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A GENETIC ALGORITHM FOR MULTICOMMODITY STOCHASTIC-FLOW NETWORKS SYSTEM WITH UNRELIABLE NODES

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Abstract.
Many real-life systems can be modelled as stochastic-flow networks, where resource flows traverse through unreliable nodes and arcs. This paper addresses the problem of optimizing resource flow allocation and control strategies in multicommodity stochastic-flow networks under budget constraints. To evaluate the reliability of such networks, we develop a genetic algorithm (GA) that identifies every vector with limited capacity meeting the specified demand and budget constraints. The system reliability is then resolute based on these vectors, ensuring the maximization of the probability that sink nodes' demands are satisfied as resource flows transmit from source nodes. The proposed GA is designed to efficiently seek the optimal allocation strategy, balancing computational complexity with effectiveness. To demonstrate the efficacy of the algorithm, a numerical example is presented. Results show that the GA not only achieves high reliability but also exhibits better temporal efficiency, making it applicable to larger and more complex networks.
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1 Introduction

A Multicommodity network flow systems play a important role in modelling and optimizing various real-world systems, such as transportation, communication, logistics, and energy networks. These systems often involve the movement of multiple types of resources (commodities) through interconnected nodes and arcs. However, the reliability of such systems is frequently compromised by uncertainties, such as failures or disruptions in nodes and arcs, making it challenging to ensure optimal flow and meet demand at sink nodes. Stochastic-flow networks (SFNs) are essential models for analyzing and optimizing real-life systems where resources must flow through complex structures with inherent uncertainties. Such networks are widely encountered in transportation, communication, supply chains, and energy systems, where components like nodes and arcs may fail or function unreliably. These uncertainties significantly impact the performance and reliability of the network, especially in scenarios involving multiple types of resources, often referred to as multicommodity stochastic-flow networks.

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Operational research provides powerful tools and methodologies for analyzing complex systems, enabling decision-makers to optimize processes, allocate resources effectively, and handle uncertainties in real-world applications. Among the many models studied in this field, multicommodity stochastic-flow networks (MSFNs) have emerged as a critical framework for representing systems where multiple types of resources (commodities) traverse interconnected nodes and arcs under uncertain conditions.

In MSFNs, the uncertainty arises from the stochastic nature of system components, such as unreliable nodes and arcs whose capacities are subject to random fluctuations or failures. This uncertainty poses significant challenges for ensuring the network can meet demand at sink nodes while maintaining efficient use of resources. The inherent complexity of these networks, particularly when dealing with multiple commodities, necessitates advanced techniques to evaluate reliability and design optimal flow allocation strategies.

This research focuses on developing a novel approach to optimize MSFNs by addressing two key aspects:

1. **Reliability Evaluation**: Determining the probability that the network can satisfy requirement at sink nodes, considering the stochastic capacity of arcs and nodes.

2. **Optimization**: Identifying resource allocation and control strategies that maximize reliability while adhering to budgetary and capacity constraints.

To achieve these objectives, we propose a computational framework based on advanced optimization techniques tailored for stochastic environments. The framework evaluates system reliability by generating minimal capacity vectors that meet demand constraints and employs optimization methods to find the best allocation of resources. This approach not only addresses the computational challenges of MSFN analysis but also ensures scalability to larger and more complex networks. The paper contributes to the field by providing both theoretical insights and practical applications, including a numerical example to illustrate the methodology. The findings have significant implications for operational research, particularly in designing resilient and efficient network systems capable of operating under uncertainty.

To address this challenge, research on multicommodity network flow systems has increasingly focused on incorporating uncertainty into network models. Unreliable nodes introduce significant complexity, as they can impact the capacity and availability of multiple arcs, disrupting the flow of resources. This paper presents a study that leverages genetic algorithms (GAs) to analyses and optimize the performance of multicommodity network flow systems with unreliable nodes.

