

**SUPPRESSION OF DETONATION WAVES IN GAS SUSPENSIONS BY
A CROSS LAYER OF HETEROGENEOUS PARTICLES OF UNITARY
FUEL IN PIPES WITH ABRUPT EXPANSION**

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Abstract:

This article presents the results of numerical modeling of detonation waves in a gas suspension with a transverse layer of inhomogeneous monopropellant particles in pipes with abrupt expansion. The calculations were performed using the coarse particle method. The objective of the study is to analyze the influence of pipeline geometric parameters and the transverse inhomogeneous distribution of particle concentration on the dynamics of detonation propagation. It was found that the dimensions of the narrow and wide sections of the pipeline, as well as the height of the transverse inhomogeneity, significantly affect both the structure of the detonation wave and the stability of its propagation. The modeling revealed three characteristic regimes: stable wave propagation, complete detonation attenuation, and partial disruption with subsequent recovery. It is shown that the critical height of the transverse spatial inhomogeneity monotonically decreases with increasing relative mass content of solid particles in the gas suspension. The obtained data can be used in the design of passive detonation protection systems and in the development of explosion localization devices in pipelines.

MSC 2020 Codes: 76T99.**Key words:** mathematical model, detonation wave, unitary fuel, sharply expanding pipe, transverse heterogeneity, mass content of particles.

1. Introduction

Detonation waves in gas suspensions pose a serious hazard in various technological processes involving the handling of powdered fuels and aerosol mixtures. One effective approach to controlling and suppressing the propagation of such waves is to utilize the geometric features of the channels and the targeted formation of heterogeneities in the mixture. In particular, the sudden expansion of the pipeline combined with a transverse layer of heterogeneous particles of monopropellant fuel can significantly affect the dynamics and stability of detonation. This paper examines the mechanisms of detonation wave suppression under such conditions and identifies the key parameters determining the effectiveness of suppression.

In [1], the influence of the oxidizer injector channel expansion angle (30° and 90°) in a rotating detonation engine (RDE) on the detonation wave and pressure increase is investigated. It is shown that the angle has little effect on thrust, but at 90° the wave speed and Mach number are approximately 5% higher. No differences in the efficiency of the pressure generator (PG) were found. In [2], the influence of the fuel-air mixture heterogeneity in a rotating detonation combustion chamber (RDE) on the detonation wave is numerically studied. It is found that with increasing heterogeneity, the wave speed, detonation efficiency and pressure increase decrease, up to the separation of the reaction zone from the shock wave. In [3], the stability of detonation waves in channels to two-dimensional and three-dimensional disturbances is analyzed. Based on the dominant mode, the detonation wave structure, cell size and number, minimum channel dimensions, and conditions for their abrupt change were predicted. The results are consistent with experiments and modeling. In [4], detonation propagation in annular curved channels was studied. Three regimes were identified: stable, unstable, and damped. It was found that the regime depends on the channel width, cell size, and radius ratio. A regime map was constructed, confirmed by experiments and modeling. In [5], stationary detonation in circular arcs was numerically investigated under various conditions on the outer surface; it was found that the limiting angular velocity is the same for both types of limiters.

In [6], detonation propagation in an acetylene-air mixture in a narrow channel was experimentally studied. Four regimes were identified depending on the excess fuel. The conditions and characteristics of detonation reinitiation were determined. In [7], an analytical model of detonation suppression during a sharp expansion of the channel was presented. By combining the geometric dynamics of shock waves according to Witham and the equations of shock evolution by Radulescu, a critical value of the channel width at which detonation transfer is possible was obtained. The calculated data are in good agreement with experiments and numerical modeling for a $\text{H}_2/\text{O}_2/\text{Ar}$ mixture. In [8], detonation diffraction from a cylindrical tube through various geometries was studied numerically and experimentally. Critical conditions for transfer to the receiving chamber were identified, including the influence of the d/λ ratio (d is the inner diameter, λ is the size of the detonation cell) and the presence of obstacles. It is shown that the configuration with double reflection is effective even at $d/\lambda=2.2$. In [9–10], the exit of a detonation wave from a flat channel into a region with linear expansion is investigated. Numerical results are presented for different expansion angles. Three regimes of detonation propagation are distinguished: supercritical, critical and subcritical. In [11], one-dimensional unsteady propagation of detonation in an inhomogeneous $\text{H}_2\text{--O}_2$ medium is numerically studied taking into account detailed chemistry. It is found that the amplitude and frequency of inhomogeneities affect the transition from regular to chaotic pulsations, and small inhomogeneities are capable of stabilizing detonation. In [12], methane-oxygen detonation in tubes of different diameters is studied. Galloping regimes ($\approx 0.7\text{DCJ}$ and DCJ) are revealed. It is found that a decrease in pressure enhances low velocity, and the tube diameter and the stability of the mixture affect its manifestation.

