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VERTEX MAGIC AND EDGE MAGIC LABELINGS OF GRAPHS WITH GROUP ELEMENTS

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Abstract

Given a graph G with n vertices and m edges and an abelian group A of order equal to max(n,m). By labeling of A^*-vertex magic labeling and A^*-edge magic labeling we mean a pair one-to-one maps, one from V(G) to A, and another one from E(G) to A, rather than the bijection maps considered in the literature from V(G)∪E(G) to A, and new results were obtained.

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Key words: Magic Labeling, Vertex Magic Labeling, Edge Magic Labeling, Abelian Groups

1. INTRODUCTION AND PRELIMINARIES

By *labeling* of vertices (resp. edges) of a graph, we mean a one-to-one map taking the vertices onto the integers 1, 2, . . . , number of vertices (resp. number of edges). If a one-to-one map takes both vertices and edges onto the integers 1, 2, . . . , number of vertices+number of edges, then such a labeling is sometimes called *total.*

Edge magic graphs have been investigated by authors such as Sedlacek [20], Wallis [24], Stewart [22], and Kotzig and Rosa [13]. A total labelling is considered edge magic if it meets the requirement that the sum of the label on an edge and the label of its end points is constant regardless of the choice of edges. A total labeling is said to be *edge antimagic* if it satisfies the condition that the sum of the label on an edge and the label of its end points is distinct for distinct edges. Refer papers by Baca et al. [2–4]. Enomoto et al. [9] have introduced the name *super edge-magic* for magic labelings in the sense of Kotzig and Rosa, with the added property that the v vertices receive the smaller labels, $\{1, 2, \ldots, v\}$.

A total labeling is said to be *vertex magic* if it satisfies the condition that the the labels on a vertex and labels of the edges incident with that vertex is constant regardless of the choice of vertices. Gray and Mac Dougall [10] studied vertex-magic labeling of regular graphs. A total labeling is said to be *vertex anti magic* if it satisfies the condition that the sum of the labels on a vertex and labels on the edges incident with that vertex is distinct for distinct vertices. This definition was considered by Baca et al. [5]. In a book by Hartsfield and Ringel [11], a vertex magic is defined by considering the sum of the labels of the edges incident with a vertex.

A labeling is said to be *distance magic* if it satisfies the condition that the sum ofthe labels on the vertices that are adjacent with a vertex is constant regardless of the choice of vertices. Authors including Mirka Miller et al. [16], Rozman et al. [19] and Miklavič et al. [15] studied distance magic labeling on some special classes of graphs. A labeling is said to be *distance antimagic* if the sum of the labels on the vertices that are adjacent with a vertex is distinct for distinct vertices. Refer papers by Arumugam et al. [1], Simanjuntak et al. [21] and Risma et al. [17].

Lih [14] proposed magic labelings of planar graphs in 1983. He traced the origins of this concept back to the 13th century in China, and the labels extended to faces in addition to edges and vertices. A graceful labeling of a graph with m edges is a labeling of its vertices with some subset of the integers from 0 to m inclusive, such that no two vertices share a label, and each edge is uniquely identified by the absolute difference between its end points, such that this magnitude lies between 1 and m inclusive. A graph which permits a graceful labeling is termed a graceful graph. In a 1967 study on graph labelling, Alexander Rosa [18] first referred to this kind of labelling as β-labeling.The Ringel-Kotzig conjecture, often known as the elegant tree conjecture and named for Gerhard Ringel and Anton Kotzig, is a significant hypothesis in graph theory. It makes the assumption that all trees are elegant. The elegant tree hypothesis has an endless number of similar variations, according to Tao-Ming Wang et al. [23]. See Joseph Gallian's survey report [12] for a thorough explanation of labelings.

Despite these numerous labelings using integers, labelings with group elements occupies a significant place in the recent literature. Cichacz et al. [6] considered a labeling of vertices with an abelian group elements and applied the distance magic and anti magic criteria therein and obtained new results. The graphs considered by Cichacz et al. is named as *A-distance anti magic labeling* of G, where A is an abelian group. Dalibor Froncek [8] explored new result on Z_n-distance magic labeling of the cartesian product of cycles. Combe et al. [7] considered edge magic (anti magic) and vertex magic (anti magic) labelings of graphs over finite abelian groups.

