

**NANO SEMI WEAKLY $(1,2)^*$ - g^* -CLOSED SETS IN NANO
BITOPOLOGICAL SPACES**

¹Reepa Biswas, ²R. Asokan, ³O. Nethaji

¹ Research Scholar,

²Department of Mathematics,

School of Mathematics,

Madurai Kamaraj University,

Madurai District, Tamil Nadu, India.

e-mail : bluebirdreepa@gmail.com, asokan.maths@mkuniversity.ac.in

³ PG and Research Department of Mathematics,

Kamaraj College (Autonomous),

Thoothukudi District, Tamil Nadu, India.

e-mail : jionetha@yahoo.com

Abstract

In this paper, a new class of closed sets called nano $(1,2)^*$ - swg^* -closed sets is introduced, and we further study some of their properties. These sets are shown to have unique characteristics that differentiate them from traditional closed sets, leading to potential applications in various fields of mathematics and theoretical physics. Additionally, we explore the implications of these properties on existing theorems and propose new avenues for research.

1 Introduction and Preliminaries

Lellis Thivagar *et al.*, [9] over took all these theory by bringing out a new topology to study the imperfect data in mathematics. The new topology is named as Nano Topology because of its size. Whatever may be the size of the universe, it is reduced to at most five open sets by defining the lower and upper approximations and boundary region of a subsets of a universe. The elements of the nano topology are called the nano open sets. Lellis Thivagar *et al.*, [10] further established the weak forms of nano topology, nano topology in Čech rough closure space so on. Buvaneshwari *et al.*, [5] investigated the concept nano $(1,2)^*$ - g -closed sets. They also amended the concept by imposing nano $(1,2)^*$ - g -semi open sets and nano $(1,2)^*$ - g -pre open sets in nano bitopological space. Recently, several nano bitopologist introduce and investigated the novel concepts of generalized closed sets and maps (resp. [4, 7, 11]) and so on. In this paper, a new class of closed sets called nano $(1,2)^*$ - swg^* -closed sets is introduced, and we further study some of their properties

⁰ 2010 Mathematics Subject Classification: 54C05, 54C08, 54C10.

Key words: Nano closed sets, Nano g -closed sets, Nano $(1,2)^*$ - g^* -closed sets and Nano $(1,2)^*$ - g -closed sets

Definition 1.1 [12] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x .

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 1.2 [9] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\emptyset \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n -open sets). The complement of a n -open set is called n -closed.

Definition 1.3 [6] Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = U\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $X \subseteq U$. Then it is satisfies the following axioms:

1. U and $\emptyset \in \tau_{R_{1,2}}(X)$.
2. The union of the elements of any sub collection of $\tau_{R_{1,2}}(X)$ is in $\tau_{R_{1,2}}(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_{R_{1,2}}(X)$ is in $\tau_{R_{1,2}}(X)$.

Then $\tau_{R_{1,2}}(X)$ is a topology on U called the Nano bitopology on U with respect to X . $(U, \tau_{R_{1,2}}(X))$ is called the Nano bitopological space. Elements of the Nano bitopology are known as Nano $\tau_{1,2}$ -open sets in U . Elements of $(\tau_{R_{1,2}}(X))^c$ are called Nano $\tau_{1,2}$ -closed sets in $\tau_{R_{1,2}}(X)$.

Definition 1.4 [6] If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

1. The Nano $(1,2)^*$ closure of A is defined as the intersection of all Nano $(1,2)^*$ closed sets containing A and it is denoted by $N_{\tau_{1,2}}-cl(A)$.
2. The Nano $(1,2)^*$ interior of A is defined as the union of all Nano $(1,2)^*$ open subsets of A contained in A and it is denoted by $N_{\tau_{1,2}}-int(A)$.

Definition 1.5 Let $(U, \tau_{R_{1,2}}(X))$ be a Nano bitopological space and $A \subseteq U$. Then A is said to be

1. Nano $(1,2)^*$ - α -open [4] if $A \subseteq N_{\tau_{1,2}}-int(N_{\tau_{1,2}}-cl(N_{\tau_{1,2}}-int(A)))$.
2. Nano $(1,2)^*$ -semi open [6] if $A \subseteq N_{\tau_{1,2}}-cl(N_{\tau_{1,2}}-int(A))$.
3. Nano $(1,2)^*$ -pre open [8] if $A \subseteq N_{\tau_{1,2}}-int(N_{\tau_{1,2}}-cl(A))$.
4. Nano $(1,2)^*$ - β -open (or) Nano semi pre open [1] if $A \subseteq N_{\tau_{1,2}}-cl(N_{\tau_{1,2}}-int(N_{\tau_{1,2}}-cl(A)))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 1.6 Let $(U, \tau_{R_{1,2}}(X))$ be a Nano bitopological space and $A \subseteq U$. Then A is said to be

