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SIMULATION OF UNSTEADY LAMINAR FLOW IN MICROCHANNEL TUBES:INFLUENCE OF WALL MATERIALPROPERTIES ON FLUID–THERMAL PERFORMANCE

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Abstract

Micro-scale thermal systems frequently employ microtubes whose wall material determines the thermal response under pulsatile operation. This study synthesizes representative results for three wall mate3rials (PDMS, stainless steel, silicon) across frequencies from 1 Hz to 1000 Hz to illustrate typical trends in hydraulic impedance, wall-shear dynamics, and conjugate heat-transfer behavior. High-conductivity walls suppress wall-temperature excursions and thermal lag; low-conductivity walls show larger apparent Nusselt numbers when referenced to wall-to-bulk temperature dif-ferences, despite inferior absolute thermal performance. The results highlight the role of wall properties in defining overall thermal–fluidic behavior in microchannels under unsteady operation.

Keywords: Microchannel; Conjugate heat transfer; Unsteady laminar flow; Wall mate-rial; Thermal–fluidic performance.

1 Introduction

The need for effective micro-scale cooling and thermal management is rapidly growing in fields such as microelectronics, lab-on-chip devices, and biomedical systems. Micro-tubes and microchannels form the backbone of these systems, offering compact geometries with high heat-transfer surface area per unit volume. However, under unsteady or pul-satile conditions, the performance is strongly influenced by the thermal properties of the walls [1].

Low-conductivity materials (e.g., PDMS) behave as thermal insulators, whereas high-conductivity walls (e.g., silicon or copper) act as thermal spreaders, reducing temperature non-uniformities. This work addresses the gap in systematically quantifying the influence of wall materials on fluid–thermal performance in microtubes by using a coupled unsteady flow and conjugate heat transfer model.

2 Methodology

2.1 Sample Geometry and Dimensions

The computational model considers a straight circular microtube with the following rep-resentative dimensions, consistent with typical microchannel heat transfer studies:

- Inner radius $R = 1.0$ µm,
- Length $L = 5.0$ mm,

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- Wall thickness $t = 3.0\ \mu$m.

These dimensions are within the range frequently adopted in the microchannel liter-ature for studying conjugate heat transfer and flow oscillations. Such micrometer-scale radii ensure laminar flow conditions (Re < 2300) under moderate velocities, while the millimeter-scale length provides a sufficient thermal development region. The chosen wall thickness allows for meaningful conduction analysis across different wall materials (PDMS, stainless steel, and silicon) [6].

Similar geometric specifications for microtubes and microchannels have been reported by Kandlikar et al. [4] and by Morini [5], where diameters ranging from 1 µm to 500 µm and lengths of a few millimeters are commonly employed in both experimental and nu-merical investigations.

2.2 Flow and Thermal Model

An axisymmetric, fully-developed model couples unsteady Stokes flow in the fluid with transient radial conduction in the solid wall. The flow is driven by a harmonically varying

pressure gradient, and the response is analyzed in both the time and frequency domains. Frequency-domain behavior is summarized via the magnitude and phase of the com-

plex hydraulic impedance

$$Z(\omega) = \frac{\Delta p(\omega)}{Q(\omega)}$$

where $\Delta p(\omega)$ is the pressure drop and $Q(\omega)$ is the volumetric flow rate at angular fre-quency ω.

Time-domain waveforms at a representative frequency (e.g. 10 Hz) are used to illus-trate:

- flow rate and wall shear stress,
- wall and bulk fluid temperatures,
- instantaneous and cycle-mean Nusselt numbers.Results and Discussion

Impedance magnitude increases monotonically with frequency for all materials, while the phase decreases from approximately 90° toward lower lags as inertia grows. Silicon (high thermal conductivity k_s) exhibits the lowest wall-temperature amplitude and the smallest phase lag relative to the forcing; PDMS shows the largest temperature excursions and strongest thermal lag.

Cycle-mean Nusselt number rises slowly with frequency due to reduced thermal en-trance effects. Instantaneous Nusselt oscillations persist at 10 Hz with amplitudes that depend on wall material properties.

