International Journal of Applied Mathematics

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

THERMOHALINE CONVECTION IN MICROPOLAR FERROMAGNETIC FLUID WITH SORET EFFECT AND SPARSELY DISTRIBUTED POROUS MEDIUM: A BRINKMAN MODEL

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Abstract

This theoretical investigation on a horizontal micropolar ferromagnetic fluid layer with Soret effect is studied. The Brinkman porous medium is taken to analyze in the study. The geometry is considered for fluid layer whose portion of the bottom is isothermally heated from below and it is salted from above. The theory of linear stability is taken in to account to reduce the non-linear effects. The normal mode method is used to analyze the system and two free boundaries are considered. The exact solution is calculated to investigate the entire system. The thermal magnetic Rayleigh number N_{SC} is examined analytically for sufficient large values of M_1. An attempt is taken to analyze the porous effect on the convective system in various situations.

Keywords Porous medium, Brinkman model, Micropolar ferromagnetic fluid, Thermohaline convection, Soret effect.

1. Introduction

The thermohaline convective effects, associated with double diffusion were first discovered experimentally in the laboratory and archived excellent results of the numerous works by Sterm [1], Turner and Stommel [2], Turner [3] and Huppert and Turner [4]-[5]. The phenomenon on double diffusive convection wasdescribed by Turner [6]. A finite amplitude instability is studied in thermohaline convection by Veronis [7] and Brekke [8] described a

Received: August 09, 2025

International Journal of Applied Mathematics

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

density gradient column of sucrose solutions on zone electrophoresis with the support of convection. The thermohaline convection in the Boussinesq approximation with two dimensional non-linearity has been studied byVeronis [9]. Then Knobloch et al. [10] utilized five mode truncation originally suggested by Veronis [7]. Vaidyanathan etal. [11] explained the two component ferroconvective system in thermohaline convection.

The fluid flow phenomenon in a porous effect is a wonderful study due to its natural occurrence. Nield and Bejan [12] surveyed the double diffusive convection with porous effect and the same effect is considered on ferrothermohaline convection of sparse particles suspension by Vaidyanathan et al. [13].

Eringen [14]-[15] presented an excellent theory of micropolar fluids and the same given by Eringen [16] in order to explain the physical systems. In monograph of Rosensweig [17], ferrofluid is colloidal suspension of nanoparticle of magnetic behavior and this is considered as microrotation effect on particle. With use of this concept, investigations have been taken by treating the ferrofluid as a micropolar ferromagnetic fluid. The magnetization parameters behaviorsshown by Abraham [18] on convection in the micropolar ferrofluid. Sunil and Bharti [19] have studied the porous effect on micropolar ferromagnetic fluid. Reena and Rana [20] have studied the thermosolutal convection of micropolar rotating fluids in a porous effect. They used the Darcy model. Chand [21] studied the porous effect on triple-diffusive convection in micropolar ferromagnetic fluid.

In this present work, an infinite horizontal micropolar ferromagnetic fluid layer heated from below and salted from above with the effect of an isotropic porous. The fluid layer is of thickness d and the fluid is considered as an electrically non-conducting incompressible one. The temperature and salinity at the bottom and top surfaces $z=\mp d/2$ are $T_0 \pm (DT)/2$ and $S_0 \mp (DS)/2,$ respectively and $b_t(=|dT/dz|)$ and $b_s(=|dS/dz|)$ are maintained.

2. Mathematical formulation of problem

2.1 Basic equations

The continuity equation is

$$\nabla .\, \mathbf{q} = 0 \tag{1}$$

The momentum equation for a Brinkman model is

$$\rho_0\left(\frac{\partial}{\partial t}+(\mathbf{q}.\nabla)\right)\mathbf{q} = -\nabla p+\rho\,\mathbf{g} \;+\nabla.(\mathbf{HB})-\frac{1}{k}(\zeta+\eta)\mathbf{q}+2\zeta(\nabla\times\boldsymbol{\omega})+(\zeta+\eta)\nabla^2\,\mathbf{q} \tag{2}$$

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

The internal angular momentum equation is

$$\rho_0 I\left(\frac{\partial}{\partial t}+(\mathbf{q}.\nabla)\right)\boldsymbol{\omega}=2\zeta(\nabla\times\mathbf{q}-2\boldsymbol{\omega})+\mu_0(\mathbf{M}\times\mathbf{H})+(\lambda'+\eta')\nabla(\nabla.\boldsymbol{\omega})+\eta'\nabla^2\boldsymbol{\omega} \quad (3)$$

The temperature equation is

$$\left[\rho_0 C_{v,H}-\mu_o\mathbf{H}.\left(\frac{\partial\mathbf{M}}{\partial T}\right)_{v,H}\right]\frac{DT}{Dt}+\rho_s C_s\left(\frac{\partial T}{\partial t}\right)+\mu_0 T\left(\frac{\partial\mathbf{M}}{\partial T}\right)_{v,H}.\frac{D\mathbf{H}}{Dt}$$
$$=K_1\nabla^2 T+\delta(\nabla\times\boldsymbol{\omega}).\nabla T+\phi \quad (4)$$

The mass flux equation is

$$\rho_0(\partial/\partial t+\mathbf{q}\cdot\nabla)S=K_S\nabla^2 S+S_T\nabla^2 T \quad (5)$$

Using Maxwell's equation, we can assume that the magnetization is

$$\mathbf{M}=\frac{\mathbf{H}}{H}M(H,T,S) \quad (6)$$

The linearized magnetic equation in term of H_0, T_a and S_a is

$$M=M_0+\chi(\mathrm{H}-H_a)-K(T-T_0)+K_2(S-S_a) \quad (7)$$

The density equation is

$$\rho=\rho_0[1-\alpha_t(T-T_a)+\alpha_S(S-S_a)] \quad (8)$$

where **q**- velocity of fluid, ρ_0 - mean density of the clean fluid, *p*- pressure, ρ - density of the fluid, **g** - gravitational field, **H**-magnetic field, **B**-magnetic induction, ζ -coupling viscosity, ω - microrotation, η-shear viscosity coefficient, *k*-porous effect, *I*-moment of inertia, **M**-magnetization, λ'-bulk spin viscosity, η'-shear spin viscosity, -effective heat capacity at constant volume, -specific heat solid material, μ_0-viscosity of the fluid when the applied magnetic field is absent, -thermal diffusivity, *T*-temperature, δ-micropolar heat conduction coefficient, *S*-solute concentration, -concentration diffusivity, -Soret coefficient, -uniform magnetic field, -average temperature, -average salinity, α_t -thermal expansion coefficient and α_s -analogous solvent coefficient.

2.2 Basic state

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

The basic state quantities are

$$\left.\begin{aligned}&\mathbf{q}=\mathbf{q}_b=\ (0,\ \ 0,\ \ 0),\quad \boldsymbol{\omega}=\boldsymbol{\omega}_b=(0,\ \ 0,\ \ 0), T_b=T_0-\beta_t z,\\ &S_b=S_0-\beta_S z,\quad \rho(z)=\rho_0[1+\alpha_t\beta_t z-\alpha_S\beta_S z],\ p=p_b(z),\ \mathbf{H}_b(z)=\left[H_0-\frac{K\beta_t\,z}{1+\chi}+\frac{K_2\beta_S\,z}{1+\chi}\right]\mathbf{k},\\ &\mathbf{M}_b(z)=\left[M_0+\frac{K\beta_t\,z}{1+\chi}-\frac{K_2\beta_S\,z}{1+\chi}\right]\mathbf{k}\ \text{and}\ \ M_0+H_0=H_0^{ext}\end{aligned}\right\} \tag{9}$$

where subscript $b-$ the basic state and $\hat{\mathbf{k}}-$ unit vector vertical direction.

