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Indoor Localization Using Signal-Based Techniques: A Review of Fingerprinting and Radio-Based Hybrid Systems

Dr. Bhagwan Sahay Meena[1]

[1]Associate Professor, Department of Computer Science, Assam University Silchar

Abstract

Signal-based indoor localization leverages radio signals from WiFi, Bluetooth Low Energy (BLE), Ultra-Wideband (UWB), and cellular networks to determine positions in GPS-denied indoor environments where walls block satellite signals and cause multipath fading. These techniques enable applications like asset tracking in warehouses, patient navigation in hospitals, and real-time personnel monitoring in smart buildings, offering meter-level accuracy without extensive sensor hardware. Fingerprinting methods dominate by creating radio maps during offline phases, where Received Signal Strength Indicator (RSSI) or Channel State Information (CSI) is collected at reference points and matched online via probabilistic models or machine learning. This paper reviews fingerprinting and hybrid radio techniques that address key challenges like signal instability from human movement, non-line-of-sight propagation, and environmental noise through Gaussian processes, kernel interpolation, and neural networks. Recent models integrate deep learning such as convolutional neural networks for CSI patterns and recurrent networks for temporal RSSI sequences—to boost robustness, achieving sub-meter errors in dynamic settings. Hybrid frameworks fuse UWB for precise ranging, BLE beacons for low-power coverage, and WiFi for ubiquitous access, often using Kalman or particle filters to combine propagation models like trilateration with fingerprint matching. Strengths include scalability in large spaces and adaptability via online learning, but limitations persist: high fingerprinting costs, sensitivity to access point failures, and computational demands for real-time inference. The review analyzes performance metrics from studies showing 0.5-2m accuracies, highlights trade-offs in cost versus precision, and identifies future directions like federated learning for privacy-preserving updates and edge computing for low-latency deployment in diverse environments.

Keywords: Indoor Localization, Fingerprinting, BLE, WiFi, RSSI, Machine Learning, UWB, Deep Learning

1 Introduction

Radio signals offer a practical way to perform indoor localization because they are available in most buildings and do not require specialized hardware. Fingerprinting is one of the most

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widely used signal-based techniques. It matches real-time signal measurements to a pre-built database of signal patterns to estimate the user's location. Although accurate, fingerprinting faces problems such as signal variability, calibration overhead, and sensitivity to environmental changes.

Recent research has focused on improving fingerprinting through statistical modeling, interpolation, and machine learning. WiFi and BLE systems now use dimensionality reduction, neural networks, and deep models to extract stable features from noisy RSSI data. Hybrid approaches combine radio signals with additional information such as UWB ranging or motion models to improve positioning in cluttered environments. This paper reviews these developments, covering both classical methods and the latest learning-based techniques.

2 Fingerprinting Method

Fingerprinting is a technique used in indoor localization that involves mapping the unique characteristics of signals (e.g., Wi-Fi, Bluetooth, and magnetic fields) at specific locations within an indoor environment [1]. During the localization process, a device measures these signals and compares them against a pre-collected database or map of signal strengths (referred to as a fingerprint) to estimate its current position [2]. The process typically consists of two phases: offline (training) and online (localization) [3]. In the offline phase, signal characteristics, such as Received Signal Strength Indicator (RSSI) values, are measured at predefined reference points within the environment and stored in a database as fingerprints [3]. Similarly, in [4–6], PIR sensor and RFID-based indoor localization have been discussed for different scenarios to estimate the relative location of an object. In the online phase, the device collects real-time signal measurements, which are then matched with pre-stored fingerprints using algorithms such as k-nearest neighbors (k-NN) [7] or machine learning models to determine the most likely location [3]. This two-phase approach enables fingerprinting to achieve accurate indoor positioning, provided the environment remains stable and the database is well-calibrated.

The study [8] discussed about indoor localization using BLE and smartphones. It looks at RSSI signals from BLE and smartphones both when there's a moving object nearby and when there isn't. The study found that smartphone-based indoor localization works better and gives more accurate results than BLE. The article [9] discusses the problem of positioning and tracking using Bluetooth Low-Energy (BLE) beacons in indoor environments. The model leverages BLE beacons to monitor the position of a sensor within an indoor environment. It utilizes the RSSI values from these beacons to construct a probabilistic radio map of the space, illustrating the probability of receiving specific RSSI values at various locations. The proposed model further refines this approach by utilizing a fingerprinting technique for position estimation, generating histograms of RSSI readings at specific locations, as depicted in the figure 1. However, conventional tracking algorithms, such as the Kalman filter and its variants, are unsuitable because the fingerprint histograms do not exhibit a Gaussian-like structure. This is likely due to signal reflections from various objects and surfaces, as illustrated in Figure 12a. Moreover, achieving finer localization requires a large number of fingerprints, which is time-consuming and impractical to collect.

To solve this problem, the authors propose using affine Wasserstein histogram interpolation [10], which allows estimating a radio map with fewer fingerprint samples, making the process more practical and efficient. They also use the sequential Monte Carlo (SMC) method [11] for filtering. The Wasserstein histogram comes from optimal transport theory and is used to measure the distance between two probability distributions. Unlike other methods, their radio