The purpose of this paper is to define and study labeling of a given graph G with elements of a given abelian group A. Let $V(G)$ and $E(G)$, respectively, denote the vertex

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set and edge set of G. Here we define vertex labeling as a one-to-one map, denoted ϕ_V, from $V(G)$ to A. In the same vein, we define edge labeling as a one-to- one map, denoted ϕ_E, from $E(G)$ to A. We apply the notion of A-vertex magic and A-edge magic in ϕ_V and ϕ_E and get new results.We use theliteral A^* to differentiate these labeling from A-vertex (resp. edge) magic labelings.

Some of the basics and notations of graph theory and group theory considered in this paper are the following:

1. Graphs considered in section 2 are simple graphs with n vertices and m edges;
2. Groups considered in section 2 are abelian groups and order of these groups are equal to max(n,m);
3. We assume the familiarity of the reader with the definitions of path, cycle, and star graphs;
4. A path of length n is denoted by P_n, a cycle of length n is denoted by C_n, and a star having n vertices is denoted by S_n.

Definition 1.1. A group (G,.) is said to be abelian group if a.b=b.a for all a,b∈G.

Definition 1.2. The order of an element b in a group G is the smallest positive integer m such that $b^m = e$, where e is the identity element of the group.

Definition 1.3. An element of a group is an involution if it has order 2, that is, an involution is an element a such that $a \neq e$ and a^2=e.

2. MAIN RESULTS

In this section, we define A^*-vertex magic labeling and A^*-edge magic labeling. Here we consider a pair one-to-one maps, one from $V(G)$ to A, and another one from $E(G)$ to A, rather than the bijection maps considered in the literature from $V(G) \cup E(G)$ to A. Here comes the definition

Definition 2.1. Let G be a graph with n vertices and m edges. Let (A,·) be an abelian group of order max(n,m). An A^*-vertex magic labeling of G is a pair of one-to-one maps

$\phi_V: V(G) \to A$

and

$\phi_E: E(G) \to A$

satisfying the following condition:

$$\phi_V(v). \prod_{uv \in E(G)} \phi_E(uv) = a$$

for some fixed a∈A for all vertex v∈V(G). In notations, denote

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$$w_V(\phi_V(v)) = \phi_V(v). \prod_{uv \in E(G)} \phi_E(uv)$$

Then $w_V(\phi_V(v)) = a$ for a fixed $a \in A$ and for every $v \in V(G)$. A graph that admits A^*-vertex magic labeling is said to be A^*-vertex magic graph and the element a is said to be magic constant.

Example 2.2. Following graph serves as an example for Definition 2.1:

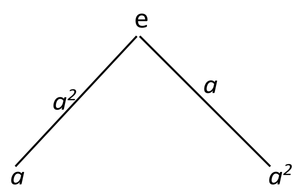

Figure 1

With $G=P_3$, and $A=\{e,a,a^2\}$ cyclic group of order 3. Here we have $w_V(a)=a \cdot a^2=a^3=e$, $w_V(e)=e \cdot a \cdot a^2=a^3=e$ and $w_V(a^2) =a^2 \cdot a=a^3=e$.

Hence the above graph is A^*-vertex magic.

Example 2.3. Following graph serves as an example for Definition 2.1:

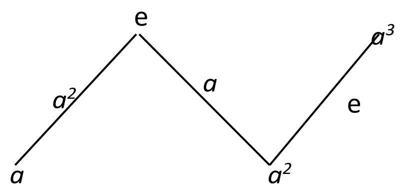

Figure 2

With $G=P_4$, and $A=\{e,a,a^3,a^4\}$ cyclic group of order 4. Here we have $w_V(a)=a \cdot a^2=a^3=e$, $w_V(e)=e \cdot a \cdot a^2=a^3=e$, $w_V(a^2)=a^2 \cdot a.e=a^3=e$ and $w_V(a^3)=a^3.e= a^3=e$. Hence the above graph is A^*-vertex magic.

Example 2.4. Following graph serves as not an example for Definition 2.1:

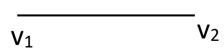

Figure 3

With $G=P_2$, and $A=\{e,a\}$ group of order 2.

