International Journal of Applied Mathematics
Volume 38 No. 11s 2025,
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

RELIABILITY, AVAILABILITY AND PROFIT ANALYSIS OF THE OXYGEN PLANT SUPPLY SYSTEM FOR HOSPITALS

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Abstract

This paper deals with the evaluation of reliability measures of oxygen plant supply system for hospitals. The oxygen supply system from the oxygen plant to the patient consists of six subsystems, namely sterilizing filter, oxygen booster pump, oxygen supply units, oxygen flow meter, gas mixer, and ventilator. The subsystem 'oxygen supply unit' contains two identical units: one is operative and other keep in cold standby. The distribution of failure times of each subsystem is taken as exponential, while the distribution of repair times of each subsystem is considered arbitrary. Supplementary variable technique and Laplace transform are used to evaluate the reliability, availability, mean time to failure, and profit of the oxygen plant supply system. The numerical results of the reliability measures and profit function are also obtained by taking particular values of the various parameters and repair cost.

Keywords: Availability. Oxygen plant. Reliability. Supplementary variable technique.

1. Introduction

Oxygen is an essential medicine used in various healthcare contexts to treat multiple conditions, including traumatic injuries, heart failure, asthma, pneumonia, maternity and pediatric care. The urgency of delivering oxygen supplies has escalated due to the unprecedented global demand, which has significantly increased with the COVID-19 pandemic. The COVID-19 pandemic revealed scarcity of medical oxygen which indicated weakness in the healthcare systems of many developing countries.

To confront these challenges, oxygen cylinders were used to increase the oxygen level as an attempt to improve the condition and aid the recovery of a patient. However, these heavy oxygen cylinders must be transported from industries or markets to hospitals in the heavy vehicles and may be time consuming as well as costly. Hospitals incur significant expenses in transporting the medical oxygen cylinders to ensure a consistent and reliable oxygen supply.

A medical oxygen gas plant can solve this problem as it is installed within hospital premises to produce continuous oxygen supply to the patients. Oxygen plants can produce oxygen on-site and reduce the reliance on the external suppliers which ensuring consistent and reliable solution of the problem. This also fulfills the oxygen's demand, reducing risk and improve patients' safety in the hospitals. That too leads the cost cutting and enhances availability and profitability. Through Reliability analysis, we can find out how to increase the availability and profitability of the complex system. These measures might help management figure out the impact of increasing or decreasing system or subsystem failure or repair rates. It is not always possible to rule out the possibility of any system or subsystem failing since these failures might sometimes be inherent. Cox [1] introduced the effective supplementary variable technique for studying non-Markovian systems for assessing the system's availability and reliability.

Reliability analysis have been carried out by several researchers for most of industrial systems. Kumar et al. [2] used the supplementary variable approach to study the role of reliability advancements on the sugar industry's refining system. A thermal power plant's steam and power generation systems' availability discussed by Arora and Kumar [3].Wang and Chiu [4] performed a cost-benefit study on availability systems incorporating warm standby units. The study of reliability models for systems with both internal and external redundancy were

Received: August 06, 2025

proposed by Zhang et al. [5]. Kumar et al. [6] studied the deterioration of the units following repair in a system that was redundant with a single standby. Shakuntla et al. [7] discussed the reliability of polytube manufacturing plant. Suleiman et al. [8] assessed a complex thermal power plant's performance stochastically. Kadyan et al. [9] presented a single-unit reliability model with arbitrary distributions and diverse meteorological circumstances. The reliability measures of a solar photovoltaic system consisting of subsystems connected in series were evaluated using the supplemental variable technique described by Kumar et al. [10]. Kadyan and Kumar [11] discussed feeding system of the sugar industry. Availability of a butter oil production system was evaluated using a mathematical model studied by Mehta et al. [12]. Kumar and Kadyan [13] discussed reliability modelling and failure mechanism of distillery plan. The condensed milk production system in a milk plant was studied by Kumari et al. [14] using the supplementary variable technique. Saini et al. [15] discussed the performance analysis of continuous casting system in steel industry. Gayathri [16] used a supplementary variable technique to evaluate the availability and reliability of a cement-producing plant. Maihulla et al. [17] analyzed reliability and availability of an industrial system with two subsystems arranged in series-parallel.

A few researchers analyzed the reliability of the healthcare system such as Sagayaraj [18] determined the reliability of the healthcare system using a fault-tree analysis approach. Vidya and Vijaya [19] discussed the reliability analysis in healthcare imaging applications. Abd Rahman et al. [20] reviewed the critical device reliability assessment in healthcare services. Yousofnrjad and Es'haghi [21] evaluated the reliability of a medical oxygen supply system by fault tree analysis based on intuitionistic fuzzy sets.

A systematic review of the literature suggests that hardly a researcher discusses reliability measures related to the oxygen plant supply system. Therefore, this study is an attempt to fill a gap. This study discusses reliability modelling and studies the failure mechanism of oxygen plant supply systems using the supplementary variable technique. We have incorporated a standby unit to improve the efficiency of the system and ensure continuous supply.

The subsequent sections of this paper are organized as follows: Section 2, the system is described, together with the model's assumptions, state specifications, and notations associated to the suggested system model. Section 3 describes the model analysis used for determining several system performance metrics, including profit, mean time to system failure (MTSF), system availability, and system reliability, while Section 4 presents a numerical study. Section 5 presents the findings and a discussion of the special cases that include the availability of the system and the profit analysis of the users. Finally, Section 6 concludes this investigation by exploring potential prospects.

2. System Description/working, Assumptions and Notations

In an oxygen plant supply system, an oxygen plant generates 99% pure O2, and then a sterilizing filter helps avoid micro-bacterial particles from the oxygen gas. Then, the oxygen booster pump increases the oxygen pressure and flows to the oxygen supply unit, where the oxygen supply unit has one direct oxygen supply unit to the hospital and a second unit as a cold standby. When the direct supply fails, the standby units immediately supply the oxygen. Then, the oxygen flow meter regulates oxygen flow, and it helps to flow oxygen normally to patients. A gas mixer is used blend oxygen and air, to deliver a specific concentration of oxygen to a patient, ensuring the correct oxygen levels for their needs. After that, ventilator is used to support over the breathing process by delivering oxygen-rich air to the lungs. As described, the oxygen plant supply system contains six subsystems (Figure 1), and the descriptions of the subsystems are as follows:

International Journal of Applied Mathematics
Volume 38 No. 11s 2025,
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

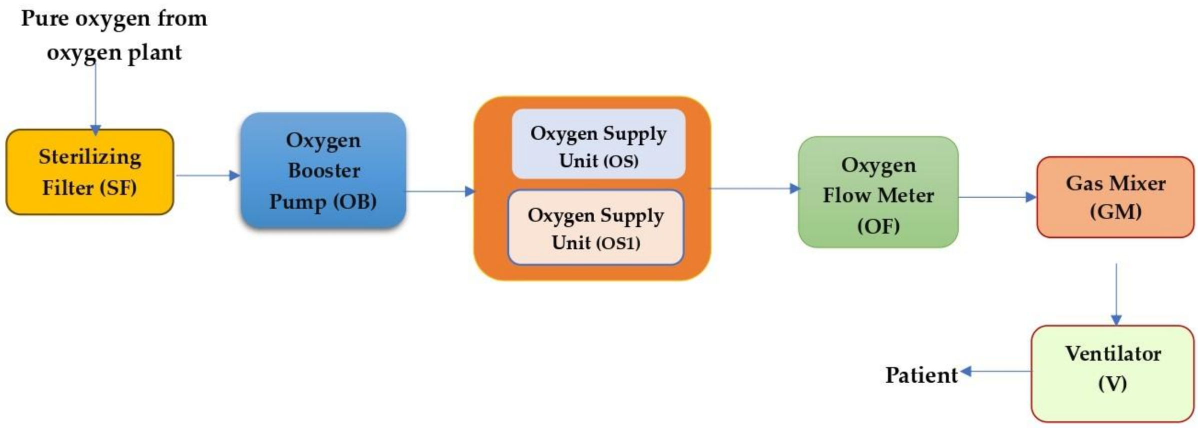

Figure 1. Process flow diagram of medical oxygen plant supply system

1. Sterilizing filter (SF): Subsystem SF is used to stop micro-bacterial particles and impurities of pure oxygen gas from oxygen plant. If the subsystem SF fails, the entire system will completely fail.
2. Oxygen booster pump (OB): Subsystem OB works to increase oxygen pressure and flow to supply units for hospitalised patients. The system will fail completely, if the subsystem OB fails to work.
3. Oxygen supply (OS) units: Subsystem OS consists with two units: one unit is working, and the second unit is kept on cold standby. If one unit supplies oxygen to a patient fails, the second backup unit works immediately. If both the units of subsubsytem OS fail, then the system will fail completely.
4. Oxygen flow (OF) meter: Subsystem OF meter regulates and measures oxygen flow, helping us monitor oxygen absorption status. If subsystem OF fails, the entire system will fail.
5. Gas mixer (GM): Subsystem GM mixes air and pure oxygen gas from the plant in a gas mixer chamber and supplies it to the patient. Failure of subsystem GM causes the complete failure of the system.
6. Ventilator (V): Subsystem V is used to supply oxygen to patients who are unable to inhale it. A ventilator is a lifesaving device for any patient. If subsystem V fails, the system will fail completely.