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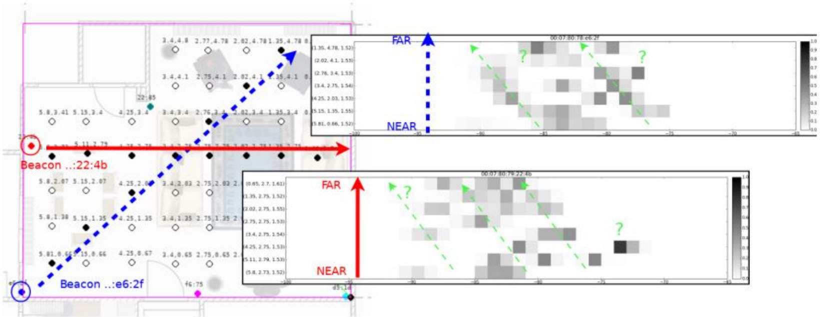
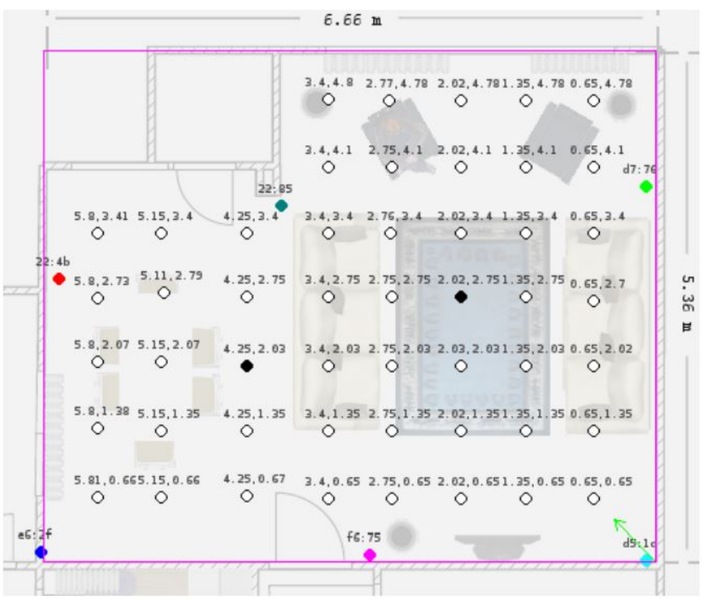
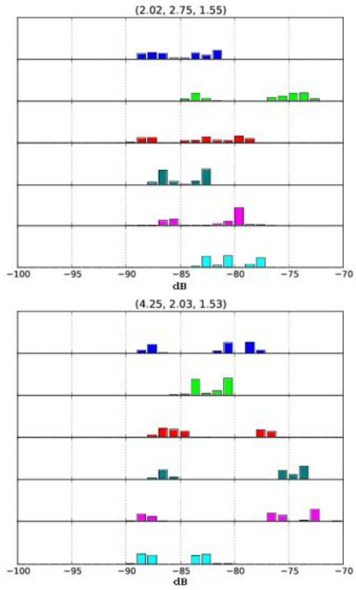


Figure 1: Variation of histograms corresponding to changes in distance. [9]



(a) A detailed example of dense fingerprinting in an indoor setting



(b) Sample fingerprint histograms at various locations for different beacons

Figure 2: Example of fingerprint samples and their respective positions [9]

map model represents the interpolated histograms as a combination of nearby histograms. This makes the method fully driven by the data itself and helps maintain the original features of the histograms despite noise from signal scattering and reflections.

For the tracking problem, the authors propose a hidden Markov model (HMM). The RSSI values belong to a discrete set ranging from -120,...,-60, with the size adjusted according to the maximum values of the captured RSSI. Each data instance is represented as D_t^b, where t is the time stamp and b is the index of the transmitting beacon. As shown in the figure 3, where A is a subset of $\mathbb{R}^2$, the position $P = (x, y)$ within this set A forms the latent variables.

The authors apply a diffusion motion technique to represent the transition density, operating under the assumption that the robot's current position is close to its previous location. They contribute to the field by using Wasserstein interpolation to estimate the observation density and compare it with other regression and interpolation methods. Additionally, they present three observation models: Nearest Fingerprint (NF), k-Nearest Fingerprint (kNF), and Artificial Neural Network (ANN).

The observation density estimators were evaluated using different sets of fingerprint loca-

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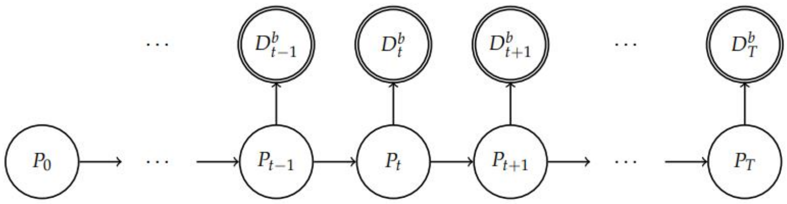

Figure 3: Tracking Graphical Model. [9]

tions. To simulate a person's movement, synthetic trajectories were created, and corresponding observation data were generated based on those paths. The experiments involved gathering BLE RSSI data and turning it into fingerprints. The researchers then searched for the best model parameters and chose a smaller set of fingerprint locations to work with. They ran the estimators on a detailed grid, and the results showed that having high-resolution fingerprint data gives the best map for tracking users. However, using fewer fingerprints makes the system easier to set up and calibrate. Wasserstein interpolation-based techniques perform well, especially with a reduced number of fingerprints.

In [12] the multidimension reduction algorithms: ISO-MAP and Multi-Dimension Scaling(MSD) has been discussed on Three Dimensional wireless sensor network. It shows that the ISO-MAP gives better accuracy than MDS after dimension reduction. Zhuang Y. et. al. [13] presents an algorithm for smartphone-based indoor localization using received signal strength (RSS) signals from Bluetooth Low Energy (BLE) beacons. It focuses on overcoming the challenges of indoor environments, such as multipath, shadowing, and reflection, which complicate accurate distance estimation. The main techniques for BLE localization are proximity detection, trilateration (which uses distance measurements), and fingerprinting [14]. The proposed method combines several approaches polynomial regression models (PRM), fingerprinting, outlier detection, and extended Kalman filtering (EKF)—to boost the accuracy of location tracking. To get better estimates for location and distance, the algorithm uses separate PRM models for each BLE advertising channel. Each channel has its own radio map database that is used during the fingerprinting process. Moreover, the method introduces a new way to combine PRM, fingerprinting, EKF, and outlier detection together. The system also features a two-level outlier detection step to make it more reliable.

