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THE FORCING ACCURATE DOMINATION NUMBER OF A GRAPH

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Abstract

Let D be a γ_a-set of G and G be a connected graph with at least two vertices. If D is the unique minimum γ_a-set including T, then a subset $T \subseteq D$ is said to be a forcing subset for D. A minimum forcing subset for D is a forcing subset for D with minimum cardinality. The cardinality of a minimum forcing subset of D is the forcing accurate domination number of D, denoted by $f_{\gamma a}(D)$. The minimum of all γ_a- sets D in G is taken to be the forcing accurate domination number of G, which is represented by the formula $f_{\gamma a}(G) = \min\{f_{\gamma a}(D)\}$. This concept's general qualities are investigated. The forcing accurate domination number a few common graphs are established. It is demonstrated that a connected graph G exists for every pair of integers a and b with $0 \leq a \leq b$ and $b \geq 2$, such that $f_{\gamma a}(G) = a$ and $\gamma_a(G) = b$.

Keywords: forcing accurate domination number, accurate domination number, domination number.

AMS Subject Classification: 05C69.

Introduction

A finite, undirected connected graph with no loops or many edges is referred to as a graph with the formula $G = (V, E)$. The letters n and m, respectively, stand for the order and size of G. We refer to [5] for the fundamental terms used in graph theory. If uv is an edge of G, then two vertices u and v are said to be adjacent. If $uv \in E(G)$,, then u is $v's$ neighbour, and the set of $v's$ neighbours is denoted by

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$N(v)$ and defined by $deg(v) = |N(v)|$. An end vertex is a vertex of degree 1. We refer to [5] for the fundamental terms used in graph theory.

If for every vertex u in a vertex subset $D \subset V(G)$ of G, there exists a vertex $v \in V(G) \backslash D$ such that $uv \in E(G)$, then D is referred to as a dominating set. The minimum cardinality of such dominating sets of a graph G is indicated by the domination number, or $\gamma(G)$, of that graph. As a result, a minimum dominating set of a graph G is frequently referred to as a γ-set. If $V - D$ lacks a dominating set of cardinality $|D|$, then a dominating set D of a graph G is an accurate dominating set. The minimum cardinality of an accurate dominating set is represented by the accurate domination number, $\gamma_a(G)$ of G. As a result, a minimum accurate dominating set of a graph G is frequently referred to as a γ_a-set. Kulli and Kattimani suggested this idea in [8], and it was subsequently explored in [5, 3, 6, 9].

Domination's forcing notion was first established and investigated in [4], and it was then further examined in [1,2,7]. A highly intriguing idea in graphs is the forcing sets. The executive committee in the management of an institution is made up of senior members who get along well with the other employees. Some executive committee members might also serve on other significant committees. Members may occasionally only be allowed to serve on one committee at a time. In fact, this brings up the idea of the forcing dominating set. The domination number and detour domination number of a graph are studied in [10, 11,12].

The Forcing Accurate Domination Number of a Graph

Definition 2.1. Let D be a γ_a-set of G and G be a connected graph with at least two vertices. If D is the unique minimum γ_a-set including T, then a subset $T \subseteq D$ is said to be a forcing subset for D. A minimum forcing subset for D is a forcing subset for D with minimum cardinality. The cardinality of a minimum forcing subset of D is the forcing accurate domination number of D, denoted by $f_{\gamma a}(D)$. The minimum of all γ_a- sets D in G is taken to be the forcing accurate domination number of G, which is represented by the formula $f_{\gamma a}(G) = \min\{f_{\gamma a}(D)\}$.

Example 2.2. The only five γ- sets of the graph G shown in Figure 2.1 are $D_1 = \{v_1, v_3\}$, $D_2 = \{v_2, v_4\}$, $D_3 = \{v_2, v_5\}$, $D_4 = \{v_3, v_5\}$, $D_5 = \{v_1, v_4\}$. D_i is not an

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accurate dominating set of G since $\langle V - D_i \rangle$ contains γ-sets for each i $(1 \leq i \leq 5)$ and as a result, $\gamma_a(G) \geq 3$.

Now the nine γ_a-sets of G are $S_1 = \{v_1, v_2, v_5\}$, $S_2 = \{v_1, v_2, v_3\}$, $S_3 = \{v_2, v_3, v_4\}$, $S_4 = \{v_3, v_4, v_5\}$, $S_5 = \{v_4, v_5, v_1\}$, $S_6 = \{v_2, v_4, v_5\}$, $S_7 = \{v_1, v_3, v_4\}$, $S_8 = \{v_1, v_2, v_4\}$, $S_9 = \{v_1, v_3, v_5\}$ such that $f_{\gamma a}(S_i) = 3$ for all i $(1 \leq i \leq 9)$ so that $f_{\gamma a}(G) = 3$ and $\gamma_a(G) = 3$.