2.1 Assumptions

1. Initially, all the system units have good condition and work efficiently.
2. Repairmen always available for instant repair.
3. The units of system work as new after their repair.
4. There is no simultaneous failure between the subsystems.
5. Failure time distribution of subsystems of oxygen supply system is taken as exponential while repair time distribution is arbitrary

2.2 *State-specification*

$\boldsymbol{S_0/S_1}$**:** The plant is in good condition and working during initial state/working in good condition because of cold standby unit is in operative mode.
$\boldsymbol{S_2/S_7}$**:** The plant fails due to the failure of sterilizing filter/fails due to the failure of sterilizing filter even cold standby unit of subsystem oxygen supply unit is in working state.
$\boldsymbol{S_3/S_8}$**:** The plant fails due to the failure of booster pump /fails due to the oxygen booster pump even standby unit of subsystem oxygen supply unit is in working mode.
$\boldsymbol{S_4/S_{10}}$**:** The plant fails due to the failure of subsystem oxygen flow meter/fails due to the oxygen flow meter even standby unit of subsystem oxygen supply unit is in working state.
$\boldsymbol{S_5/S_{11}}$**:** The plant fails due to the failure of gas mixer/fails due to the failure of gas mixer even standby unit of the oxygen supply unit is in state of working.
$\boldsymbol{S_6/S_{12}}$**:** The plant fails due to the failure of ventilator/fails due to the failure of ventilator even standby unit of the oxygen supply unit is in operative mode.

$\boldsymbol{S_9}$**:** The plant fails due to the failure of direct oxygen supply unit and cold standby unit.

2.3 Notations

$SF, OB, OS_1, OS_2, OF, GM, V$: Subsystem unit as sterilizing filter/oxygen booster pump/ oxygen supply unit/ oxygen supply unit (standby unit) / oxygen flow meter / gas mixer / ventilator, respectively.

$\overline{SF}, \overline{OB}, \overline{OS_1}, \overline{OS_2}, \overline{OF}, \overline{GM}, \bar{V}$: Failed units of sterilizing filter / oxygen booster pump/ oxygen supply unit / oxygen supply unit (standby unit)/ oxygen flow meter / gas mixer / ventilator, respectively.

$\alpha_{SF}, \alpha_{OB}, \alpha_{OS}, \alpha_{OF,}\alpha_{GM,}\alpha_V$:Failure rate of sterilizing filter / oxygen booster pump/ oxygen supply unit/ oxygen flow meter / gas mixer/ ventilator respectively.

$\beta_{SF}(x), \beta_{OB}(x), \beta_{OS}(x), \beta_{OF}(x), \beta_{GM}(x), \beta_V(x)$: Repair rate of sterilizing filter / Oxygen booster pump/ Oxygen supply unit/ oxygen flow meter/ gas mixer / ventilator respectively.

$p_0(t)$: Probability density that system is operating with full capacity in initial state 0.

$p_i(x,t)\Delta t$: The probability of the system being in the i^{th} state at time t and having an elapsed repair time x, where $i = 1,2,\dots,12$.

$p(s)$: Laplace transformation of function $p(t)$.

s : Laplace transform variable.

3. Model analysis

The system model has six units: a sterilizing filter, oxygen booster pump, oxygen supply unit, oxygen flow meter, gas mixer, and ventilator in the oxygen plant supply system. The repairman is always available for instant system repair, and only one failure is allowed at a time between the subsystems. The system consists of thirteen states (S_0 to S_{12}). Initially, the system is working at full capacity without any failure but if any unit fails, it is called a failure state ($S_2$to S_{12}). At the initial state S_0 where six units are in working condition but these six units can be failed in state S_1 to state S_6 and every units have their failure rate (α) and repair rates (β) and repairman is always available for repair these failed units with the time. But oxygen supply units fails in state S_0, one standby unit comes immediately in working state S_1 and repaireman repairs this unit during stanby unit is working. Same as above state, six units are working with standby unit in state $S_1$and these unit can also be failed in state S_7 to state S_{12} and same as above units have their failure rate and repair rate. The failed unit is repaired instant at a time without stopping production during repair and when primary oxygen supply unit is repaired, it will work again with the state S_0in the model and production will always continuing. Based on all assumptions and notations, the transition diagram of all states of the oxygen plant supply system, namely operating and failed states, is shown in Figure 2.

3.1 Mathematical Modeling and Analysis of the oxygen Plant supply system

Based on this transition diagram (Figure 2) of the oxygen plant supply system, the following set of difference-differential equations can be formulated by using the supplementary variable technique as follows:

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Volume 38 No. 11s 2025,
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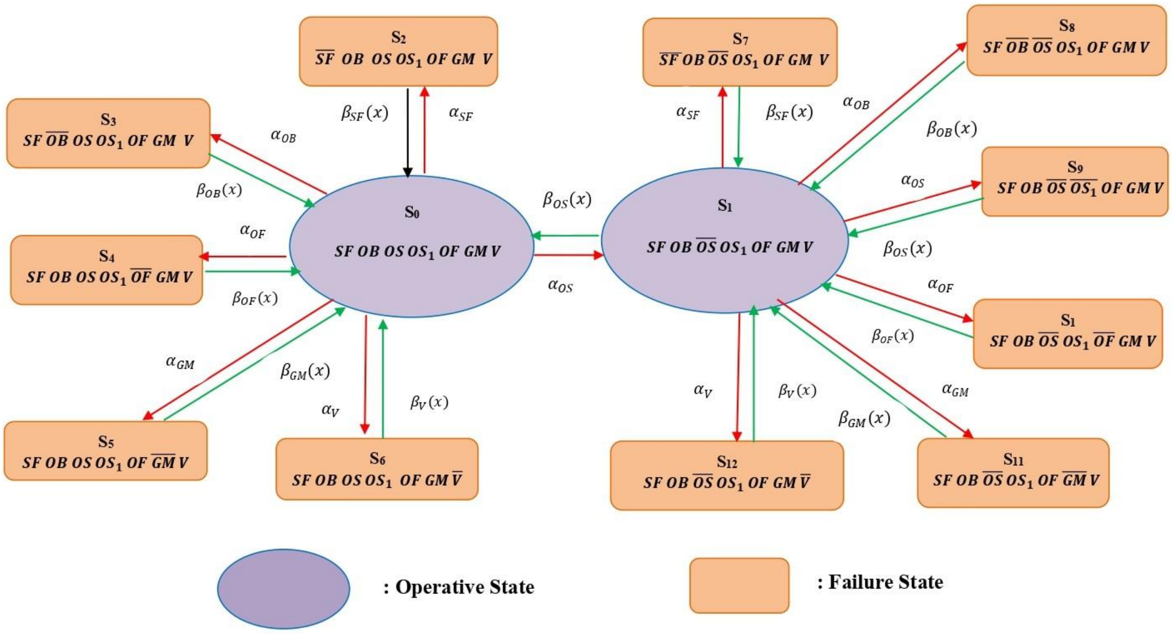