The system overview as shown in the figure 4, illustrates the flow of data processing. The RSS values from BLE beacons' advertisement channels are initially smoothed before undergoing channel-separate processing through the fingerprinting and PRM models. These processes generate location and distance estimates, which may vary across the different advertisement channels. To address this, the first level of outlier detection is employed, improving the "FP + PRM-derived distance estimates" through statistical analysis. These enhanced estimates are then input into the EKF, which computes the target's current location. A second level of outlier detection is subsequently applied to the EKF output to refine the results further. This robust outlier detection, coupled with the EKF process, ensures that the final BLE localization solution is accurate and reliable. Additionally, the final localization result is fed back into the radio map database, enhancing the fingerprinting performance.

For distance estimation, the radio propagation model (PM) is commonly used, especially in outdoor line-of-sight (LOS) scenarios. The log-normal shadowing model [15, 16], as shown in equation 1 is widely applied

$$P(d) = P(d_0) - 10\gamma log_{10}(d/d_0) + X_\sigma \tag{1}$$

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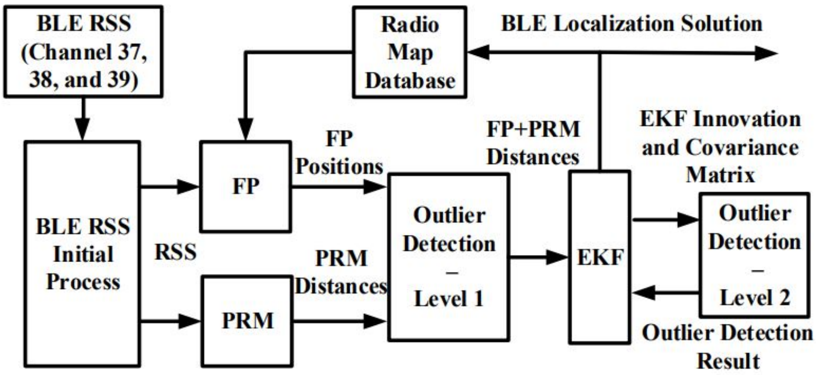

Figure 4: Flow of data processing [13]

where, γ represents the path-loss exponent, $P(d_0)$ is the RSS at the reference distance d_0, $P(d)$ is the RSS at distance d_1 and X_σ is a Gaussian random variable representing shadow fading.

However, in indoor environments, PM can be inaccurate due to the mixture of LOS and non-line-of-sight (NLOS) signals, which have different propagation parameters. To address this, the paper proposes the polynomial regression model (PRM) to better capture the RSS-distance relationship. PRM assumes an nth-degree polynomial relationship between RSS and distance, where the coefficients are estimated during the calibration process. The PRM model is given by

$$\hat{d}_{\text{PRM}} = \sum_{i=0}^{n} c_i . RSS^i \tag{2}$$

where, c_i are the coefficients of the polynomial, RSS is the measured value, and $\hat{d}_{\text{PRM}}$ is the estimated distance.

In the fingerprinting approach, a BLE radio map database is built by surveying specific locations and recording their RSS values. To fill in the gaps between these surveyed points, Gaussian Process Regression (GPR) is applied. Separate databases are created for each advertising channel. During positioning, the search area for fingerprinting is narrowed down to an ellipse, which is defined based on the previous position estimated by the extended Kalman filter (EKF), represented by a matrix:

$$P = \begin{bmatrix} \sigma_E^2 & \sigma_{EN} \\ \sigma_{NE} & \sigma_N^2 \end{bmatrix} \tag{3}$$

where, σ represents variances, N & E are used for North and East respectively, NE used for North-East and σ_{NE} and σ_{EN} are NorthEast & EastNorth covariances, respectively. The major and minor axes and azimuth of the ellipse are calculated using the location covariance matrix from EKF.

The first level of outlier detection is applied by comparing the three PRM-derived distances and three FP-derived locations for each observed BLE beacon. This process produces enhanced FP + PRM distance estimates for each beacon. The EKF is then utilized to estimate the target's location by combining current measurements with previous estimates. After the EKF computation, the second level of outlier detection is applied using statistical testing on the EKF's innovation sequence. Given that the measurement and process noises are assumed to be zero-mean, white, and Gaussian distributed, the innovation sequence should also follow this distribution, allowing for further refinement of the localization estimate.

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Field experiments were conducted in an office environment with two deployments of BLE beacons: dense (20 beacons) and sparse (8 beacons). The performance of the PRM-based distance estimation was evaluated, and the polynomial degree was selected based on a trade-off between distance accuracy and computation load. The results showed that the PRM outperformed the PM in distance estimation, and the 90% distance estimation error using PRM was reduced by 18.42% over the PM.

Wi-Fi indoor localization systems often use machine learning approaches where the goal is to match a user's fingerprint to a set of predefined grid points in a radio map. To enhance performance and reduce computation time, principal component analysis (PCA) [17, 18] is applied. A method proposed in [19] was developed and tested on an Android smartphone using IEEE 802.11 WLAN. Experiments were conducted in real indoor environments under both static and dynamic conditions. The effectiveness of the method was evaluated using several classifiers including k-nearest neighbors [20], decision trees [21], random forests [22], and support vector machines [23, 24]. The optimization problem was developed to find the closest vector from the same class, and the solution was obtained using Lagrangian multipliers. The Gini diversity index [25] is used as the node's data subset splitting rule in decision trees. It is obtain by the following equation:

$$Gini(t) = \sum_{i=1}^{N} p(i/t)(1 - p(i/t)) = 1 - \sum_{i=1}^{N} [p(i/t)]^2 \tag{4}$$