Genetic algorithms (GA) have been used by many researchers in the design and implementation of network reliability in the past few years. For example, Kumar et al. [18] developed a Computer Network Expansion Methodology using GA-based optimization for solving three network design problems: namely, reliability, diameter, and average distance. In Dengiz et al. [9], GA were applied to resolve the problems with the design of all-terminal networks, taking reliability and cost into account. In Pierre and Legault [23], the k-connectivity concept was the measure for reliability in the design of distributed computer network topologies. Chen et al. [5]

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used GA to determine a distributed system's S-node set reliability optimization. Kumar et al. [17] used a Pareto Converging GA to simultaneously solve multi-objective genetic optimization representing costs and delay of a computer communication network. Gen et al. [11] discuss about Genetic algorithms and their applications. Mrabet et al. [21] developed a Capacity Optimization on Networks Based on Genetic Algorithm. Several recent studies by Rizwanullah and collaborators have contributed significantly to the field of optimization, inventory management, and supply chain analysis. Meena and Rizwanullah [20] developed a two-warehouse inventory model for non-instantaneous deteriorating items under the effects of inflation and preservation technology. Yadav and Rizwanullah [28] proposed fuzzy-based combinatorial optimization approaches for the travelling salesman problem using two optimal methods. Bhuranda and Rizwanullah [4] addressed the stochastic travelling salesman problem through a genetic algorithm with a roulette wheel method. Furthermore, Singh and Rizwanullah [27] explored sustainable supplier selection in green supply chain frameworks by applying the fuzzy TOPSIS method. Collectively, these works highlight diverse applications of fuzzy logic, metaheuristics, and multi-criteria decision-making in tackling real-world operational research problems.

Recent studies published in the Journal of Applied Data Analysis and Modern Stochastic Modelling have explored various aspects of stochastic systems and applied mathematical modelling. Belopolskaya [29] investigated stochastic models for systems of forward Kolmogorov equations, providing insights into probabilistic dynamics. Gliklikh and Ryazantsev [30] examined the completeness of flows generated by stochastic algebraic-differential equations under the rank-degree condition, contributing to the theoretical development of stochastic analysis. Additionally, Ilolov and Rahmatov [31] addressed inverse problems in geothermic, emphasizing practical applications of stochastic and analytical techniques in geophysical contexts.

Optimization problems can be effectively solved by genetic algorithms in complex and uncertain systems due to their ability to explore large solution spaces efficiently and adaptively. By employing a GA-based approach, we aim to identify optimal flow allocation strategies that maximize the probability of satisfying sink node demands while accounting for node unreliability. The proposed method evaluates system reliability and provides a robust framework for resource allocation under uncertainty. Chung at al. [6] in the context of multi-resource allocation with a focus on reliability in a stochastic flow network, the impacts of this study involve the improvement of a GA framework for analyzing multicommodity network flow systems, the integration of node unreliability into the optimization process, and the demonstration of the algorithm's performance through a numerical example. The findings highlight the GA's capability to provide reliable and computationally efficient solutions, making it a valuable tool for managing resource flows in uncertain network environments.

2 Problem Formulation

Let G = (A, N, M, S, T) denotes a multi-source stochastic-flow network, where $A = \{a_i | 1 \leq i \leq j\}$ is the set of n directed arcs and $N = \{a_i | j + 1 \leq i \leq j + p\}$ is the set of nodes except source nodes and sink nodes. Let $M = (M_1, M_2 \dots M_{j+p})$ with M_i being the maximal capacity of a_i. In particular, we let $S = \{S_1, S_2 \dots S_\alpha)$ be the set of source nodes, and $T = \{T_1, T_2 \dots T_\beta)$

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be the set of sink nodes. Let $R = \{r_{\gamma,n} | 1 \leq \gamma \leq m, 1 \leq n \leq \alpha\}$ with $r_{\gamma,n}$ being the maximal quantity of resource γ that source node S_n can supply. Let $D = \{d_{\gamma,l} | 1 \leq \gamma \leq m, 1 \leq l \leq \theta\}$ with $d_{\gamma,l}$ being the demand of resource γ at sink node T_l.Let $X = (X_1, X_2 \dots X_{j+p})$ be the current state vector, where x_i is the current capacity of arc ae and is an integer-valued random variable. Furthermore, the various components' capabilities are statistically independent.

We suppose that all of the nodes in G are perfect and reliable and that during flow transmission no flow would disappear or be created at arcs and nodes, except at source and sink nodes. We also assume that the capacities of the arcs in g are statistically independent random variables, taking on nonnegative integral values. Table1 illustrates one possible instant of the probability mass functions of the arc capacities for the stochastic-flow network in Fig.1. Note that the probability mass functions of all possible capacities for each arc sum to 1.