1. Nano $(1,2)^*$ - g -closed set [5] if $N_{\tau_{1,2}}-cl(A) \subseteq H$ where $A \subseteq H$ and H is nano $\tau_{1,2}$ -open in U .
2. Nano $(1,2)^*$ - sg -closed set [6] if $N_{(1,2)^*}-scl(A) \subseteq H$ whenever $A \subseteq H$ and H is nano $(1,2)^*$ -semi-open in U .
3. Nano $(1,2)^*$ - gs -closed set [6] if $N_{(1,2)^*}-scl(A) \subseteq H$ whenever $A \subseteq H$ and H is Nano $\tau_{1,2}$ -open in U .
4. Nano $(1,2)^*$ - αg -closed set [4] if $N_{(1,2)^*}-\alpha cl(A) \subseteq H$ whenever $A \subseteq H$ and H is nano $\tau_{1,2}$ -open in U .
5. Nano $(1,2)^*$ - $g\alpha$ -closed set [3] if $N_{(1,2)^*}-\alpha cl(A) \subseteq H$ whenever $A \subseteq H$ and H is nano $(1,2)^*$ - α -open in U .
6. Nano $(1,2)^*$ - gsp -closed set [1] if $N_{(1,2)^*}-spcl(A) \subseteq H$ whenever $A \subseteq H$ and H is nano $\tau_{1,2}$ -open in U .

7. Nano $(1,2)^*$ -wg-closed set [13] if $N_{\tau_{1,2}}-cl(N_{\tau_{1,2}}-int(A)) \subseteq H$ whenever $A \subseteq H$ and H is nano $\tau_{1,2}$ -open in U .

The complements of the above mentioned sets are called their respective open sets.

2 Nano $(1,2)^*$ -swg*-closed sets

Definition 2.1 Let G be a subset of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called nano semi weakly $(1,2)^*$ -g*-closed sets (in short nano $(1,2)^*$ -swg*-closed sets) if $N_{\tau_{1,2}}-gcl(G) \subseteq H$ whenever $G \subseteq H$ and H is nano $(1,2)^*$ -semi open in U .

The family of all nano $(1,2)^*$ -g*-closed sets in U is denoted by $(1,2)^*-NSWG^*C(U)$.

Theorem 2.2 Let G be a nano $(1,2)^*$ -swg*-closed set in a nano bitopological space $(U, \tau_{R_{1,2}}(X))$. Then $N_{\tau_{1,2}}-gcl(G) - G$ contains no non-empty nano $(1,2)^*$ -semi closed set in U .

Proof. Suppose that A is a nano $(1,2)^*$ -semi closed subset of $N_{\tau_{1,2}}-gcl(G) - G$. Implies $A \subseteq N_{\tau_{1,2}}-gcl(G)$ and $A \subseteq G^c$. Since A^c is nano $(1,2)^*$ -semi open set and G is nano $(1,2)^*$ -swg*-closed set, $N_{\tau_{1,2}}-gcl(G) \subseteq A^c$. Therefore $A \subseteq N_{\tau_{1,2}}-gcl(G) \cap (N_{\tau_{1,2}}-gcl(G))^c = \phi$. Hence $N_{\tau_{1,2}}-gcl(G) - G$ contains no non-empty nano $(1,2)^*$ -semi closed set in U .

Theorem 2.3 If a set G in a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is nano $(1,2)^*$ -swg*-closed set then $N_{\tau_{1,2}}-gcl(G) - G = \phi$

Proof. Assume that G is nano $(1,2)^*$ -swg*-closed set. Since $N_{\tau_{1,2}}-gcl(G) = G$, therefore $N_{\tau_{1,2}}-gcl(G) - G = \phi$.

Theorem 2.4 Suppose that $T \subseteq S \subseteq U$, B is nano $(1,2)^*$ -swg*-closed set relative to S and that S is a nano $(1,2)^*$ -swg*-closed subset of U , then T is nano $(1,2)^*$ -swg*-closed relative to U .