Figure 2 Transition diagram of the oxygen plat supply system

$$\left[\frac{d}{dt} + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V\right] p_0(t) = \int_0^\infty \beta_{SF}(x) p_2(x,t)dx + \int_0^\infty \beta_{OB}(x) p_3(x,t)dx + \int_0^\infty \beta_{OS}(x) p_1(x,t)dx + \int_0^\infty \beta_{OF}(x) p_4(x,t)dx + \int_0^\infty \beta_{GM}(x) p_5(x,t)dx + \int_0^\infty \beta_V(x) p_6(x,t)dx \tag{1}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V + \beta_{OS}(x)\right] p_1(x,t) = \beta_{SF}(x)p_7(x,t) + \beta_{OB}(x)p_8(x,t) + \beta_{OS}(x)p_9(x,t) + \beta_{OF}(x)p_{10}(x,t) + \beta_{GM}(x)p_{11}(x,t) + \beta_V(x)p_{12}(x,t) + \alpha_{OS}p_0(t) \tag{2}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \beta_{SF}(x)\right] p_2(x,t) = \alpha_{SF}p_0(t) \tag{3}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \beta_{OB}(x)\right] p_3(x,t) = \alpha_{OB}p_0(t) \tag{4}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \beta_{OF}(x)\right] p_4(x,t) = \alpha_{OF}p_0(t) \tag{5}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \beta_{GM}(x)\right] p_5(x,t) = \alpha_{GM}p_0(t) \tag{6}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \beta_V(x)\right] p_6(x,t) = \alpha_V p_0(t) \tag{7}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \beta_{SF}(x)\right] p_7(x,t) = \alpha_{SF}p_1(x,t) \tag{8}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \beta_{OB}(x)\right] p_8(x,t) = \alpha_{OB}p_1(x,t) \tag{9}$$

$$\left[\frac{\partial}{dt} + \frac{\partial}{dx} + \beta_{OS}(x)\right] p_9(x,t) = \alpha_{OS}p_1(x,t) \tag{10}$$

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$\left[\frac{\partial}{dt}+\frac{\partial}{dx}+\beta_{OF}(x)\right]p_{10}(x,t)=\alpha_{OF}p_1(x,t) \tag{11}$$

$$\left[\frac{\partial}{dt}+\frac{\partial}{dx}+\beta_{GM}(x)\right]p_{11}(x,t)=\alpha_{GM}p_1(x,t) \tag{12}$$

$$\left[\frac{\partial}{dt}+\frac{\partial}{dx}+\beta_{V}(x)\right]p_{12}(x,t)=\alpha_{V}p_1(x,t) \tag{13}$$

Initial conditions,

$$\left.\begin{array}{l} p_0(0)=1 \\ p_i(t)=0, \quad \text{where } i=1,2,\dots,12. \end{array}\right\} \tag{14}$$

Boundary conditions,

$$\left.\begin{array}{llll} p_1(0,t)=\alpha_{OS}p_0(t); & p_2(0,t)=\alpha_{SF}p_0(t); & p_3(0,t)=\alpha_{OB}p_0(t); & p_4(0,t)=\alpha_{OF}p_0(t) \\ p_5(0,t)=\alpha_{GM}P_0(t); & p_6(0,t)=\alpha_{V}p_0(t); & p_7(0,t)=\alpha_{SF}p_1(t); & p_8(0,t)=\alpha_{OB}p_1(t) \\ p_9(0,t)=\alpha_{OS}p_1(t); & p_{10}(0,t)=\alpha_{OF}p_1(t); & p_{11}(0,t)=\alpha_{GM}p_1(t); & p_{12}(0,t)=\alpha_{V}p_1(t) \end{array}\right\} \tag{15}$$

Eq. (1) is a linear first-order differential equation, while Eqs. (2)-(13) are linear partial differential equations. The set of Eqs. (1)-(13), along with the initial condition (14) and boundary condition (15), are referred to as Chapman Kolmogorov difference differential equations. By applying the Laplace transformation to these equations, we can obtain the probabilities $p_i(s)$, where i ranges from 0 to 12.

$$(s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_{V})p_0(s)=1+\int_0^\infty \beta_{SF}(x)p_2(x,s)dx+\int_0^\infty \beta_{OB}(x)p_3(x,s)dx+\int_0^\infty \beta_{OS}(x)p_1(x,s)dx+\int_0^\infty \beta_{OF}(x)p_4(x,s)dx+\int_0^\infty \beta_{GM}(x)p(x,s)dx+\int_0^\infty \beta_{V}(x)p_6(x,s)dx \tag{16}$$

$$\left\{\frac{d}{dx}+s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_{V}+\beta_{OS}(x)\right\}p_1(x,s)=\beta_{SF}(x)p_7(x,s)+\beta_{OB}(x)p_8(x,s)+\beta_{OS}(x)p_9(x,s)+\beta_{OF}(x)p_{10}(x,s)+\beta_{GM}(x)p_{11}(x,s)+\beta_{V}(x)p_{12}(x,s)+\alpha_{OS}p_0(s) \tag{17}$$

$$\left[\frac{d}{dx}+s+\beta_{SF}(x)\right]p_2(x,s)=\alpha_{SF}p_0(s) \tag{18}$$

$$\left[\frac{d}{dx}+s+\beta_{OB}(x)\right]p_3(x,s)=\alpha_{OB}p_0(s) \tag{19}$$

$$\left[\frac{d}{dx}+s+\beta_{OF}(x)\right]p_4(x,s)=\alpha_{OF}p_0(s) \tag{20}$$

$$\left[\frac{d}{dx}+s+\beta_{GM}(x)\right]p_5(x,s)=\alpha_{GM}p_0(s) \tag{21}$$

$$\left[\frac{d}{dx}+s+\beta_{V}(x)\right]p_6(x,s)=\alpha_{V}p_0(s) \tag{22}$$

$$\left[\frac{d}{dx}+s+\beta_{SF}(x)\right]p_7(x,s)=\alpha_{SF}p_1(x,s) \tag{23}$$

$$\left[\frac{d}{dx}+s+\beta_{OB}(x)\right]p_8(x,s)=\alpha_{OB}p_1(x,s) \tag{24}$$

$$\left[\frac{d}{dx}+s+\beta_{OS}(x)\right]p_9(x,s)=\alpha_{OS}p_1(x,s) \tag{25}$$

$$\left[\frac{d}{dx}+s+\beta_{OF}(x)\right]p_{10}(x,s)=\alpha_{OF}p_1(x,s) \tag{26}$$

$$\left[\frac{d}{dx}+s+\beta_{GM}(x)\right]p_{11}(x,s)=\alpha_{GM}p_1(x,s) \tag{27}$$

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 38 No. 11s 2025,
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$\left[\frac{d}{dx} + s + \beta_V(x)\right] p_{12}(x,s) = \alpha_V p_1(x,s) \tag{28}$$

Laplace of boundary conditions

$$\left.\begin{array}{lll} p_1(0,s) = \alpha_{OS} p_0(s); & p_2(0,s) = \alpha_{SF} p_0(s); & p_3(0,s) = \alpha_{OB} p_0(s) \\ p_4(0,s) = \alpha_{OF} p_0(s); & p_5(0,s) = \alpha_{GM} p_0(s); & p_6(0,s) = \alpha_V p_0(s) \\ p_7(0,s) = \alpha_{SF} p_1(s); & p_8(0,s) = \alpha_{OB} p_1(s); & p_9(0,s) = \alpha_{OS} p_1(s) \\ p_{10}(0,s) = \alpha_{OF} p_1(s); & p_{11}(0,s) = \alpha_E p_1(s); & p_{12}(0,s) = \alpha_V p_1(s) \end{array}\right\} \tag{29}$$

$$p_0(s) = \frac{1}{s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V} \left[\begin{array}{c} [1 + \int_0^\infty \beta_{SF}(x) p_2(x,s)dx + \int_0^\infty \beta_{OB}(x) p_3(x,s)dx + \int_0^\infty \beta_{OS}(x) p_1(x,s)dx + \\ \int_0^\infty \beta_{OF}(x) p_4(x,s)dx + \int_0^\infty \beta_{GM}(x) p_5(x,s)dx + \int_0^\infty \beta_V(x) p_6(x,s)dx] \end{array}\right] \tag{30}$$

$$p_1(s) = \int_0^\infty e^{-\int_0^\infty (s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V+\beta_{OS}(x))dx} \Big\{\alpha_{OS} p_0(s) + \int_0^\infty [\beta_{SF}(x) p_7(x,s) + \beta_{OB}(x) p_8(x,s) + \beta_{OS}(x) p_9(x,s) + \beta_{OF}(x) p_{10}(x,s) + \beta_{GM}(x) p_{11}(x,s) + \beta_V(x) p_{12}(x,s)]\, e^{\int_0^\infty (s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V+\beta_{OS}(x))dx}\Big\} dx \tag{31}$$

$$p_2(s) = \alpha_{SF} p_0(s) \int_0^\infty e^{-\int_0^\infty (s+\beta_{SF}(x)dx} \{1 + \int_0^\infty e^{\int_0^\infty (s+\beta_{SF}(x)dx} dx\} dx \tag{32}$$

$$p_3(s) = \alpha_{OB} p_0(s) \int_0^\infty e^{-\int_0^\infty (s+\beta_{OB}(x)dx} \{1 + \int_0^\infty e^{\int_0^\infty (s+\beta_{OB}(x)dx} dx\} dx \tag{33}$$