Our objective is to determine the best strategy for distributing and managing resource flow across various arcs and nodes. To comply with the flow-conservation law, traditional integer programming would require the inclusion of complex constraints. In this approach, we leverage the concept of MPs to simplify the original problem and minimize the number of constraints. As a result, we divide the task into two stages. First, we identify the optimal allocation pattern f across all MPs, where f is a vector representing the flow quantity for each MP. Second, for each arc and node, we apply the formula (1) to aggregate the flow quantities from each MP that intersects with the respective arc or node. This allows us to determine the optimal allocation for all arcs and nodes.

In order to compare two path vectors, we define that a vector X is less than or equal to another vector Y, $(X \leq Y)$ if $X_{\gamma,l} \leq Y_{\gamma,l}$ for all γ, l and that $X < Y$ if $X_{\gamma,l} \leq Y_{\gamma,l}$ for all γ, l with $X_{\gamma,l} < Y_{\gamma,l}$ for some γ and l. We also define two functions, the structure function $\phi(X)$ and the system-state function $\psi(X)$ to characterize a state vector. The structure function $\phi(X) = 1$ if the stochastic-flow network can transmit one unit of resource from at least one of the source nodes to at least one of the sink nodes under X, 0, other wise. The system-state function $\psi(X)$ on the other hand, is defined to be the system demand of the stochastic-flow network, in form of a vector, at the sink nodes under the state vector X .Then, a state vector X is a path vector if $\phi(X) = 1$ and X is a minimal path vector if $\phi(X) = 0$ for all $Y < X$. Further, if X is a minimal path vector, the set $\{a_l | x_l = \sum_{\gamma=1}^{m} X_{\gamma,l} > 0, l = 0,1, \dots, j\}$ is a minimum path set (MP). We suppose that MPs are identified in a stochastic-flow network. Furthermore, a state vector X is a path vector to demand d, if $\phi(X) = d$ and X is a minimal path vector to demand d, denoted by d d- MP, if $\psi(y) < \psi(X)$ for all $Y < X$.

We make every f as the solution of model and objective function is the probability of successfully transmitting resources according to f . For each flow allocation pattern f, we can construct a vector $X_f = (x_{f1}, x_{f1}, \dots, x_{fi}, \dots, x_{f(j+p)})$

$$x_{fi} = \sum_{n=1}^{\alpha} \sum_{l=1}^{\beta} \sum_{k=1}^{k_{n,l}} \sum_{\gamma=1}^{m} \{f_{n,l,k,\gamma} | a_i \in MPs_{n,l,k}\} \quad (1)$$

Make Ω be the set of all generated X_f. $X = (x_1, x_2, \dots, x_i, \dots, x_{j+p})$ is network's capacity vector. It's element x_i is the capacity of arc or node a_e and is a random variable that obeys a known distribution. To arc or node a_i , the probability that transmission is successful is Pr $\{x_i \geq x_{fi}\}$ and we use B_i represents $\{x_i \geq x_{fi}\}$. So, the objective function of Network reliability is

$$R(X_f) = \Pr\{B_1 \cap B_2 \cap \dots \cap B_{j+p}\} = \prod_{i=1}^{j+p} \Pr\{B_i\}$$

The model of the problem is as follows:

$$MaxR(X_f) \qquad \forall X_f \in \Omega \qquad (2)$$

$$\sum_{n=1}^{\alpha} \sum_{k=1}^{k_{n,l}} f_{n,l,k,\gamma} = d_{\gamma,f} \qquad \forall \gamma = 1, \dots, m; l = 1, \dots, \beta \qquad (3)$$

$$\sum_{n=1}^{\beta} \sum_{k=1}^{k_{n,l}} f_{n,l,k,\gamma} \leq r_{\gamma,l} \qquad \forall \gamma = 1, \dots, m; n = 1, \dots, \alpha \qquad (4)$$

$$x_{fi} \leq M_i \qquad i = 1, \dots, j+p \qquad (5)$$

$$f_{i,j,k,w} \in N^* \qquad \forall \gamma = 1, \dots, m; l = 1, \dots, \beta \qquad (6)$$

$$n = 1, \dots, \alpha; \;\; k = 1,2, \dots, k_{n,l}$$

N^* is collection of nonnegative integers.