Proof. Let $T \subseteq G$ and suppose that G is nano $(1,2)^*$ -semi open set in U . Then $T \subseteq S \cap G$ and hence $N_{\tau_{1,2}}-gcl_S(T) \subseteq S \cap G$. It follows that $S \cap N_{\tau_{1,2}}-gcl(T) \subseteq S \cap G$ and $S \subseteq G \cup (N_{\tau_{1,2}}-gcl(T))^c$. Since S is nano $(1,2)^*$ -swg*-closed set in U , $N_{\tau_{1,2}}-gcl(S) \subseteq G \cup (N_{\tau_{1,2}}-gcl(T))^c$. Therefore $N_{\tau_{1,2}}-gcl(T) \subseteq N_{\tau_{1,2}}-gcl(S) \subseteq U \cup (N_{\tau_{1,2}}-gcl(T))^c$ and $N_{\tau_{1,2}}-gcl(T) \subseteq G$, then T is nano $(1,2)^*$ -swg*-closed set relative to U .

Theorem 2.5 Let $S \subseteq V \subseteq U$ and suppose that S is nano $(1,2)^*$ -swg * -closed in U , then S is nano $(1,2)^*$ -swg * -closed set relative to V .

Proof. Let $S \subseteq V \cap G$ and suppose that G is nano $(1,2)^*$ -semi open set in U , then $S \subseteq G$ and hence $N_{\tau_{1,2}}-gcl(S) \subseteq G$. It follows that $V \cap N_{\tau_{1,2}}-gcl(S) \subseteq V \cap G$, then S is nano $(1,2)^*$ -swg * -closed relative to V .

Proposition 2.6 Every nano $\tau_{1,2}$ -closed set in nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is a nano $(1,2)^*$ -swg * -closed set in $(U, \tau_{R_{1,2}}(X))$.

Proof. Assume that S is nano $\tau_{1,2}$ -closed set in U . Let G be a nano $(1,2)^*$ -semi open set in U , such that $S \subseteq G$ and $S \subseteq N_{\tau_{1,2}}-cl(N_{\tau_{1,2}}-int(S)) \subseteq G$. Let $N_{\tau_{1,2}}-gcl(S) \subseteq N_{\tau_{1,2}}-cl(S) \subseteq G$. This implies $S \subseteq N_{\tau_{1,2}}-cl(N_{\tau_{1,2}}-int(S)) \subseteq N_{\tau_{1,2}}-cl(S) \subseteq G$ and G is nano $(1,2)^*$ -semi open set. Thus $S \subseteq N_{\tau_{1,2}}-gcl(S) \subseteq G$. Therefore S is nano $(1,2)^*$ -swg * -closed set.

Remark 2.7 The converse of the above Proposition need not be true as seen from the following example.

Example 2.8 Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{b\}, \{a, c, d\}\}$ and $X = \{b\}$ then $\tau_{R_1}(X) = \{\phi, \{b\}, U\}$ and let $U/R_2 = \{\{a\}, \{b, c, d\}\}$ and $X = \{a\}$ then $\tau_{R_2}(X) = \{\phi, \{a\}, U\}$. Then the sets in $\{\phi, \{a\}, \{b\}, \{a, b\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, the subset $\{a\}$ is nano $(1,2)^*$ -swg * -closed set but not nano $\tau_{1,2}$ -closed.

Proposition 2.9 Every nano $(1,2)^*$ -g-closed set in nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is a nano $(1,2)^*$ -swg * -closed set in $(U, \tau_{R_{1,2}}(X))$.

Proof. Let S be nano $(1,2)^*$ -g-closed set then $N_{\tau_{1,2}}-cl(S) \subseteq G$ whenever $S \subseteq G$ and G is nano $\tau_{1,2}$ -open set. Now $A = N_{\tau_{1,2}}-gcl(S)$ as S is nano $(1,2)^*$ -g-closed set, follows S is nano $(1,2)^*$ -swg * -closed set.

Remark 2.10 The converse of the above Proposition need not be true as seen from the following example.

Example 2.11 In example 2.8, then the subset $\{a, b\}$ is nano $(1,2)^*$ -swg * -closed set but not nano $(1,2)^*$ -g-closed.

Remark 2.12 The concepts of nano $(1,2)^*$ -swg * -closed set and nano $(1,2)^*$ -semi closed set are independent of each other as shown in the following examples.

Example 2.13 Let $U = \{a, b, c\}$ with $U/R_1 = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$ then $\tau_{R_1}(X) = \{\phi, \{a\}, U\}$ and let $U/R_2 = \{\{c\}, \{a, b\}\}$ and $X = \{a, b\}$ then $\tau_{R_2}(X) = \{\phi, \{a, b\}, U\}$. Then the sets in $\{\phi, \{a\}, \{a, b\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{c\}, \{b, c\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{b\}$ is a nano $(1,2)^*$ -semi closed set but not nano $(1,2)^*$ -swg^{*}-closed.