$$p_4(s) = \alpha_{OF} p_0(s) \int_0^\infty e^{-\int_0^\infty (s+\beta_{OF}(x)dx} \{1 + \int_0^\infty e^{\int_0^\infty (s+\beta_{OF}(x)dx} dx\} dx \tag{34}$$

$$p(s) = \alpha_{GM} p_0(s) \int_0^\infty e^{-\int_0^\infty (s+\beta_{GM}(x)dx} \{1 + \int_0^\infty e^{\int_0^\infty (s+\beta_{GM}(x)dx} dx\} dx \tag{35}$$

$$p_6(s) = \alpha_V p_0(s) \int_0^\infty e^{-\int_0^\infty (s+\beta_V(x)dx} \{1 + \int_0^\infty e^{\int_0^\infty (s+\beta_V(x)dx} dx\} dx \tag{36}$$

$$p_7(s) = \int_0^\infty e^{-\int_0^\infty (s+\beta_{SF}(x)dx} \{\alpha_{SF} p_1(s) + \int_0^\infty \alpha_{SF} p_1(x,s) e^{\int_0^\infty (s+\beta_{SF}(x)dx} dx\} dx \tag{37}$$

$$p_8(s) = \int_0^\infty e^{-\int_0^\infty (s+\beta_{OB}(x)dx} \{\alpha_{OB} p_1(s) + \int_0^\infty \alpha_{OB} p_1(x,s) e^{\int_0^\infty (s+\beta_{OB}(x)dx} dx\} d \tag{38}$$

$$p_9(s) = \int_0^\infty e^{-\int_0^\infty (s+\beta_{OS}(x)dx} \{\alpha_{OS} p_1(s) + \int_0^\infty \alpha_{OS} p_1(s)\, e^{\int_0^\infty (s+\beta_{OS}(x)dx} dx\} dx \tag{39}$$

$$p_{10}(s) = \int_0^\infty e^{\int_0^\infty -(s+\beta_{OF}(x)dx} \{\alpha_{OF} p_1(s) + \int_0^\infty \alpha_{OF} p_1(x,s)\, e^{\int_0^\infty (s+\beta_{OF}(x)dx} dx\} dx \tag{40}$$

$$p_{11}(s) = \int_0^\infty e^{-\int_0^\infty (s+\beta_{GM}(x)dx} \{\alpha_{GM} p_1(s) + \int_0^\infty \alpha_{GM} p_1(x,s)\, e^{\int_0^\infty (s+\beta_{GM}(x)dx} dx\} dx \tag{41}$$

$$p_{12}(s) = \int_0^\infty e^{-\int_0^\infty (s+\beta_V(x)dx} \{\alpha_V p_1(s) + \int_0^\infty \alpha_V p_1(x,s) e^{\int_0^\infty (s+\beta_V(x)dx} dx\} dx \tag{42}$$

It is noticing that

$$p_0(s) + p_1(s) + p_2(s) + p_3(s) + p_4(s) + p_5(s) + p_6(s) + p_7(s) + p_8(s) + p_9(s) + p_{10}(s) + p_{11}(s) + p_{12}(s) = \frac{1}{s} \tag{43}$$

3.2 *Evaluation of Laplace transformation of Up and Down state probabilities*

The Laplace transforms of the probabilities that the system is in the Up state ($i.e.$ good) and the Down state ($i.e.$ failed) are as stated:

$$p_{up}(s) = p_0(s) + p_1(s)$$

International Journal of Applied Mathematics
Volume 38 No. 11s 2025,
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$$p_{up}(s) = \frac{1}{s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V}\left[\begin{array}{c}[1+\int_0^\infty \beta_{SF}(x)p_2(x,s)dx + \int_0^\infty \beta_{OB}(x)p_3(x,s)dx + \int_0^\infty \beta_{OS}(x)p_1(x,s)dx + \\ \int_0^\infty \beta_{OF}(x)p_4(x,s)dx + \int_0^\infty \beta_{GM}(x)p_5(x,s)dx + \int_0^\infty \beta_V(x)p_6(x,s)dx]\end{array}\right]$$

$$+ \int_0^\infty e^{-\int_0^\infty (s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V+\beta_{OS}(x))dx}\left\{\alpha_{OS}p_0(s) + \int_0^\infty [\beta_{SF}(x)p_7(x,s) + \beta_{OB}(x)p_8(x,s) + \beta_{OS}(x)p_9(x,s) + \beta_{OF}(x)p_{10}(x,s) + \beta_{GM}(x)p_{11}(x,s) + \beta_V(x)p_{12}(x,s)]\, e^{\int_0^\infty (s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V+\beta_{OS}(x))dx}\right\}dx$$

$$p_{down}(s) = p_2(s) + p_3(s) + p_4(s) + p_5(s) + p_6(s) + p_7(s) + p_8(s) + p_9(s) + p_{10}(s) + p_{11}(s) + p_{12}(s)$$

3.3 *Reliability of the system*

The reliability ($R(t)$) of a systempertains to the probability that the system will function efficiently within a specified period. The differential-difference equations for the system reliability are given by

$$[\frac{d}{dt} + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V]\, p_0(t) = 0 \quad (44)$$

$$[\frac{d}{dt} + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V]\, p_1(t) = \alpha_{OS}p_0(t) \quad (45)$$

Taking Laplace transforms of Eqs. (44) and (45), we get

$$[s + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V]\, p_0(s) = 1 \quad (46)$$

$$[s + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V]\, p_1(s) = \alpha_{OS}p_0(s) \quad (47)$$

The solution of Eqs. (46)-(47) are given as

$$p_0(s) = \frac{1}{(s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V)} \quad (48)$$

$$p_1(s) = \frac{\alpha_{OS}}{(s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V)^2} \quad (49)$$

$$R(s) = p_0(s) + p_1(s)$$

$$R(s) = \frac{1}{(s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V)} + \frac{\alpha_{OS}}{(s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V)^2} \quad (50)$$

Taking inverse Laplace transform of Eq. (50), we get

$$R(t) = \alpha_{OS}t \exp(-(\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V)t) + \exp(-(\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V)t) \quad (51)$$

3.4 *Mean time to Failure of System*

Mean time to system failure (MTTF) is defined as the expected time for which the system is in operation before it completely fails. The MTTF is defined as:

$$MTTF = \int_0^\infty R(t)\, dt$$

$$MTTF = \int_0^\infty (\alpha_{OS}t \exp(-(\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V)\mathrm{t}) + \exp(-(\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V)\mathrm{t})\, dt$$

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$$MTTF = \frac{1}{(\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_{V})} + \frac{\alpha_{OS}}{(\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_{V})^2} \tag{52}$$

3.5 Particular case

To show the importance of results and characterize the behavior of availability and profit of the oxygen plant supply system, we assume there pair times are exponentially distributed. For this particular case system of Eqs. (1) - (28) reduces as follows:

$$\left[\frac{d}{dt} + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_{V}\right] p_0(t) = \beta_{SF}p_2(t) + \beta_{OB}p_3(t) + \beta_{OS}p_1(t) + \beta_{OF}p_4(t) + \beta_{GM}p_5(t) + \beta_{V}p_6(t) \tag{53}$$

$$\left[\frac{d}{dt} + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_{V} + \beta_{OS}(x)\right] p_1(t) = \beta_{SF}p_7(t) + \beta_{OB}p_8(t) + \beta_{OS}p_9(t) + \beta_{OF}p_{10}(t) + \beta_{GM}p_{11}(t) + \beta_{V}p_{12}(t) + \alpha_{OS}p_0(t) \tag{54}$$

$$\left[\frac{d}{dt} + \beta_{SF}\right] p_2(t) = \alpha_{SF}p_0(t) \tag{55}$$

$$\left[\frac{d}{dt} + \beta_{OB}\right] p_3(t) = \alpha_{OB}p_0(t) \tag{56}$$

$$\left[\frac{d}{dt} + \beta_{OF}\right] p_4(t) = \alpha_{OF}p_0(t) \tag{57}$$

$$\left[\frac{d}{dt} + \beta_{GM}\right] p_5(t) = \alpha_{GM}p_0(t) \tag{58}$$

$$\left[\frac{d}{dt} + \beta_{V}\right] p_6(t) = \alpha_{V}p_0(t) \tag{59}$$

$$\left[\frac{d}{dt} + \beta_{SF}\right] p_7(t) = \alpha_{SF}p_1(t) \tag{60}$$

$$\left[\frac{d}{dt} + \beta_{OB}\right] p_8(t) = \alpha_{OB}p_1(t) \tag{61}$$

$$\left[\frac{d}{dt} + \beta_{OS}\right] p_9(t) = \alpha_{OS}p_1(t) \tag{62}$$

$$\left[\frac{d}{dt} + \beta_{OF}\right] p_{10}(t) = \alpha_{OF}p_1(t) \tag{63}$$