Suppose $\phi_V(v_1) = e$ and $\phi_V(v_1) = a$

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Case(i): Suppose $\phi_E(v_1v_2) = e$. Then $w_V(v_1) = e.e = e$ and $w_V(v_2) = a.e = a$

Case(ii): Suppose $\phi_E(v_1v_2) = a$. Then $w_V(v_1) = e.a = a$ and $w_V(v_2) = a.a = e$

So P_2 is not A^*-vertex magic graph.

Theorem 2.5. Let n be an odd number and let A be a cyclic group of order n. Then C_n is A^*-vertex magic.

Proof. Let $v_0,\dots,v_{n-1}$ be the vertices of C_n in cyclic pattern. Let $A=\{e,a,\dots,a^{n-1}\}$ be the cyclic group of order n. Define the vertex labeling ϕ_V as follows:

$$\phi_V(v_i) = \begin{cases} a^{n-2i+1} & if\ i \in \left\{0,1,\dots\frac{n-1}{2}\right\} \\ a^{2n-2i+1} & if\ i \in \left\{\frac{n+1}{2},\dots,n-1\right\} \end{cases}$$

Now give the edge labeling ϕ_E as follows:

For $i\in\{0,1,\dots,n-2\}$,

$\phi_E(v_iv_{i+1}) = a^i$

and

$\phi_E(v_0v_{n-1}) = a^{n-1}$.

Now

$$w_V(\phi_V(v_0)) = \phi_V(v_0).\phi_E(v_0v_1).\phi_E(v_0v_{n-1}) = a^{n+1}.a^0.a^{n-1} = a.e.a^{n-1} = e$$

For $i \in \left\{1,\dots\frac{n-1}{2}\right\}$

$$w_V(\phi_V(v_i)) = \phi_V(v_i).\phi_E(v_iv_{i+1}).\phi_E(v_{i-1}v_i) = a^{n-2i+1}.a^i.a^{i-1} = a^n = e\ ,$$

for $i \in \left\{\frac{n+1}{2},\dots,n-2\right\}$,

$$w_V(\phi_V(v_i)) = \phi_V(v_i).\phi_E(v_iv_{i+1}).\phi_E(v_{i-1}v_i) = a^{2n-2i+1}.a^i.a^{i-1} = a^{2n} = e$$

and

$$\begin{aligned} w_V(\phi_V(v_{n-1})) = \phi_V(v_{n-1}).\phi_E(v_{n-2}v_{n-1}).\phi_E(v_{n-1}v_0) &= a^{2n-2(n-1)+1}.a^{n-2}.a^{n-1} \\ &= a^{2n} = e \end{aligned}$$

Hence C_n is A^*-vertex magic.

Theorem 2.6. Let n be an odd positive integer. Let A be an abelian group of order n. Then S_n is A^*-vertex magic.

Proof. Let v be the vertex in S_n having degree $n-1$, and let $v_1,\dots,v_{n-1}$ are the vertices having degree 1.

Since the order of A is odd, for every $a \neq e$ in A, the element a and a^{-1} are distinct.

Now define labeling as follows:

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$$\phi_V(v) = e$$

For i=1,2,…,n-1

$\phi_V(v_i) = a$ and $\phi_E(v_i v) = a^{-1}$

Now we have

$$w_V(\phi_V(v)) = \phi_V(v).\phi_E(v_1 v).\phi_E(v_2 v)\dots\phi_E(v_{n-1} v) = e.\prod_{\substack{a\in A\\ a\neq e}} a = e$$

Further for i=1,2,…,n-1

$$w_V(\phi_V(v_i)) = \phi_V(v_i).\phi_E(v_i v) = a.a^{-1} = e$$

Hence S_n is A^*-vertex magic.

Theorem 2.7. Let A be an abelian group. If there is a A^*-vertex magic labeling of G, say (ϕ_V,ϕ_E) with magic constant μ such that the range of ϕ_V or ϕ_E has no involution, then there exists another A^*-vertex magic labeling with magic constant μ^{-1}, say (ψ_V,ψ_E) such that $\phi_V(v)\neq\psi_V(v)$ for every $v\in V(G)$ and $\phi_E(ab)\neq\psi_E(ab)$ for every $ab\in E(G)$.