In [13], the limits of detonation propagation in $\text{CH}_4\text{--O}_2$ mixtures in macro- and microchannels were investigated. It was shown that in macrochannels, the $\lambda = \pi D$ criterion is valid for unstable mixtures, while in microchannels, wall losses and boundary conditions play an important role, indicating a difference in the mechanisms of detonation breakdown. In [14], the formation of an overcompressed detonation wave in methane-oxygen mixtures in a channel of variable cross section was experimentally studied. The dependences of the front velocity and pressure on the mixture composition were determined. In [15], the influence of the geometry of expanding tubes and the longitudinal inhomogeneity of particles on detonation in a gas suspension was numerically

investigated. Three regimes were identified, the greatest suppression being achieved with a linearly decreasing particle concentration. In [16], the problem of detonation diffraction and the determination of the critical diameter at which the wave is either transmitted or attenuated were considered. Analytical and semi-analytical models based on wavefront curvature, decay rate, and energy criteria were constructed, which are consistent with experimental data. The one-dimensional model based on wavefront curvature proved to be the most accurate and computationally efficient.

2. Formulation of the problem

We consider a pipeline consisting of two sections: a narrow section and a wide section. The narrow section is completely filled with a homogeneous gas suspension of monitary propellant, while the wide section exhibits transverse non-uniform particle distribution. Initially, a triangular shock wave is generated near the left end of the pipe (zone 0), initiating combustion in zone 1—the narrow section of the pipeline. The objective of the problem is to numerically simulate the formation of a stationary heterogeneous detonation wave in the narrow section and its subsequent propagation to the wide section (zone 2). Particular attention is given to analyzing the influence of transverse non-uniformity of the monitary propellant concentration on the stability and behavior of the detonation wave.

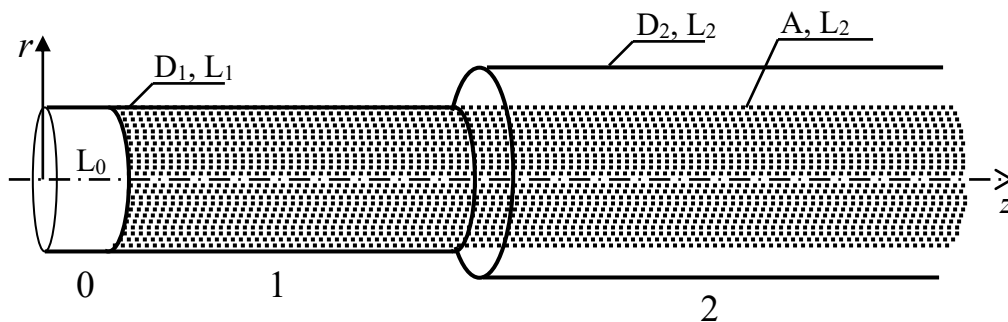


Fig. 1. Schematic representation of the problem

3. Statement of the problem

The system of equations for the two-dimensional plane unsteady motion of gas and particles of a unitary fuel has the form [17, 18, 19, 20]:

mass conservation equations

$$\frac{\partial \rho_1}{\partial t} + \frac{1}{r} \frac{\partial(\rho_1 v_{1,r} r)}{\partial r} + \frac{\partial(\rho_1 v_{1,z})}{\partial z} = J_{21}, \quad (1)$$