2.3 Perturbed state and equations

In this analysis, a small thermal disturbance is made. The perturbation of **M** and **H** can be taken as $[M_1^{'},\ M_2^{'},\ M_0(z)+M_3^{'}]$ and $[H_1^{'},\ H_2^{'},\ H_0(z)+H_3^{'}]$. The quantities are

$$\left.\begin{aligned}&\mathbf{q}=\mathbf{q}_b+\mathbf{q}',\ \boldsymbol{\omega}=\boldsymbol{\omega}_b+\boldsymbol{\omega}',\ \rho=\rho_b+\rho',\ \ p=p_b(z)+p',\\ &T=T_b(\mathrm{z})+T',\ \ S=S_b(\mathrm{z})+S',\ \mathbf{H}=\mathbf{H}_b(z)+\mathbf{H}',\ \ \mathbf{M}=\mathbf{M}_b(z)+\mathbf{M}'\end{aligned}\right\} \tag{10}$$

the superscript ' denotes perturbed state.

The density equation (8) can be manipulated is

$$\rho'=\rho_0(-\alpha_t T'+\alpha_s S') \tag{11}$$

Then the linearized perturbation equations of Eq. (2) become

$$\rho_0\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\mu_0(M_0+H_0)\frac{\partial H_1^{'}}{\partial z}+2\zeta\Omega_1^{'}+\left(\zeta+\eta\right)\nabla^2 u-\frac{1}{k}\left(\zeta+\eta\right)u, \tag{12}$$

$$\rho_0\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\mu_0(M_0+H_0)\frac{\partial H_2^{'}}{\partial z}+2\zeta\Omega_2^{'}+\left(\zeta+\eta\right)\nabla^2 v-\frac{1}{k}\left(\zeta+\eta\right)v, \tag{13}$$

$$\begin{aligned}\rho_0\frac{\partial w}{\partial t}=&-\frac{\partial p}{\partial z}+\mu_0(M_0+H_0)\frac{\partial H_3^{'}}{\partial z}-\mu_0 K\beta_t H_3^{'}+\frac{\mu_0 K^2\beta_t T'(1-S_T)}{1+\chi}\\ &-\frac{\mu_0 KK_2\beta_t S^{'}}{1+\chi}+\mu_0 K_2\beta_S H_3^{'}-\frac{\mu_0 KK_2\beta_s(1-S_T)T'}{1+\chi}+\frac{\mu_0 K_2^2\beta_S S^{'}}{1+\chi}\\ &+\rho_0 g\alpha_t\theta-\rho_0 g\alpha_t S^{'}+2\zeta\Omega_3^{'}+\left(\zeta+\eta\right)\nabla^2 w-\frac{1}{k}\left(\zeta+\eta\right)w,\end{aligned} \tag{14}$$

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Eqs. (3)-(5) can be calculated to perturbed equations as

$$\rho_0 I\left(\frac{\partial \Omega_3^{'}}{\partial t}\right) = -2\zeta\left(\nabla^2 w + 2\Omega_3^{'}\right) + \eta'\nabla^2\Omega_3^{'}, \tag{15}$$

$$\rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) = \left[\rho C_2 \beta_t - \left(\frac{\mu_0 K^2 T_0^2 \beta_t}{1+\chi}\right) + \left(\frac{\mu_0 K K_2 T_0 \beta_S}{1+\chi}\right)\right] w + K_1 \nabla^2 \theta - \delta \beta_t \Omega_3^{'}, \tag{16}$$

$$\frac{\partial S}{\partial t} + \beta_s w = K_S \nabla^2 S' + S_T \nabla^2 T^{'}, \tag{17}$$

3. Normal mode analysis technique

We consider the perturbation parameters are of the form

$$f(x,y,z,t) = f(z,t) exp[ik_x x + ik_y y], \tag{18}$$

where $f(z, t)$ represents $w(z, t)$, $T(z, t)$, $\phi(z,t)$ and $S(z, t)$.

With the use of Eqs. (12) - (14), the vertical component of Eq. (2) is

$$\begin{aligned}
&\left(\rho_0 \frac{\partial}{\partial t} + \frac{1}{k}(\xi+\eta)\right)\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w \\
&= \mu_0 K \beta_t k_0^2 \frac{\partial \phi}{\partial z} - \left(\frac{\mu_0 K^2 \beta_t (1-S_T)}{1+\chi}\right) k_0^2 T' + \left(\frac{\mu_0 K K_2 \beta_t}{1+\chi}\right) k_0^2 S' - \mu_0 K_2 \beta_S k_0^2 \frac{\partial \phi}{\partial z} \\
&\quad + \left(\frac{\mu_0 K K_2 \beta_t}{1+\chi}\right) k_0^2 S' - \mu_0 K_2 \beta_S k_0^2 \frac{\partial \phi}{\partial z} + \left(\frac{\mu_0 K K_2 \beta_S (1-S_T)}{1+\chi}\right) k_0^2 T' - \left(\frac{\mu_0 K_2^2 \beta_S}{1+\chi}\right) k_0^2 S' \\
&\quad - \rho_0 g \alpha_t k_0^2 T^{'} + \rho_0 g \alpha_S k_0^2 S + 2\zeta\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\Omega_3^{'} + \left(\zeta+\eta\right)\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)^2 w,
\end{aligned} \tag{19}$$

From Eq. (3) after performing mathematical manipulation, one gets

$$\rho_0 I \frac{\partial \Omega_3^{'}}{\partial t} = -2\zeta\left[\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w + 2\Omega_3^{'}\right] + \eta'\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\Omega_3^{'}, \tag{20}$$

Eq. (4) can be calculated as

Received: August 09, 2025

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$\rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right)$$
$$= \left[\rho C_2 \beta_t - \left(\frac{\mu_0 K^2 T_0^2 \beta_t}{1+\chi}\right) + \left(\frac{\mu_0 K K_2 T_0 \beta_S}{1+\chi}\right)\right] w - \delta \beta_t \Omega_3' + K_1\left(\frac{\partial^2}{\partial z} - k_0^2\right)\theta \tag{21}$$

The Salinity equation is

$$\left(\frac{\partial S}{\partial t}\right) + \beta_S w = K_S \left(\frac{\partial^2}{\partial z^2} - k_0^2\right) S' + S_T \left(\frac{\partial^2}{\partial z^2} - k_0^2\right) T', \tag{22}$$

Using the investigation of Vaidyanathan et al. [11], one gets

$$(1+\chi)\frac{\partial^2 \phi}{\partial z^2} - \left(1 + \frac{M_0}{H_0}\right) k_0^2 \phi - K \frac{\partial T'}{\partial z} + K_2 \frac{\partial S}{\partial z} + S_T K \frac{\partial T'}{\partial z} = 0, \tag{23}$$

Then, Eqs. (19)-(23) become non-dimensional equations

$$\left(\frac{\partial}{\partial t^*} + \frac{1}{k}(1+N_1)\right)(D^2 - a^2) w^*$$
$$= R^{1/2} M_1 D\phi^* - (1 + M_1(1 - S_T)(1 + M_5)) a R^{1/2} T^*$$
$$+ a R^{1/2} M_1 M_5 D\phi^* + a R_S^{1/2}(1 + M_4) S^* - a R^{1/2} M_1 M_5 T^* + a R_S^{1/2}\left(\frac{M_4}{M_5}\right) S^*$$
$$+ 2N_1 (D^2 - a^2)\Omega_3'^* + (\zeta + \eta)(D^2 - a^2)^2 w^*, \tag{24}$$

$$I' \frac{\partial \Omega_3'^*}{\partial t} = -2N_1 \left[\left(D^2 - a^2\right) w^* + 2\Omega_3'^*\right] + N_3'\left(D^2 - a^2\right)\Omega_3'^*, \tag{25}$$

$$\left[P_r' \frac{\partial T^*}{\partial t^*} - P_r M_2 \frac{\partial}{\partial t^*}(D\phi^*)\right] = (D^2 - a^2) T^* + a R^{1/2}(1 - M_2 - M_2 M_5) w^* - a R^{1/2} N_5' \Omega_3'^*, \tag{26}$$

$$P_r \frac{\partial S^*}{\partial t^*} = \tau (D^2 - a^2) S^* - a R_S^{1/2} M_6 w^* + S_T \left(\frac{M_5}{M_6}\right)\left(\frac{R}{R_s}\right)^{1/2} (D^2 - a^2) T^*, \tag{27}$$

$$D^2 \phi^* - M_3 a^2 \phi^* - (1 - S_T) D T^* + \frac{M_5}{\tau}\left(\frac{R}{R_S}\right)^{1/2} D S^* = 0, \tag{28}$$

