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BINARY π GENERALIZED SEMI CLOSED SETS IN BINARY TOPOLOGICAL SPACES

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Abstract: In this paper, a new class of sets called binary πgs-closed sets is introduced and its properties are studied. Moreover the notions of binary semi limit point and binary semi-derived sets are introduced.

1 Introduction and Preliminaries

Nithyanantha Jothi, et al.[2] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A,B) where $A \subseteq X$ and $B \subseteq Y$. In this paper, a new class of sets called binary πgs-closed sets is introduced and its properties are studied. Moreover the notions of binary semi limit point and binary semi-derived sets are introduced.

Let X and Y be any two nonempty sets. A binary topology [2] from X to Y is a binary structure $\mathcal{M} \subseteq \mathbb{P}(X) \times \mathbb{P}(Y)$ that satisfies the axioms namely

1. (ϕ,ϕ) and $(X,Y) \in \mathcal{M}$,
2. $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1,B_1) \in \mathcal{M}$ and $(A_2,B_2) \in \mathcal{M}$, and
3. If $\{(A_\alpha,B_\alpha): \alpha \in \delta\}$ is a family of members of $\mathcal{M}$, then $(\cup_{\alpha\in\delta} A_\alpha, \cup_{\alpha\in\delta} B_\alpha) \in \mathcal{M}$.

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If $\mathcal{M}$ is a binary topology from X to Y then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$. If $Y = X$ then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary topological space.

Definition 1.1 *[2] Let X and Y be any two nonempty sets and let (A, B) and $(C, D) \in \mathbb{P}(X) \times \mathbb{P}(Y)$. We say that $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.*

Definition 1.2 *[2] Let $(X, Y, \mathcal{M})$ be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary closed in $(X, Y, \mathcal{M})$ if $(X \backslash A, Y \backslash B) \in \mathcal{M}$.*

Proposition 1.3 *[2] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$.*

Let $(A, B)^{1*} = \cap \{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \cap \{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.

Proposition 1.4 *[2] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1*} = \cup \{A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $(A, B)^{2*} = \cup \{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.*

Definition 1.5 *[2] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of (A, B), denoted by $b\text{-}cl(A, B)$ in the binary space $(X, Y, \mathcal{M})$ where $(A, B) \subseteq (X, Y)$.*

Definition 1.6 *[2] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ defined in proposition 1.4 is called the binary interior of of (A, B), denoted by $b\text{-}int(A, B)$. Here $((A, B)^{1*}, (A, B)^{2*})$ is binary open and $((A, B)^{1*}, (A, B)^{2*}) \subseteq (A, B)$.*

Definition 1.7 *[2] Let $(X, Y, \mathcal{M})$ be a binary topological space and let $(x, y) \subseteq (X, Y)$. The binary open set (A, B) is said to be a binary neighbourhood of (x, y) if $x \in A$ and $y \in B$.*

Proposition 1.8 *[2] Let $(A, B) \subseteq (C, D) \subseteq (X, Y)$ and $(X, Y, \mathcal{M})$ be a binary topological space. Then, the following statements hold:*

1. $b\text{-}int(A, B) \subseteq (A, B)$.
2. If (A, B) is binary open, then $b\text{-}int(A, B) = (A, B)$.
3. $b\text{-}int(A, B) \subseteq b\text{-}int(C, D)$.

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4. $b\text{-}int(b\text{-}int(A,B)) = b\text{-}int(A,B)$.
5. $(A,B) \subseteq b\text{-}cl(A,B)$.
6. If (A,B) is binary closed, then $b\text{-}cl(A,B) = (A,B)$.
7. $b\text{-}cl(A,B) \subseteq b\text{-}cl(C,D)$.
8. $b\text{-}cl(b\text{-}cl(A,B)) = b\text{-}cl(A,B)$.

Definition 1.9 *A subset* (A,B) *of a binary topological space* $(X,Y,\mathcal{M})$ *is called*

1. a binary semi open set [3] if $(A,B) \subseteq b\text{-}cl(b\text{-}int(A,B))$.
2. a binary pre open set [1] if $(A,B) \subseteq b\text{-}int(b\text{-}cl(A,B))$.
3. a binary regular open set [4] if $(A,B) = b\text{-}int(b\text{-}cl(A,B))$.
4. a binary π-open [7] if the finite union of binary regular-open sets.

The complements of the above mentioned sets are called their respective closed sets.

Definition 1.10 *A subset* (A,B) *of a binary topological space* $(X,Y,\mathcal{M})$ *is called*

1. a binary g-closed set [5] if $b\text{-}cl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary open.

2. a binary gp-closed set [8] if $b\text{-}pcl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary open.

3. a binary gs-closed set [6] if $b\text{-}scl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary semi open.