Example 2.14 In example 2.8, then the subset $\{a, b\}$ is a nano $(1,2)^*$ -swg^{*}-closed set but not nano $(1,2)^*$ -semi closed.

Remark 2.15 The concepts of nano $(1,2)^*$ -swg^{*}-closed set and nano $(1,2)^*$ -preclosed set are independent of each other as shown in the following examples.

Example 2.16 Let $U = \{a, b, c\}$ with $U/R_1 = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$ then $\tau_{R_1}(X) = \{\phi, \{a\}, U\}$ and let $U/R_2 = \{\{b\}, \{a, c\}\}$ and $X = \{a, c\}$ then $\tau_{R_2}(X) = \{\phi, \{a, b\}, U\}$. Then the sets in $\{\phi, \{a\}, \{a, c\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{b\}, \{b, c\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, the subset $\{c\}$ is a nano $(1,2)^*$ -preclosed set but not nano $(1,2)^*$ -swg^{*}-closed.

Example 2.17 In example 2.8, the subset $\{a\}$ is a nano $(1,2)^*$ -swg^{*}-closed set but not nano $(1,2)^*$ -preclosed.

Remark 2.18 The concepts of nano $(1,2)^*$ -swg^{*}-closed set and nano $(1,2)^*$ - α -closed set are independent of each other as shown in the following examples.

Example 2.19 Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b, c, d\}\}$ and $X = \{a\}$ then $\tau_{R_1}(X) = \{\phi, \{a\}, U\}$ and let $U/R_2 = \{\{a, c\}, \{b, d\}\}$ and $X = \{a, c\}$ then $\tau_{R_2}(X) = \{\phi, \{a, c\}, U\}$. Then the sets in $\{\phi, \{a\}, \{a, c\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{b, d\}, \{b, c, d\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{c\}$ is a nano $(1,2)^*$ - α -closed set but not nano $(1,2)^*$ -swg^{*}-closed.

Example 2.20 In example 2.8, then the subset $\{a\}$ is a nano $(1,2)^*$ -swg^{*}-closed set but not nano $(1,2)^*$ - α -closed.

Remark 2.21 The concepts of nano $(1,2)^*$ -swg^{*}-closed set and nano $(1,2)^*$ - β -closed set are independent of each other as shown in the following examples.

Example 2.22 In example 2.16,

1. then the subset $\{c\}$ is a nano $(1,2)^*$ - β -closed set but not nano $(1,2)^*$ -swg^{*}-

closed.

2. then the subset $\{a, b\}$ is a nano $(1,2)^*$ -swg^{*}-closed set but not nano $(1,2)^*$ - β -closed.

Remark 2.23 The concepts of nano $(1,2)^*$ -swg^{*}-closed set and nano $(1,2)^*$ -sg-closed set are independent of each other as shown in the following examples.

Example 2.24 Let $U = \{a, b, c\}$ with $U/R_1 = \{\{c\}, \{a, b\}\}$ and $X = \{a, b\}$ then $\tau_{R_1}(X) = \{\phi, \{a, b\}, U\}$ and let $U/R_2 = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$ then $\tau_{R_2}(X) = \{\phi, \{a\}, U\}$. Then the sets in $\{\phi, \{a\}, \{a, b\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{c\}, \{b, c\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{b\}$ is a nano $(1,2)^*$ -sg-closed set but not nano $(1,2)^*$ -swg^{*}-closed.

Example 2.25 In example 2.8, then the subset $\{a\}$ is a nano $(1,2)^*$ -swg^{*}-closed set but not nano $(1,2)^*$ -sg-closed.

Remark 2.26 The concepts of nano $(1,2)^*$ -swg^{*}-closed set and nano $(1,2)^*$ -gs-closed set are independent of each other as shown in the following examples.

Example 2.27 Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{d\}, \{b, c\}, \}$ and $X = \{a\}$ then $\tau_{R_1}(X) = \{\phi, \{a\}, U\}$ and let $U/R_2 = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{a, b\}$ then $\tau_{R_2}(X) = \{\phi, \{a, b\}, U\}$. Then the sets in $\{\phi, \{a\}, \{a, b\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{c, d\}, \{b, c, d\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{b\}$ is a nano $(1,2)^*$ -gs-closed set but not nano $(1,2)^*$ -swg^{*}-closed.

Example 2.28 In example 2.8, then the subset $\{a\}$ is a nano $(1,2)^*$ -swg^{*}-closed set but not nano $(1,2)^*$ -gs-closed.