$$\frac{\partial \rho_2}{\partial t} + \frac{1}{r} \frac{\partial(\rho_2 v_{2,r} r)}{\partial r} + \frac{\partial(\rho_2 v_{2,z})}{\partial z} = -J_{21},$$

equations of conservation of mass of inert gas and gaseous combustion products of a gas mixture

$$\frac{\partial \rho_{1,l}}{\partial t} + \frac{1}{r} \frac{\partial(\rho_{1,l} v_{1,r} r)}{\partial r} + \frac{\partial(\rho_{1,l} v_{1,z})}{\partial z} = 0, \quad \frac{\partial \rho_{1,2}}{\partial t} + \frac{1}{r} \frac{\partial(\rho_{1,2} v_{1,r} r)}{\partial r} + \frac{\partial(\rho_{1,2} v_{1,z})}{\partial z} = J_{21}, \quad (2)$$

equations of conservation of the number of dispersed particles

$$\frac{\partial n_2}{\partial t} + \frac{1}{r} \frac{\partial(n_2 v_{2,r} r)}{\partial r} + \frac{\partial(n_2 v_{2,z})}{\partial z} = 0, \quad \alpha_2 = \frac{1}{6} \pi d_2^3 n_2, \quad (3)$$

equations of conservation of phase momentum

$$\begin{aligned} \frac{\partial(\rho_1 v_{1,r})}{\partial t} + \frac{1}{r} \frac{\partial(\rho_1 v_{1,r} v_{1,r} r)}{\partial r} + \frac{\partial(\rho_1 v_{1,r} v_{1,z})}{\partial z} + \frac{\partial p}{\partial r} &= -F_{2,r} + J_{21} v_{2,r} \\ \frac{\partial(\rho_1 v_{1,z})}{\partial t} + \frac{1}{r} \frac{\partial(\rho_1 v_{1,r} v_{1,z} r)}{\partial r} + \frac{\partial(\rho_1 v_{1,z} v_{1,z})}{\partial z} + \frac{\partial p}{\partial z} &= -F_{2,z} + J_{21} v_{2,z}, \end{aligned} \quad (4)$$

$$\frac{\partial(\rho_2 v_{2,r})}{\partial t} + \frac{1}{r} \frac{\partial(\rho_2 v_{2,r} v_{2,r} r)}{\partial r} + \frac{\partial(\rho_2 v_{2,r} v_{2,z})}{\partial z} = F_{2,r} - J_{21} v_{2,r},$$

$$\frac{\partial(\rho_2 v_{2,z})}{\partial t} + \frac{1}{r} \frac{\partial(\rho_2 v_{2,r} v_{2,z} r)}{\partial r} + \frac{\partial(\rho_2 v_{2,z} v_{2,z})}{\partial z} = F_{2,z} - J_{21} v_{2,z},$$

equations of heat fluxes to the particle phases

$$\frac{\partial(\rho_2 e_2)}{\partial t} + \frac{1}{r} \frac{\partial(\rho_2 e_2 v_{2,r} r)}{\partial r} + \frac{\partial(\rho_2 e_2 v_{2,z})}{\partial z} = Q\eta(-J_{21}) - J_{21} e_2, \quad (5)$$

equations of conservation of total energy of the mixture

$$\sum_{i=1}^3 \left[\frac{\partial(\rho_i E_i)}{\partial t} + \frac{1}{r} \frac{\partial(\rho_i E_i + \alpha_i p) v_{i,r} r}{\partial r} + \frac{\partial(\rho_i E_i + \alpha_i p) v_{i,z}}{\partial z} \right] = 0, \quad (6)$$

$$\rho_1 = \sum_{k=1}^2 \rho_{1,k}, \quad \rho_{1,k} = \rho_{1,k}^0 \alpha_1, \quad \rho_i = \rho_i^0 \alpha_i, \quad \rho_1^0 = \sum_{k=1}^2 \rho_{1,k}^0,$$