$$\left[\frac{d}{dt} + \beta_{GM}\right] p_{11}(t) = \alpha_{GM}p_1(t) \tag{64}$$

$$\left[\frac{d}{dt} + \beta_{V}\right] p_{12}(t) = \alpha_{V}p_1(t) \tag{65}$$

Initial Conditions,

$$p_0(0) = 1; \qquad p_i(t) = 0, \text{ where } \quad i = 1, 2, \ldots, 12$$

In a hospital, oxygen plant supply system is expected to operate at full capacity for long period of time, that is why the steady states availability of the system is calculated by taking $\frac{d}{dt} = 0$ as $t \to 0$, $p_i(t) = p_i$ in every equation from (53) to (65). After that, we obtained the subsequent set of steady- state probabilities;

$$\left.\begin{array}{llll} p_1 = \frac{\alpha_{OS}}{\beta_{OS}}P_0; & p_2 = \frac{\alpha_{SF}}{\beta_{SF}}P_0; & p_3 = \frac{\alpha_{OB}}{\beta_{OB}}P_0; & p_4 = \frac{\alpha_{OF}}{\beta_{OF}}P_0; \\ p_5 = \frac{\alpha_{GM}}{\beta_{GM}}P_0; & p_6 = \frac{\alpha_{V}}{\beta_{V}}P_0; & p_7 = \frac{\alpha_{SF}}{\beta_{SF}}\frac{\alpha_{OS}}{\beta_{OS}}P_0; & p_8 = \frac{\alpha_{OB}}{\beta_{OB}}\frac{\alpha_{OS}}{\beta_{OS}}P_0; \\ p_9 = \frac{\alpha_{OS}}{\beta_{OS}}\frac{\alpha_{OS}}{\beta_{OS}}P_0; & p_{10} = \frac{\alpha_{OF}}{\beta_{OF}}\frac{\alpha_{OS}}{\beta_{OS}}P_0; & p_{11} = \frac{\alpha_{GM}}{\beta_{GM}}\frac{\alpha_{OS}}{\beta_{OS}}P_0; & p_{12} = \frac{\alpha_{V}}{\beta_{V}}\frac{\alpha_{OS}}{\beta_{OS}}P_0 \end{array}\right\} \tag{66}$$

Using normalizing condition $\sum_{i=0}^{i=12} p_i = 1$, we get

$$p_0 = [1 + (1 + \frac{\alpha_{OS}}{\beta_{OS}})(\frac{\alpha_{SF}}{\beta_{SF}} + \frac{\alpha_{OB}}{\beta_{OB}} + \frac{\alpha_{OS}}{\beta_{OS}} + \frac{\alpha_{OF}}{\beta_{OF}} + \frac{\alpha_{GM}}{\beta_{GM}} + \frac{\alpha_V}{\beta_V})]^{-1} \quad (67)$$

3.6 Availability of the system

The long run Availability of system (A_v) of the oxygen plant supply system is given as:

$$A_v = p_0 + p_1$$

$$A_v = \left(1 + \frac{\alpha_{OS}}{\beta_{OS}}\right) p$$

$$A_v = \left(1 + \frac{\alpha_{OS}}{\beta_{OS}}\right) [1 + (1 + \frac{\alpha_{OS}}{\beta_{OS}})(\frac{\alpha_{SF}}{\beta_{SF}} + \frac{\alpha_{OB}}{\beta_{OB}} + \frac{\alpha_{OS}}{\beta_{OS}} + \frac{\alpha_{OF}}{\beta_{OF}} + \frac{\alpha_{GM}}{\beta_{GM}} + \frac{\alpha_V}{\beta_V})]^{-1} \quad (68)$$

3.7 Profit Analysis

Hospital requires running the oxygen plant all the time for patients, it should profitable because of opportunity to sell excess oxygen to externally, tax benefit & subsidies, cost for repair the units and pay for labor of plant and as well as for hospital. Profit incurred to the considered oxygen plant supply system model for distinct value of subsystems failure and repair rates in steady state is obtained using Eq. (68) as:

$$\text{Profit} = K_1 A_v - C_R$$

$$\text{Profit} = K_1[\left(1 + \frac{\alpha_{OS}}{\beta_{OS}}\right)\left[1 + \left(1 + \frac{\alpha_{OS}}{\beta_{OS}}\right)\left(\frac{\alpha_{SF}}{\beta_{SF}} + \frac{\alpha_{OB}}{\beta_{OB}} + \frac{\alpha_{OS}}{\beta_{OS}} + \frac{\alpha_{OF}}{\beta_{OF}} + \frac{\alpha_{GM}}{\beta_{GM}} + \frac{\alpha_V}{\beta_V}\right)\right]^{-1}] - C_R \quad (69)$$

Where A_v is steady state availbility of oxygen plant supply system, K_1 is overall revenue of uptime per unit; and$C_R$is overall repair cost of the oxygen plant supply system units.

4. Numerical Analysis

The numerical results are computed for reliability, availability, and expected profit of oxygen plant supply system. Numerical results are presented in Tables from 1 to 5.

Table 1: Effect of failure rate of oxygen supply unit (α_{OS}) on Reliability ($R(t)$) of the oxygen plant supply system

$\alpha_{SF} = 0.001, \alpha_{OB} = 0.002, \alpha_{OF} = 0.004, \alpha_{GM} = 0.003, \alpha_V = 0.001$				
Time (t)	α_{OS}= **0.001**	α_{OS}= **0.002**	α_{OS}= **0.003**	α_{OS}= **0.004**
2	**0.97824**	**0.97823**	**0.97822**	**0.97821**
4	0.95695	0.95692	0.95689	0.95683
6	0.93611	0.93606	0.93598	0.93587
8	0.91573	0.91564	0.91550	0.91530
10	0.89579	0.89566	0.89544	0.89514
12	0.87628	0.87609	0.87579	0.87536
14	0.85719	0.85694	0.85654	0.85598
16	0.83851	0.83820	0.83768	0.83697
18	0.82024	0.81985	0.81922	0.81834
20	0.80236	0.80189	0.80113	0.80008

Table 2: Effect of failure rates of subsystems on the oxygen supply system's Availability

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ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$\beta_{SF} = 0.1, \beta_{OB} = 0.2, \beta_{OS} = 0.3, \beta_{OF} = 0.2, \beta_{GM} = 0.15, \beta_V = 0.1$						
α_{OS} ↓	$\alpha_{SF} = 0.001$ $\alpha_{OB} = 0.002$ $\alpha_{OF} = 0.004$ $\alpha_{GM} = 0.003$ $\alpha_V = 0.001$	$\boldsymbol{\alpha_{SF}} = \mathbf{0.002}$ $\boldsymbol{\alpha_{OB}} = \mathbf{0.002}$ $\alpha_{OF} = 0.004$ $\alpha_{GM} = 0.003$ $\alpha_V = 0.001$	$\alpha_{SF} = 0.001$ $\boldsymbol{\alpha_{OB}} = \mathbf{0.003}$ $\alpha_{OF} = 0.004$ $\alpha_{GM} = 0.003$ $\alpha_V = 0.001$	$\alpha_{SF} = 0.001,$ $\alpha_{OB} = 0.002,$ $\boldsymbol{\alpha_{OF}} = \mathbf{0.005}$ $\alpha_{GM} = 0.003$ $\alpha_V = 0.001$	$\alpha_{SF} = 0.001$ $\alpha_{OB} = 0.002$ $\alpha_{OF} = 0.004$ $\boldsymbol{\alpha_{GM}} = \mathbf{0.004}$ $\alpha_V = 0.001$	$\alpha_{SF} = 0.001$ $\alpha_{OB} = 0.002$ $\alpha_{OF} = 0.004$ $\alpha_{GM} = 0.003$ $\boldsymbol{\alpha_V} = \mathbf{0.002}$
0.001	**0.93457**	**0.92592**	**0.93022**	**0.93022**	**0.92878**	**0.92592**
0.002	0.93454	0.92589	0.93019	0.93019	0.92875	0.92589
0.003	0.93449	0.92584	0.93015	0.93015	0.92871	0.92584
0.004	0.93443	0.92578	0.93008	0.93008	0.92864	0.92578
0.005	0.93434	0.92569	0.92999	0.92999	0.92856	0.92569
0.006	0.93424	0.92559	0.92989	0.92989	0.92845	0.92559
0.007	0.93412	0.92547	0.92977	0.92977	0.92833	0.92547
0.008	0.93397	0.92533	0.92963	0.92963	0.92819	0.92533
0.009	0.93382	0.92518	0.92948	0.92948	0.92804	0.92518
0.010	0.93364	0.92501	0.92930	0.92930	0.92787	0.92501