3 Purpose Genetic Algorithm

Genetic algorithms were chosen as the optimization method due to their adaptability and effectiveness, which have been demonstrated across numerous NP-hard problems. As a meta-heuristic, they draw inspiration from the process of natural evolution. Initially introduced by Holland [26], De Jong [16], and Goldberg [12] for tackling continuous non-linear optimization challenges, genetic algorithms have since been expanded to address combinatorial problems by various researchers. Alhijawi and Awajan[1] also provide insights into genetic algorithms, including their theoretical foundations, genetic operators, solutions, and applications. In this approach, the search space consists of potential solutions, represented as strings known as chromosomes. Each chromosome is assigned a fitness value based on the objective function. A group of chromosomes, along with their corresponding fitness values, forms a population. The population at any specific iteration of the algorithm is referred to as a generation.

The GA repetitive loop consists of three primary steps:

1) The selection of current generation strings to be parents of the following generation, with a preference for more fit strings. This is how reproduction selection works.

2) Crossover is the technique of crossing two chosen strings to create new children's strings. Chromosome components are likely to be disturbed during the process of producing a child.

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We refer to this process as mutation. Reproduction is formed up of both crossover and mutation.

3) Using each new solution's goal function, the fitness value is determined. A flowchart of the algorithm is included after a discussion of the phases in the GA technique used in this study.

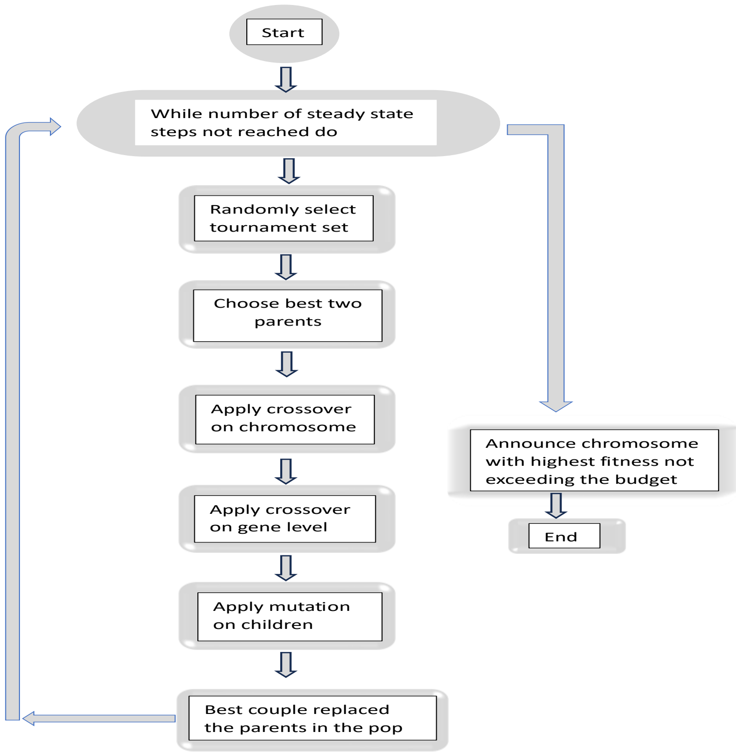

Fig. 1. Steady-state GA implication

We utilize integer encoding, where a flow pattern is represented as a chromosome, and the flow amount at each MP is encoded as a gene. The chromosome's structure is defined as follows:

$$(f_{1,1,1,1,},\dots,f_{n,l,k_{n,l},m,1},\dots,f_{n,l,k_{n,l},m},\dots,f_{\alpha,\beta,k_{\sigma,\beta},m,})$$

3.1 Operators

Algorithm adopt binary tournament selection as selection operator and compulsively keep elitist to next generation. We improve arithmetic crossover operator to adapt to our problem. The operator is as follows:

$$f_X^{t+1} = \lceil \theta\, f_Y^t \rceil + \lfloor (1-\theta) f_X^t \rfloor$$

$$f_Y^{t+1} = \lceil \theta\, f_X^t \rceil + \lfloor (1-\theta) f_Y^t \rfloor \qquad (7)$$

We design a chaos mutation operator acts on chromosomes. Operation is as follows:

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$$\begin{cases} \mu^{t+1} = \rho\mu^{t}(1-\theta) \\ chaos^{t+1}(-1,1) = (\mu^{t+1}-0.5)*2 \\ f_n^{t+1}(l) = f_n^{l}(l) + \lceil \alpha(l) \rceil\, chaos^{t+1}(-1,1) \end{cases} \tag{8}$$

Let $\rho = 4$ and $chaos(-1,1)$ represents a chaos sequence values in $(-1,1)$. $f_n(l)$ is the l^{th} element of chromosomes f_n , $\alpha(l)$ is the mutation scale of element $f_n(l)$.