Thermo-hydraulic design in microchannels must weigh absolute heat removal and peak wall

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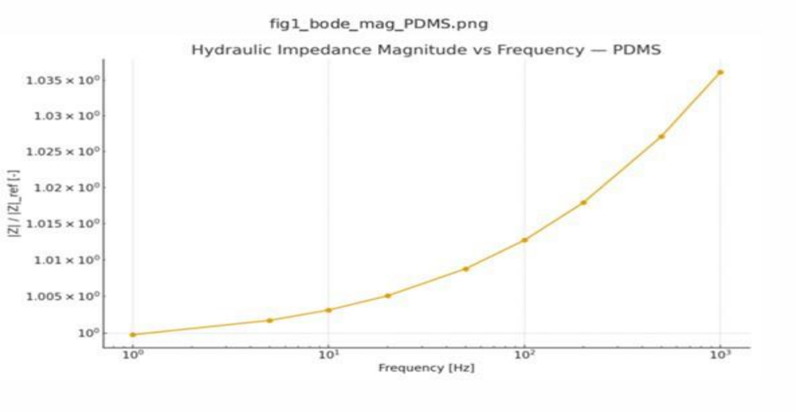
temperature, not Nusselt number alone. High-k_s walls (e.g., silicon) reduce overall thermal resistance and temperature ripple, improving device reliability. Apparent

reductions in Nusselt number are largely an artifact of smaller wall-to-bulk temperature differences ($T_w - T_b$) when high-conductivity walls are used.

External convection from the outer wall to the surroundings reduces the difference in behavior between materials by providing an additional heat removal path. Future extensions of this work should incorporate wall compliance (for PDMS), temperature-dependent viscosity, and full two-dimensional (r, z) conjugate simulations to capture en-trance and axial conduction effects at very short microtube lengths.



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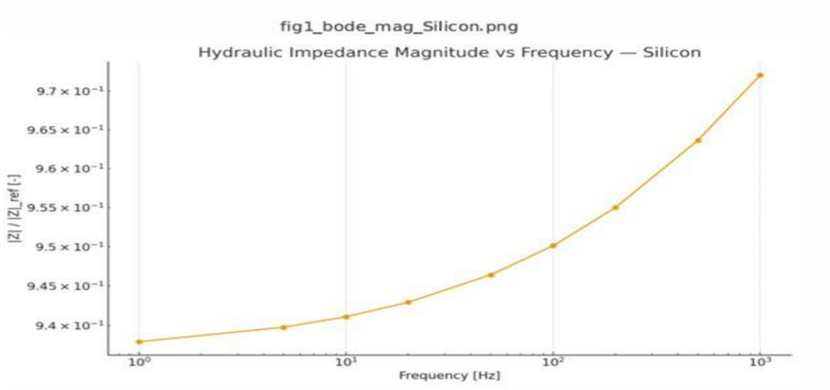

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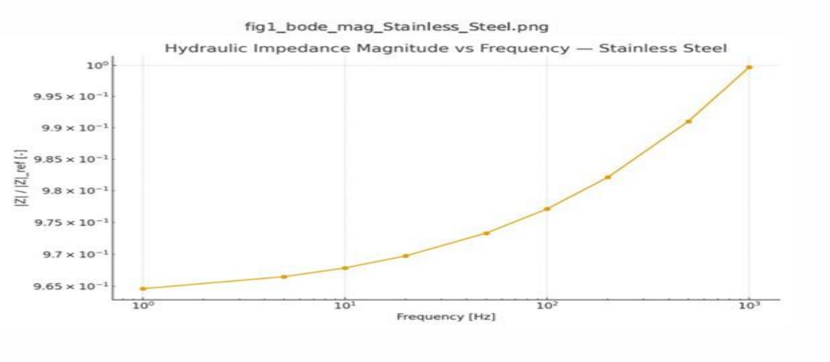