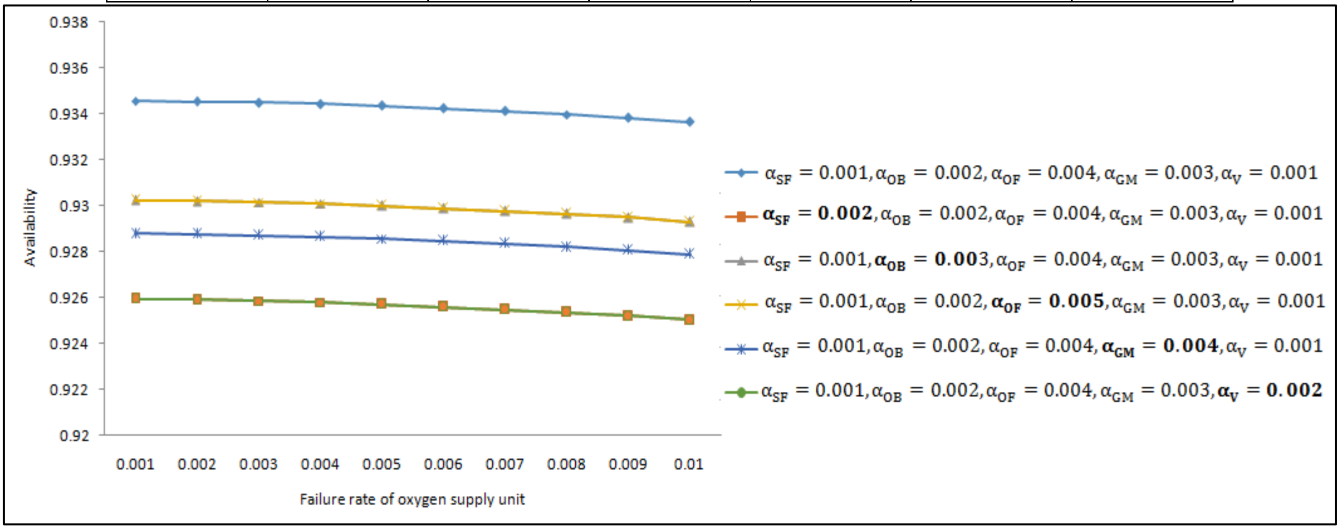

Figure 3: Effect of failure rates of subsystem on availability of oxygen plant supply system

Received: August 06, 2025

International Journal of Applied Mathematics
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ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Table 3: Effect of failure rates of subsystems on profit of oxygen supply system

$\beta_{SF} = 0.1, \beta_{OB} = 0.2, \beta_{OS} = 0.3, \beta_{OF} = 0.2, \beta_{GM} = 0.15, \beta_V = 0.1, K_1 = 5000, C_R = 500$						
α_{OS} ↓	α_{SF}=0.001 α_{OB}=0.002 α_{OF}=0.004 α_{GM}=0.003 α_V=0.001	$\boldsymbol{\alpha_{SF}}$**=0.002** α_{OB}=0.002 α_{OF}=0.004 α_{GM}=0.003 α_V=0.001	α_{SF}=0.001 $\boldsymbol{\alpha_{OB}}$**=0.003** α_{OF}=0.004 α_{GM}=0.003 α_V=0.001	α_{SF}=0.001 α_{OB}=0.002 $\boldsymbol{\alpha_{OF}}$**=0.005** α_{GM}=0.003 α_V=0.001	α_{SF}=0.001 α_{OB}=0.002 α_{OF}=0.004 $\boldsymbol{\alpha_{GM}}$**=0.004** α_V=0.001	α_{SF}=0.001 α_{OB}=0.002 α_{OF}=0.004 α_{GM}=0.003 $\boldsymbol{\alpha_V}$**=0.002**
0.001	**4172.85**	**4129.6**	**4151.1**	**4151.1**	**4143.9**	**4129.6**
0.002	4172.7	4129.45	4150.95	4150.95	4143.75	4129.45
0.003	4172.45	4129.2	4150.75	4150.75	4143.55	4129.2
0.004	4172.15	4128.9	4150.4	4150.4	4143.2	4128.9
0.005	4171.7	4128.45	4149.95	4149.95	4142.8	4128.45
0.006	4171.2	4127.95	4149.45	4149.45	4142.25	4127.95
0.007	4170.6	4127.35	4148.85	4148.85	4141.65	4127.35
0.008	4169.85	4126.65	4148.15	4148.15	4140.95	4126.65
0.009	4169.1	4125.9	4147.4	4147.4	4140.2	4125.9
0.010	4168.2	4125.05	4146.5	4146.5	4139.35	4125.05

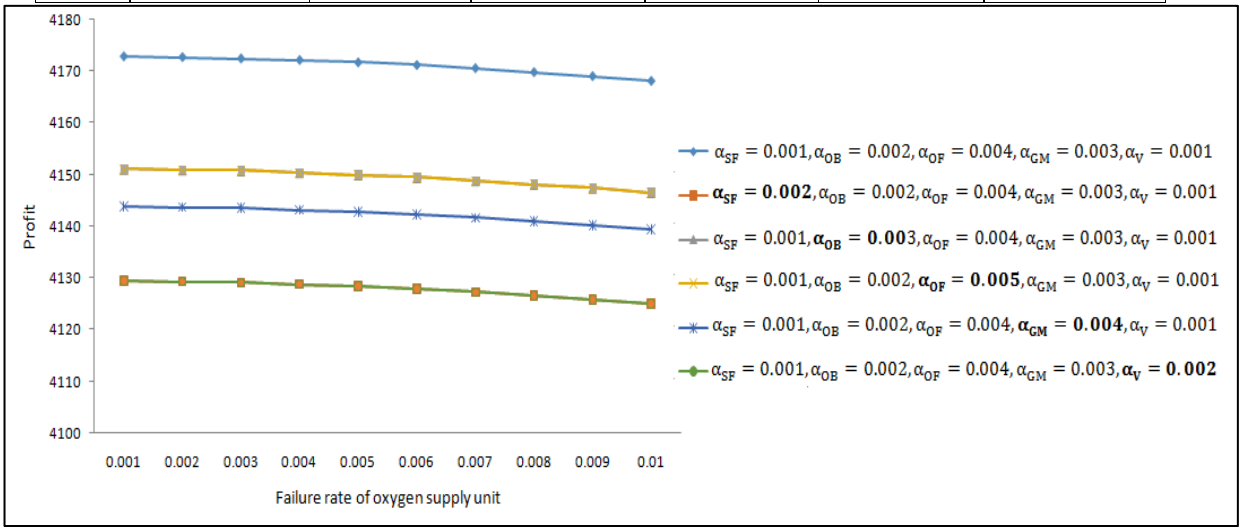

Figure 4: Effect of failure rates of subsystem on profit of oxygen plant supply system

Table 4 Effect of repair rates of subsystems on the oxygen supply system's Availability.

$\alpha_{SF} = 0.001, \alpha_{OB} = 0.002, \alpha_{OS} = 0.001, \alpha_{OF} = 0.004, \alpha_{GM} = 0.003, \alpha_V = 0.001$						
β_{OS} ↓	$\boldsymbol{\beta_{SF}}$**=0.1** β_{OB}=0.2	$\boldsymbol{\beta_{SF}}$**=0.15** β_{OB}=0.2	β_{SF}=0.1 $\boldsymbol{\beta_{OB}}$**=0.25**	β_{SF}=0.1 β_{OB}=0.2	β_{SF}=0.1 β_{OB}=0.2	β_{SF}=0.1 β_{OB}=0.2

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

	β_{OF}=0.25 β_{GM}=0.15 β_V=0.1	β_{OF}=0.25 β_{GM}=0.15 β_V=0.1	β_{OF}=0.25 β_{GM}=0.15 β_V=0.1	$\boldsymbol{\beta_{OF}}$**=0.35** β_{GM}=0.15 β_V=0.1	β_{OF}=025 $\boldsymbol{\beta_{GM}}$**=0.25** β_V=0.1	β_{OF}=025 β_{GM}=0.15 $\boldsymbol{\beta_V}$**=0.2**
0.1	0.93799	0.94094	0.93976	0.94204	0.94509	0.94242
0.15	0.93805	0.94099	0.93981	0.94209	0.94514	0.94247
0.2	0.93806	0.94101	0.93983	0.94210	0.94516	0.94248
0.25	0.93807	0.94102	0.93984	0.94211	0.94517	0.94249
0.3	**0.93808**	**0.94102**	**0.93984**	**0.94212**	**0.94517**	**0.94250**
0.35	0.93808	0.94102	0.93984	0.94212	0.94517	0.94250
0.4	0.93808	0.94102	0.93984	0.94212	0.94517	0.94250
0.45	0.93808	0.94102	0.93985	0.94212	0.94518	0.94250
0.5	0.93808	0.94103	0.93985	0.94212	0.94518	0.94250
0.55	0.93808	0.94103	0.93985	0.94212	0.94518	0.94250