3.2 Constraints handling

We employ a penalty parameter method to address constraints in this problem. Solutions that violate the constraints are assigned a penalty value, which reduces their fitness score. As a result, these infeasible solutions have a lower possibility of being nominated for the next generation when the binary tournament selection method is applied. Over time, this process ensures that infeasible solutions are progressively eliminated.

The penalty-parameter of node j t that can't satisfy constraint (3) is:

$$Pt_l(f) = \sum_{\gamma=1}^{m} \left| \sum_{n=1}^{\alpha} \sum_{k=1}^{k_{n,l}} f_{n,l,k,\gamma} - d_{\gamma,l} \right| \qquad l = 1, \dots, \theta \tag{9}$$

The penalty-parameter of node i s that can't satisfy constraint (4) is:

$$Ps_n(f) = \sum_{\gamma=1}^{m} \max\left(\sum_{l=1}^{\beta} \sum_{k=1}^{k_{n,l}} f_{n,l,k,\gamma} - r_{\gamma,n}, 0\right) \qquad n = 1, \dots, \alpha \tag{10}$$

The penalty-parameter of arc or node e a that can't satisfy constraint (5) is:

$$Pa_i(f) = \max\left(x_{fi} - M_{i}, 0\right) \qquad i = 1, \dots, j+p \tag{11}$$

4 Fitness function

A quadratic penalty function is combined into the total cost of all network links to account for networks that fail to meet the minimum reliability criteria, forming the basis of the objective function. The penalty function's goal is to guide the optimization process towards feasible, nearly optimum solutions. Allowing infeasible solutions to enter the population is important because good solutions are frequently the product of breeding between feasible and infeasible solutions, and even when both parents are feasible, the genetic algorithm reproduction process does not guarantee feasible children [8], particularly in cases involving highly constrained problems where constraints are likely to be active. In genetic algorithms, the problem is framed as maximization, as the fitness, F(x), typically improves alongside the objective function. Consequently, the fitness is defined in this study as:

The last fitness function with penalty parameter is:

$$F(f) = R(X_f) - \sum_{l=1}^{\beta} \lambda t_l Pt_l(f) - \sum_{n=1}^{\alpha} \lambda s_n Ps_n(f) - \sum_{i=1}^{j+p} \lambda a_i Pa_i(f) \tag{13}$$

$\lambda t_l, \lambda s_n, \lambda a_i$ represent distinct weights assigned to various penalty functions, and they adhere to the following conditions:

$$\sum_{l=1}^{\beta} \lambda t_l + \sum_{n=1}^{\alpha} \lambda s_n + \sum_{i=1}^{j+p} \lambda a_i = 1 \tag{14}$$

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If chromosome satisfies the constraints, fitness function $F(f) = R(X_f)$

5 Algorithm

A chromosome is chosen according to its fitness value, and the program selects new parents using the roulette wheel selection process (Hassan [13]).

Start

Enter the network's data, including the number of edges, their reliability, demand d, and path-ways.

Set the GA parameters.

Generate primary population.

Determine Fit$(f)$for each chromosome after monitoring the initial population.

while $g <= g_{max}$, do

while $\rho <= \rho_{max}$, do

Using the roulette wheel selection method, choose two parents. In accordance with λ and μ, respectively, use crossover and mutation to create a new child.

$\rho = \rho + 1$

End do.

Calculate the current population.

$g := g + 1$

End do.