4. a binary πg-closed [7] if $b\text{-}cl(A,B) \subseteq (U,V)$, whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary π-open.

5. a binary rg-closed set [4] if $b\text{-}cl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary regular open.

6. a binary π generalized pre closed set (briefly, binary πgp-closed set) [8] if $b\text{-}pcl(A,B) \subseteq (G,H)$ whenever $(A,B) \subseteq (G,H)$ and (G,H) is binary π-open.

The complements of the above mentioned sets are called their respective open sets.

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2 Binary πgs-closed sets

Definition 2.1 *A subset (A,B) of (X,Y) in a binary topological space $(X,Y,\mathcal{M})$ is called a binary π generalized pre closed set (briefly, binary πgs-closed set) if $b\text{-}scl(A,B) \subseteq (G,H)$ whenever $(A,B) \subseteq (G,H)$ and (G,H) is binary π-open.* The complement of a binary πgs-closed set is binary πgs-open.

Definition 2.2 *A subset (A,B) of (X,Y) in a binary topological space $(X,Y,\mathcal{M})$ is called a binary strong $\mathcal{NP}$-set if $b\text{-}int(b\text{-}cl(A,B)) = b\text{-}cl(b\text{-}int(A,B))$.*

Proposition 2.3 *Let $(X,Y,\mathcal{M})$ be a topological space and $(A,B) \subseteq (X,Y)$. The following properties hold:*

1. (A,B) is gs-closed $\Rightarrow$ (A,B) is πgs-closed,
2. (A,B) is πg-closed $\Rightarrow$ (A,B) is πgs-closed.

Remark 2.4 *From the Proposition 2.3, we have the the following diagram holds for any subset of a binary topological space $(X,Y,\mathcal{M})$.*

$$\begin{tikzcd}
 & \text{binary pre-closed} \arrow[r] & \text{binary } gp\text{-closed} \arrow[r] & \text{binary } \pi gp\text{-closed} \\
\text{binary } \pi\text{-closed} \arrow[r] & \text{Binary closed} \arrow[r] \arrow[u] \arrow[d] & \text{binary } g\text{-closed} \arrow[r] \arrow[u] \arrow[d] & \text{binary } \pi g\text{-closed} \arrow[u] \arrow[d] \\
 & \text{binary semi-closed} \arrow[r] & \text{binary } gs\text{-closed} \arrow[r] & \text{binary } \pi gs\text{-closed}
\end{tikzcd}$$

None of the implications is reversible as seen from the following Example.

Example 2.5 *Let $X = \{1,2\}$, $Y = \{a,b\}$ and $\mathcal{M} = \{(\phi,\phi),(\phi,\{b\}),(\{1\},\{a\}),(\{1\},Y),(X,Y)\}$. In the binary topological space $(X,Y,\mathcal{M})$, then the subset $(\{1\},\{a\})$ is binary πgs-closed but it is neither binary πg-closed nor binary πgp-closed.*

Example 2.6 *Let $X = \{a,b\}$, $B = \{1,2\}$ and $\mathcal{M} = \{(\phi,\phi),(\phi,\{2\}),(\{a\},\{1\}),(\{a\},Y),(X,\{1\}),(X,Y)\}$. In the binary topological space $(X,Y,\mathcal{M})$, then the subset $(\phi,\{1\})$ is binary πgs-closed but not binary gs-closed.*

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Theorem 2.7 *A subset (A, B) of (X, Y) in a binary topological space $(X, Y, \mathcal{M})$ is binary regular open if and only if it is both binary π-open and binary πgs-closed.*

Proof. Since every binary regular open set is binary π-open. For the second claim note that binary regular open sets are even binary semi-closed.

Conversely, since (A, B) is binary π-open and binary πgs-closed in a binary topological space $(X, Y, \mathcal{M})$. $b\text{-}scl(A, B) \subseteq (A, B)$ since (A, B) is binary π-open and binary πgs-closed. Thus $b\text{-}int(b\text{-}cl(A, B) \subseteq (A, B)$. Since (A, B) is binary open, then (A, B) is clearly binary preopen and thus $(A, B) \subseteq b\text{-}int(b\text{-}cl(A, B)$. Therefore $b\text{-}int(b\text{-}cl(A, B) \subseteq (A, B) \subseteq b\text{-}int(b\text{-}cl(A, B)$ or equivalently $(A, B) = b\text{-}int(b\text{-}cl(A, B))$, which follows that (A, B) is binary regular open.

Corollary 2.8 *If (A, B) is binary π-open and binary πgs-closed, then (A, B) is binary semiclosed and hence binary gs-closed.*

Proof. By assumption and Theorem 2.7, (A, B) is binary regular open. Thus (A, B) is binary semiclosed. Since every binary semiclosed set is binary gs-closed, (A, B) is binary gs-closed.