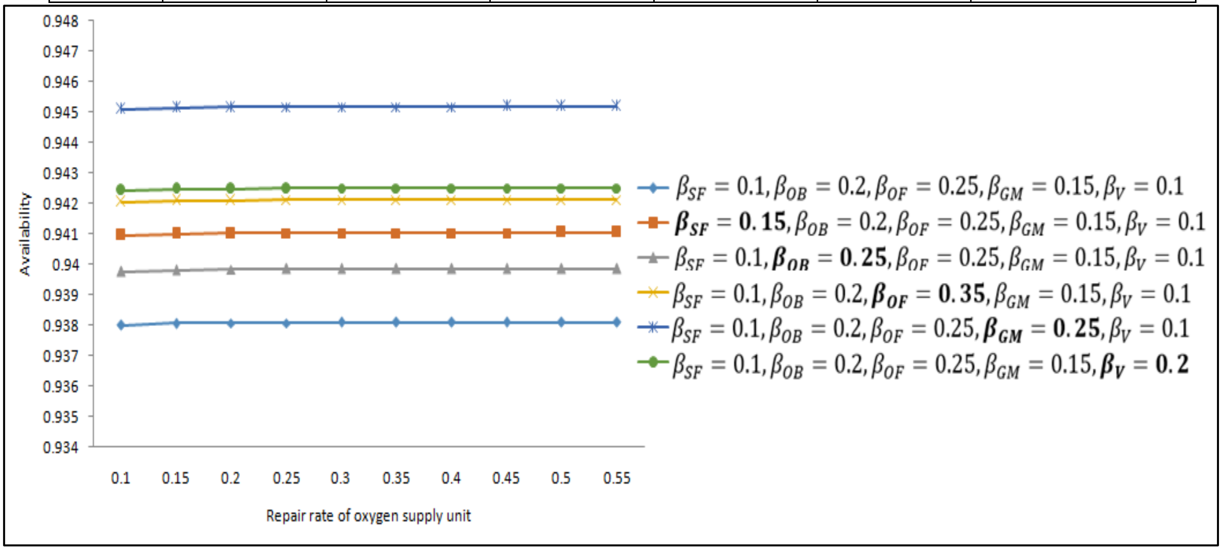

Figure 5: Effect of failure rates of subsystem on availability of oxygen plant supply system

Table 5: Effect of repair rates of subsystems on profit of oxygen supply system

$\alpha_{SF} = 0.001, \alpha_{OB} = 0.002, \alpha_{OS} = 0.001, \alpha_{OF} = 0.004, \alpha_{GM} = 0.003, \alpha_V = 0.001 K_1 = 5000, C_R = 500$						
β_{OS} ↓	$\boldsymbol{\beta_{SF}}$**=0.1** β_{OB}=0.2 β_{OF}=0.25 β_{GM}=0.15 β_V=0.1	$\boldsymbol{\beta_{SF}}$**=0.15** β_{OB}=0.2 β_{OF}=0.25 β_{GM}=0.15 β_V=0.1	β_{SF}=0.1 $\boldsymbol{\beta_{OB}}$**=0.25** β_{OF}=0.25 β_{GM}=0.15 β_V=0.1	β_{SF}=0.1 β_{OB}=0.2 $\boldsymbol{\beta_{OF}}$**=0.35** β_{GM}=0.15 β_V=0.1	β_{SF}=0.1 β_{OB}=0.2 β_{OF}=025 $\boldsymbol{\beta_{GM}}$**=0.25** β_V=0.1	β_{SF}=0.1 β_{OB}=0.2 β_{OF}=025 β_{GM}=0.15 $\boldsymbol{\beta_V}$**=0.2**
0.1	4189.95	4204.7	4198.8	4210.2	4225.45	4212.1

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ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

0.15	4190.25	4204.95	4199.05	4210.45	4225.7	4212.35
0.2	4190.3	4205.05	4199.15	4210.5	4225.8	4212.4
0.25	4190.35	4205.1	4199.2	4210.55	4225.85	4212.45
0.3	**4190.4**	**4205.1**	**4199.2**	**4210.6**	**4225.85**	**4212.5**
0.35	4190.4	4205.1	4199.2	4210.6	4225.85	4212.5
0.4	4190.4	4205.1	4199.2	4210.6	4225.85	4212.5
0.45	4190.4	4205.1	4199.25	4210.6	4225.9	4212.5
0.5	4190.4	4205.15	4199.25	4210.6	4225.9	4212.5
0.55	4190.4	4205.15	4199.25	4210.6	4225.9	4212.5

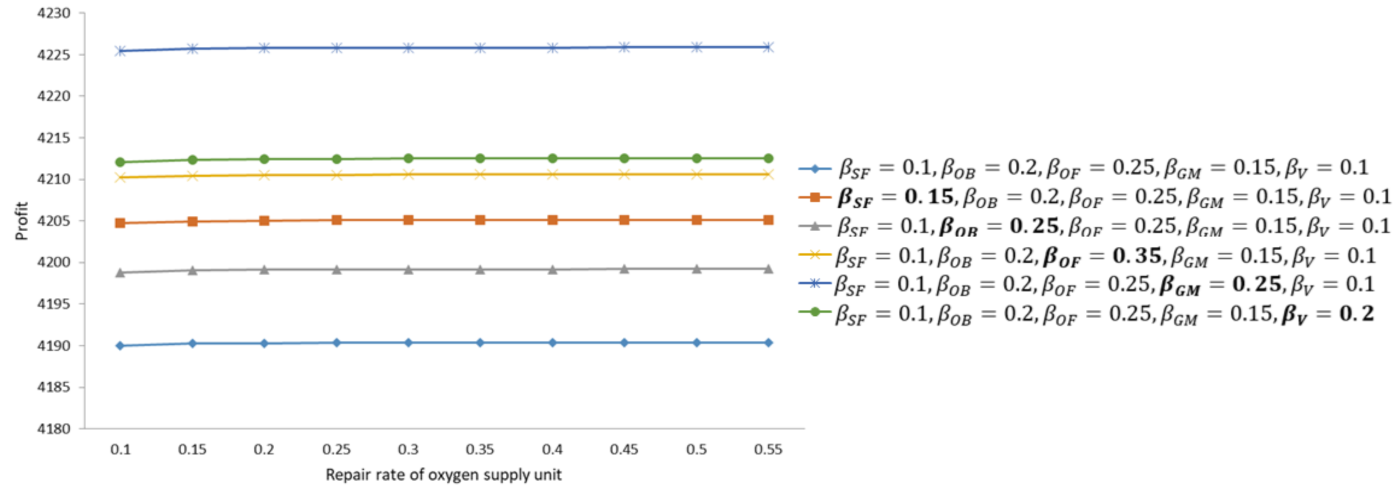

Figure 6: Effect of repair rates of subsystems on profit of oxygen supply system

5. Results and Discussion

The Table 1 reveal the effect of the failure rate of oxygen supply units (α_{OS}) on system reliability. It can be observed that the reliability of the system decreases with the increase in failure rates of oxygen supply unit over a certain time.

Tables 2 and 3 demonstrate how the availability and profit of the oxygen plant supply system are affected by the failure rates of the subsystem. It is observed that availability and profit of the oxygen plant supply system decrease with the increase of failure rates of the subsystems-$SF, OB, OS, OF, GM,$ and V. The analysis carried out reveals that the effect of failure rates of subsystems $SF, GM,$ and V are more as compared to the failure rates of subsystems- OB and OF. The failure rates of subsystems SF and V reduces from 0.002 to 0.001 the availability increases 0.93% and profit increases 1%. Similarly, on reducing the failure rate of gas mixer from 0.004 to 0.003 the availability increases 0.6% and profit increases 0.7%. And, on reducing the failure rate of oxygen booster pump from 0.003 to 0.002 and oxygen flow meter from 0.005 to 0.004 the availability increases 0.5% and profit increases 0.5%. It is found that reducing the failure rate of sterilizing filter, ventilator and gas mixer unit can more improve the system availability and profit.