End

Network

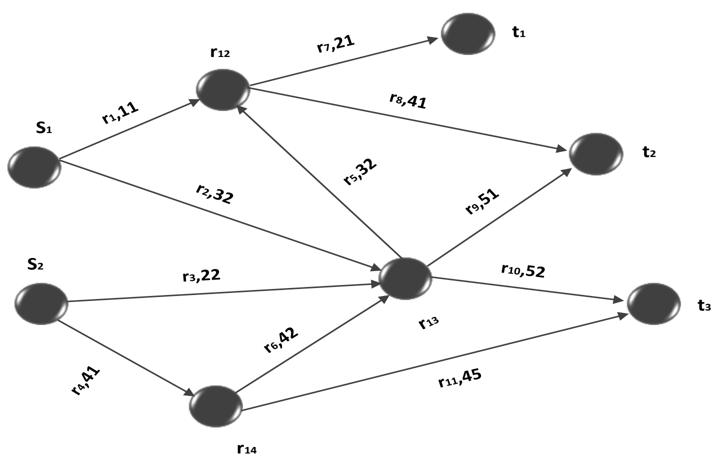

Fig. 2.A two-source and three-silk Stochastic-flow network with unreliable nodes

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Let $P = \{p_{n,l,k} | n = 1, \ldots, \sigma, l = 1, \ldots, \theta, k = 1,2, \ldots, k_{n,l}\}$ be the collection of the MPs since source nodes to sink nodes in a stochastic-flow network, where $p_{n,l,k}$ and $k_{n,l}$ represent the k[th] MP from s_n to s_l and the number of MPs from s_n to t_l, respectively. Let $k = \sum_n \sum_l k_{n,l}$ to be the total number of the MPs, and $(f_{1,1,1,1,,\ldots,}f_{n,l,k_{n,l},m,1,\ldots,}f_{n,l,k_{n,l},m,\ldots,}f_{\alpha,\beta,k_{\sigma,\beta},m,})$ be a flow pattern through P, where $f_{n,l,k_{n,l},m}$ is the non-negative integral f low of resource m through $p_{n,l,k}$. Taking the stochastic-flow network in Fig. 2, for representative, there are thirteen MPs present.

We use the network shown in Figure 1 to illustrate the proposed genetic algorithm. This network contains two source nodes and two sink nodes. Let $M = (11,32,22,41,32,42,21,41,51,52,45)$, $R = (r_1, r_2, r_3, r_4) = (10,13,14,19)$, $D = (d_{11}, d_{12}, d_{13}, d_{21}, d_{22}, d_{23}) = (7,8,6,8,7,8)$. Table 1 shows the probability distribution of each arc's capacity in the stochastic flow network. In this example, existed MPs are as follows:

$MPs_{111} = \{a_1, a_{12}, a_7\}$, $MPs_{112} = \{a_1, a_{13}, a_5, a_{12}, a_7\}$,$MPs_{121} = \{a_1, a_{12}, a_8\}$

$MPs_{122} = \{a_2, a_{13}, a_9\}$, $MPs_{123} = \{a_2, a_{13}, a_5, a_{12}, a_8\}$, $MPs_{131} = \{a_2, a_{13}, a_{10}\}$

$MPs_{211} = \{a_3, a_{13}, a_5, a_{12}, a_7\}$, $MPs_{212} = \{a_4, a_{14}, a_6, a_{13}, a_5, a_{12}, a_7\}$,

$MPs_{221} = \{a_3, a_{13}, a_9,\}MPs_{222} = \{a_4, a_{14}, a_6, a_{13}, a_9\}$,

$MPs_{231} = \{a_3, a_{13}, a_{10}\}$,$MPs_{232} = \{a_4, a_{14}, a_6, a_{13}, a_{10}\}MPs_{233} = \{a_4, a_{14}, a_{11}\}$

Table 1. shows the probability distribution of each arc's capacity in the stochastic flow network