$$v_i^2 = v_{i,r}^2 + v_{i,z}^2,$$

$$\alpha_1 + \alpha_2 = 1, \quad E_i = e_i + 0.5v_i^2, \quad (i=1; 2).$$

In equations (1) – (6), $\rho_i, \rho_i^0, \alpha_i, v_i, e_i, E_i$ respectively denotes the average and true densities, volumetric content, mass velocity, specific internal and total energies of the i -th phase ($i=1$ – gas, $i=2$ – particles); $\rho_{1,1}, \rho_{1,2}$ and $\rho_{1,1}^0, \rho_{1,2}^0$ – are the average and true densities of the gas phase components; $v_{i,r}$ and $v_{i,z}$ – are the velocity components of v_i ; n_2 and d_2 – are the number and diameter of particles; p – is the pressure of the gas mixture; $F_{2,r}$ and $F_{2,z}$ – are the components of the interphase friction force; Q – is the heat exchange in the gas suspension; J_{21} – is the intensity of interphase mass transfer; η – is the Heaviside function.

Closing relations, initial and boundary conditions are specified similarly to [21, 22].

The numerical solution of the problem was carried out using the large particle method [23].

All calculations were performed for an initiating shock wave length of $z_f = 0.4$ m. In the main series of calculations, it was assumed that the shock wave Mach number was $M_0 = 9$. The lengths L_0, L_1 and L_2 were assumed to be 0.4, 25, and 50 m.

In the calculations, the stepwise distribution of the concentration of monopropellant particles along the pipeline radius was used in the form [15].

$$\varepsilon(z, r, A) = 1 \quad (0 \leq r \leq r_0, \quad r_0 = (1 - A)r, \quad 0 \leq A \leq 1) \quad (7)$$

The parameter r_0 denotes the distance from the axis of symmetry ($r=0$) to the upper boundary of the particle layer ($r=r_0$).

4. Calculation results

Figure 2 shows the envelopes of the maximum pressures arising behind the combustion wave front, along the symmetry axis ($r=0$) and on the wall of the wide section of the pipeline ($r=R_2$). The following parameters were used in the

calculations: the particle diameter of the unitary fuel $d_{20} = 3 \text{ }\mu\text{m}$, the initial relative mass content of particles in the mixture $m_{20} = 2$, the parameter of the transverse non-uniformity of the particle distribution $A = 0.4$. The radii of the narrow and wide sections of the pipeline are equal, respectively, to $R_1 = 0,1 \text{ m}$ and $R_2 = 0,3 \text{ m}$. Parameter A characterizes the degree of transverse heterogeneity in the distribution of monetary propellant particles: for $A < 1$, particles are concentrated closer to the axis of symmetry, while their concentration approaches zero near the walls. This leads to the intensity of chemical reactions and, consequently, the energy release near the pipeline walls being significantly lower than in the central part of the flow. Analysis of the graphs shows that the maximum pressure behind the detonation wave is observed along the axis of symmetry (in the central part of the flow). This is explained by the fact that the majority of monetary propellant particles, actively involved in combustion and the detonation process, are concentrated in this region. Conversely, the pressure near the pipeline walls is significantly lower, since the particle concentration there is low, meaning the energy released during combustion is also lower.

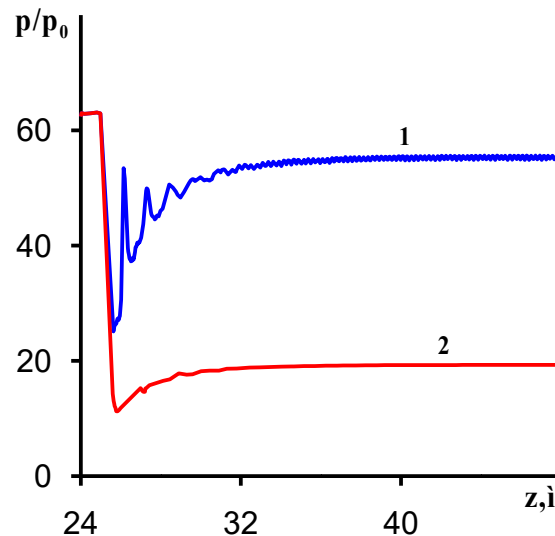


Fig. 2. Envelopes of maximum pressures behind combustion waves on the axis of symmetry (1) and the wall in the wide part of the pipeline: $d_{20} = 3 \text{ }\mu\text{m}$; $m_{20} = 2$; $A = 0.4$; $R_1 = 0,1 \text{ m}$, $R_2 = 0,3 \text{ m}$.