Proof. Let (ϕ_V,ϕ_E) be a A^*-vertex magic labeling G with magic constant μ. Define (ψ_V,ψ_E) as follows:

$\psi_V(v)=[\phi_V(v)]^{-1}$

and

$\psi_E(uv)=[\phi_E(uv)]^{-1}$

We observe that (ϕ_V,ϕ_E) is not identically with (ψ_V,ψ_E) at every vertex $v\in V(G)$ and every edge $ab\in E(G)$. For if $\psi_V(v) = \phi_V(v)$ for some v or $\psi_E(uv) =\phi_E(ab)$ for some $ab \in E(G)$, then we can realize an element, say g, in the range of ϕ_V or ϕ_E such that $g^2=e$, which is impossible.

Now the weight with respect to (ψ_V,ψ_E) can be calculated as follows:

$$w_V(\psi_V(u)) = \psi_V(u).\prod_{uv\in E(G)}\psi_E(uv) = [\phi_V(u)]^{-1}.\prod_{uv\in E(G)}[\phi_E(uv)]^{-1}$$
$$= \left[\phi_V(u).\prod_{uv\in E(G)}\phi_E(uv)\right]^{-1} = \mu^{-1}$$

Now the result follows.

Definition 2.8. Let A be an abelian group. Let (ϕ_V,ϕ_E) be a A^*-vertex magic labeling of a graph G with magic constant μ. Let $g\neq e\in G$. Then the g-translation of A^*-vertex magic labeling with respect to g, denoted (ϕ_V^g, ϕ_E^g) is defined by

$$\phi_V^g(v) = g.\phi_V(v)$$

and

$$\phi_E^g(uv) = g.\phi_E(uv)$$

Theorem 2.9. Let A be an abelian group. Let g be an involution in A.Then g-translation of A*-vertex magic labeling is an A*-vertex magic labeling if and only if either degree of each vertex of G is odd or degree of each vertex of G is even.

Proof. Suppose (ϕ_V,ϕ_E) is a A^*-vertex magic labeling for G with magic constant μ.

Let (ϕ_V^g, ϕ_E^g) be the translation of (ϕ_V,ϕ_E) with respect to g. Let $u \in V(G)$. Then we have

$$w_V\left(\phi_V^g(v)\right) = \phi_V^g(v).\prod_{uv\in E(G)} \phi_E^g(uv) = g.\phi_V(u).\prod_{uv\in E(G)} [g.\phi_E(uv)]$$
$$= g.g^{\deg(u)}.\phi_V(u).\prod_{uv\in E(G)} \phi_E(uv) = g^{\deg(u)+1}.\mu$$

Clearly $w_V\left(\phi_V^g(v)\right)$ is μ or $g.\mu$ if and only if all vertices of G are of odd degree or even degree. Now the result follows.

Theorem 2.10. Let A be a finite abelian group. Let g be an element of A which is not an involution. Let G be a graph with every vertices of degree less than |A|−1. Then the translation of A*-vertex magic labeling with respect to g is A*-vertex magic labeling if and only if G is regular.

Proof. Suppose (ϕ_V,ϕ_E) is a A^*-vertex magic labeling for G with magic constant μ.

Let (ϕ_V^g, ϕ_E^g) be the translation of (ϕ_V,ϕ_E) with respect to g. Let $u \in V(G)$. Then we have from the proof of the theorem 2.9

$$w_V\left(\phi_V^g(v)\right) = g^{\deg(u)+1}.\mu$$

Clearly $g^{\deg(u)+1}.\mu$ is a magic constant if and only if the degrees of all vertices of G are same. That is G is regular.

Definition 2.11. Let G be a graph with n vertices and m edges. Let (A,·) be an abelian group of order max(n,m). An A*-edge magic labeling of G is a pair of one-to-one maps

ϕ_V:V(G)⟶A

and

ϕ_E:E(G)⟶A

satisfying the following condition:

$$\phi_V(u).\phi_E(uv).\phi_V(v) = a$$

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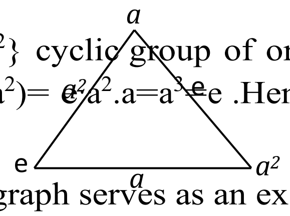
for some fixed a∈A for all edges uv. In notations, if one defines

$$\mathrm{w_E}(\phi_\mathrm{E}(\mathrm{uv})) = \phi_\mathrm{V}(\mathrm{u}).\,\phi_\mathrm{E}(\mathrm{uv}).\,\phi_\mathrm{V}(\mathrm{v}) = \mathrm{a}$$

for some fixed a∈A for all edges uv. A graph that admits A*-edge magic labeling is said to be A*-edge magic graph.