Arc	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
a1	0.001	0.002	0.003	0.005	0.008	0.009	0.010	0.012	0.015	0.010	0.895
a2	0.001	0.001	0.003	0.005	0.005	0.010	0.011	0.017	0.018	0.020	0.025
a3	0.001	0.001	0.002	0.002	0.003	0.005	0.007	0.009	0.016	0.021	0.024
a4	0.001	0.001	0.002	0.003	0.004	0.005	0.007	0.008	0.009	0.011	0.015
a5	0.001	0.002	0.002	0.004	0.005	0.006	0.007	0.009	0.014	0.017	0.020
a6	0.001	0.001	0.002	0.002	0.003	0.004	0.005	0.007	0.009	0.011	0.015
a7	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.007	0.008
a8	0.001	0.001	0.002	0.003	0.005	0.008	0.009	0.011	0.013	0.014	0.015
a9	0.001	0.001	0.002	0.005	0.008	0.010	0.012	0.015	0.015	0.017	0.020
a10	0.001	0.002	0.005	0.005	0.007	0.008	0.010	0.012	0.015	0.015	0.016
a11	0.001	0.001	0.002	0.002	0.003	0.004	0.005	0.008	0.009	0.010	0.018
a12	0.001	0.002	0.009	0.012	0.020	0.040	0.050	0.060	0.806	0.000	0.000
a13	0.001	0.002	0.002	0.005	0.010	0.012	0.015	0.017	0.020	0.025	0.891

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Arc	11	12	13	14	15	16	17	18	19	20	
a_1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
a_2	0.031	0.853	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
a_3	0.025	0.030	0.035	0.040	0.060	0.719	0.000	0.000	0.000	0.000	
a_4	0.017	0.020	0.027	0.870	0.000	0.000	0.000	0.000	0.000	0.000	
a_5	0.022	0.025	0.030	0.035	0.010	0.761	0.000	0.000	0.000	0.000	
a_6	0.017	0.018	0.019	0.020	0.022	0.844	0.017	0.017	0.000	0.000	
a_7	0.009	0.011	0.013	0.014	0.014	0.015	0.000	0.000	0.019	0.020	
a_8	0.017	0.020	0.030	0.081	0.000	0.000	0.000	0.000	0.000	0.000	
a_9	0.022	0.025	0.030	0.817	0.000	0.000	0.000	0.000	0.000	0.000	
a_{10}	0.020	0.884	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
a_{11}	0.015	0.016	0.017	0.019	0.020	0.857	0.000	0.000	0.000	0.000	
a_{12}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
a_{13}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	

Each generation consists of 200 chromosomes. The primary population is created randomly, and the genetic algorithm is applied iteratively. The algorithm is executed across various generations, with a portion of the outcomes displayed in Table 2.

In Table 2, the $F(f)$ column represents the best fitness value identified up to that point. The columns are indicating the generation in which the optimal solution was discovered, while the mean column shows the average fitness value for that specific generation. The sequence of the best solutions identified is as follows:

$f = (4,1,2,1,5,1,2,0,2,1,2)$,$f = (4,0,2,0,2,3,0,3,4,0,1)$,$f(2,3,1,2,6,0,2,0,3,5,2)$,

$f = (3,3,2,2,5,0,0,1,2,2,1)$,$f = (5,1,1,2,1,3,2,0,4,1,4)$,$f(6,2,1,0,4,1,0,1,2,3,3)$. After determining the optimal allocation pattern f across all MPs, we calculate the optimal allocation and control strategies for each arc and node using formula (1). As no similar heuristic algorithms addressing this specific problem were identified, we have not conducted a comparative performance analysis with other methods.

The Genetic Algorithm (GA) approach in this study leverage's reliability approximations to balance computational efficiency with solution accuracy. Ideally, computationally demanding techniques such as Monte Carlo simulations (or exact network reliability calculations) would be applied exclusively to the optimal network design. However, because GA is iterative and requires evaluating numerous candidate networks throughout its search process, this ideal scenario is not feasible. To address this, a multi-stage screening process is integrated into the GA for evaluating candidate networks.

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Initially, all new network designs undergo a connectivity check to verify the presence of a spanning tree, as described in [14]. For designs that satisfy this criterion, the 2-connectivity metric [25] is assessed by examining node degrees. Networks passing both of these initial filters then proceed to an upper-bound reliability calculation, RU(x), using Jan's method [24]. This reliability upper bound is incorporated into the objective function (Equation 3) for all networks, excepting the current global solution, (X_{BEST}). Candidates with RU(x) ≥ Ro and lowest cost identified so far are further evaluated using an efficient Monte Carlo simulation method developed by Yeh et al. [22], to achieve a more precise reliability estimate. For all networks considered in this paper, the Monte Carlo simulations are performed for 3000 iterations per network. Deniz et al. [6] have also explored GA-based network design, emphasizing considerations for all-terminal reliability.

Table 2. $F(f)$ column represents the best fitness value identified up to that point.