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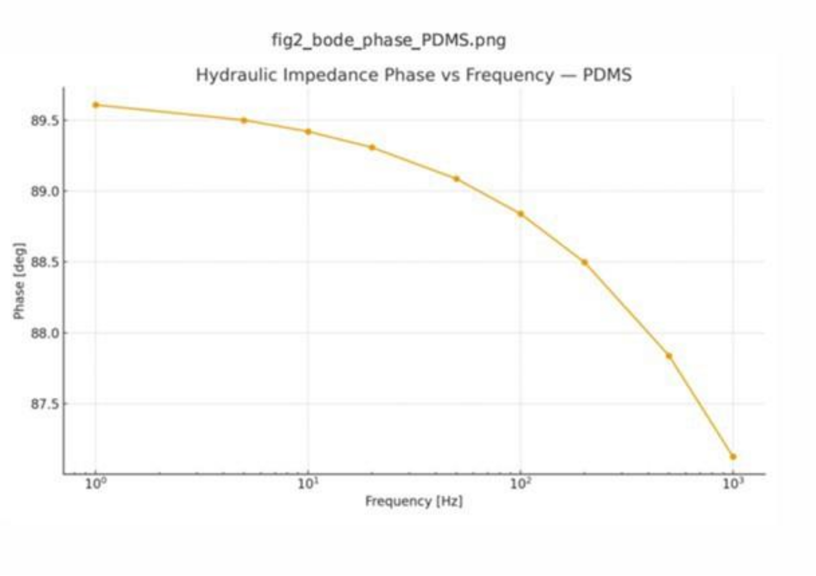

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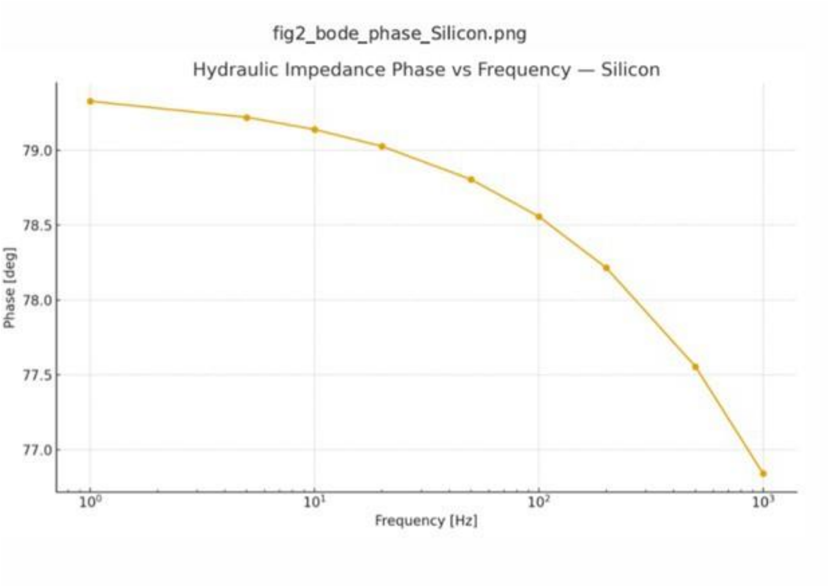

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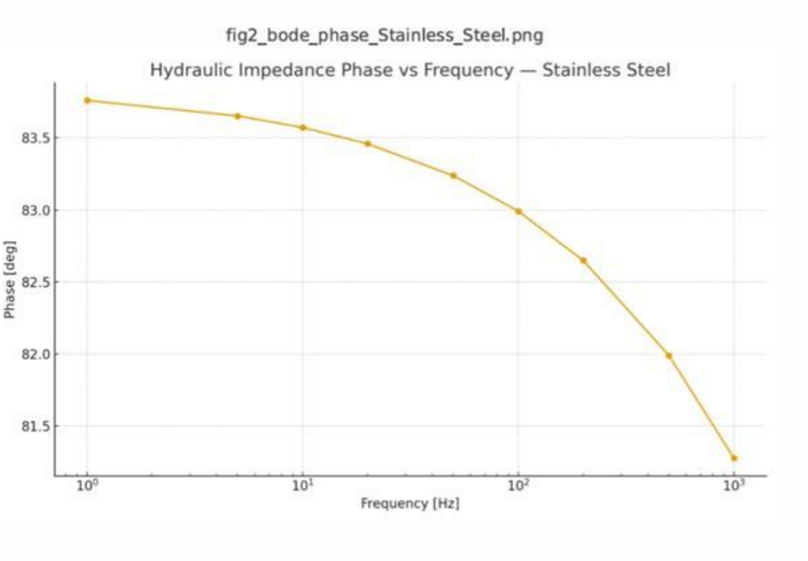

