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LABELING ON SPECIFIC STRUCTURED GRAPHS

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Abstract
In this article, we Investigation HMC labelling of some special graphs such as Flower Fl_n, Friendship and Butterfly $B_{n,m}$ Maps.

Keywords: HMC labeling, Flower graph Fl_n, Friendship graph F_n, Butterfly graph $B_{n,m}$,

INTRODUCTION

In which paper all Maps are finite, Simple and unobtrusive. Symbols V (G) and E (G) represent the apex and edge of the G graph. Previously, there are so many labeling concepts are studied on many graphs [5]. For example K. Chithralakshmi and K. Nagarajan have proved that Star $K_{1,n}$ is geometric mean cordial labelling [4]. Flower graph Fl_n and Friendship graph F_n are prime labeling is proved by S. Ashokkumar and S. Maragathavalli [2]. S. Meena and P. Kavitha investigated that the Butterfly graph $B_{n,m}$ is prime labelling [6]. Path, Star, Bistar graphs are HMC labeling proved by J. Gowri and J. Jayapriya [8]. Here we proved that Flower graph Fl_n, Friendship graph F_n, and Butterfly graph $B_{n,m}$ are HMC (Harmonic Mean Cordial) labelling.

PRELIMINARIES

Definition 1:
A simple graph G = (V, E) is said to be HMC (Harmonic Mean Cordial) labeling if there exists a function $f: V \to \{1,2\}$ such that the induced edge function $g: E \to \{1,2\}$ defined by $uv = \left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor, f(u), f(v) \neq 0$ for each edge and $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_g(x)$ denotes the number of edges labeled with x where $x \in \{0,1,2\}$ respectively. A graph which admits a HMC (Harmonic Mean Cordial) labeling is called HMC(Harmonic Mean Cordial) graph.

FLOWER GRAPH

Graph theory, one of the most dynamic areas of discrete mathematics, provides a powerful framework for studying relationships and connections between various entities. Among the many classes of graphs studied for their unique topological and structural properties, the flower graph stands out due to its aesthetic symmetry and versatile structure.

It is possible to visualize a flower graph as a set of cycles (or petals), how to which a central node is connected, having the shape of a flower. Flower graphs and their labelling patterns have recently emerged as an important field of study because of their use in network design, system of data transmission, biological modeling, and communication theory.

Definition 2: The resultant graph is found by joining together the pendent peaks to the helminth peak H_n is referred to as a flower graph Fl_n. It is of three kinds; the apex is of degree *2n* and the *n* of degree *4n* verticals and *n* verticals of degree 2.

Theorem 1: Flower graph Fl_n admits a HMC (Harmonic mean cordial) labeling.

Proof:
Let G = (V, E) = Fl_n be the flower graph where $V = \{v, v_1, v_2, \dots \dots, v_n, u, u_1, u_2, \dots \dots, u_n\}$ be the vertices of G. Let V be the apex vertex of degree 2n, $v_1, v_2, \dots \dots, v_n$ be the vertices of degree 4 and $u_1, u_2, \dots \dots, u_n$ of Fl_n. Then the flower graph has 2n+1 vertices and 4n edges.
Define a HMC labeling $f: V \to \{1,2\}$ as follows

$$f(v) = 2$$
$$f(v_i) = 2, 1 \leq i \leq n$$
$$f(u_i) = 1, 1 \leq i \leq n$$

Then, we have $v_f(1) = n, v_f(2) = n + 1 \; and \; e_g(1) = 2n = \; e_g(2)$. Hence the flower Fl_n admits HMC labeling.

FRIENDSHIP GRAPH

Graph theory is an effective mathematical instrument on the study of structures that are composed of elements that are connected. The friendship graph is one of the most investigated and interesting graph families that is a perfect representation of the concept of commonality among people or objects.
The friendship graph, or the Dutch Windmill graph, is a result of a social interpretation, which states that when each pair of individuals have one friend in common, then their friendship relationships can be denoted by a certain kind of graph, the friendship graph.
The importance of this structure is that it is not only symmetrical and simple but also has a large variety of applications in social network analysis, communication systems and in combinatorial optimization. It is also the basis of the study of graph labelling where different mathematical relationships are given to the vertices and the edges.
Definition 3: The friendship graph F_n is obtained from n copies of cycle C_3 is united by one – single point.

Theorem 1: The friendship graph F_n confess a HMC labeling for all $n \geq 1$

Proof:
Let G = (V, E) = F_n be the friendship graph with n copies of cycle C_3, where $V = \{v_1, v_2, \dots \dots, v_n\}$ be the vertices of G. Let V be the apex vertex $v_1, v_2, \dots \dots, v_{2n}$ be the other vertices and $e_1, e_2, \dots \dots, e_{3n}$ be the edges of F_n.

Define $f: V \to \{1,2\}$ as follows:
$f(V) = 2$ (apex vertex)

$$f(V_i) = 1, 1 \leq i \leq n$$
$$f(V_i) = 2, n + 1 \leq i \leq 2n$$

From the above defined pattern, we have $v_f(1) = \text{n}, v_f(2) = n + 1 \; and \; e_g(1) = \; e_g(2)$ or $e_g(1) = e_g(2) + 1$. In this manner friendship graph $F_n$has a HMC labeling.

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BUTTERFLY GRAPH

Graph theory is a subdiscipline of discrete mathematics which deals with networks of interrelated objects. Among others, the butterfly graph is one of the most interesting in graph theory, as it is symmetrical, simple, and has been used in a variety of way in communication networks, combinatorial designs, and data flow systems. The butterfly graph also has the name bowtie graph as a consequence of its similarity to the shape of a butterfly or a bowtie, two triangular subgraphs are attached at a single vertex. Its distinctive dual-triangle form contributes to it being a very good model to study graph labelling techniques including graceful, mean, cordial and harmonic labelling.

Definition 4: Butterfly graph is formed by two n-cycles which have a common vertex with a number of pendent edges which are attached to the common vertex in an arbitrary number m, where m and n are positive integers.

Theorem 1: If m is any positive integer, then the butterfly graph $B_{n,n}$ is HMC for n ≥ 2.

Proof:

Let G be a butterfly graph. Let the vertex set be $V(G) = \{u_1, u_2, u_3, u_4, u_{2n-1}, v_1, v_2, \dots . v_m\}$ and the edge set $E = \{e_i, e_j\}$ where $e_i = \{u_i, v_i\}$ and $e_{ij} = (u_i, u_j)$

Define $f: V \rightarrow \{1,2\}$ as follows

$f(u_i) = 2, 1 \leq i \leq n$

$f(u_i) = 1, n + 1 \leq i \leq 2n - 1$ and

$f(v_i) = 1, l = 2m - 1$

$f(v_i) = 2, i = 2m, where\ m = 1,2,3, \dots ..$

Then the above defined function , we have $v_f(1) = v_f(2) or\ v_f(2) = v_f(1) + 1\ and\ e_f(1) = e_g(2)\ or\ e_g(1) = e_g(2) + 1$, we see that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.

Example:

HMC labeling of $B_{5,5}$ is given below

Conclusion

The Flower, Butterfly and Friendship graphs are crucial network models, which depict structural beauty and practicality in communication, biological, computational and social networks. A balanced distribution of vertex and edge labels is also possible when these graphs are given the labeling Harmonic Mean Cordial (HMC) which represents a kind of harmony between the interacting states. This balance indicates the equal distribution of loads, the stable flow of energy or data and the equal distribution of resources. Therefore, HMC labeling is a common mathematical construct that connects abstract graph theory to systems optimization, providing stable and efficient models of its application in a variety of fields of science and engineering.

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