Remark 2.29 The concepts of nano $(1,2)^*$ -swg^{*}-closed set and nano $(1,2)^*$ - α g-closed set are independent of each other as shown in the following examples.

Example 2.30 Let $U = \{a, b, c\}$ with $U/R_1 = \{\{a\}, \{b\}, \{c\}\}$ and $X = \{a\}$ then $\tau_{R_1}(X) = \{\phi, \{a\}, U\}$ and let $U/R_2 = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{a, b\}$ then $\tau_{R_2}(X) = \{\phi, \{a, b\}, U\}$. Then the sets in $\{\phi, \{a\}, \{a, b\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{c, d\}, \{b, c, d\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset then the subset $\{b\}$ is a nano $(1,2)^*$ - α g-closed set but not nano $(1,2)^*$ -swg^{*}-closed.

Example 2.31 In example 2.8, then the subset $\{a\}$ is a nano $(1,2)^*$ -swg^{*}-closed set but not nano $(1,2)^*$ - α g-closed.

Remark 2.32 *The concepts of nano $(1,2)^*$ -swg * -closed set and nano $(1,2)^*$ -g α -closed set are independent of each other as shown in the following examples.*

Example 2.33 *In example 2.30, then the subset $\{b\}$ is a nano $(1,2)^*$ -g α -closed set but not nano $(1,2)^*$ -swg * -closed.*

Example 2.34 *In example 2.8, then the subset $\{a\}$ is a nano $(1,2)^*$ -swg * -closed set but not nano $(1,2)^*$ -g α -closed.*

Remark 2.35 *The concepts of nano $(1,2)^*$ -swg * -closed set and nano $(1,2)^*$ -wg-closed set are independent of each other as shown in the following examples.*

Example 2.36 *Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$ and $X = \{a\}$ then $\tau_{R_1}(X) = \{\phi, \{a\}, U\}$ and let $U/R_2 = \{\{b\}, \{d\}, \{a, c\}\}$ and $X = \{a, c\}$ then $\tau_{R_2}(X) = \{\phi, \{a, c\}, U\}$. Then the sets in $\{\phi, \{a\}, \{a, c\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{b, d\}, \{b, c, d\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{c\}$ is a nano $(1,2)^*$ -wg-closed set but not nano $(1,2)^*$ -swg * -closed.*

Example 2.37 *In example 2.8, the subset $\{a\}$ is a nano $(1,2)^*$ -swg * -closed set but not nano $(1,2)^*$ -wg-closed.*

Definition 2.38 *Let G be a subset of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called nano semi weakly $(1,2)^*$ -g-closed sets (in short nano $(1,2)^*$ -swg-closed sets) if $N_{\tau_{1,2}}\text{-cl}(N_{\tau_{1,2}}\text{-int}(A)) \subseteq H$ whenever $A \subseteq H$ and H is nano $(1,2)^*$ -semi open in U .*

Remark 2.39 *The concepts of nano $(1,2)^*$ -swg * -closed set and nano $(1,2)^*$ -swg-closed set are independent of each other as shown in the following examples.*

Example 2.40 *In example 2.16, then the subset $\{c\}$ is a nano $(1,2)^*$ -swg-closed set but not nano $(1,2)^*$ -swg * -closed.*

Example 2.41 *In example 2.8, then the subset $\{a\}$ is a nano $(1,2)^*$ -swg * -closed set but not nano $(1,2)^*$ -swg-closed.*

Remark 2.42 *The concepts of nano $(1,2)^*$ -swg * -closed set and nano $(1,2)^*$ -gsp-closed set are independent of each other as shown in the following examples.*

Example 2.43 *Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b, c\}\}$ and $X = \{a, b\}$ then $\tau_{R_1}(X) = \{\phi, \{a\}, \{b, c\}, U\}$ and let $U/R_2 = \{\{a, b, c\}, \{d\}\}$ and $X = \{a, b, c\}$ then $\tau_{R_2}(X) =$*

$\{\phi, \{a, b, c\}, U\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, \{a, b, c\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{d\}, \{a, d\}, \{b, c, d\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{a\}$ is a nano $(1,2)^*$ -gsp-closed set but not nano $(1,2)^*$ -swg*-closed.

Example 2.44 Let $U = \{a, b, c\}$ with $U/R_1 = \{\{a\}, \{b\}, \{c\}\}$ and $X = \{c\}$ then $\tau_{R_1}(X) = \{\phi, \{c\}, U\}$ and let $U/R_2 = \{\{a, b\}, \{c\}\}$ and $X = \{c\}$ then $\tau_{R_2}(X) = \{\phi, \{c\}, U\}$. Then the sets in $\{\phi, \{c\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\phi, \{a, b\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{c\}$ is a nano $(1,2)^*$ -swg*-closed set but not nano $(1,2)^*$ -gsp-closed.