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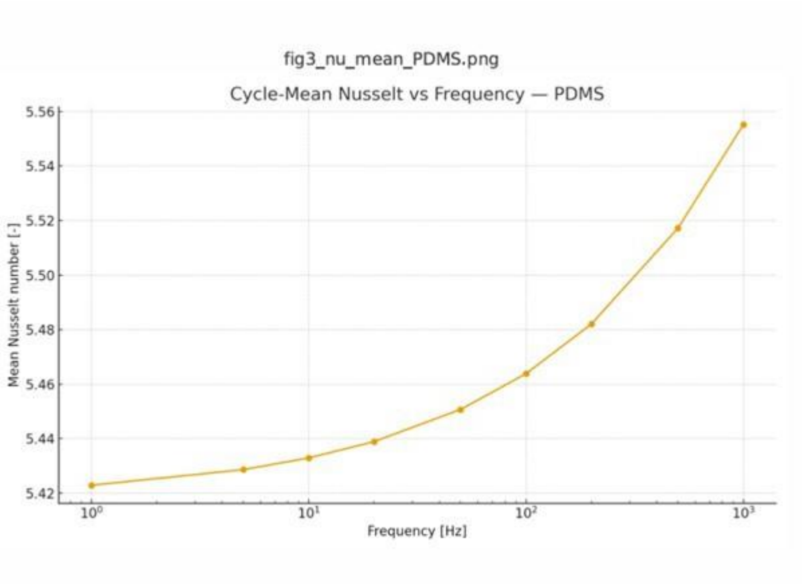

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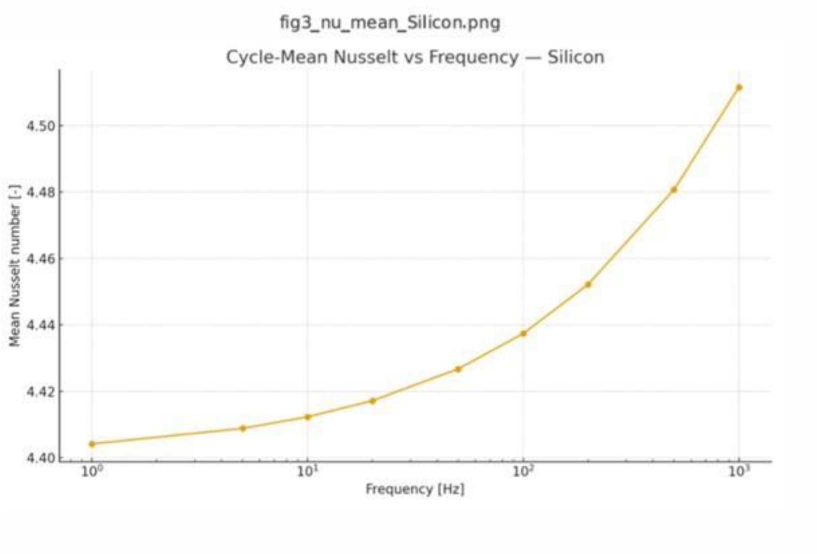

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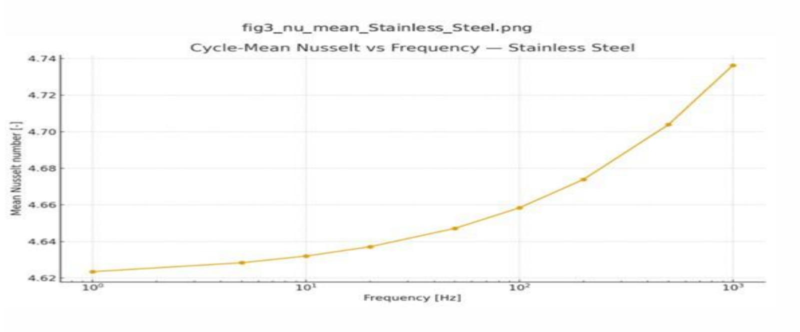

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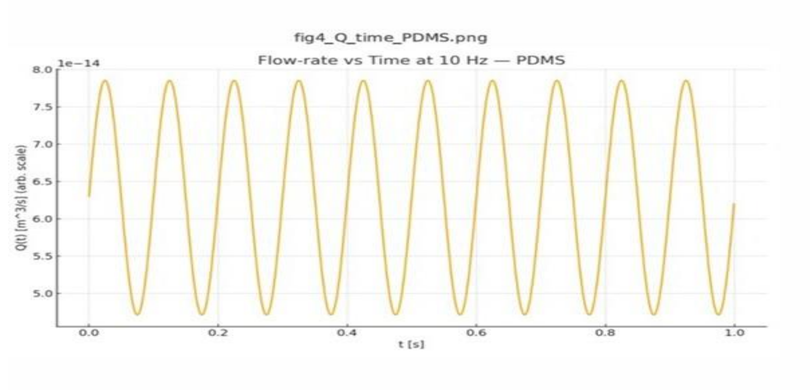