Support vectors are responsible for identifying the tested data class, and the measured fingerprint is classified according to the sign of the maximal margin hyperplane. The measured fingerprint is classified using sign of:

$$f(x) = \sum_{i=1}^{M} \alpha_i y_i K(x_i, x) + b \tag{5}$$

where, the parameters α_i and b are determined by solving an optimization problem, where i represents the dataset entry and includes both the class label and the corresponding fingerprint for M access points (APs). Multiclass classification problems are converted into a combination of two-class problems using one-against-all or one-against-one approaches. The proposed system comprises two stages: offline and online. Figure 5 shows the two stages and their components. During the offline phase, RSSI fingerprints are gathered to create a radio map, which is then simplified using PCA to transform the data into an uncorrelated form. The PCA method extracts the key details from the radio map while significantly reducing the size of the data matrix with minimal information loss. Principal Components (PCs) are chosen based on the variance they explain, and an eigenspace is formed using RSSI readings collected from each location. Experiments took place in a hall with 45 grid points, with RSSI data recorded via an Android app. The method's performance was evaluated using KNN and Random Forest (RF) classifiers, showing that KNN with K = 20 and RF with 15 trees performed best. The number of PCs selected was determined by assessing each PC's information significance, minimizing mean error, and considering the cumulative eigenvalue percentage. Using this method, computational complexity dropped by 70% with RF in static mode and by 33% with KNN. Both classifiers improved their position accuracy. In dynamic mode, the percentages of position error within 3 meters were 79% for linear SVM, 79% for KNN at K = 2, 77% for decision tree, and 75% for RF, all with PCA applied. Precision was measured by the minimum error distance covering 100% of cumulative errors.

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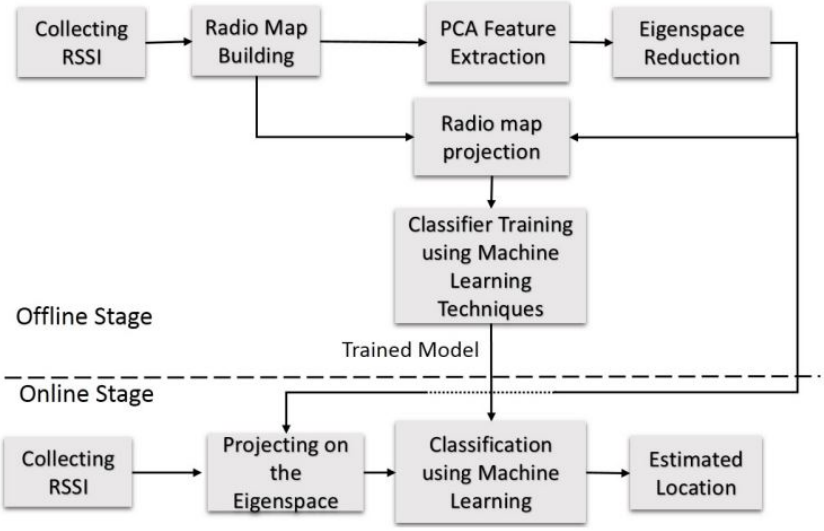

Figure 5: System Architecture providing overview of Online and Offline stages [19]

In their paper [26], Zhang et al. propose an efficient machine learning-based approach for indoor localization, combining Principal Component Analysis (PCA) with a Grid Search-optimized Support Vector Machine (SVM). The motivation behind this approach is to improve the localization accuracy and computational efficiency in indoor environments where high-dimensional RSS data from multiple Access Points (APs) can introduce redundancy and noise. These issues increase the complexity of traditional localization algorithms, making real-time applications challenging. To address these challenges, the authors first apply PCA to reduce the dimensionality of the RSS data. The PCA transformation aims to retain the most significant variations in the data while discarding the redundancy and noise. Specifically, PCA reduces a large set of interrelated variables to a smaller set of uncorrelated principal components, while preserving 80-90% of the original data variance. The covariance matrix as given in equation 6 represented by C is constructed from the RSS data.

$$C = AA^T \tag{6}$$

where, $A = (r_2, r_2, ..., r_{N_p})$ represents the RSS measurements at each anchor point. The matrix decomposition is performed using Singular Value Decomposition (SVD), where $A = U\Lambda V^T$ The eigenvectors corresponding to the largest eigenvalues are selected to form a new subspace that minimizes computational costs. The formula for cumulative contribution rate is given in equation 7.

$$\omega = \sum_{N_k}^{i=1} \lambda_1 / \sum \lambda_1 \tag{7}$$

it is used to ensure that enough principal components are retained to capture the essential information from the high dimensional data.

After dimensionality reduction, the authors implement a kernel SVM to model the relationship between the reduced RSS data and the corresponding geographical locations. SVM is a powerful machine-learning technique known for its excellent generalization capabilities. The classification function is defined as in equation 8.

$$f(\hat{r}_i) = w^T \hat{r}_i + b \tag{8}$$

where w represents the weight vector and b is the bias term. The goal is to maximize the

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geometric margin (as shown in equation 9)

$$\tilde{\gamma} = p_i \frac{f\hat{r_i}}{\parallel w \parallel} = \frac{\hat{\gamma}}{\parallel w \parallel} \tag{9}$$

between different classes, they ensure that the decision boundary separates the data as widely as possible. A penalty term C is introduced to control the trade-off between margin size and classification error, allowing the model to tolerate some misclassifications in noisy data environments. The parameters of the SVM are optimized using a Grid Search (GS) algorithm, which systematically searches for the best values of C and kernel parameters to minimize localization error.

The authors test their approach in an office building, deploying 126 anchor points to collect RSS data from 16 APs. The mapping of anchor points is shown in figure 6. The training and testing points are represented by "$\times$" and $\circ$ respectively. In the offline stage, 100 RSS measurements are taken at each anchor point, and PCA is applied to reduce the data dimensionality. The SVM model is then trained using a subset of the data, while the remaining data is used for testing. The results show that the proposed PCA-SVM approach localization error of $1.37 meters$. This is lower than the errors for standard SVM ($1.76m$), Backpropagation Neural Network(BPNN) ($2.13m$), and k-Nearest Neighbor(KNN) ($3.08m$). Additionally, the computational efficiency is enhanced, with the localization speed improved by up to 21% in some cases.