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

where the non-dimensional numbers used are

$$w^* = \frac{wd}{\nu}, t^* = \frac{\nu t}{d^2}, T^* = \left(\frac{K_1 a R^{1/2}}{\rho_0 C_{v,H} \beta_t \nu d}\right)\theta, z^* = \frac{z}{d}, a = k_0 d, D = \frac{\partial}{\partial z^*}, S^* = \left(\frac{K_S a R_S^{1/2}}{\rho_0 C_{v,H} \beta_S \nu d}\right) S,$$

$$\Omega_3^{'*} = \frac{\Omega_3}{\nu} d^3, M_1 = \frac{\mu_0 K^2 \beta_t}{(1+\chi)\rho_0 g \alpha_t}, M_2 = \frac{\mu_0 K^2 T}{(1+\chi)\rho_0 C_{v,H}}, N_1 = \frac{\zeta}{\eta}, P_r = \frac{\nu}{K_1}\rho C_2$$

$$M_3 = \frac{1 + M_0 / H_0}{(1+\chi)}, M_4 = \frac{\mu_0 K^2 \beta_s}{(1+\chi)\rho_0 g \alpha_s}, M_5 = \frac{K_2 \beta_s}{K \beta_t}, M_6 = \frac{K_S}{K_1}, \tau = \rho_0 C_{v,H}\left(\frac{K_S}{K_1}\right), N_3^{'} = \frac{\eta'}{\eta d^2},$$

$$I' = \frac{I}{d^2}, R_S = \frac{\rho_0 C_{v,H} \beta_s \alpha_s g d^4}{\nu K_S}, R = \frac{\rho_0 C_{v,H} \beta_t \alpha_t g d^4}{\nu K_1}, \phi^* = \left(\frac{(1+\chi) K_1 a R^{1/2}}{\rho_0 C_{v,H} K \beta_t \nu d^2}\right)\phi, \nu = \frac{\mu}{\rho_0},$$

$$P_r^{'} = \frac{\nu}{K_1}\rho C_1, k^* = \frac{k}{d^2}, N_5^{'} = \frac{\delta^{'}}{\rho C_2 d^2},$$

4. Linear stability theory

In this, one can study both the stationary and oscillatory instabilities. Also, in this moment the linear theory is used. The free boundary conditions are

$$w^* = D^2 w^* = T^* = D\phi^* = S^* = \Omega_3^* = 0 \quad \text{at} \quad z = \pm 1/2, \tag{29}$$

The exact solutions satisfying above Eq. (29) are

$$w^* = C_1 e^{\sigma t^*} cos\pi z^*, T^* = C_2 e^{\sigma t^*} cos\pi z^*, \; S^* = C_3 e^{\sigma t^*} cos\pi z^*, D\phi^* = C_4 e^{\sigma t^*} cos\pi z^*,$$
$$\phi^* = \frac{C_4}{\pi} sin\pi z^*, \Omega_3^* = C_5 e^{\sigma t^*} cos\pi z^*, \tag{30}$$

where C_1, C_2, C_3, C_4 and C_5 are constants. In this part, all the partial derivatives and asterisks are removed with use of exact solutions to find the solution of the system of homogeneous equations in Eqs. (24) - (28). Using Eq. (30) in Eqs. (24) -(28), we get

$$\left[\left(\left(\sigma + \frac{1}{k}(1+N_1)\right)(\pi^2 + a^2) + (1+N_1)(\pi^2+a^2)^2\right)\right] C_1 - aR^{1/2}\left[1 + M_1(1-S_T)(1+M_5)\right] C_2$$
$$+ aR_S^{1/2}(1 + M_4 + M_4 M_5^{-1}) C_3 + aR^{1/2} M_1 (1+M_5) C_4 - 2N_1(\pi^2 + a^2) C_5 = 0, \tag{31}$$

$$2N_1(\pi^2 + a^2) C_1 - \left[4N_1 + (\pi^2 + a^2) N_3^{'} + I'\sigma\right] C_5 = 0, \tag{32}$$

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$aR^{1/2}(1-M_2-M_2M_5)C_1-(\pi^2+a^2+P_r\sigma)C_2+P_r\sigma M_2C_4-aR^{1/2}N_5^{'}C_5=0, \tag{33}$$

$$aR_S^{1/2}M_6C_1+S_T\left(\frac{M_5}{M_6}\right)\left(\frac{R}{R_s}\right)^{1/2}(\pi^2+a^2)C_2+\left[\tau(\pi^2+a^2)+\sigma P_r\right]C_3=0, \tag{34}$$

$$R_S^{1/2}\pi^2(1-S_T)C_2-R^{1/2}\pi^2(M_5/M_6)C_3-R_S^{1/2}(\pi^2+a^2M_3)C_4=0, \tag{35}$$

To obtain the Eigen function, the determinant of co-efficient of C_1, C_2, C_3, C_4 and C_5 is equal to zero. Eqs. (31) - (35) have been calculated as

$$Z_1\sigma^4+Z_2\sigma^3+Z_3\sigma^2+Z_4\sigma+Z_5=0, \tag{36}$$

$$\begin{aligned}
Z_1 &= x_1x_9P_rP_r^{'}I'\\
Z_2 &= x_9P_rI'\left(x_1^2(x_9P_r^{'}+1)+\frac{P_rx_9}{k}\right)+x_1x_9P_r^{'}(I'+x_1\tau),\\
Z_3 &= x_1^2x_9P_r^{'}\left(a^2x_2R-x_1^3x_9-\frac{x_1^2}{k}\right)-a^2x_4x_7x_8P_rRI'-P_r^{'}(x_1x_9+aM_6I'R_s)-2x_1x_9P_r^{'}\tau,\\
Z_4 &= ax_4x_8R((ax_1x_6+x_6M_6)I'+x_5M_6P_r^{'})+ax_4x_7x_8P_rR(2x_1N_1N_5^{'}-ax_5)\\
&\quad -x_1^3x_5x_9\tau P_r^{'}+x_9(y_2P_r-2x_1^3N_1P_r^{'}\tau)-ax_3x_5(x_1I'(1+x_5R_s))-x_5M_6R_sP_r^{'},\\
Z_5 &= ax_4x_8(M_6(x_1x_2+I'P_r^{'})-2x_1^2x_6N_1N_5^{'})+ax_1x_4x_7x_8R\tau\\
&\quad (2x_1N_1N_5^{'}-ax_5)-aM_6R_sI'P_r^{'}+x_1x_9y_1\tau(1+x_5)\\
&\quad -a^2x_3x_5(x_1x_5R_s(x_6+M_6)-2x_1^2x_6N_1N_5^{'}),\\
x_1 &= \pi^2+a^2,\ x_2=1+(1-S_T)x_4,\ x_3=1+M_4+M_4/M_5\\
x_4 &= M_1(1+M_5),\ x_5=4N_1+x_1N_3^{'},\ x_6=S_TM_5/M_6,\\
x_7 &= (1-S_T)\pi^2,\ x_8=\pi^2M_5/M_6,\ x_9=\pi^2+a^2M_3,\\
y_1 &= a^2x_2R-x_1^3x_9-\frac{x_1^2x_9}{k},\ y_2=a^2x_2RN_5^{'}-2x_1^2N_1.
\end{aligned}$$