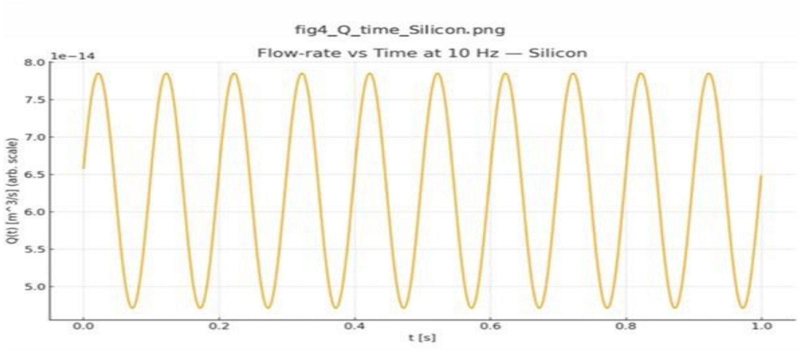

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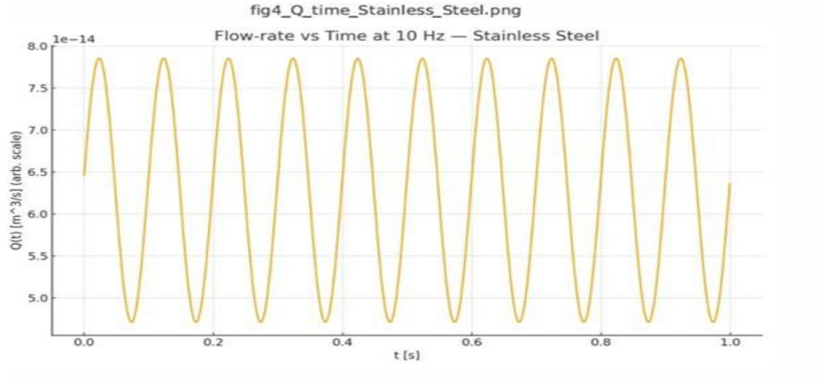

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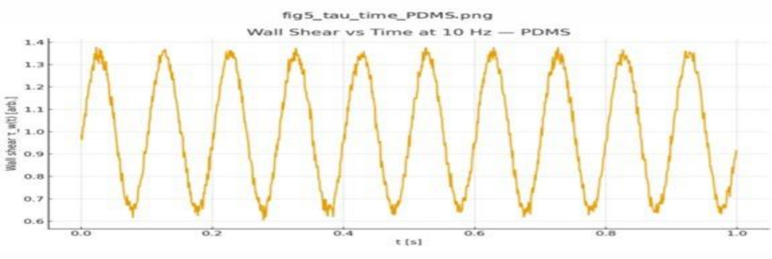

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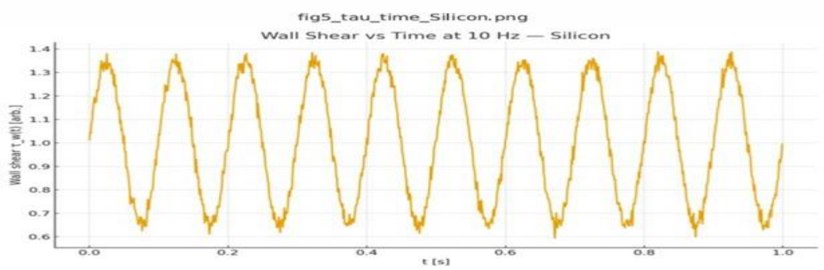