The influence of transverse spatial non-uniformity of the particle distribution of a unitary fuel on the propagation of detonation waves in a pipeline with an

expanding cross-section filled with a non-uniform gas suspension is considered. Fig. 3 shows the envelope of maximum pressures behind the combustion front along the symmetry axis of the wide section of the pipeline. The following parameters were used in the calculations: $d_{20} = 15 \text{ } \mu\text{m}$, $m_{20} = 2$, $R_1 = 0,2 \text{ m}$, $R_2 = 0,5 \text{ m}$. Three variants of the degree of non-uniformity of the particle distribution were considered, characterized by the parameter A , which determines the width of the layer within which the bulk of the solid phase is concentrated: curve 1 – $A=0.4$, curve 2 – $A=0.2$, curve 3 – $A=0$ (uniform distribution). Analysis of the obtained results shows that the degree of transverse non-uniformity has a noticeable effect on the structure of the pressure distribution behind the detonation wave. At lower values of A (more pronounced heterogeneity), particles are concentrated in the central part of the channel, with virtually no particles near the walls. This leads to intense combustion developing primarily near the axis of symmetry, where the solid fuel concentration is higher, while in the peripheral zones, the energy released during combustion is significantly lower. This results in an uneven pressure distribution across the pipeline cross-section, with the maximum pressure occurring near the axis. Conversely, as A increases, when the particles are distributed more evenly along the radius, the pressure becomes more uniform, which favorably impacts the stability and symmetry of the detonation wave front.

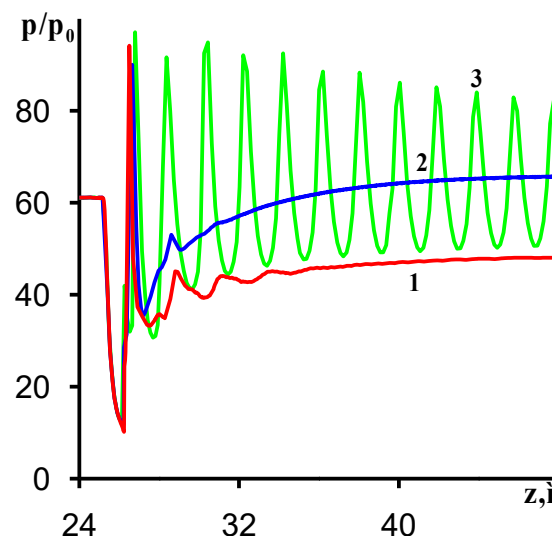


Fig. 3. The influence of parameter A on the distribution of the maximum pressure behind the combustion front along the axis of symmetry: $d_{20} = 15 \text{ } \mu\text{m}$; $m_{20} = 2$; $R_1 = 0,2 \text{ m}$, $R_2 = 0,5 \text{ m}$; $A=0.4$ (1), $A=0.2$ (2), $A=0$ (3)

In order to determine the detonation mode in a wide section of a pipeline, a numerical study was conducted to examine the effect of the particle diameter d_{20} of a monolithic fuel at a fixed height of transverse spatial non-uniformity in the solid phase distribution. The following radii were used in the calculations: $R_1 = 0,2$ m, $R_2 = 0,5$ m. The graphs show the results for three variants: Curves 1 correspond to the particle diameter $d_{20} = 30$ μm , curves 2 to $d_{20} = 15$ μm , and curves 3 to $d_{20} = 3$ μm . Two cases of transverse non-uniformity are considered separately: Fig. 4(a) considers the case $A = 0.4$, while Fig. 4(b) considers $A = 0.2$. Analysis of the obtained results shows that the particle diameter of the monolithic fuel has a significant effect on the structure and stability of the detonation wave. With increasing particle diameter (for example, to 30 μm), a decrease in combustion intensity is observed, which is associated with increased particle inertia and deterioration of heat and mass transfer between the phases. This, in turn, leads to a weakening of the detonation wave and its possible attenuation. Conversely, with a decrease in particle diameter to 3 μm , the interaction of particles with the gas phase improves, accelerating the heating and ignition of the solid fuel, which contributes to the formation of a more stable and powerful detonation wave. A comparison of Fig. 4(a) and 4(b) also shows that with a more uniform particle distribution (higher A value), the detonation wave front forms faster and more uniformly than with pronounced transverse inhomogeneity. This is due to the fact that with a more uniform distribution of solid fuel, its mass involved in combustion is higher in the near-surface zones, which contributes to the stabilization of the wave across the entire cross-section of the channel.