The novelty of this approach lies in its integration of PCA for dimensionality reduc-

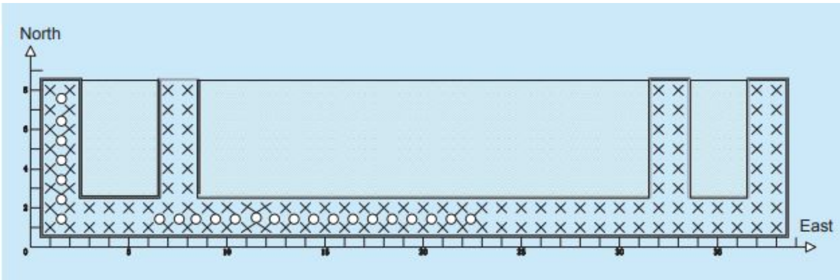

Figure 6: Experimental area [26]

tion with Grid Search-optimized SVM for localization, which sets it apart from other machine learning-based localization methods. While traditional approaches like KNN and BPNN are sensitive to noisy data and require extensive computational resources, the proposed PCA-SVM model offers a more efficient and accurate solution. By dynamically adjusting the SVM parameters using Grid Search, the authors achieve a better balance between localization accuracy and computational complexity, making their method well-suited for real-time indoor localization applications.

Rizk et al. [27] propose CellinDeep, a robust and accurate indoor localization system based on deep learning [28]. The system aims to achieve fine-grained indoor localization using cellular signals, addressing challenges posed by the limitations of WiFi-based and sensor-based approaches, which are either limited by device compatibility or coarse-grained accuracy. CellinDeep specifically leverages the deep learning framework to model the complex, non-linear relationships between RSS from different cellular towers and the user's location. The primary goal is to provide accurate localization even in environments where cellular signals are noisy and prone to variability.

The CellinDeep system works in two main steps: offline training and online tracking. In

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the offline step, it collects geo-tagged RSS data from various cell towers at different reference points in the area. This data is then used to train a deep neural network (DNN). To deal with limited training samples and noisy signals, the authors introduced two data augmentation techniques called Random Augmenter and Lower-bound Cropper. These help create more training data artificially, which reduces the need for extensive data collection and helps prevent overfitting during training. Additionally, dropout regularization and early stopping methods are included to make the model more robust and better at generalizing, as shown in Figure 7. Mathematically, CellinDeep employs a DNN classification model to estimate the user's posi-

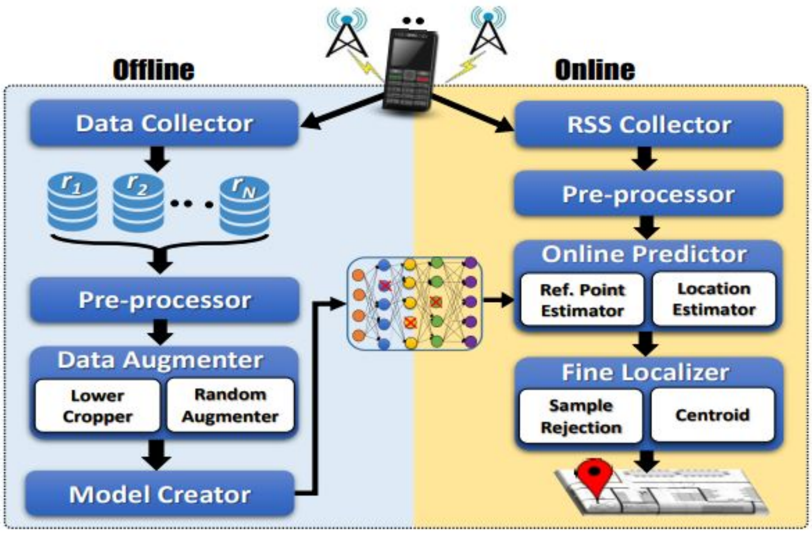

Figure 7: Architecture for CellinDeep [27]

tion. The input layer consists of the RSS measurements from multiple cell towers, represented as a vector $s = (s_1, s_2, ..., s_q)$ where each element corresponds to a tower. The output layer uses the softmax function to classify the user into one of the reference points, converting logits (scores for each reference point) into probabilities using the equation 10:

$$p(a_{ij}) = \frac{e^{a_{ij}}}{\sum_{j=1}^{j=q} e^{a_{ij}}} \tag{10}$$

where $p(a_{ij})$ is the probability that the RSS sample corresponds to reference point j. The system refines this classification by using a spatial smoothing technique, which estimates the user's continuous location by computing a weighted average of the most likely reference points, thus improving the accuracy beyond discrete reference point classification.

Experimentally, the CellinDeep system was tested in an indoor environment with 51 reference points. It achieved a median localization accuracy of 0.78 meters, outperforming state-of-the-art cellular-based systems like SkyLoc [29, 30] and SVM-based systems by more than 350%. The data augmentation techniques significantly contributed to this performance improvement, with both the lower cropper and Random Augmenter methods enhancing localization accuracy by generating more diverse training samples (as shown in figure 8). Additionally, CellinDeep reduced energy consumption by 93.45% compared to WiFi-based methods, making it not only more accurate but also more more power efficient.

