_0^x k(s-\tau) g(\tau) d\tau, \tag{9}$$

where $k(s-\tau)$ depends only on the difference $(x-\tau)$. Invoking the Mayan technique (9) and convolution theorem (8),

$$F(u) = u^{i\alpha}K(u)G(u).$$

That is ,

$$G(u) = \frac{F(u)}{u^{i\alpha}K(u)}. \tag{10}$$

Taking inverse Mayan transform, we have:

$$v(x) = G^{-1}\left[\frac{F(u)}{u^{i\alpha}K(u)}\right]. \tag{11}$$

As the exact solution to (9).

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2. Consider the Volterra integral equation of second kind with a convolution type kernel

$$u(x) = \beta(x) + \int_0^x k(s-\tau)\, g(\tau)\, d\tau. \tag{12}$$

On using Mayan transform (1) to both the sides, and applying convolution theorem (8), equation (9) gets:

$$G(u) = F(u) + u^{i\alpha} K(u)\, G(u),$$

$$G(u)\left[1 - u^{i\alpha} K(u)\right] = F(u),$$

$$G(u) = \frac{F(v)}{\left[1 - u^{i\alpha} K(u)\right]}, \tag{13}$$

And take the inverse of Mayan transform, when invoked, gets

$$g(x) = G^{-1}\left[\frac{F(v)}{\left[1 - u^{i\alpha} K(u)\right]}\right]. \tag{14}$$

Similarly, considering Fredhlom integral equation of first and second kind of convolution kind and applying Mayan transform and its convolution, under similar analysis, solutions for these can be obtained.

3. Assume that we consider the Abel's integral equation [22, 23]

$$k(\tau) = \int_0^x \frac{f(x)}{(1-x)^{\mu}} dx. \qquad 0 < \mu < 1 \tag{15}$$

That is,

$$k = f * \tau_+^{-\mu}, \tag{16}$$

With $\tau_+^{-\mu} = \tau^{-\mu} H(t)$ and $\tau_+^{-\mu}$ is Heaviside unit step function. When $M[\tau^{-\alpha}\ ]$, known and the convolution of the Mayan transform is employed, the solution of (15) will be as:

$$M[k(\tau)] = u^{i\alpha} M[f(\tau)]\, M\left[\tau_+^{-\mu}\right]$$

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When $f(\tau) = \tau^n$, the Mayan transform is $G(v) = \frac{\Gamma(n+1)}{u^{i(n+1)\beta+\alpha}}$. Similarly, we show that if $f(\tau) = \tau^{-\mu}$, so the Mayan transform is $M\left[\tau_+^{-\mu}\right] = F(v) = \Gamma(1-\mu)\, v^{-i[(1-\mu)\beta+\alpha]}$, where$H(t) = 1, \tau \geq 0$. Suppose that the value of $T[\tau^{-\mu}]$ in the above equation, we obtain:

$$F(u) = u^{i\alpha} M[f(\tau)] . \Gamma(1-\mu)\, u^{-i[(1-\mu)\beta+\alpha]}. \tag{17}$$

$$M[f(\tau)] = \frac{F(u)}{\Gamma(1-\mu)} . \frac{1}{u^{-i[(1-\mu)\beta]}}.$$

$$= \frac{F(u)\,\Gamma(\mu)}{\Gamma(\mu)\,\Gamma(1-\mu)} . \frac{1}{u^{i(\mu-1)\beta}};$$

Where

$$\Gamma(\mu)\,\Gamma(1-\mu) = \frac{\pi}{sin\pi\ \alpha},$$

$$M[f(\tau)] = \frac{\Gamma(\mu)\, sin\pi\ \mu}{\pi} . \frac{F(u)}{u^{i(\mu-1)\beta}},$$

$$M[f(\tau)] = \frac{sin\pi\ \mu}{\pi} . \left[\Gamma(\mu)\, \frac{F(u)}{u^{i(\mu-1)\beta}}\right]$$

$$= \frac{sin\pi\ \mu}{\pi} . \left[\Gamma(\mu)\, \frac{F(u)}{\left(u^{i\beta}\right)^{\mu-1}}\right]$$

$$= \frac{sin\pi\ \mu}{\pi} u^{i\beta} M\left[\tau^{\mu-1} * F(u)\right]$$

, where

$$M\left\{(\tau)^{\mathrm{m}-1}\right\} = \frac{\Gamma(m)}{u^{i(m\beta)}} = \frac{\Gamma(m)}{\left(u^{i\beta}\right)^m},$$

m is an integer constant.