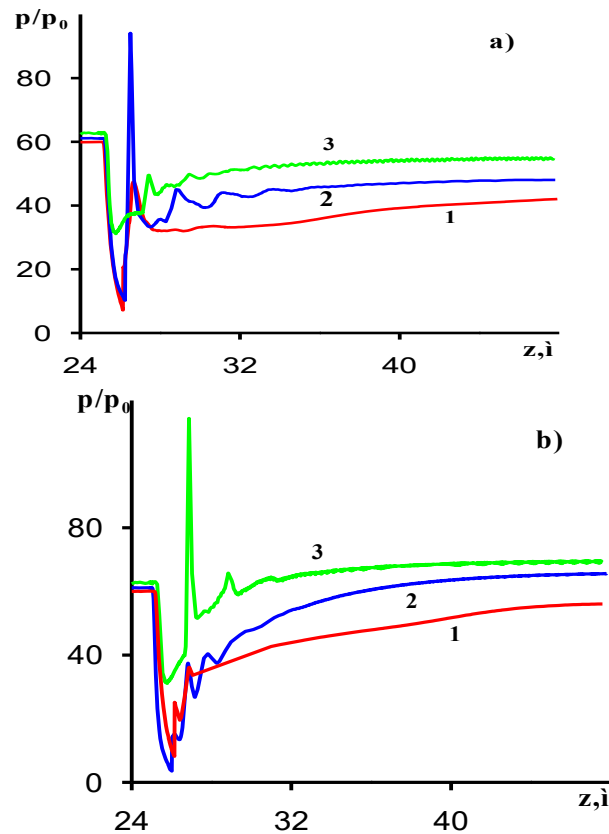


Fig. 4. The influence of the particle diameter d_{20} on the distribution of the maximum pressure behind the combustion front along the symmetry axis: $d_{20} = 30 \mu\text{m}$ (1), $d_{20} = 15 \mu\text{m}$ (2), $d_{20} = 3 \mu\text{m}$ (3); $A=0.4$ (a), $A=0.2$ (b); $m_{20} = 2$; $R_1 = 0,2 \text{ m}$, $R_2 = 0,5 \text{ m}$

According to the data presented in Fig. 5, for fixed initial particle diameters of monotary fuel $d_{20} = 5 \mu\text{m}$ and $d_{20} = 15 \mu\text{m}$, as well as radii of the narrow section of the pipeline $R_1 = 0,1 \text{ m}$ and the wide section $R_2 = 0,3 \text{ m}$, it was established that the critical height of transverse spatial heterogeneity decreases monotonically with increasing relative particle mass fraction in the suspension. Physically, this effect is associated with the fact that an increase in the concentration of the dispersed phase leads to more intense absorption of heat and mechanical energy by the shock wave front due to heat transfer to the particles and their inertial drag. At the same time, the total mass of the system increases, and the rate of chemical reaction in the gas phase decreases due to heat removal for heating the solid phase. As a result, the energy support of the detonation front decreases, making it more sensitive to transverse heterogeneity in the particle distribution. The decrease in the critical height of inhomogeneity

with increasing mass content of particles means that even a relatively small transverse non-uniformity in the distribution of the dispersed phase can lead to the interruption of the detonation wave at a high concentration of suspended matter.