4.1 The case of stationary instability

For steady state, we have σ = 0 at the marginal stability. The critical magnetic Rayleigh number R_{sc} has been calculated from Eq. (36) as

$$R_{sc}=Nr/Dr, \tag{37}$$

International Journal of Applied Mathematics

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

where

$$Nr = (\pi^2 + a^2)^2 \begin{pmatrix} (4N_1 + (\pi^2 + a^2)N_3^{'})(1 + N_1) \\ \left(\dfrac{1}{k} + \pi^2 + a^2\right) + 4(\pi^2 + a^2)N_1^2 \end{pmatrix}$$
$$- \begin{pmatrix} (1 + M_4 + M_4 / M_5)((4N_1 + (\pi^2 + a^2)N_3^{'}) \\ (M_6 + S_T M_5 / M_6) + 2(\pi^2 + a^2)S_T M_6 N_5^{'}) \end{pmatrix} \frac{a^2 R_s}{\tau}$$

$$Dr = a^2(1 + (1 - S_T)M_1(1 + M_5))(4N_1 + (\pi^2 + a^2)(N_3^{'} - 2N_1 N_5^{'}))$$
$$- \left(\frac{a^2 \pi^2 M_1 (1 + M_5)}{\pi^2 + a^2 M_3} \right)$$
$$\begin{pmatrix} (4N_1 + (\pi^2 + a^2)N_3^{'})(1 - S_T + (S_T (M_5 / M_6)^2 + M_6) / \tau) \\ -2a^2 N_1 N_5^{'} (\pi^2 + a^2)(1 - S_T + (S_T (M_5 / M_6)^2) / \tau) \end{pmatrix}$$

when M_1 is very large, one can gets N_{sc} ($= M_1 R_{sc}$)

$$N_{sc} = Nr / Dr_1, \tag{38}$$

where

$$Dr_1 = a^2(1 + (1 - S_T)(1 + M_5))(4N_1 + (\pi^2 + a^2)(N_3^{'} - 2N_1 N_5^{'}))$$
$$- \left(\frac{a^2 \pi^2 (1 + M_5)}{\pi^2 + a^2 M_3} \right)$$
$$\begin{pmatrix} (4N_1 + (\pi^2 + a^2)N_3^{'})(1 - S_T + (S_T (M_5 / M_6)^2 + M_6) / \tau) \\ -2a^2 N_1 N_5^{'} (\pi^2 + a^2)(1 - S_T + (S_T (M_5 / M_6)^2) / \tau) \end{pmatrix}$$

Setting $S_T = 0$ in Eq. (38), one gets

$$N_{sc} = Nr / Dr, \tag{39}$$

where

$$Nr = (\pi^2 + a^2)^2 \begin{pmatrix} (4N_1 + (\pi^2 + a^2)N_3^{'})(1 + N_1) \\ \left(\dfrac{1}{k} + \pi^2 + a^2\right) + 4(\pi^2 + a^2)N_1^2 \end{pmatrix}$$
$$- \left((1 + M_4 + M_4 / M_5)((4N_1 + (\pi^2 + a^2)N_3^{'})M_6 \right) \frac{a^2 R_s}{\tau}$$

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$Dr = a^2(1+M_5)(4N_1 + (\pi^2 + a^2)(N_3^{'} - 2N_1N_5^{'})) \\ -\left(\frac{a^2\pi^2(1+M_5)}{\pi^2 + a^2M_3}\right) \\ \left(\begin{array}{l}(4N_1 + (\pi^2 + a^2)N_3^{'})(1+M_6/\tau)) \\ -2a^2N_1N_5^{'}(\pi^2 + a^2)\end{array}\right)$$

which is an expression of N_{sc} of Sekar and Raju [22].When $S_T = 0$, $\varepsilon = 1, N_1 = 0, N_3^{'} = 1$ and $N_5^{'} = 0$ in Eq. (38), we get

$$N_{sc} = Nr / Dr, \tag{40}$$

$$N_{SC} = \frac{(\pi^2 + a^2M_3)(\pi^2 + a^2)^2\left(\frac{1}{k} + \pi^2 + a^2\right) - a^2M_6R_s(1 + M_4 + M_4/M_5)/\tau}{a^2(1+M_5)\left[1 - \pi^2(1 + M_5/\tau)\right]}$$

which is an expression of N_{sc} of Vaidyanathan et al. [13].

4.2 The case of oscillatory instability

Using $\sigma = i\sigma_1$ in Eq. (36), it gives the Rayleigh number R_{oc} and following Refs. [22] and [23],

$$R_{OC} = \frac{A_2A_6\sigma_1^6 + (A_1A_6 + A_2A_5 + A_3A_8)\sigma_1^4 + (A_2A_4 + A_1A_5 + A_3A_7)\sigma_1^2 + A_1A_4}{(A_1 + A_2\sigma_1^2)^2 - A_3^2\sigma_1^2}$$

where

$$A_1 = a^2x_4x_8(2x_1^2x_6N_1N_5^{'} - (x_5x_7 + I'P_r^{'})M_6 \\ + x_1x_4x_7x_8\tau(ax_5 - 2x_1N_1N_5^{'}) + ax_1x_2x_5x_9 - ax_1x_2x_9N_5^{'}\tau)$$

$$A_2 = -a^2I'P_r(x_4x_7x_4 + x_2)$$

$$A_3 = a^2x_4x_8(x_6I'(x_2 + M_6) + x_5M_6P_r^{'}) + ax_4x_7x_8P_r^{'}(ax_5 - 2x_1N_1N_5^{'}) \\ + a^2x_1x_4x_7x_8I'\tau + a^2x_2x_9(x_5P_r + x_1\tau I') + a^2x_2P_rN_5^{'}$$

$$A_4 = x_1^2x_5x_9\tau\left(\frac{x_4}{k} + x_1^2x_4\right) + a^2x_1x_3x_5x_6R_s(x_5(1+M_6) + 2x_1N_1N_5^{'})$$

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$$A_5 = x_1 x_9 I' P_r \left(\frac{x_4}{k} + x_1 x_9 \right) + x_1 x_5 x_9 P_r^{'} \tau + x_1^2 (1 + x_9 P_r^{'})(x_5 P_r + x_1 \tau I')$$
$$+ 2 x_1 N_1 P_r P_r^{'} + a^2 x_3 x_5 M_6 P_r^{'} I' R_s$$
$$A_6 = -x_1 P_r P_r^{'}$$
$$A_7 = -x_1^3 x_5 x_9 (1 + x_9 P_r^{'}) \tau - (x_5 P_r + x_1 I' \tau) + 2 x_1 N_1 (2 x_1^2 + P_r^{'})$$
$$- a^2 x_3 x_5 (x_1 x_6 I' R_s + x_1 I' M_6) + x_6 M_6 R_s P_r^{'}$$
$$A_8 = x_1 x_9 P_r^{'} (x_5 P_r + x_1 I' \tau) - P_r I' (x_1^2 P_r^{'} (1 + x_9))$$
$$\sigma_1^2 = \frac{B_2 \pm (B_2^2 - 4 B_1 B_3)^{1/2}}{2 B_1}$$
$$B_1 = A_2 A_8 - A_3 A_6,\ B_2 = A_3 A_5 - A_1 A_8 - A_2 A_7,\ B_3 = A_1 A_7 - A_3 A_4.$$

5. Results and Discussion

A linear analytical study has been worked out to investigate the onset of thermohaline convection on micropolar ferromagnetic fluid in Soret and porous effects with uniform angular velocity. The stationary and oscillatory instabilities are calculated using free-free boundary conditions. In the present study, we attempted to analyze the Soret and porous effects on thermohaline convection in micropolar ferromagnetic fluid and Brinkman model is used. The magnetic numbers M_1 and M_2 are the assumed to 1000 and 0, respectively. M_3 is taken from 5 to 25 (Vaidyanthan et al. [13]) and k ranges from 0.1 to 0.9 (large permeability). τ ranges from 0.05 to 0.11 (Vaidyanthan et al. [13]) and R_s ranges from -500 to 500. The Soret coefficient S_T is considered values from 0-.002 to 0.002 (Sekar et al. [23]). The magnetization parameters $M_4$and M_6 are assumed to be 0.1 and $M_5 = 0.5$ (Sekar and Raju[22]). Moreover, N_1, $N_3^{'}$ and $N_5^{'}$ are taken to be non-negative values which is presented by Eringen [15] and he assumed the clausius-Duhem inequality.