CellinDeep differs from existing systems by its use of deep learning to capture complex dependencies between cellular signals. Traditional approaches, such as K-Nearest Neighbors (KNN) and SVM, are less effective at handling noisy data and typically assume independence between cell towers, which results in less accurate localization. In contrast, the deep learning

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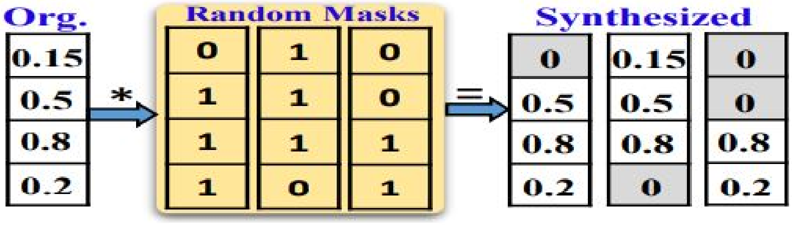

Figure 8: Random Augmenter methods to generate diverse training samples [27]

model in CellinDeep can capture and leverage the inter-dependencies among cell towers to improve localization accuracy, even in challenging indoor environments.

In paper [31] Gadhgadhi at el. proposes an innovative approach to improve the accuracy of indoor positioning systems (IPS) using machine learning techniques, specifically artificial neural networks (ANNs), for localization based on the RSSI. Traditional RSSI-based methods rely on deterministic approaches, converting signal strength to distance using trilateration. However, these methods face challenges due to RSSI's instability caused by environmental factors such as obstacles, interference, and noise, leading to significant inaccuracies.

The authors propose a machine learning model that leverages neural networks to estimate position coordinates using RSSI data. The approach is tested on data collected from four beacons, with the system trained on a subset of three beacons for comparison with a baseline decision tree approach from a previous study. By integrating neural networks, the authors aim to reduce the error margin in distance and position estimation. Two different ANN architectures were evaluated: one with three input RSSI measures and another with four. The performance of these neural networks was compared to traditional methods like polynomial approximation and decision trees.

For implementation, the researchers employed MATLAB's Neural Network Toolbox. They used a feed-forward neural network with one hidden layer and varying neuron counts in the hidden layer as shown in the figure 9. Training data consisted of RSSI measures from three beacons and corresponding ground truth coordinates for the mobile device. The system was trained using 14 sets of RSSI measures, where 10 were used for training, and the rest were split between validation and testing.

A critical aspect of the methodology is converting RSSI to distance using the Friis trans-

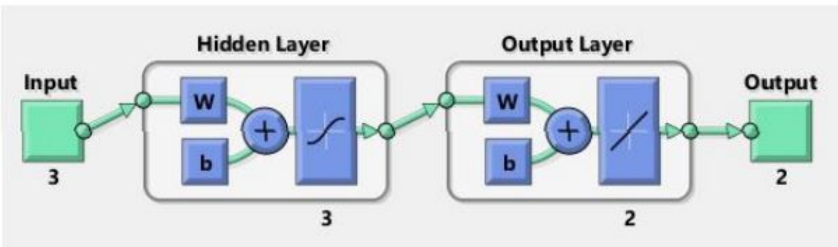

Figure 9: Feed-Forward neural network [31]

mission equation 11 which relates the distance between a transmitter and receiver to their respective power and gain values.

$$d_c = \sqrt{\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 P_r}} \tag{11}$$

where, d_c is the distance between transmitter and receiver, P_t and P_r are the transmitted and received power, G_t and G_r represent the gains of the transmit and receive antennas, λ is the

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wavelength of the signal. The researchers then estimated position coordinates through multi-lateration, using circles corresponding to the distances from each beacon (as shown in Figure 10).

The results demonstrate that the ANN-based model significantly outperforms the decision

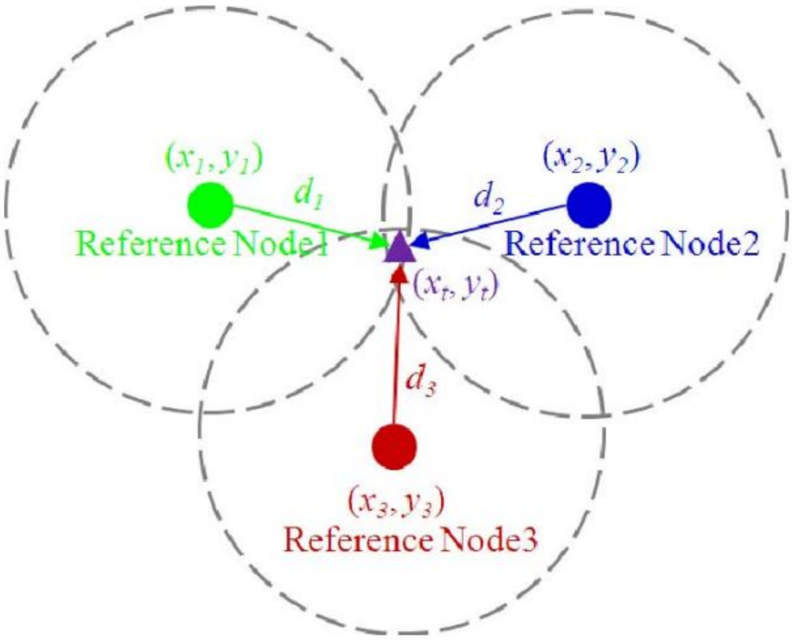

Figure 10: multilateration method to estimate coordinates of unknown node [31]

tree method in accuracy. The decision tree approach yielded a maximum error of 1.74 meters and a mean error of 0.7 meters. In contrast, the neural network architecture with three neurons in the hidden layer achieved an average error of 0.26 meters. The four-input neural network architecture improved performance, with an average error of 0.076 meters. Notably, the neural network's superior performance indicates that increasing the number of input beacons enhances localization accuracy, which is also evident from the error reductions compared to earlier methods.

This study shows that using machine learning, particularly ANNs, can significantly improve indoor localization systems' accuracy by reducing error margins compared to traditional filtering or decision tree approaches. The research opens avenues for further exploration, such as applying this methodology in more complex environments with obstacles, where traditional analytical methods struggle to maintain accuracy due to line-of-sight requirements.