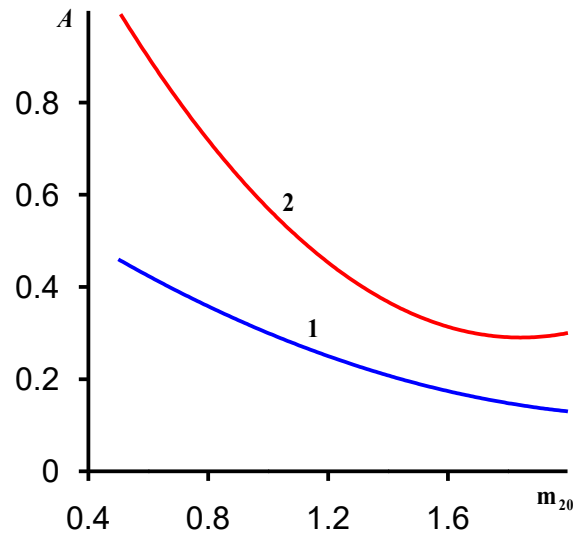


Fig. 5. Dependence of the critical pipeline height ratio on the relative content of monolithic fuel particles. 1 - $d_{20} = 5 \mu m$, 2 - $d_{20} = 15 \mu m$.

5. Conclusion

Numerical modeling has shown that the transverse heterogeneity of the particle distribution of monopropellant fuel is a key factor determining the stability and efficiency of heterogeneous detonation propagation in pipelines with sudden expansion. It has been established that decreasing the heterogeneity height enhances combustion localization and axial pressure, but reduces the completeness of particle involvement in the reaction. Particle diameter and the degree of their transverse distribution are critical for creating stable detonation conditions. Numerical calculations have confirmed that both particle diameter and the degree of their transverse distribution are critical for the stable propagation of heterogeneous detonation in pipelines with variable cross-sections. This is important to consider when developing systems that utilize detonation in gas suspensions as a combustion mechanism or explosion protection. Integral dependences of the critical heterogeneity height on the relative solids content are determined. The results confirm the possibility of controlled detonation suppression by introducing controlled transverse heterogeneity. The identified patterns can be used in the design of power plants and explosion protection systems based on gas suspensions.

References

- [1] K. Nakajima, K. Matsuoka, N. Itouyama, etc, Effect of channel expansion angle near injector outlet on a rotating detonation engine performance, *Shock Waves*, 34(2024), 215–224. DOI: 10.1007/s00193-024-01173-6
- [2] P. Raj, J.Meadows, Influence of fuel inhomogeneity on detonation wave propagation in a rotating detonation combustor, *Shock Waves*, 34(2024), 429–449. doi.org/10.1007/s00193-024-01180-7
- [3] A.N. Kudryavtsev, S.P. Borisov, Stability of Detonation Waves Propagating in Plane and Rectangular Channels, *Combustion, Explosion, and Shock Waves*, 56(2020), 92 – 99. DOI: 10.1134/S0010508220010116
- [4] X. Shi, R. Hencel, J. Crane, M. Fotia, H. Wang, Geometric stability of detonation propagation in curved channels, *Shock Waves*, 35(2025), 157–167. doi.org/10.1007/s00193-024-01212-2
- [5] M. Short, J.J.Quirk, C. Chiquete, C.D. Meyer, Detonation propagation in a circular arc: reactive burn modeling, *J. Fluid Mech*, 835(2018), 970–998. doi.org/10.1017/jfm.2017.751
- [6] S.V. Golovastov, G.Yu. Bivol, V.V. Golub, An experimental study of detonation propagation and re-initiation for acetylene–air mixtures in a narrow channel, *Shock Waves*, 31(2021), 49-61. doi.org/10.1007/s00193-020-00985-6
- [7] M.I. Radulescu, R. Mével, Q. Xiao, S. Gallier, On the self-similarity of diffracting gaseous detonations and the critical channel width problem, *Physics of Fluids*, 33(2021), doi.org/10.1063/5.0054219
- [8] R. Sorin, R. Zitoun, B. Khasainov, D. Desbordes, Detonation diffraction through different geometries, *Shock Waves*, 19(2009), 11–23. doi 10.1007/s00193-008-0179-1
- [9] A.V. Fedorov, T.A. Khmel, S.A. Lavruk, Exit of a heterogeneous detonation wave into a channel with linear expansion. I. Propagation regimes, *Combustion, Explosion, and Shock Waves*, 53(2017), 585-595. doi: 10.1134/S0010508217050136.
- [10] A.V. Fedorov, T.A. Khmel, S.A. Lavruk, Exit of a heterogeneous detonation wave into a channel with linear expansion. II. Critical