Figs. 1-2 show the variation of N_{SC} versus N_1 for different values of k, $S_T = -0.002$ and $S_T = 0.002$, respectively. This gives that k has a stabilizing effect. The nature of the effect of isotropic has a destabilizing behavior which was studied by Sekar et al. [23]. But, influence of coupling effect on convective system, the system gets to an opposite behavior of destabilization. Also, when $S_T = -0.002$ and $k = 0.9$, the system gets lowest thermal energy.

Figs. 3-4 represent the variation of a_c versus N_1 for different k, $S_T = -0.002$ and $S_T = 0.002$, respectively. When N_1 increased from 0.2 to 0.8, a_c decreased in the lowest and highest values of S_T (-0.002 and 0.002). This trend is opposite to N_{sc} which is depicted in Figs. 1-2.

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Fig. 5 displays the variation of N_{SC} versus coupling parameter N_1 for different medium permeability k with heavy salt. When the S_T (= 0.002) and R_s (= 500) on the convective system for different moment, N_{sc} gets smallest values. The system has null effect when k = 0.1 and N_1 = 0.6. That is N_{sc} has a zero value in this moment. But the system has high thermal energy only when k = 0.7 and N_1 = 0.6. It is shown that the N_{SC} coincides in many physical situations and the system is in oscillatory position in the different values of k.

Figs. 6-7 represent the plot of N_{SC} versus $N_3^{'}$ for different k, $S_T = -0.002$ and $S_T = 0.002$, respectively. These figures show clearly that $N_3^{'}$ has a destabilizing behavior for increasing of k. When k = 0.1, the system gets high energy because of lowest medium permeability and noticed that the increasing of k from 0.3 to 0.9, the system observed low energy.

In Fig. 8, the variation of N_{SC} versus spin diffusion parameter $N_3^{'}$ for various k, S_T = 0.002 and R_S = 500 is analyzed. This figure indicates a decrease in cell shape as $N_3^{'}$ takes values from 2 to 8 when k =0.3, 0.5 and k =0.7. But, as $N_3^{'}$ from 2 to 4, N_{SC} increases and it then decreases phenomenonally for k = 0.1 and k = 0.9.

Figs. 9-10 show the variation of N_{SC} with respect to $N_5^{'}$ for various k, $S_T = -0.002$ and S_T = 0.002, respectively. $N_5^{'}$ has always a stabilizing flow and both figures have analyzed for S_T $=-0.002$ and 0.002.

In Fig. 11, the variation of N_{SC} versus micropolar heat conduction parameter $N_5^{'}$ is analysed. It is clear from this figure that there is competition between stabilizing and destabilizing behaviors on system for different k. The micropolar heat conduction parameter $N_5^{'}$ increases, N_{SC} increases exponentially only when k = 0.1. The system has stabilizing behavior in this situation. When $N_5^{'}$ increases from 0.1 to 0.5, N_{SC} decreases and it then increases suddenly. It is shown parabolic form of cell shape when k = 0.3. But, for the values of k = 0.5, 0.7 and 0.9, N_{SC} gets increased, as $N_5^{'}$ increased from 0.1 to 0.5 and then decreased. Therefore, there is competition between stabilizing and destabilizing behaviors in Fig. 11.

Fig. 12 analysed for the salinity Rayleigh number R_S for various values of k. When R_S increased from -500 to 500, N_{SC} decreased uniformly and converged. When R_S = 500, N_{SC} gets zero value. Therefore the system has a destabilizing behavior and converged to null effect.

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

In Fig. 13, the variation of N_{SC} versus S_T for different k is investigated. When S_T increased from -0.002 to 0.002, N_{SC} increased. Therefore the system has a stabilizing behavior. Fig. 14 is analysed for M_3 with the various values of k. When M_3 increased from 5 to 25, N_{SC} decreased. Therefore the system has destabilizing behavior.

6. Conclusion

In the present investigation, the results of a theoretical study of Soret effect on thermohaline convection in a micropolar ferromagnetic fluid saturating a porous medium is considered with free-free boundary conditions. We conclude that the destabilizing behavior of permeability of medium, non-buoyancy magnetization, salinity Rayleigh number and ratio of mass transport to heat transport is analyzed. The coupling and micropolar heat conduction parameters have a stabilizing behavior, which is dominating the effect of the permeability of the porous medium and non-buoyancy magnetization parameter which are depicted in Figs. 2-4 and Figs. 9-12. The spin diffusion parameter is studied for destabilizing behavior which is depicted in Figs. 6-8. But although in heavy arrangements of salt on the system, the system gets equilibrium state or otherwise it is changing the effect of stabilizing and destabilizing behaviors.

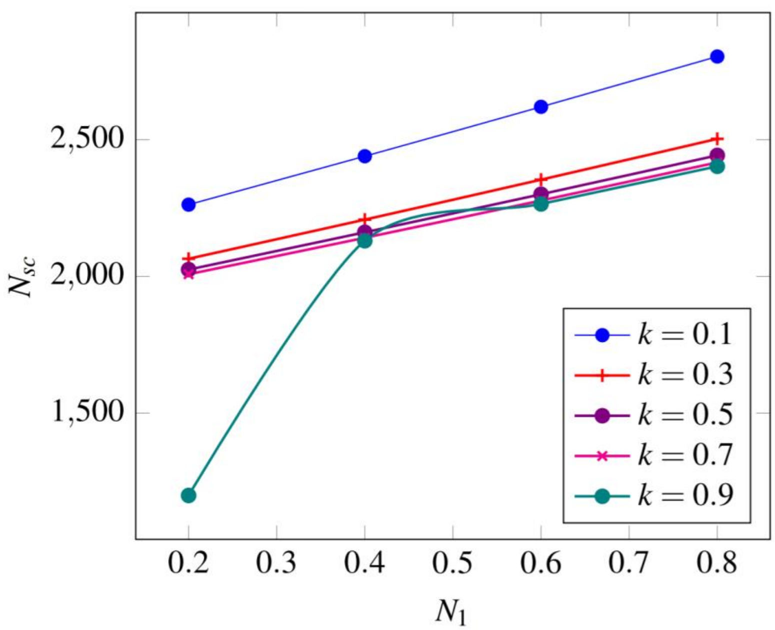


Figure 1: Variation of N_{sc} versus N_1 for various k, $M_3 = 5$, $S_T = -0.002$, $R_S = -500$, $\tau = 0.03$, $N_3' = 2$ and $N_5' = 0.2$.

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

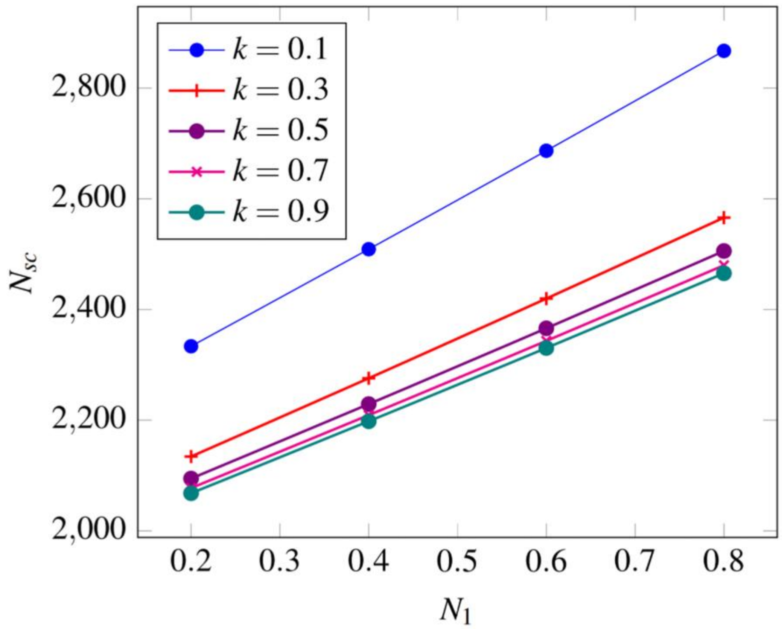


Figure 2: Variation of N_{sc} versus N_1 for various *k*, $S_T = 0.002$, $R_S = -500$, $M_3 = 5$, $\tau = 0.03$, $N_3' = 2$ and $N_5' = 0.2$.