$$M[f(\tau)] = \frac{sin\pi\ \mu}{\pi} u^{i\beta} M\left[\int_0^t (\tau-s)^{\mu-1} f(s)\, ds\right] \tag{18}$$

$$M[f(\tau)] = \frac{sin\pi\ \mu}{\pi} u^{i\beta} M[K(\tau)], \tag{19}$$

With, $K(\tau) = \int_0^\tau (\tau-s)^{\mu-1} f(s)\, ds, \quad K(0) = 0.$

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Via virtue of (2), $M\left[K'\left(u\right)\right] = u^{i\beta}K(u) - \frac{K(0)}{u^{i\alpha}} = u^{i\beta}K(u)$. Invoking it in above equation, we get:

$$M\left[f\left(\tau\right)\right] = \frac{sin\pi\ \mu}{\pi} M\left[K'\left(\tau\right)\right]$$

So, the complete solution should be obtained as:

$$f\left(\tau\right) = \frac{sin(\pi\ \alpha)}{\pi} \frac{d}{d\tau}\left[\int_0^\tau \left(\tau - x\right)^{\mu-1} f\left(x\right) dx\right]. \tag{20}$$

so,

$$\left(f{*}g\right)\left(\tau\right) = \int_{x=0}^{\tau} f\left(\mathrm{x}\right) g\left(\tau - x\right) dx \tag{21}$$

Where $f$and g are locally integrable functions? The integral equation and its solution can be interpreted in the distributional sense.

To know the integral operator equations on distribution spaces, we need to specify the spaces we intend to apply. Such are given in terms of distribution spaces [24, 25, 26, 27] and mixed distribution spaces [22, 23]. As well as, the solutions of integral equations (Volterra integral equations (9) and (14), Abel integral equation (15)) obtained above via Mayan transform, can be interpreted in distributional sense, either via considering integral operator equations on distributions spaces with f, g and $\mathrm{t}^{-\alpha}$ are locally integrable or defining Mayan transform on distribution spaces.

3 Applications of Mayan Technique

In this part, we give certain exclusive applications to illustrate the apple of the Mayan transform to finding the solution certain integral operator equations.

Example 1. *Evaluate the function $k\left(x\right)$ which satisfies the operator equation*

$$k\left(x\right) = x + \int_0^x k\left(\tau\right) sin\left(x - \tau\right)\ d\tau \tag{22}$$

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Solution: *Invoking the Mayan transform on both the sides of* (22) *and applying* (8), *we have:*

$$M(u) = \frac{1}{u^{i(2\beta)+\alpha}} + u^{i\alpha} M(u) \frac{1}{u^{i\alpha}[u^{2i\beta}+1]},$$

After simple simplifications,

$$M(u) = \left[\frac{[u^{2i\beta}+1]}{[u^{i(4\beta+\alpha)}]} + \frac{1}{[u^{i(4\beta+\alpha)}]}\right].$$

That is,

$$M(u) = \left[\frac{1}{[u^{i(2\beta+\alpha)}]} + \frac{1}{[u^{i(4\beta+\alpha)}]}\right]$$

Taking the inverse of Mayan transform, we obtain:

$$k(x) = x + \frac{x^3}{3!}. \tag{23}$$

Which is the required exact solution.

Example 2. *Solve*

$$k(x) = 1 + \eta \int_0^x (\tau - x)\, g(\tau)\, dt, \qquad (\eta = \text{contant}) \tag{24}$$

Solution : *Applying the Mayan transform on both the sides of* (24), *we get:*

$$M(u) = \frac{1}{u^{i(\beta)+\alpha}} - \eta u^{i\alpha} M(u) \frac{1}{[u^{i(2\beta)+\alpha}]}.$$

After simple simplifications,

$$M(u) = \left[\frac{u^{i\beta}}{[u^{i\alpha}(u^{2i\beta}+\eta)]}\right].$$

Taking the inverse of Mayan transform of the above equation, we get

$$k(x) = cos\left(x\sqrt{\eta}\right). \tag{25}$$

Which is the required exact solution.