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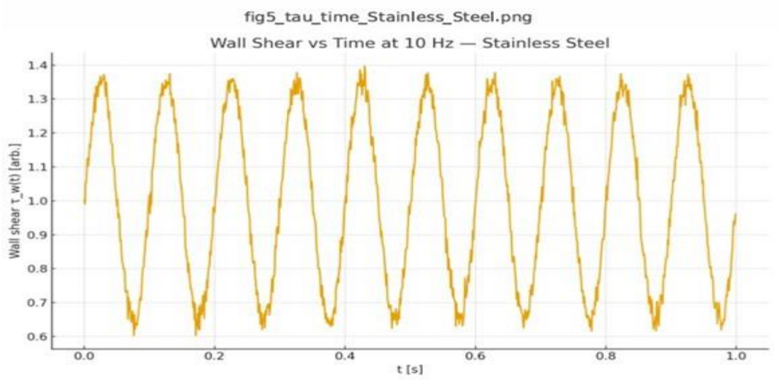

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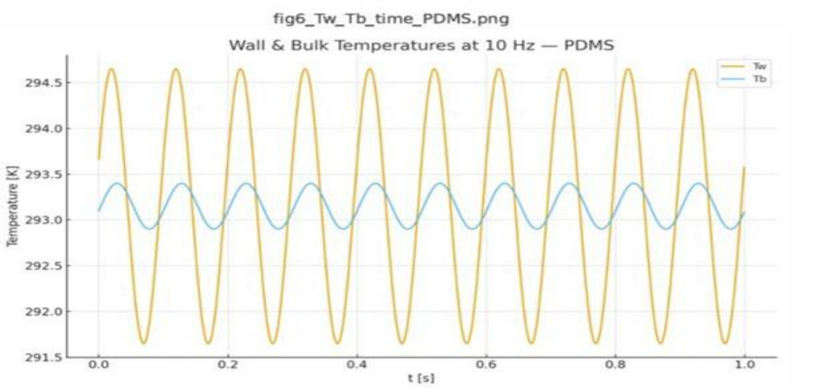

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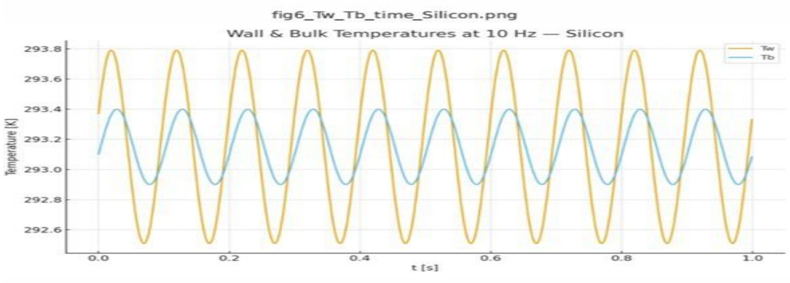

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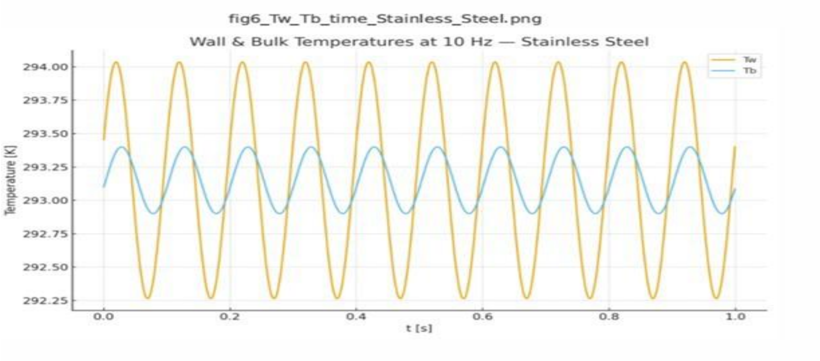

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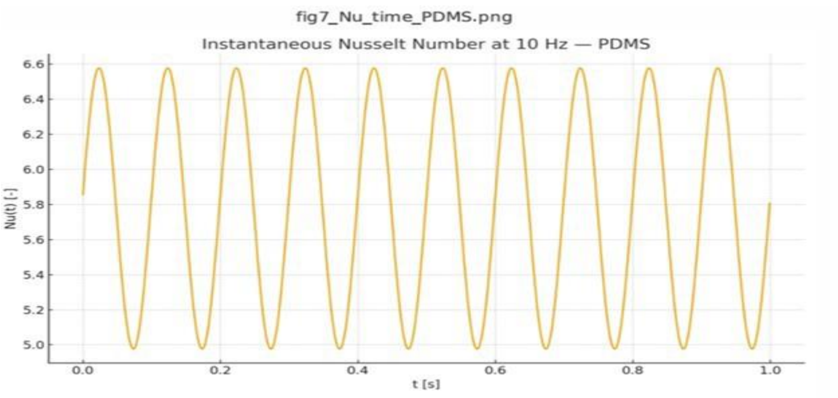