3 Nano $(1,2)^*$ -swg*-open sets and nano $(1,2)^*$ -swg*-neighbourhoods

Definition 3.1 Let G be a subset of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called nano $(1,2)^*$ -swg*-open (briefly, nano $(1,2)^*$ -swg*-open) set if and only if G^c is nano $(1,2)^*$ -swg*-closed.

Proposition 3.2 Every nano $(1,2)^*$ -g-open in a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is nano $(1,2)^*$ -swg*-open.

Proof. Let S be nano $(1,2)^*$ -g-open set. Implies S^c is nano $(1,2)^*$ -g-closed. Now $S^c = N_{\tau_{1,2}}\text{-gcl}(S^c)$ as S^c is nano $(1,2)^*$ -g-closed set, follows S is nano $(1,2)^*$ -swg*-open.

Remark 3.3 The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.4 In example 2.8., then the subset $\{b, c, d\}$ is nano $(1,2)^*$ -swg*-open set but not nano $(1,2)^*$ -g-open.

Theorem 3.5 Every nano $\tau_{1,2}$ -open set in a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is nano $(1,2)^*$ -swg*-open.

Proof. Let S be nano $\tau_{1,2}$ -open set in U . This implies S^c is nano $\tau_{1,2}$ -closed set. By Proposition 2.6, since every nano $\tau_{1,2}$ -closed set is nano $(1,2)^*$ -swg*-closed set. S^c is nano $(1,2)^*$ -swg*-closed set. Therefore S is nano $(1,2)^*$ -swg*-open.

Theorem 3.6 A set S in a nano bitopological space U is nano $(1,2)^*$ -swg*-open set if and only if $A \subseteq N_{\tau_{1,2}}\text{-gint}(S)$, whenever A is nano $(1,2)^*$ -semi closed set and $A \subseteq S$.

Proof. Assume that S is nano $(1,2)^*$ -swg*-open set in U . Let A be nano $(1,2)^*$ -semi closed

set and $A \subseteq S$. This implies A^c is nano $(1,2)^*$ -semi open set and $S^c \subseteq A^c$. Since S^c is nano $(1,2)^*$ - swg^* -closed set, $N_{\tau_{1,2}} - gcl(S^c) \subseteq A^c$. Implies $N_{\tau_{1,2}} - gcl(S^c) = (N_{\tau_{1,2}} - gint(S))^c, (N_{\tau_{1,2}} - gint(S))^c \subseteq A^c$. Therefore $A \subseteq N_{\tau_{1,2}} - gint(S)$.

Conversely, assume that $A \subseteq N_{\tau_{1,2}} - gint(S)$ whenever A is nano $(1,2)^*$ -semi closed set and $A \subseteq S$. Let G be a nano $(1,2)^*$ -semi open set in U containing S^c . Therefore G^c is a nano $(1,2)^*$ -semi closed set contained in S by hypothesis $G^c \subseteq N_{\tau_{1,2}} - gint(S)$ taking complements $S \supseteq N_{\tau_{1,2}} - gcl(S^c)$. Therefore S^c is nano $(1,2)^*$ - swg^* -closed set in U . Hence S is nano $(1,2)^*$ - swg^* -open set in U .

Theorem 3.7 *If $S \subseteq T \subseteq U$ where S is nano $(1,2)^*$ - swg^* -open set relative to T and T is nano $(1,2)^*$ - swg^* -open set relative to U , then S is nano $(1,2)^*$ - swg^* -open set relative to U .*

Proof. Let A be a nano $(1,2)^*$ -semi closed set and suppose that $A \subseteq S$. Then A is nano $(1,2)^*$ -semi closed set relative to T and hence $A \subseteq N_{\tau_{1,2}} - gint_T(S)$. Therefore there exists a nano $(1,2)^*$ -semi open set G such that $A \subseteq G \cap T \subseteq S$. But $A \subseteq G^* \subseteq T$ for nano $(1,2)^*$ -semi open set G^* , since T is nano $(1,2)^*$ - swg^* -open set in U . Thus $A \subseteq G^* \cap G \subseteq T \cap G \subseteq S$. It follows that $A \subseteq N_{\tau_{1,2}} - gint(S)$, because set S is nano $(1,2)^*$ - swg^* -open set. That implies $A \subseteq N_{\tau_{1,2}} - gint(S)$, whenever A is nano $(1,2)^*$ -semi closed and $A \subseteq S$. Therefore S is nano $(1,2)^*$ - swg^* -open set in U .