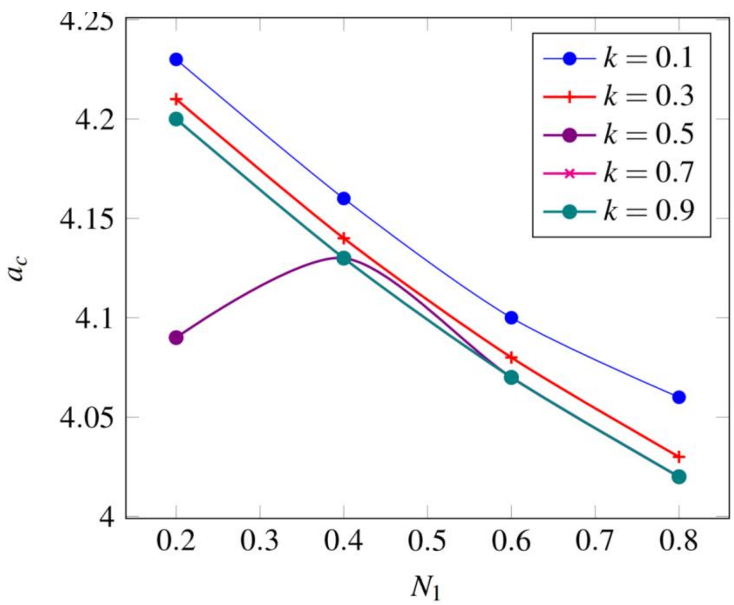


Figure 3: Variation of a_c versus N_1 for various *k*, $S_T = -0.002$, $R_S = -500$, $M_3 = 5$, $\tau = 0.03$, $N_3' = 2$ and $N_5' = 0.2$.

Received: August 09, 2025

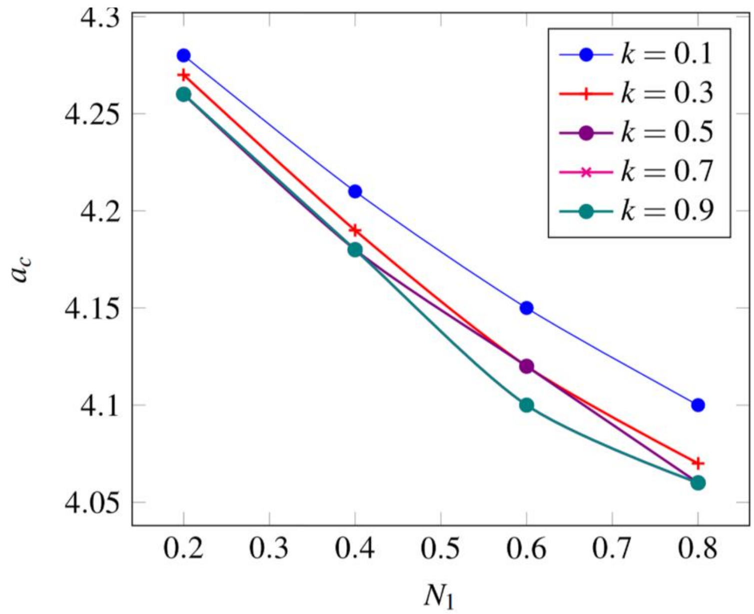


Figure 4: Variation of a_c versus N_1 for various k, $S_T = 0.002$, $R_S = -500$, $M_3 = 5$, $\tau = 0.03$, $N_3' = 2$ and $N_5' = 0.2$.

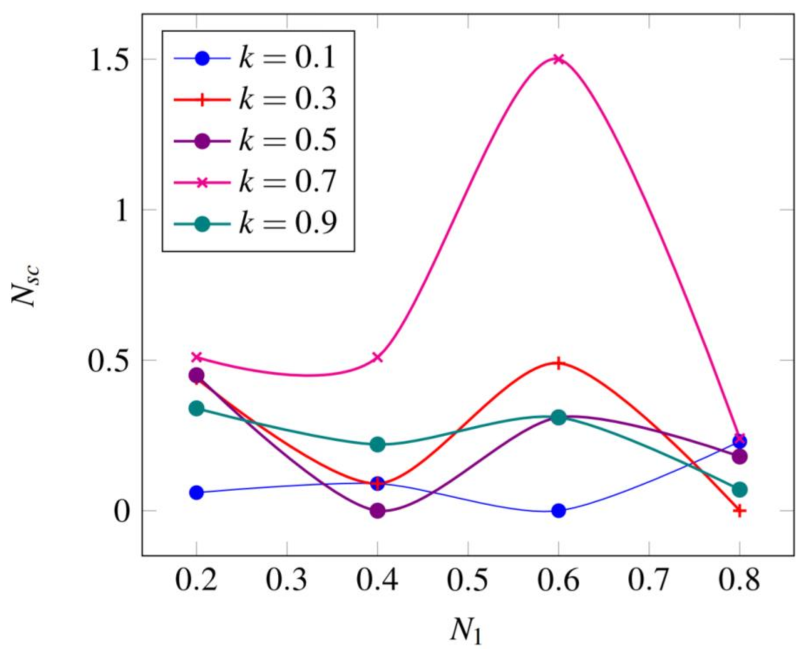


Figure 5: Variation of N_{sc} versus N_1 for various k, $S_T = 0.002$, $R_S = 500$, $M_3 = 5$, $\tau = 0.03$, $N_3' = 2$ and $N_5' = 0.2$.

International Journal of Applied Mathematics

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

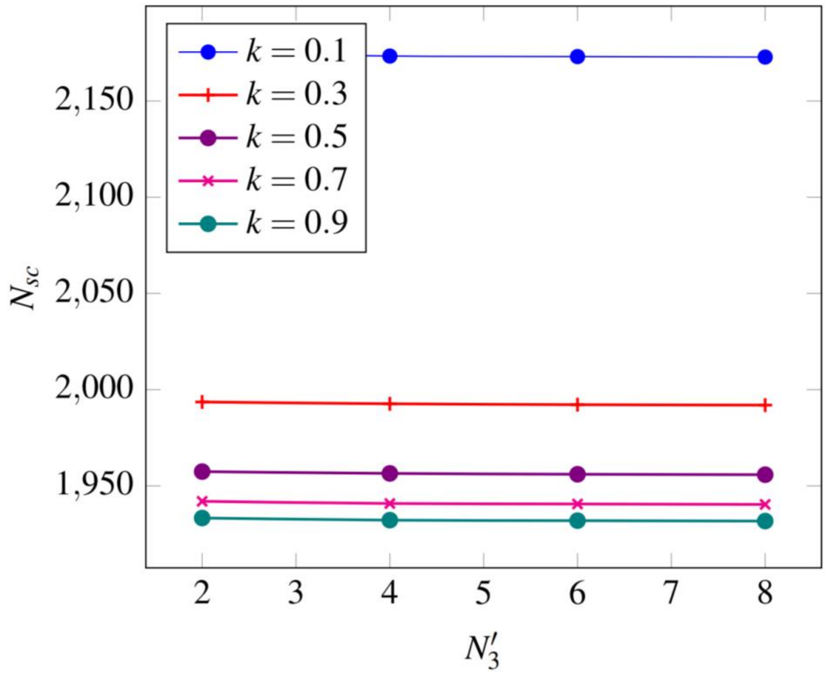
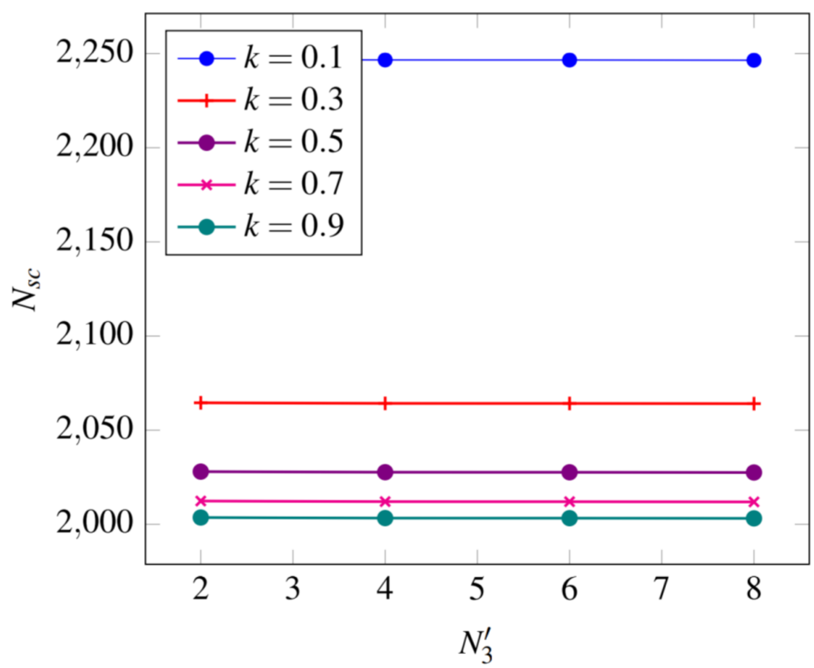