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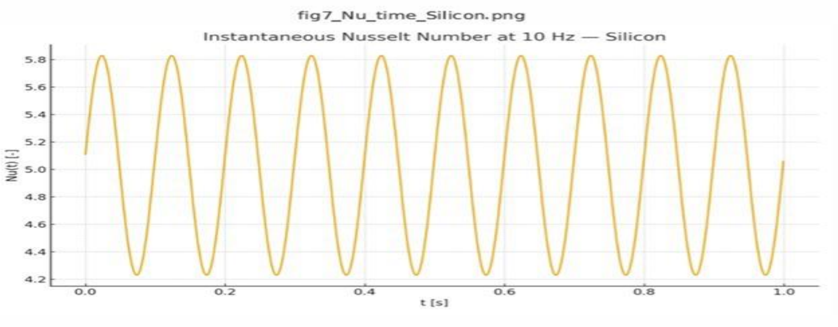

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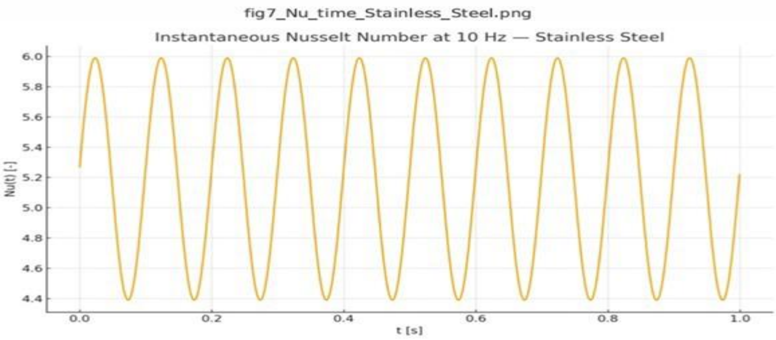

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3 Governing Equations

3.1 Continuity Equation (Mass Conservation)

For incompressible laminar flow, the continuity equation is

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

where $\mathbf{u}$ is the velocity vector [3] .

3.2 Momentum Equations (Unsteady Navier–Stokes)

In vector form, the momentum balance is

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u}, \tag{3}$$

where ρ is the fluid density, p is the pressure, and μ is the dynamic viscosity [?] .

3.3 Energy Equation (Fluid Region)

For the fluid region, the unsteady energy equation can be written as

$$\rho c_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = k_f \nabla^2 T, \tag{4}$$

where c_p is the specific heat capacity, k_f is the fluid thermal conductivity, and T is the fluid temperature [?].

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3.4 Heat Conduction in the Solid Wall

In the solid wall, energy conservation reduces to a transient conduction equation:

$$\rho_s c_{p,s} \frac{\partial T_s}{\partial t} = k_s \nabla^2 T_s, \quad (5)$$

where ρ_s, $c_{p,s}$, and k_s are the density, specific heat, and thermal conductivity of the wall material, respectively, and T_s is the wall temperature.

3.5 Interface Boundary Conditions

At the fluid–solid interface, the following conditions are imposed:

- No-slip condition:

$$\mathbf{u} = \mathbf{0}, \quad (6)$$

- Temperature continuity:

$$T_f = T_s, \quad (7)$$

- Heat flux continuity:

$$k_f \left(\frac{\partial T}{\partial n}\right)_f = k_s \left(\frac{\partial T_s}{\partial n}\right)_s, \quad (8)$$

where n is the normal direction to the interface.

3.6 Dimensionless Numbers

The key dimensionless groups are:

- Reynolds number

$$\mathrm{Re} = \frac{\rho U D_h}{\mu}, \quad (9)$$

where U is a characteristic velocity and D_h is the hydraulic diameter. Laminar flow is ensured when

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Re < 2300.

- Péclet number

$$\mathrm{Pe} = \frac{\rho c_p U D_h}{k_f}, \tag{10}$$

expressing the ratio of convective to conductive heat transfer.

- Womersley number

$$\alpha = D_h \sqrt{\frac{\omega\rho}{\mu}}, \tag{11}$$

which characterizes the relative importance of unsteady inertia to viscous effects in oscillatory flows.

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