Propagation Condition, Combustion, Explosion, and Shock Waves, 54(2018), 585-595.

doi: 10.1134/S0010508218010112.

- [11] W.J. Ma, C. Wang, W.H. Han, Effect of concentration inhomogeneity on the pulsating instability of hydrogen–oxygen detonations, Shock Waves, 30(2020), 703–711. doi.org/10.1007/s00193-020-00976-7
- [12] W. Cao, H.D. Ng, J.H.S. Lee, Near-limit detonations of methane–oxygen mixtures in long narrow tubes, Shock Waves, 30(2020), 713–719. doi.org/10.1007/s00193-020-00940-5
- [13] B. Zhang, H. Liu, B. Yan, Investigation on the detonation propagation limit criterion for methane–oxygen mixtures in tubes with different scales, Fuel 239,239(2019), 617–622. doi.org/10.1016/j.fuel.2018.11.062
- [14] G.Yu Bivol, S.V. Golovastov and V.V. Golub, Formation of an overdriven detonation wave in the flow of methane–oxygen mixtures in a variable cross section channel, High Temperature, 55(2017), 561-566. doi: 10.7868/S0040364417030036
- [15] U.A. Nazarov, Numerical investigation into the propagation of detonation waves in inhomogeneous gas suspensions of a monofuel in abruptly expanding tubes, Journal of Engineering Physics and Thermophysics, 96 (2023), 937–944. doi:10.1007/s10891-023-02756-3.
- [16] J. Klein, J.R. Klein, O. Samimi-Abianeh, One-dimensional model predictions for the detonation diffraction critical tube diameter, Shock Waves, 35(2025), 329-347. doi.org/10.1007/s00193-025-01235-3
- [17] V. F. Burnashev, U.A. Nazarov, M. T. Shodmonkulov and J. K. Haydarov, Numerical Study of the Propagation of Detonation Waves in Gas Suspensions of Unitary Fuel in Sharply Expanding Rough Tubes, International Journal of Mechanical Engineering and Robotics Research, 13(2024), 578-585. doi: 10.18178/ijmerr.13.6.
- [18] A.G. Kutushev, B.F. Burnashev, and U.A. Nazarov, Numerical study of detonation waves in a monofuel-gas mixture in sharply expanding pipes, Combustion, Explosion, and Shock Waves, 49 (2013), 418–423.

- [19] V.F. Burnashev, U.A. Nazarov and Khuzhaerov B.Kh, Detonation Waves in Polydisperse Gas Suspensions of Monofuel in Tubes with an Abrupt Expansion, *Journal of Fluid Dynamics*. 51(2016), 507–512.
- [20] V.F. Burnashev, U.A. Nazarov, H. Akhadkulov, M.T. Shodmonkulov and J.K. Haydarov, Numerical Study of Detonation Waves in Gas Suspensions with Spatially Inhomogeneous Particle Concentration Distribution in Sharply Expanding Pipes, *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 123(2024), 41-52. <https://doi.org/10.37934/arfmts.123.1.4152>.
- [21] U.A. Nazarov, Interruption of Detonation Wave Propagation in Monofuel–Air Mixtures by a Layer of Inhomogeneous Inert Particles, *Combustion, Explosion, and Shock Waves*, 57 (2021), 693–703. doi: 10.1134/S0010508221060083.
- [22] R.I. Nigmatullin, *Dynamics of Multiphase Media*, Nauka, Moscow (1987) [in Russian].
- [23] O.M. Belotserkovskii and Yu. M. Davydov, *Method of Large Particles in Gas Dynamics*, Nauka, Moscow (1982) [in Russian].