Example 2.12. The following graph serves as an example for the definition 2.11:

Figure 4



With G=C_3, and A={e,a,a^2} cyclic group of order 3. Here w_E(e)=a^2. e .a=a^3 =e and w_E(a)=e.a.a^2=a^3=e and w_E(a^2)= e.a^2.a=a^3=e .Hence the above graph is A*-edge magic graph.

Example 2.13. Following graph serves as an example for Definition 2.11:

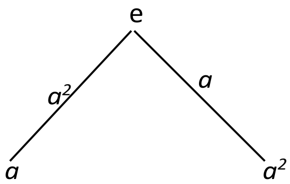


Figure 5

With G=P_3, and A={e,a,a^2} cyclic group of order 3. Here we have w_E(a)=a·a^2=a^3=e and w_E(a^2) =a^2·a=a^3=e.

Hence the above graph is A*-edge magic graph.

Theorem 2.14. Let n be an odd number and let A be a cyclic group of order n. Then C_n is A*-edge magic.

Proof. Let $v_0,\ldots,v_{n-1}$ be the vertices of C_n in cyclic pattern. Let $A=\{e,a,\ldots,a^{n-1}\}$ be the cyclic group of order n. Define the vertex labeling ϕ_V as follows:

$\phi_V(v_0)=e,$

and for $i\in\{1,\ldots,n-1\}$,

$\phi_V(v_i)=a^i.$

Now give the edge labeling ϕ_E as follows:

$$\phi_E(v_iv_{i+1}) = \begin{cases} a^{n-(2i+1)} & if\ i \in \left\{0,1,\ldots\frac{n-1}{2}\right\} \\ a^{2n-(2i+1)} & if\ i \in \left\{\frac{n+1}{2},\ldots,n-1\right\} \end{cases}$$

For $i \in \left\{0,1,\ldots\frac{n-1}{2}\right\}$

$$w_E(\phi_E(v_iv_{i+1})) = \phi_V(v_i).\,\phi_E(v_iv_{i+1}).\,\phi_V(v_{i+1}) = a^i.\,a^{n-(2i+1)}.\,a^{i+1} = e$$

And for $i \in \left\{\frac{n+1}{2},\ldots,n-1\right\}$,

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$$w_E\big(\phi_E(v_iv_{i+1})\big) = \phi_V(v_i).\phi_E(v_iv_{i+1}).\phi_V(v_{i+1}) = a^i.a^{2n-(2i+1)}.a^{i+1} = e$$

Hence C_n is A^*-edge magic.

Theorem 2.15. Let n be an odd positive integer. Let A be an abelian group of order n. Then S_n is A^*-edge magic.

Proof. Let v be the vertex in S_n having degree $n-1$, and let $v_1,\ldots,v_{n-1}$ are the vertices having degree 1.

Since the order of A is odd, for every $a \neq e$ in A, the element a and a^{-1} are distinct.

Now define labeling as follows:

$$\phi_V(v) = e$$

For i=1,2,…,n-1

$\phi_V(v_i) = a$ and $\phi_E(v_iv) = a^{-1}$

Now we have for i=1,2,…,n-1

$$w_V\big(\phi_E(v_iv)\big) = \phi_V(v_i).\phi_E(v_iv).\phi_V(v) = a.a^{-1}.e = e$$

Hence S_n is A^*-edge magic.

3. CONCLUSION AND SCOPE

In this paper we have introduced the concept of A^*-vertex magic labeling and A^*-edge magic labeling. We have obtained a few basic results on graphs which admit such labelings. This concept is wide open for further investigation. One can investigate whether standard families of graphs are these labelings. Finding necessary or sufficient condition for a graph to admit these labelings and study of A^*-vertex magic graphs and A^*-edge magic graphs with reference to various graph operations are other possible directions for future research.

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