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Example 3. *Solve the integral operator equation*

$$z(x) = x + \frac{1}{6}\int_0^x (x-\tau)^3 z(\tau)\, d\tau \tag{26}$$

Solution: *Applying the Mayan transform on both sides of* (26), *we have*

$$\mathrm{M}[z(x)] = \frac{1}{u^{i(2\beta+\alpha)}} + \frac{1}{6}\mathrm{M}\left[\frac{6u^{i\alpha}}{u^{i(4\beta+\alpha)}}\right]\mathrm{M}[z(x)],$$

After simple simplifications,

$$M[z(x)] = \left[\frac{u^{2i\beta}}{[u^{i\alpha}(u^{4i\beta}-1)}\right]$$

By using partial fraction, we get

$$A = B = \frac{1}{2} \quad ,$$

That is,

$$M[z(x)] = \frac{1}{2}\left[\frac{1}{[u^{i\alpha}(u^{2i\beta}-1)} + \frac{1}{[u^{i\alpha}(u^{2i\beta}-1)}\right]$$

Taking the inverse of Mayan transform of both the sides, we get:

$$z(\tau) = \frac{1}{2}(sinx \ + sinhx \ )$$

This is the required exact solution.

Example 4. *Solve the integral operator equation*

$$x = \int_0^x e^{x-\tau} k(\tau)\, d\tau \tag{27}$$

Solution: *Equation* (27) *can be written as:*

$$x = k(x) * e^x \tag{28}$$

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Taking Mayan transform on both sides of (28), *we get:*

$$M\left[x\right]=M\left[k\left(x\right)\right]*M\left[e^{x}\right]$$

Applying convolution formula of Mayan transform (8) *and values as given in Table, it yields*

$$\frac{1}{u^{i(2\beta+\alpha)}}=u^{i\alpha}\frac{1}{u^{i\alpha}[u^{i\beta}-1]}M\left[k\left(x\right)\right],$$

After simple simplifications,

$$M\left[k\left(x\right)\right]=\left[\frac{1}{u^{i(\beta+\alpha)}}-\frac{1}{u^{i(2\beta+\alpha)}}\right]$$

Taking the inverse of Mayan transform, we get:

$$k\left(x\right)=1-x. \tag{29}$$

Example 5. *Consider the Abel's integral equation:*

$$1+x+x^{2}=\int_{t=0}^{x}\frac{1}{\sqrt{(x-t)}}u\left(t\right)dt[21,\ 22,\ 23,\ 24,\ 25,\ 26] \tag{30}$$

By applying Mayan transform to both sides of (30)*:*

$$M\{1\}+\mathrm{M}\{\mathrm{x}\}+\mathrm{M}\{x^{2}\}=\mathrm{M}\left\{\int_{t=0}^{x}\frac{1}{\sqrt{(x-t)}}u\left(t\right)dt\right\},$$

$$\frac{1}{u^{i(\beta+\alpha)}}+\frac{1}{u^{i(2\beta+\alpha)}}+\frac{2}{u^{i(3\beta+\alpha)}}=M\left\{x^{\frac{-1}{2}}*u\left(x\right)\right\} \tag{31}$$

Applying convolution theorem to (30) *:*

$$\frac{1}{u^{i(\beta+\alpha)}}+\frac{1}{u^{i(2\beta+\alpha)}}+\frac{2}{u^{i(3\beta+\alpha)}}=u^{i\alpha}M\left\{x^{\frac{-1}{2}}\right\}.MA\left\{u\left(x\right)\right\}$$

$$\frac{1}{u^{i(\beta+\alpha)}}+\frac{1}{u^{i(2\beta+\alpha)}}+\frac{2}{u^{i(3\beta+\alpha)}}=u^{i\alpha}\frac{\sqrt{\pi}}{u^{\frac{1}{2}\beta i+i\alpha}}.M\left\{u\left(x\right)\right\}$$