Theorem 2.9 *In a binary topological space $(X, Y, \mathcal{M})$, for a subset $(A, B) \subseteq (X, Y)$ the following statements are equivalent:*

1. (A, B) is binary π-clopen;
2. (A, B) is binary π-open, a binary strong $\mathcal{NP}$-set and binary πgs-closed.

Proof. (1) ⇒ (2) is obvious.

(1) ⇒ (2). By Theorem 2.7, (A, B) is binary regular-open. Since (A, B) is a binary strong $\mathcal{NP}$-set, $(A, B) = b\text{-}int(b\text{-}cl(A, B)) = b\text{-}cl(b\text{-}int(A, B))$. So (A, B) is binary regular-closed. Which proves that (A, B) is binary π-closed and hence (A, B) is binary π-clopen.

Remark 2.10 *The union of two binary πgs-closed sets need not be πgs-closed in a binary topological space $(X, Y, \mathcal{M})$.*

Example 2.11 *In Example 2.5, $(A, B) = (\phi, \{a\})$ and $(C, D) = (\phi, \{b\})$ are binary πgs-closed but $(A, B) \cup (C, D) = (\phi, Y)$ is not binary πgs-closed.*

Remark 2.12 *The intersection of two binary πgs-closed sets is also a binary πgs-closed in a binary topological space $(X, Y, \mathcal{M})$.*

Example 2.13 *In Example 2.5,* $(A,B)=(\{1\},\{a\})$ *and* $(C,D)=(\{2\},\{a\})$ *are binary* πgs*-closed. Then* $(A,B)\cap(C,D)=(X,\{a\})$ *is also a binary* πgs*-closed.*

Theorem 2.14 *Let* (A,B) *be binary* πgs*-closed in* $(X,Y,\mathcal{M})$*. Then* $b\text{-}scl(A,B)-(A,B)$ *does not contain any non-empty binary* π*-closed set.*

Proof. Let (F,G) be a binary π-closed set such that $(F,G)\subseteq b\text{-}scl(A,B)-(A,B)$. Then $(F,G)\subseteq(X,Y)-(A,B)$ implies $(A,B)\subseteq(X,Y)-(F,G)$. Therefore $b\text{-}scl(A,B)\subseteq(X,Y)-(F,G)$. That is $(F,G)\subseteq(X,Y)-b\text{-}scl(A,B)$. Hence $(F,G)\subseteq b\text{-}scl(A,B)\cap(X,Y)-b\text{-}scl(A,B)=\phi$. Which proves that $(F,G)=\phi$.

Theorem 2.15 *If* (A,B) *is binary* πgs*-closed and* $(A,B)\subseteq(C,D)\subseteq b\text{-}scl(A,B)$*, then* (C,D) *is binary* πgs*-closed.*

Proof. Let (A,B) be binary πgs-closed and $(C,D)\subseteq(U,V)$ where (U,V) is binary π-open. Then $(A,B)\subseteq(C,D)$ implies $(A,B)\subseteq(U,V)$. Since (A,B) is binary πgs-closed, $b\text{-}scl(A,B)\subseteq(U,V)$. $(C,D)\subseteq b\text{-}scl(A,B)$ implies $b\text{-}scl(C,D)\subseteq b\text{-}scl(A,B)$. Therefore $b\text{-}scl(C,D)\subseteq(U,V)$ and hence (C,D) is binary πgs-closed.

Definition 2.16 *Let* $(X,Y,\mathcal{M})$ *be a binary topological space and* $(A,B)\subseteq(X,Y)$*. A point* $(x,y)\in(X,Y)$ *is said to be a binary semi-limit point of* (A,B) *if any binary semiopen set containing* (x,y) *contains a point of* (A,B) *different from* (x,y)*.*

Definition 2.17 *Let* $(X,Y,\mathcal{M})$ *be a binary topological space and* $(A,B)\subseteq(X,Y)$*. The set of all binary semi-limit points of* (A,B) *is said to be a binary semi-derived set of* (A,B) *and is denoted by* $D_{bs}[A,B]$*.*

Similarly, $(x,y)\in D[A,B]$ iff any binary open set (U,V) such that $(x,y)\in(U,V)$ meets $(A,B)-\{x,y\}$.