Figure 6: Variation of N_{sc} versus N_3' for various k, $S_T = -0.002$, $R_S = -500$, , $\tau = 0.03$, $N_1 = 0.2$ and $N_5' = 0.2$.



Received: August 09, 2025

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Figure 7: Variation of N_{sc} versus N_3' for various k, $S_T = 0.002$, $R_S = -500$, $M_3 = 5$, $\tau = 0.03$, $N_1 = 0.2$ and $N_5' = 0.2$.

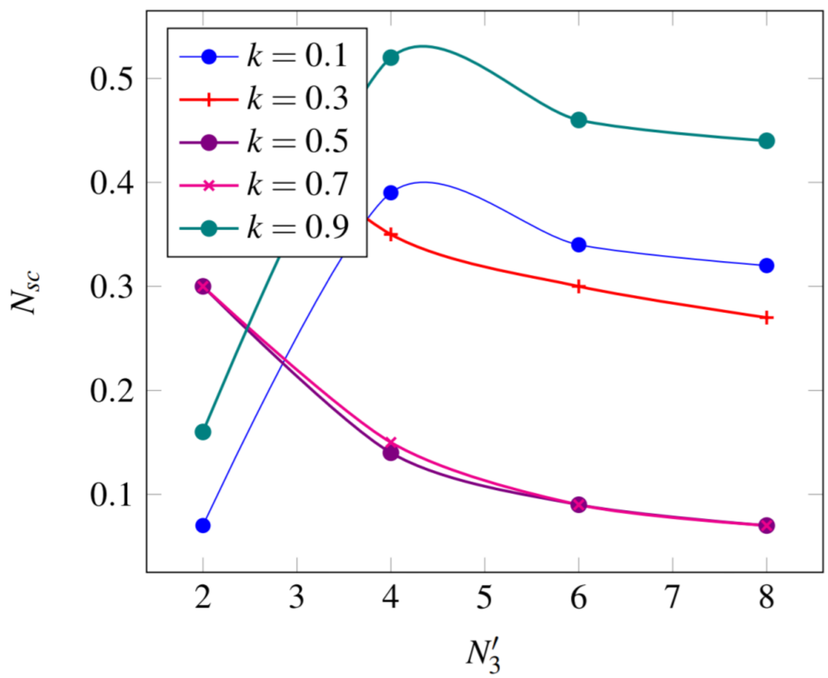


Figure 8: Variation of N_{sc} versus N_3' for various k, $S_T = 0.002$, $R_S = 500$, $M_3 = 5$, $\tau = 0.03$, $N_1 = 0.2$ and $N_5' = 0.2$.

International Journal of Applied Mathematics

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

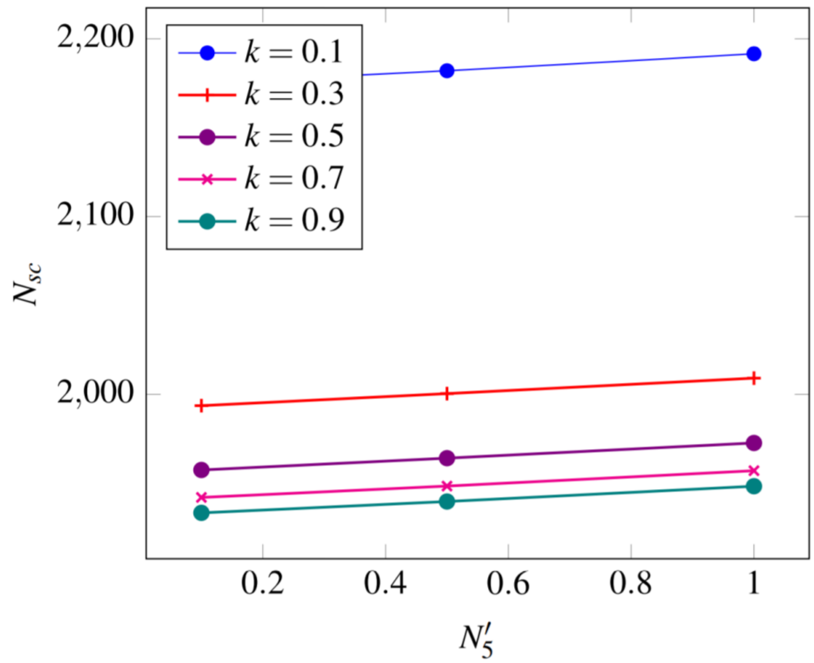
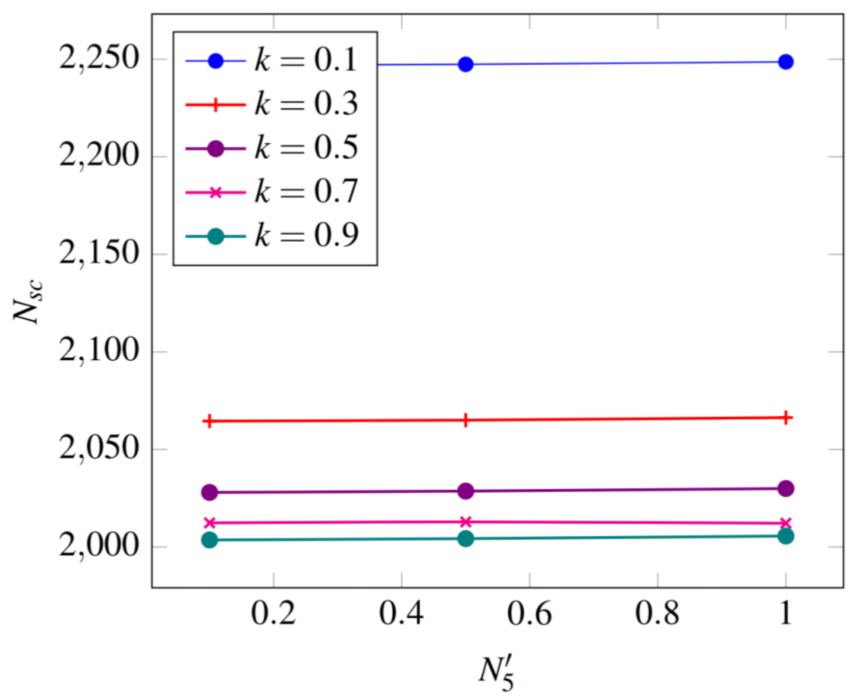


Figure 9: Variation of N_{sc} versus N_5' for various k, $S_T = -0.002$, $R_S = -500$, , $\tau = 0.03$, $N_1 = 0.2$ and $N_3' = 2$.



International Journal of Applied Mathematics

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Figure 10: Variation of N_{sc} versus N_5' for various k, $S_T = 0.002$, $R_S = -500$, $M_3 = 5$, $\tau = 0.03$, $N_1 = 0.2$ and $N_3' = 2$.