3 Radio-Signal Hybrid Indoor Localization Systems

In [32], an indoor navigation app was created for Android using Bluetooth Low Energy (BLE) beacons to figure out where the user is, combined with a modified Dijkstra's Shortest Path Algorithm [33] to find the best route. The app offers two ways to navigate: one shows pre-recorded images guiding the user, and the other uses a built-in compass to point toward the next spot. The positioning system [34] involves clear steps: first, it measures and cleans the RSSI signals from each beacon to get accurate data. Then, it calculates the distance to each beacon using the average RSSI and a path loss model. If the user is between several beacons, the system estimates their spot by dividing the space proportionally. If only one beacon is detected

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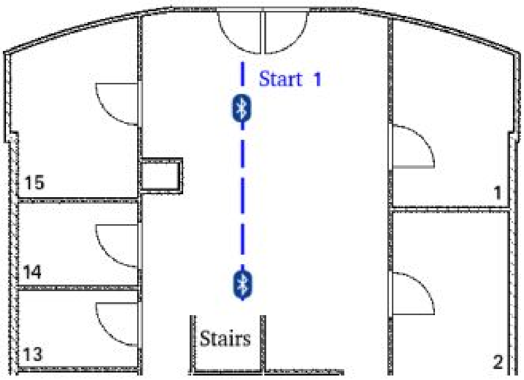
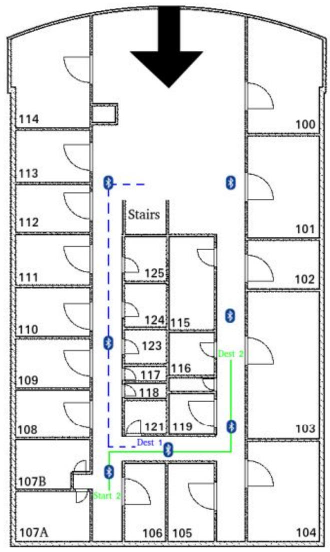
or others are far away, it assumes the user is on the opposite side of that beacon. Finally, it matches the calculated location to the closest known point in the database to finalize the user's position. The system was tested in a three-floor building with 34 positions and 33 connecting paths on each level, as shown in Figure 11. They ran 20 virtual tests—5 with fixed start and end points of the same distance, and 15 with random points to check for missing paths. The system dealt with positioning errors by only using beacons within 5 meters of the user and leaving a 2-meter gap between beacons uncovered.



(a) The ground level of the test environment.



(b) The test environment and the designated paths on the first floor

Figure 11: Testing Environment [32]

Poulose et al. [35] proposes an advanced indoor localization system utilizing Ultra-Wideband (UWB) signals combined with deep learning, particularly long short-term memory (LSTM) networks [36]. Traditional UWB-based systems rely on the time of arrival (TOA) [37] of UWB signals to estimate distances between a UWB tag and anchor nodes (ANs), which are used to calculate the user's position. However, these systems often suffer multipath effects and non-line-of-sight (NLOS) conditions, leading to position estimation errors. To overcome these limitations, the authors propose an LSTM-based deep learning model that enhances localization accuracy by learning from previous time-series distance data.

In the proposed approach, UWB signals are transmitted from a tag to several anchor nodes placed at fixed positions in the indoor environment. The distances between the tag and the anchor nodes are estimated using TOA, and these distances are then processed by a two-layer LSTM network to predict the user's position. As shown in Figure 12, the proposed system contrasts with conventional UWB localization methods by leveraging deep learning to improve accuracy. This system can handle complex environments where traditional algorithms may fail due to LOS interruptions or other environmental challenges.

The mathematical foundation of the system involves using trilateration to estimate the position of the UWB tag. The distance d_i between the UWB tag at position $p(x_p, y_p)$ and an anchor node at position $R_i(x_i, y_i)$ is calculated as:

$$d_i^2 = (x_i - x_p)^2 + (y_i - y_p)^2 \tag{12}$$

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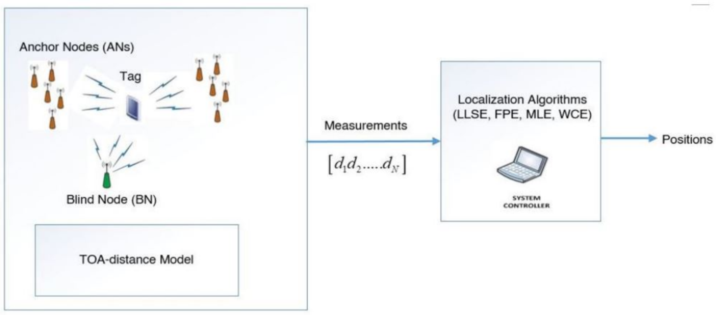
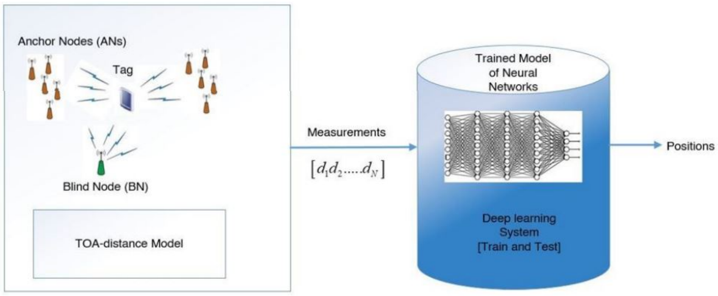


(a) Traditional UWB-Based Positioning System



(b) Proposed system using deep learning

Figure 12: Architecture of the UWB Localization System [35]

This relationship is used in conjunction with multiple anchor nodes to solve for the tag's position. The estimated distances are then refined using the LSTM network, which provides more accurate predictions by learning from temporal distance variations. The UWB localization system's overall workflow, including signal generation, transmission, reception, and TOA estimation, is depicted in Figure 13. The system's simulation is conducted in MATLAB, where UWB pulses are generated and transmitted through an indoor channel, and the distances between the tag and anchors are estimated based on TOA. The distance data serves as the input to the LSTM network, which predicts the user's position in real-time. Simulation results demonstrate the