Theorem 2.18 *Let* (A,B) *and* (C,D) *be* πgs*-closed sets in* $(X,Y,\mathcal{M})$ *such that* $D[A,B]\subseteq D_{bs}[A,B]$ *and* $D[C,D]\subseteq D_{bs}[C,D]$*. Then* $(A,B)\cup(C,D)$ *is binary* πgs*-closed.*

Proof. For any set $(E,F)\subseteq(X,Y), D[E,F]\supseteq D_{bs}[E,F]$. Therefore $D[A,B]=D_{bs}[A,B]$ and $D[C,D]=D_{bs}[C,D]$. That is $b\text{-}scl(A,B)=b\text{-}cl(A,B)$ and $b\text{-}scl(C,D)=b\text{-}cl(C,D)$. Let $(A,B)\cup(C,D)\subseteq(U,V)$ where (U,V) is binary π-open. Then $(A,B)\subseteq(U,V)$ and $(C,D)\subseteq(U,V)$. Since (A,B) and (C,D) are binary πgs-closed, $b\text{-}scl(A,B)\subseteq(U,V)$ and $b\text{-}scl(C,D)\subseteq(U,V)$. Then, $b\text{-}scl((A,B)\cup(C,D))\subseteq b\text{-}cl((A,B)\cup(C,D))=b\text{-}cl(A,B)\cup b\text{-}cl(C,D)=b\text{-}scl(A,B)\cup b\text{-}scl(C,D)\subseteq(U,V)$. So $b\text{-}scl((A,B)\cup(C,D))\subseteq(U,V)$ and

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hence $((A,B) \cup (C,D))$ is binary πgs-closed.

Remark 2.19 *$b\text{-}scl((X,Y)-(A,B)) = (X,Y) - b\text{-}sint(A,B)$, for any subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$.*

Theorem 2.20 *Let $(X,Y,\mathcal{M})$ be a binary topological space. $(A,B) \subseteq (X,Y)$ is binary πgs-open if and only if $(F,G) \subseteq b\text{-}sint(A,B)$ whenever (F,G) is binary π-closed and $(F,G) \subseteq (A,B)$.*

Proof. Necessity. Let (A,B) be binary πgs-open. Let (F,G) be binary π-closed and $(F,G) \subseteq (A,B)$. Then $(X,Y)-(A,B) \subseteq (X,Y)-(F,G)$ where $(X,Y)-(F,G)$ is binary π-open. Binary πgs-closedness of $(X,Y)-(A,B)$ implies $b\text{-}scl(X,Y)-(A,B) \subseteq (X,Y)-(F,G)$. By Remark 2.19, $b\text{-}scl(X,Y)-(A,B) = (X,Y)-b\text{-}sint(A,B)$. So $(F,G) \subseteq b\text{-}sint(A,B)$.

Sufficiency. Suppose (F,G) is binary π-closed and $(F,G) \subseteq (A,B)$ imply $(F,G) \subseteq b\text{-}sint(A,B)$. Let $(X,Y)-(A,B) \subseteq (U,V)$ where (U,V) is binary π-open. Then $(X,Y)-(U,V) \subseteq (A,B)$ where $(X,Y)-(U,V)$ is binary π-closed. By hypothesis $(X,Y)-(U,V) \subseteq b\text{-}sint(A,B)$. That is $(X,Y)-b\text{-}sint(A,B) \subseteq (U,V)$. By Remark 2.19, $b\text{-}scl(X,Y)-(A,B) \subseteq (U,V)$. So, $(X,Y)-(A,B)$ is binary πgs-closed and (A,B) is binary πgs-open.

Theorem 2.21 *If (A,B) is binary πgs-open and $b\text{-}sint(A,B) \subseteq (C,D) \subseteq (A,B)$ then (C,D) is binary πgs-open.*

Proof. Follows from the Theorem 2.15.

References

[1] S. Jayalakshmi and A. Manonmani, Binary regular beta closed sets and binary regular beta open sets in binary topological spaces, *The International Journal of Analytical and Experimental Modal Analysis*, 12(4) (2020), 494-497.

[2] S. Nithyanantha Jothi and P. Thangavelu, Topology between two sets, *Journal of Mathematical Sciences & Computer Applications*, 1(3) (2011), 95-107.

[3] S. Nithyanantha Jothi, Binary semi open sets in binary topological spaces, *International Journal of Mathematical Archieve*, 7(9) (2016), 73-76.

[4] S. Nithyanantha Jothi and P. Thangavelu, Generalized binary regular closed sets, *IRA-International Journal of Applied Sciences*, 4(2) (2016), 259-263.

[5] S. Nithyanantha Jothi and P. Thangavelu, Generalized binary closed sets in binary topological spaces, *Ultra Scientist*, 26(1)(A) (2014), 25-30.

Received: August 02, 2025

[6] C. Santhini and T. Dhivya, New notion of generalized binary closed sets in binary topological space, *International Journal of Mathematical Archive*, 9(10) (2018), 1-7.

[7] M. Vinoth and R. Asokan, Binary π-closed sets and binary πg-closed sets in binary topological spaces, *Indian Journal of Natural Sciences*, 14(81)(2023), 66686-66690.

[8] M. Vinoth, R. Asokan, V. Ramesh and R. Premkumar, Binary π generalized pre closed sets in binary topological spaces, *Asia Mathematika*, 8(2)(2024), 16-22.