Results (1000 generations)			Results (1500 generations)			Results (3000 generations)		
No.	F(f)	Mean	No.	F(f)	Mean	No.	F(f)	Mean
1	-14.256	-37.834	1	-13.245	-40.148	1	-12.215	-44.245
5	-8.4584	-29.415	5	-7.245	-24.451	24	-7.245	-25.528
10	-2.5642	-14.241	10	-4.124	-10.487	87	-2.245	-12.452
15	-1.7845	-7.245	17	-1.245	-4.354	152	-0.471	-6.152
21	-0.8747	-2.548	27	-0.0071	-2.259	451	0.0087	-2.451
47	-0.0081	-1.567	65	0.0046	-1.9885	874	0.0174	-0.9485
69	0.0051	-1.245	124	0.0415	-1.4587	1245	0.1245	-0.8465
91	0.0098	-1.145	217	0.0782	-1.2457	1746	0.1354	-0.2457
112	0.0128	-1.047	345	0.1032	-1.2178	2415	0.1492	-0.1657
188	0.0256	-1.007	587	0.1245	-1.1485	2614	0.1421	-0.1324
245	0.0862	-0.9925	841	0.1465	-0.7842	2842	0.1584	0.0212
379	0.1057	-0.8745	1054	0.15278	-0.5421	2882	0.15762	0.0387
448	0.1258	-0.7412	1307	0.15274	-0.2445	2914	0.15271	0.0398
556	0.15215	-0.5241	1396	0.15272				
787	0.15274	-0.2415						
841	0.15272	-0.2284						
910	0.15271	-0.2142						

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6 Results Analysis

Due to the use of penalty parameters and random initialization, the fitness function values may initially be negative real numbers. However, these values improve rapidly as the algorithm progresses. The fitness value for the three best solutions is consistently 0.15271. In results 1, 2, and 3, the optimal solutions are identified in the 910th, 1396th, 2914th, and 1143rd generations, respectively. How many generations required to find the optimal solution is significantly lower than the maximum generation limit. Moreover, the best solutions remain unchanged beyond the point where they are found, indicating that 0.15271 is likely the globally optimal fitness value and represents the minimum cost calculated in each generation, the optimal cost is 367 and the reliability value equal to 0.15271. The proposed genetic algorithm demonstrates high efficiency. Due to the use of penalty parameters for managing constraints, it cannot guarantee that all solutions will satisfy the constraints when the algorithm is run for fewer generations. However, the identified optimal solutions can be validated, and feasible solutions can be retained. This approach significantly reduces computational time compared to integer programming, particularly for large-scale networks.

7 Conclusion

This paper investigates a resource allocation and control problem within a stochastic-flow network. The network consists of multiple sources and sinks nodes, and various types of resources traverse the network, which includes unreliable nodes. The objective is to determine an optimal strategy for allocating and controlling resource flows across arcs and nodes. Traditional integer programming methods require complex constraints to ensure the flow-conservation law is met. To address this, the concept of MPs (Minimal Paths) is employed to simplify the problem and reduce the number of constraints. The task is divided into two steps. First, the optimal allocation pattern f is identified across all MPs, where f is a vector with elements representing the flow quantities for specific MPs. Second, for each arc and node, the flow quantities from all relevant MPs are summed, as outlined in formula (1). This process yields the optimal allocation for each arc and node. Given that solving integer programming problems becomes computationally expensive as the network size grows, a genetic algorithm is proposed as an efficient method to search for the optimal solution. This research introduced a stochastic search algorithm grounded in genetic algorithms (GAs) to address the network topology design problem, aiming to minimize costs while satisfying a reliability constraint. The evolutionary approach demonstrated several advantages, including consistent computational efficiency, robust optimization capabilities, and adaptability. As an iterative process, GAs naturally exhibits diminishing returns in performance improvement, allowing the algorithm to be terminated at any stage while still delivering high-quality solutions. Tests revealed that the computational demands remained relatively stable even as network sizes increased significantly. Across all test cases, regardless of problem type, scale, or random number seed, the algorithm consistently produced optimal or near-optimal solutions. The framework also proved highly adaptable, enabling straightforward substitution of different objectives (e.g., maximizing reliability under a cost constraint) or approaches for reliability estimation, such as the Monte Carlo approach.

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