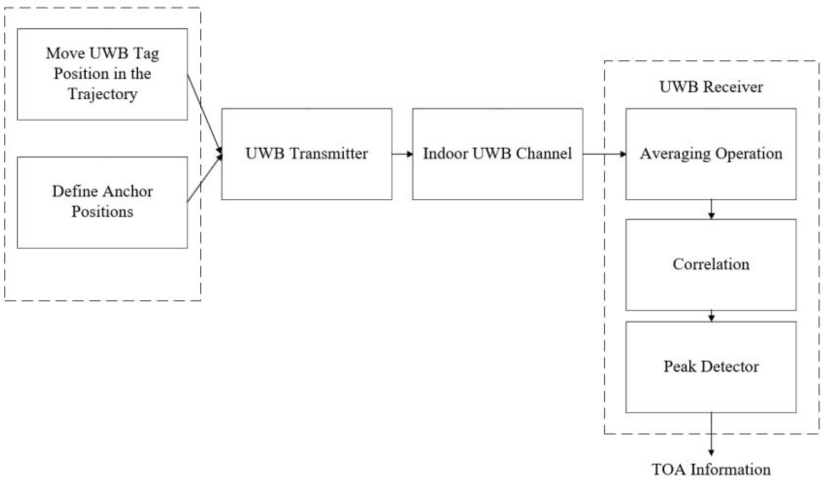


Figure 13: Schematic of a UWB Simulation Environment [35]

superiority of the LSTM-based system compared to traditional UWB localization methods such as linearized least square estimation (LLSE) and maximum likelihood estimation (MLE) [38]. The proposed system achieved a mean localization error of 7 cm, which is significantly lower than conventional methods. Additionally, as shown in Figure 14, the authors simulated UWB

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anchors and tag positions in an experiment area. The tag's motion was tracked while the system continuously estimated its position, validating the effectiveness of the LSTM model in handling real-world scenarios.

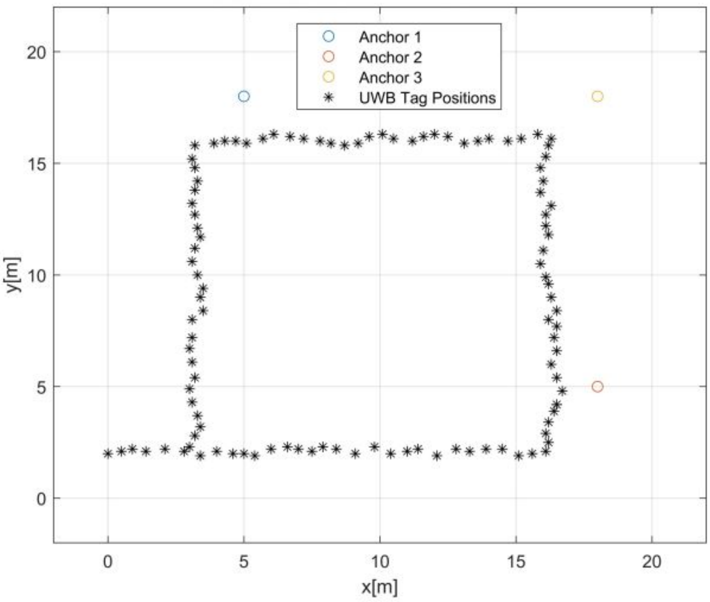

Figure 14: UWB reference points and target locations [35]

4 Discussion

Signal-based indoor localization techniques rely on characteristics of radio signals such as RSSI, channel variations, and TOA measurements. The reviewed studies demonstrate that fingerprinting achieves competitive accuracy but suffers from calibration overhead and signal instability caused by multipath effects. Approaches using histogram modeling, interpolation, and probabilistic maps attempt to improve robustness, but their performance still depends on the density and quality of the reference fingerprints.

Machine learning and deep learning have started to play an important role in addressing fingerprint variability. PCA-based dimensionality reduction reduces redundancy and improves computational efficiency, while SVMs and neural networks produce more stable location estimates compared to classical distance-based methods. Deep learning models, particularly those operating on cellular or WiFi RSSI patterns, capture nonlinear relationships that traditional algorithms cannot. They deliver higher accuracy but require larger datasets and careful training to generalize across environments.

Hybrid signal-based systems add another layer of complexity. BLE-based navigation systems integrate path-loss modeling, filtering, and graph-based routing, while UWB-LSTM models achieve highly precise localization by learning temporal patterns in TOA measurements. These systems offer strong performance but are sensitive to deployment conditions such as anchor placement, beacon power, or channel obstructions. Collectively, the literature indicates that while signal-based techniques can achieve high accuracy, their real-world reliability depends heavily on environmental stability and calibration effort.

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5 Conclusion

Signal-based indoor localization methods offer flexibility because they rely on widely available communication technologies. Fingerprinting remains the most widely adopted technique, and recent work has improved its performance using statistical modeling, interpolation, and machine learning. WiFi and BLE systems benefit from dimensionality reduction, classifier optimization, and probabilistic fusion models. Deep learning methods further enhance accuracy by learning complex signal patterns that are not captured by traditional approaches.

Hybrid systems that combine radio measurements with advanced algorithms or temporal modeling, such as UWB-LSTM architectures, push the limits of achievable accuracy in indoor environments. Still, these systems are sensitive to multipath, NLOS conditions, and deployment constraints. The main challenges remain scalability, ease of calibration, and robustness under environmental changes. Future research should pursue adaptive fingerprinting, online learning models, and hybrid frameworks that reduce calibration cost while maintaining high accuracy across diverse indoor spaces.

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