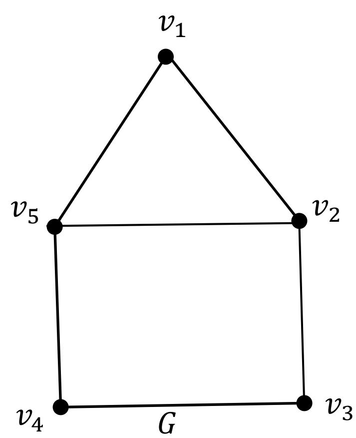

Figure 2.1

Definition 2.3. If a vertex v is a member of each γ_a- set of a connected graph G, then it is said to be G's accurate dominating vertex.

Example 2.4. The only two γ_a -sets of the path $G = P_6$ with $V(P_6) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ are $D_1 = \{v_1, v_2, v_4\}$ and $D_2 = \{v_2, v_5, v_6\}$. $S = \{v_1, v_2\}$ is the set of accurate dominating vertices of G since v_1, v_2 belongs to each γ_a-sets of G .

Observation 2.5. $0 \leq f_{\gamma a}(G) \leq \gamma_a(G) \leq n$ for each connected graph G.

Observation 2.6. Suppose G is a connected graph. Then

(a) G has a unique γ_a- set if and only if $f_{\gamma a}(G) = 0$.

(b) G has at least two γ_a-sets, one of which is a unique γ_a-set containing one of its elements if and only if $f_{\gamma a}(G) = 1$.

(c) No γ_a-set of G is the unique γ_a-set containing any of its proper subsets if and only if $f_{\gamma a}(G) = \gamma_a(G)$

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Observation 2.7. Let W represent the collection of all accurate dominating vertices in connected graph G. Then $f_{\gamma a}(G) \leq \gamma_a(G) - |W|$.

Remark 2.8. Sharp bound can be found in Observation 2.7**.** We can observe that $f_{\gamma a}(G) = 1$, $\gamma_a(G) = 3$ and $|S| = 2$ for $G = P_6$ given in Example 2.4. Accordingly $f_{\gamma a}(G) = \gamma_a(G) - |S|$. For the graph G shown in Figure 2.2, $D_1 = \{v_2, v_6, v_7\}$, $D_2 = \{v_2, v_5, v_6\}, D_3 = \{v_2, v_4, v_5\}$, $D_4 = \{v_2, v_5, v_3\}$, $D_5 = \{v_1, v_5, v_8\}$ are the only five γ_a -sets of G such that $f_{\gamma a}(D_1) = f_{\gamma a}(D_2) = f_{\gamma a}(D_4) = f_{\gamma a}(D_5) = 2$ and $f_{\gamma a}(D_3) = 1$ and $\gamma_a(G) = 3$. The set of accurate dominating vertex of G is $W = \{x\}$. Accordingly $f_{\gamma a}(G) = \gamma_a(G) - |W|$.

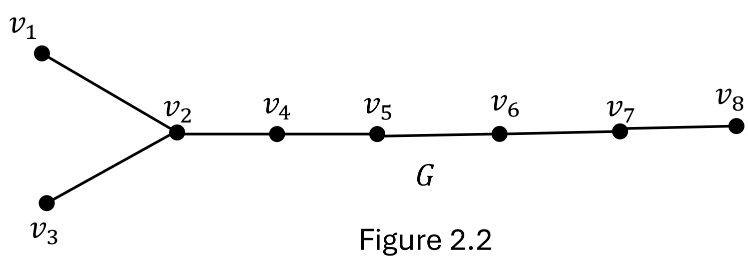

Figure 2.2

Theorem 2.9. For the star graph $G = K_{1,n-1}(n \geq 4)$, $f_{\gamma a}(G) = 1$.

Proof. Let $W = \{v_1, v_2, \ldots, v_{n-1}\}$ be set of all end vertices and x be the central vertex of G. As a result $D_i = \{x, v_i\}$ is a γ_a-sets of G for all i $(1 \leq i \leq n-1)$ such that $f_{\gamma a}(D_i) = 1$ for all i $(1 \leq i \leq n-1)$ so that $f_{\gamma a}(G) = 1$.