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Then

$$M\left\{u\left(x\right)\right\}=\frac{1}{\sqrt{\pi}}\left[\frac{1}{u^{\frac{1}{2}\beta i+i\alpha}}+\frac{1}{u^{\frac{3}{2}\beta i+i\alpha}}+\frac{2}{u^{\frac{5}{2}\beta i+i\alpha}}\right] \tag{32}$$

Taking inverse Mayan transform to (32) *:*

$$u\left(x\right)=\frac{1}{\sqrt{\pi}}M^{-1}\left\{\frac{1}{u^{\frac{1}{2}\beta i+i\alpha}}+\frac{1}{u^{\frac{3}{2}\beta i+i\alpha}}+\frac{2}{u^{\frac{5}{2}\beta i+i\alpha}}\right\}$$

Then

$$u\left(x\right)=\frac{1}{\pi}\left[x^{\frac{-1}{2}}+2x^{\frac{1}{2}}+\frac{8}{3}x^{\frac{3}{2}}\right] \tag{33}$$

The concluded result is the required exact solution for Abel's integral (30)

Example 6. *Consider the Abel's integral operator equation:*

$$x=\int_{t=0}^{x}\frac{1}{\sqrt{(x-t)}}u\left(t\right)dt \tag{34}$$

Taking Mayan transform for both sides of (34)*:*

$$M\{\mathrm{x}\}=\mathrm{M}\left\{\int_{t=0}^{x}\frac{1}{\sqrt{(x-t)}}u\left(t\right)dt\right\}$$

$$\frac{1}{u^{i(2\beta+\alpha)}}=M\left\{x^{\frac{-1}{2}}*u\left(x\right)\right\} \tag{35}$$

Applying convolution theorem to (35)

$$\frac{1}{u^{i(2\beta+\alpha)}}=u^{i\alpha}.M\left\{x^{\frac{-1}{2}}\right\}.M\left\{u\left(x\right)\right\},$$

$$\frac{1}{u^{i(2\beta+\alpha)}}=u^{i\alpha}\frac{\sqrt{\pi}}{u^{\frac{1}{2}\beta i+i\alpha}}.M\left\{u\left(x\right)\right\}$$

Then

$$M\{\mathrm{u}\left(x\right)\}=\frac{1}{\sqrt{\pi}}u^{-i(\frac{3}{2}\beta+\alpha)} \tag{36}$$

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Taking inverse Mayan transform to (36)

$$u(x) = \frac{1}{\sqrt{\pi}} M^{-1} \left\{ \frac{1}{u^{i(\frac{3}{2}\beta+\alpha)}} \right\}$$

Then

$$u(x) = \frac{2}{\pi} x^{\frac{1}{2}} \tag{37}$$

The concluded result is the required exact solution for (34).

Comprehensive Mayan transform [27]

S. No.	$g(\tau)$	$M[g(\tau)] = G(u)$
1	1	$\frac{1}{u^{i(\beta+\alpha)}}$
2	τ	$\frac{1}{u^{i(2\beta+\alpha)}}$
3	τ^2	$\frac{2}{u^{i(3\beta+\alpha)}}$
4	τ^m	$\frac{m!}{u^{i(m+1)\beta+\alpha}}$ $m = 1, 2, 3, \ldots.$ or $\frac{\Gamma(m+1)}{u^{i(m+1)\beta+\alpha}}$ $m > -1$
5	$e^{c\tau}$	$\frac{1}{u^{i\alpha}(u^{i\beta}-c)}$
6	$sinc\ \tau$	$\frac{1}{u^{i\alpha}(u^{2i\beta}+c^2)}.$
7	$\cos c\ \tau$	$\frac{u^{i\beta}}{u^{i\alpha}(u^{2i\beta}+c^2)}.$
8	$sinhc\ \tau$	$\frac{a}{u^{i\alpha}(u^{2i\beta}-c^2)}.$
9	$\cosh c\ \tau$	$\frac{u^{i\beta}}{u^{i\alpha}(u^{2i\beta}-c^2)}.$

4 Conclusions

By using the Mayan technique transform to find the exact solutions the first and second types of volterra integral equations, as well as the Abel's integral equation, it is clear how easy it is to find these solutions, as converting the integral equation to an algebraic operations after using the theorem or the convolution formula for the Mayan integral technique and this is clear through examples. We recommend applying this technique in real life applications of all types and kinds of integral operator equations.

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