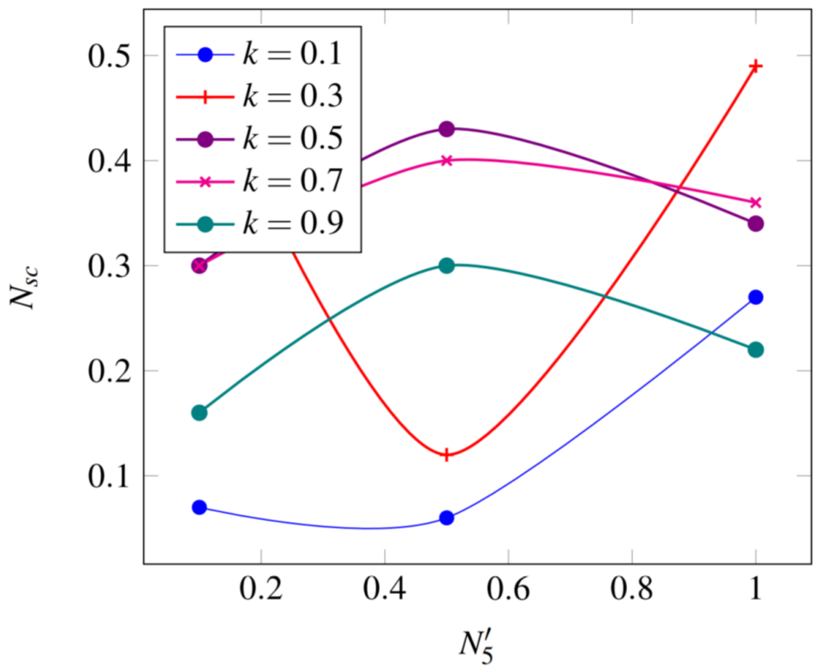


Figure 11: Variation of N_{sc} versus N_5' for various k, $S_T = 0.002$, $R_S = 500$, $M_3 = 5$, $\tau = 0.03$, $N_1 = 0.2$ and $N_3' = 2$.

International Journal of Applied Mathematics

Volume 38 No. 11s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

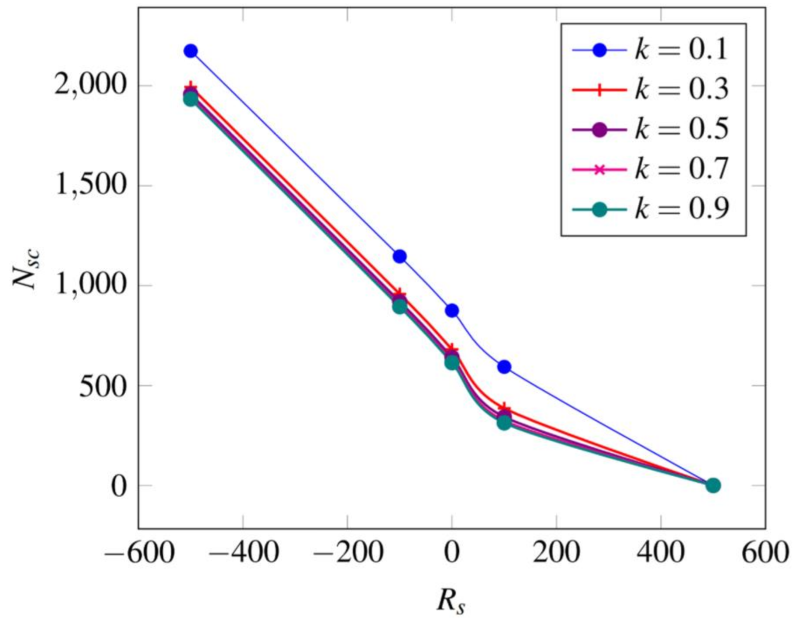


Figure 12: Variation of N_{sc} versus R_s for various k, $S_T = -0.002$, $M_3 = 5$, $\tau = 0.03$, $N_1 = 0.2$, $N_3' = 2$ and $N_5' = 0.1$.

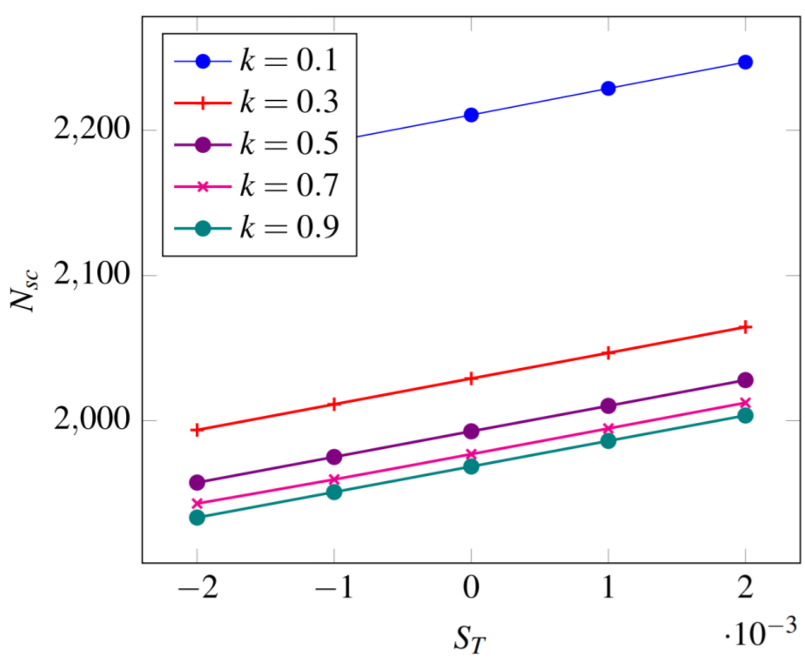


Figure 13: Variation of N_{sc} versus S_T for various k, $R_s = -500$, $M_3 = 5$, $\tau = 0.03$, $N_1 = 0.2$, $N_3' = 2$ and $N_5' = 0.1$.

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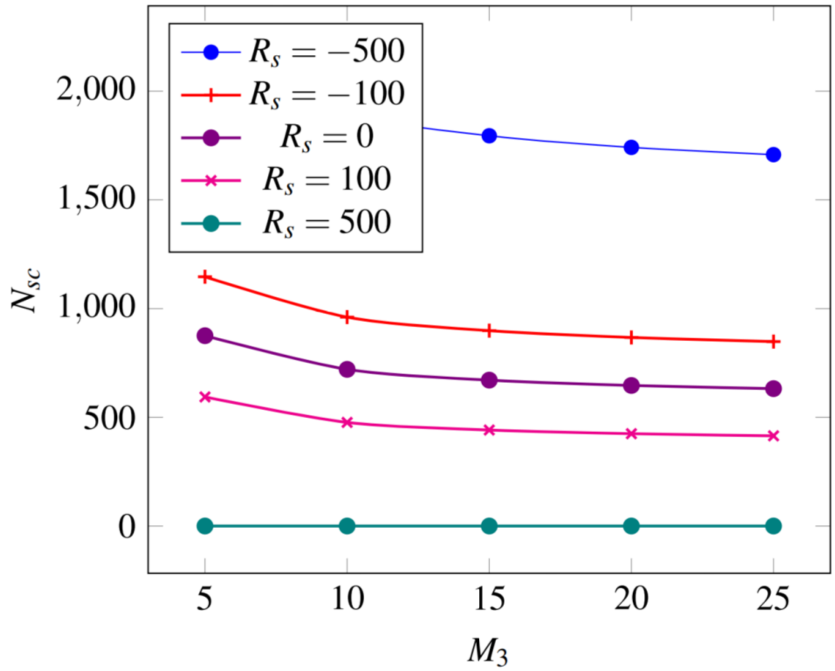

Figure 14: Variation of N_{sc} versus M_3 for various k, $R_s = -500$, $S_T = -0.002$, $\tau = 0.03$, $N_1 = 0.2$, $N_3' = 2$ and $N_5' = 0.1$.

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