■

Theorem 2.10. For the wheel graph $G = K_1 + C_{n-1}, n \geq 5$,

$$f_{\gamma a}(G) = \begin{cases} \dfrac{n+1}{2} & \text{if } n-1 \text{ is even} \\ \dfrac{n}{2} & if\ n-1 \text{ is odd} \end{cases}.$$

Proof. Let $V(K_1) = x$ and $V(C_{n-1}) = \{v_1, v_2, \ldots, v_{n-1}\}$.

Case (i) $n-1$ is even. Let $n-1 = 2k$ $(k \geq 2)$ and P be a path on the cycle C_{n-1} with length $k+1$. Let $S = M \cup \{x\}$, where $M \subset V(C_{n-1})$. Then $|S| = k+1$. There is no γ_a-sets of G in $V \backslash S$ with cardinality $k+2$ because $n = 2k+1$. As a

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result S is a γ_a-set of G so that $\gamma_a(G) = k + 2 = \frac{n-1}{2} + 2 = \frac{n+3}{2}$. x being the accurate dominating vertex of G, Observation 2.7 states that $f_{\gamma a}(G) \leq k + 2 - 1 = k + 1$. It is establish $f_{\gamma a}(G) = k + 1 = \frac{n+1}{2}$. Let T be a minimum forcing subset of S and S be a γ_a-set of G. We establish $|T| = k + 1$. Let's say that isn't the case. Next, $|T| \leq k$. Observation 2.7 says that $x \notin T$. Consequently, $T \subset V(C_{n-1})$. Let's assume, without losing generality, that $v_1 \in M$ such that $v_1 \notin T$. Let $y \in V(C_{n-1})$ such that $y \notin M$. Then $S_1 = (S - \{v_1\} \cup \{y\}$ is a γ_a-sets of G such that $T \subset S_1$. S is not a unique γ_a-set of G containing T as a result, which is a contradiction. As a result $|T| = k + 1$. This suggests that $f_{\gamma a}(G) = k + 1$. Given that this holds true for every G γ_a-set of G, $f_{\gamma a}(G) = k + 1 = \frac{n+1}{2}$.

Case (ii) $n - 1$ is odd. Let $n - 1 = 2k + 1$ $(k \geq 2)$. We may demonstrate that $f_{\gamma a}(G) = \frac{n}{2}$ using the same argument as in Case (i). ■

Theorem 2.11. For the fan graph $G = K_1 + P_{n-1}$ $n \geq 5$,

$$f_{\gamma a}(G) = \begin{cases} \frac{n+1}{2} & \text{if } n-1 \text{ is even} \\ \frac{n}{2} & \text{if } n-1 \text{ is odd} \end{cases}$$

Proof. Case (i) $n - 1$ is even. Let $n - 1 = 2k$ $(k \geq 2)$ and P be a path on the path P_{n-1} with length $k + 1$. Let $S = M \cup \{x\}$, where $M \subset V(P_{n-1})$. Then $|S| = k + 1$. There is no γ_a-sets of G in $V \backslash S$ with cardinality $k + 2$ because $n = 2k + 1$. As a result S is a γ_a-set of G so that $\gamma_a(G) = k + 2 = \frac{n-1}{2} + 2 = \frac{n+3}{2}$. x being the accurate dominating vertex of G, Observation 2.7 states that $f_{\gamma a}(G) \leq k + 2 - 1 = k + 1$. It is establish $f_{\gamma a}(G) = k + 1 = \frac{n+1}{2}$. Let T be a minimum forcing subset of S and S be a γ_a-set of G. We establish $|T| = k + 1$. Let's say that isn't the case. Next, $|T| \leq k$. Observation 2.7 says that $x \notin T$. Consequently, $T \subset V(C_{n-1})$. Let's assume, without losing generality, that $v_1 \in M$ such that $v_1 \notin T$. Let $y \in V(C_{n-1})$ such that $y \notin M$. Then $S_1 = (S - \{v_1\} \cup \{y\}$ is a γ_a-sets of G such that $T \subset S_1$. S is not a unique γ_a-set of G containing T as a result, which is a contradiction. As a

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result $|T| = k+1$. This suggests that $f_{\gamma a}(G) = k+1$. Given that this holds true for every G γ_a-set of G, $f_{\gamma a}(G) = k+1 = \frac{n+1}{2}$.