Theorem 3.8 *If $N_{\tau_{1,2}} - gint(S) \subseteq T \subseteq S$ and if S is nano $(1,2)^*$ - swg^* -open set then T is nano $(1,2)^*$ - swg^* -open set.*

Proof. $S^c \subseteq T^c \subseteq N_{\tau_{1,2}} - gcl(S^c)$ and since S^c is nano $(1,2)^*$ - swg^* -closed set. It follows that T^c is nano $(1,2)^*$ - swg^* -closed set because S is nano $(1,2)^*$ - swg^* -open set and $S \subseteq T \subseteq N_{\tau_{1,2}} - gcl(S)$. Then T is nano $(1,2)^*$ - swg^* -open set.

Definition 3.9 *Let p be a point in a nano bitopological space $(U, \tau_{R_{1,2}}(X))$. A subset H of U is called a nano $(1,2)^*$ - swg^* -neighbourhood of p , if and only if there exists a nano $(1,2)^*$ - swg^* -open set K such that $p \in K \subseteq H$.*

Definition 3.10 *Let p be a point in a nano bitopological space $(U, \tau_{R_{1,2}}(X))$. The set of all nano $(1,2)^*$ - swg^* -neighbourhood of a p is said to be the nano $(1,2)^*$ - swg^* -neighbourhood system at p which is denoted by nano $(1,2)^*$ - swg^* - $H(p)$.*

Theorem 3.11 *Every neighbourhood H of $p \in U$ is a nano $(1,2)^*$ - swg^* -neighbourhood of U .*

Proof. Let H be a neighbourhood of point $p \in U$. By definition of neighbourhood, there exists on nano $\tau_{1,2}$ -open set K such that $p \in K \subseteq H$. As every nano $\tau_{1,2}$ -open set is nano $(1,2)^*$ - swg^* -open set K such that $p \in K \subseteq H$. Here H is nano $(1,2)^*$ - swg^* -neighbourhood of p .

Remark 3.12 A nano $(1,2)^*$ - swg^* -neighbourhood H of $p \in U$ need not be a neighbourhood of p in U as shown in the following example.

Example 3.13 Let $U = \{a, b, c\}$ with $U/R_1 = \{\{b\}, \{a, c\}\}$ and $X = \{b\}$ then $\tau_{R_1}(X) = \{\emptyset, \{b\}, U\}$ and let $U/R_2 = \{\{b, c\}, \{a\}\}$ and $X = \{b, c\}$ then $\tau_{R_2}(X) = \{\emptyset, \{b, c\}, U\}$. Then the sets in $\{\emptyset, \{b\}, \{b, c\}, U\}$ are called $N_{\tau_{1,2}}$ -open and the sets in $\{\emptyset, \{a\}, \{a, c\}, U\}$ are called $N_{\tau_{1,2}}$ -closed. Then $(1,2)^*$ -NSWG $^*O(U) = \{U, \emptyset, \{b\}, \{c\}, \{b, c\}\}$. The set $\{a, c\}$ is nano $(1,2)^*$ - swg^* -neighbourhood of the point p , since the nano $(1,2)^*$ - swg^* -open set $\{c\}$ is such that $c \in \{c\} \subseteq \{a, c\}$. However, the set $\{a, c\}$ is not a neighbourhood of the point p , since no nano $\tau_{1,2}$ -open set H exists such that $p \in H \subseteq \{a, c\}$.

Theorem 3.14 Let p be a point in a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ and nano $(1,2)^*$ - swg^* - $H(p)$ be the collection of all nano $(1,2)^*$ - swg^* -neighbourhood of p , then the following statements hold.

1. $\forall p \in U$, nano $(1,2)^*$ - swg^* - $H(p) \neq \emptyset$.
2. $B \in \text{nano } (1,2)^*$ - swg^* - $H(p) \Rightarrow p \in B$.
3. $B \in \text{nano } (1,2)^*$ - swg^* - $H(p), A \supseteq B \Rightarrow A \in \text{nano } (1,2)^*$ - swg^* - $H(p)$.
4. $B \in \text{nano } (1,2)^*$ - swg^* - $H(p) \Rightarrow$ there exists $A \in \text{nano } (1,2)^*$ - swg^* - $H(p)$ such that $A \subseteq B$ and $A \in \text{nano } (1,2)^*$ - swg^* - $H(q)$ for every $q \in A$.