Tables 4 and 5 show the effect of repair rates of subsystems on availability and profit of the oxygen plant supply system. On increasing of repair rate oxygen supply unit increases little bit availability and profit of system but we can see after 0.3 the availability and profit comes same and on increasing the repair rate of gas mixer from 0.15 to 0.25 the availability increases 0.75% and profit increases 0.85%. And, on increasing the repair rate of ventilator from 0.1 to 0.2 the availability increases 0.5% and profit increases 0.5%. And on increasing 0.05 repair rate of

Received: August 06, 2025

International Journal of Applied Mathematics
Volume 38 No. 11s 2025,
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

oxygen flow meter and oxygen booster pump, the availability and profit of system increase less than 0.5%. It is observed that increasing the repair rate of gas mixer unit and ventilator can improve the availability and profit of system.

6. Conclusion

In the present paper, we have discussed the reliability, availability and profit of oxygen pant supply system for hospitals with standby unit. It is can be observed from Table 1 that reliability of the system little bit decreases with the increase of failure rate of subsystem oxygen supply unit. The failure rate of the subsystem of oxygen pant supply system is considered to be followed by exponential distribution. The results for availability and profit are presented in Tables 2 to 5.

The effect of failure rate of subsystem of oxygen plant supply system on availability and profit of has been examined for different values of various parameters and cost. It is observed that availability and profit of the oxygen supply system increase with the decreasing the failure rates of the subsystemsterilizing filter, ventilator and gas mixer unit. However, the effect of failure rate of subsystem sterilizing filter, ventilator and gas mixer unitis much more in availability and profit as compared to another subsystem. Therefore, there is need to control the failure rates of subsystem sterilizing filter, ventilator and gas mixer unit in order to make oxygen plant supply system more available and profitable for use.

However, the effect of repair rate of subsystem gas mixer unit and ventilator is much more in availability and profit as compared to another subsystem. Therefore, there is need to control the repair rates of subsystem gas mixer and ventilator in order to make oxygen plant supply system more available and profitable for use.

Hence, the study reveals that oxygen plant supply system can be made more available and profitable if hospital pays more attention of repair rate of gas mixer and ventilator and control the failure rates of sterilizing filter, ventilator and gas mixer unit.

Data Availability Statement: NA

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

In a system that becomes time-dependent for either the repair rate or the failure rate, or both, the Markovian characteristic is lost, and future events depend on both the present and the past. These occurrences are referred to as non-Markovian events. In order to transform the system from non-Markovian to Markovian, a new variable called supplemental variable (x) is introduced.

From derivation of Eqs. (1)-(13), Assuming failure rates of the system are exponential and repair rates are arbitrary. By applying supplementary variable technique, we develop the following differential-difference equations associated with the state transition diagram (Fig 2) of the system at times (t +Δt),

$$P_0(t)(t+\Delta t) = [1-\alpha_{SF}\,\Delta t-\alpha_{OB}\,\Delta t-\alpha_{OS}\,\Delta t-\alpha_{OF}\Delta t-\alpha_{GM}\Delta t-\alpha_V\,\Delta t]P_0(t)+\int_0^\infty \beta_{SF}\,(x)P_2(x,t)dx\,\Delta t+\int_0^\infty \beta_{OB}\,(x)P_3(x,t)dx\,\Delta t+\int_0^\infty \beta_{OS}\,(x)P_1(x,t)dx\,\Delta t+\int_0^\infty \beta_{OF}\,(x)P_4(x,t)dx\,\Delta t+\int_0^\infty \beta_{GM}\,(x)P_5(x,t)dx\,\Delta t+\int_0^\infty \beta_V\,(x)P_6(x,t)dx\,\Delta t+0(\Delta t)$$

$$\lim_{\Delta t\to\infty}\frac{P_0(t)(t+\Delta t)-P_0(t)}{\Delta t} = [-\alpha_{SF}-\alpha_{OB}-\alpha_{OS}-\alpha_{OF}-+\alpha_{GM}-\alpha_V]P_0(t)+\int_0^\infty \beta_{SF}\,(x)P_2(x,t)dx+\int_0^\infty \beta_{OB}\,(x)P_3(x,t)dx+\int_0^\infty \beta_{OS}\,(x)P_1(x,t)dx+\int_0^\infty \beta_{OF}\,(x)P_4(x,t)dx+\int_0^\infty \beta_{GM}\,(x)P_5(x,t)dx+\int_0^\infty \beta_V\,(x)P_6(x,t)dx+\lim_{\Delta t\to\infty}\frac{0(\Delta t)}{\Delta t}$$

International Journal of Applied Mathematics
Volume 38 No. 11s 2025,
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$\frac{d}{dt}P_0(t) = [-\alpha_{SF} - \alpha_{OB} - \alpha_{OS} - \alpha_{OF} - +\alpha_{GM} - \alpha_V]P_0(t) + \int_0^\infty \beta_{SF}(x)P_2(x,t)dx + \int_0^\infty \beta_{OB}(x)P_3(x,t)dx + \int_0^\infty \beta_{OS}(x)P_1(x,t)dx + \int_0^\infty \beta_{OF}(x)P_4(x,t)dx + \int_0^\infty \beta_{GM}(x)P_5(x,t)dx + \int_0^\infty \beta_V(x)P_6(x,t)dx$$

$$\left[\frac{d}{dt} + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V\right]P_0(t) = \int_0^\infty \beta_{SF}(x)P_2(x,t)dx + \int_0^\infty \beta_{OB}(x)P_3(x,t)dx + \int_0^\infty \beta_{OS}(x)P_1(x,t)dx + \int_0^\infty \beta_{OF}(x)P_4(x,t)dx + \int_0^\infty \beta_{GM}(x)P_5(x,t)dx + \int_0^\infty \beta_V(x)P_6(x,t)dx$$

Appendix B

Solution of partial differential Eqs. (17-26). For illustration let us consider Equation (17)

$$\left\{\frac{d}{dx} + s + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V + \beta_{OS}(x)\right\}P_1(x,s) = \beta_{SF}(x)P_7(x,s) + \beta_{OB}(x)P_8(x,s) + \beta_{OS}(x)P_9(x,s) + \beta_{OF}(x)P_{10}(x,s) + \beta_{GM}(x)P_{11}(x,s) + \beta_V(x)P_{12}(x,s) + \alpha_{OS}P_0(s)$$

After taking Initial and boundary condition

$$P_2(0) = 1, \quad P_2(x,0) = 0, \quad P_2(0,s) = \alpha_{OS}P_0(s)$$

This is first order linear partial differential equation and solved this using initial and Laplace boundary conditions. The equation is

$$\frac{d}{dx}P_1(x,s) + (s + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V + \beta_{OS}(x))\,P_1(x,s) = \beta_{SF}(x)P_7(x,s) + \beta_{OB}(x)P_8(x,s) + \beta_{OS}(x)P_9(x,s) + \beta_{OF}(x)P_{10}(x,s) + \beta_{GM}(x)P_{11}(x,s) + \beta_V(x)P_{12}(x,s) + \alpha_{OS}P_0(s)$$

Apply the Laplace Transform to each term in the equation

$L\left\{\frac{d\,P_1(x,s)}{dx}\right\} = sP_1(s) - P_1(0,s)$ and take $L\{P_1(x.s)\} = P_1(s)$

$$sP_1(s) - P_1(0,s) + (s + \alpha_{SF} + \alpha_{OB} + \alpha_{OS} + \alpha_{OF} + \alpha_{GM} + \alpha_V + \beta_{OS}(x))\,P_1(s) = \beta_{SF}(x)P_7(x,s) + \beta_{OB}(x)P_8(x,s) + \beta_{OS}(x)P_9(x,s) + \beta_{OF}(x)P_{10}(x,s) + \beta_{GM}(x)P_{11}(x,s) + \beta_V(x)P_{12}(x,s) + \alpha_{OS}P_0(s)$$

The solution of this equation is

$$P_1(s) = \int_0^\infty e^{\int_0^\infty (s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V+\beta_{OS}(x)dx} \int_0^\infty [\beta_{SF}(x)P_7(x,s) + \beta_{OB}(x)P_8(x,s) + \beta_{OS}(x)P_9(x,s) + \beta_{OF}(x)P_{10}(x,s) + \beta_{GM}(x)P_{11}(x,s) + \beta_V(x)P_{12}(x,s)]\, e^{\int_0^\infty (s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V+\beta_{OS}(x)dx}]dx + \alpha_{OS}P_0(s)[1 + \int_0^\infty e^{\int_0^\infty (s+\alpha_{SF}+\alpha_{OB}+\alpha_{OS}+\alpha_{OF}+\alpha_{GM}+\alpha_V+\beta_{OS}(x)dx}]dx$$

Similarly, we can solve other Eqs. (18)- (28).

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