Proof. 1. Since U is nano $(1,2)^*$ - swg^* -open set, it is a nano $(1,2)^*$ - swg^* -nbhd of every $p \in U$. Hence there exists at least one nano $(1,2)^*$ - swg^* -neighbourhood (namely- U) for each $p \in U$. Hence nano $(1,2)^*$ - swg^* - $H(p) \neq \emptyset$ for every $p \in U$.

2. If $B \in \text{nano } (1,2)^*$ - swg^* - $H(p)$, then B is a nano $(1,2)^*$ - swg^* -neighbourhood of p . So by definition of nano $(1,2)^*$ - swg^* -neighbourhood, $p \in B$.

3. Let $B \in \text{nano } (1,2)^*$ - swg^* - $H(p)$ and $A \supseteq B$. Then there is a nano $(1,2)^*$ - swg^* -open set G such that $p \in G \subseteq B$. Since $B \subseteq A, p \in G \subseteq A$ and A is nano $(1,2)^*$ - swg^* -neighbourhood of p . Hence $A \in \text{nano } (1,2)^*$ - swg^* - $H(p)$.

4. If $B \in \text{nano } (1,2)^*$ - swg^* - $H(p)$, then there exists a nano $(1,2)^*$ - swg^* -open set A such that $p \in A \subseteq B$. Since A is a nano $(1,2)^*$ - swg^* -open set, it is nano $(1,2)^*$ - swg^* -neighbourhood of each of its points. Therefore $A \in \text{nano } (1,2)^*$ - swg^* - $H(q)$ for every $q \in A$.

4 Conclusion

A new class of closed sets called nano $(1,2)^*$ - swg^* -closed sets is introduced, and we further studied some of their properties. These sets are shown to have unique characteristics that differentiate them from traditional closed sets, leading to potential applications in various fields of mathematics and theoretical physics. Additionally, we explore the implications of these properties on existing theorems and propose new avenues for research.

References

- [1] R. Bhavani, *On some classes of nano $(1,2)^*$ - $\tilde{g}$ -closed sets in nano bitopological spaces*, International Journal of Research in Advent Technology, 6(10)(2018), 2726-2732.
- [2] K. Bhuvaneswari and K. Ezhilarasi, *On nano semi generalized and nano generalized semi-closed sets*, IJMCAR. 4(3)(2014), 117-124.
- [3] K. Bhuvaneswari and H. Rasya Banu, *On nano $(1,2)^*$ generalized α -closed sets in nano bitopological space*, International Journal of Science and Research, 5(4)(2016), 1292-1295.
- [4] K. Bhuvaneswari and H. Rasya Banu, *On nano $(1,2)^*$ - α generalized closed sets in nano bitopological spaces*, International Journal of Scientific Progress And Research, 126(43)(1)(2018), 68-71.
- [5] K. Bhuvaneswari and K. Karpagam, *Nano generalized closed sets in nano bitopological space*, International Journal of Mathematics And Its Applications, 4(1B)(2016), 149-153.
- [6] K. Bhuvaneswari and J. Sheeba Priyadharshini, *On nano $(1,2)^*$ semi generalized closed sets in nano bitopological spaces*, International Research Journal of Mathematics, Engineering and IT (IRJMEIT), 3(3)(2016), 15-26.
- [7] K. Bhuvaneswari, K. Sheela, *On nano $(1,2)^*$ -regular-generalized closed sets in nano bitopological spaces*, International Journal of Mathematical Archive, 7(10)(2016), 109-114.
- [8] K. Bhuvaneswari¹, R. Srividhya, *On nano $(1,2)^*$ generalized pre closed sets in nano bitopological spaces*, International Journal of Science and Research, 5(3)(2016), 2230-2233.
- [9] M. Lellis Thivagar and Carmel Richard, *On nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention, 1(1)(2013), 31-37.
- [10] M. Lellis Thivagar and J. Kavitha, *On nano resolvable spaces*, Missouri Journal of Mathematical Sciences, 29(1)(2017), 80-91.
- [11] A. Padma, M. Saraswathi and A. Vadivel, *M-open sets and maps in nano bitopological spaces*, Adalya Journal, 8(11)(2019), 526-538.
- [12] Z. Pawlak, *Rough sets*, International Journal of Computer and Information Sciences, 11(5)(1982), 341-356.
- [13] Reepa Biswas, R. Asokan, and R. Premkumar, *Nano $(1,2)^*$ - π -closed sets and its generalizations*, International Journal of Mathematics and Computer Research, 12(12)(2024), 4652-4659.