Case (ii) $n-1$ is odd. Let $n-1 = 2k+1$ $(k \geq 2)$. We may demonstrate that $f_{\gamma a}(G) = \frac{n}{2}$ using the same argument as in Case (i). ■

Theorem 2.12. For the complete graph $G = K_n$ $(n \geq 4)$,

$$f_{\gamma a}(G) = \begin{cases} \frac{n}{2}+1 & \text{if } n \text{ is even} \\ \frac{n+1}{2} & \text{if } n \text{ is odd} \end{cases}.$$

Case (i) n is even. Let $n = 2k$ $(k \geq 2)$. Any set S of $G's$ $k+1$ vertices is then one of its γ_a-sets because $\gamma_a(G) = k+1 = \frac{n}{2}+1$. It follows that $f_{\gamma a}(S) = \frac{n}{2}+1$ since S is not the only γ_a-set that contains any of its proper subsets. We have $f_{\gamma a}(G) = \frac{n}{2}+1$ because this holds true for all γ_a-sets S in G.

Case (ii) n is odd. Let $n = 2k+1$ $(k \geq 2)$. We may demonstrate that $\gamma_a(G) = f_{\gamma a}(G) = \frac{n+1}{2}$ using the same argument as in Case (i). ■

Theorem 2.13. There exists a connected graph G such that $f_{\gamma a}(G) = a$ and $\gamma_a(G) = b$ for every integers a and b with $0 \leq a \leq b$ and $b \geq 2$

Proof. If $b = 2$, then $G = K_2$meets the desired characteristic, assuming $a = 0$. So let $b \geq 3$. Let a path of order b be $P: x_1, x_2, \ldots, x_{b-1}, x_b$. Let a copy of the path on two vertices be $P_i: u_i, v_i$ $(1 \leq i \leq b)$. Let G be the graph created by adding the edge $v_i x_i$ $(1 \leq i \leq b)$ between P and P_i $(1 \leq i \leq b)$. The graph G displays in Figure 2.3.
Let $S = \{v_1, v_2, \ldots, v_{b-1}, v_b\}$. Then S is an accurate dominating set of G. As a result $\gamma_a(G) \leq b$. We establish that $\gamma_a(G) = b$. Contrarily, assume that $\gamma_a(G) < b$. Then there exist an accurate dominating set D such that $|D| \leq b-1$. As a result there exists $z_i \in \{u_i, v_i, x_i\}$ $(1 \leq i \leq a)$ such that $z_i \notin D$ $(1 \leq i \leq a)$. Without loss of generality let us that $z_i \in \{u_1, v_1, x_1\}$. If $z_1 = u_1$, then neither v_1 nor x_1 is dominated by any element of D. If $z_1 = v_1$, then u_1 is not dominated by

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any element of D. If $u_1 = x_1$, then neither u_1 nor v_1 is dominated by any element of D, which is a contradiction to D a γ_a-set of G. consequently $\gamma_a(G) = b$.

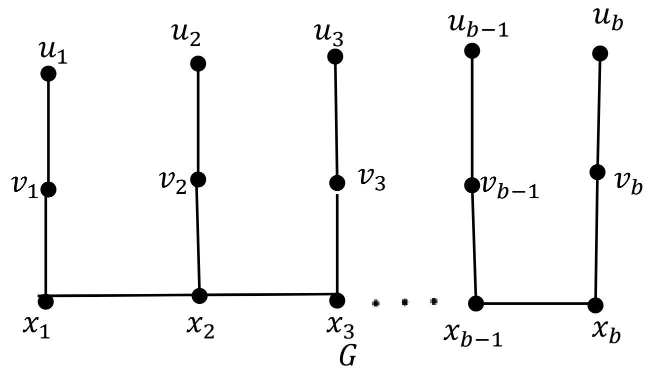

Figure 2.3

Next we then demonstrate that $f_{\gamma a}(G) = a$. First we demonstrate that S is a unique γ_a-set of G. Let's say that isn't the case. Then there exists a γ_a-set of S_1 such that $|S_1| = b$ and $S_1 \neq S$. Suppose that $S \cap S_1 = \phi$, then neither u_i nor x_i is dominated by any element of S_1 for some $(1 \leq i \leq b)$, which is a contradiction. So assume that $S \cap S_1 \neq \phi$. Without loss of generality let us assume that $v_1 \in S$ and $v_2 \notin S_1$. Then u_1 must belong to S_1. Since $v_2 \notin S_1$. Then $u_2 \in S_1$. Moreover either x_1 or $x_2 \in S_1$. Hence it follows that $|S_1| \geq b + 1$, this is a contradiction. Therefore S is the unique γ_a-set of G. Hence by Theorem, it follows that $f_{\gamma a}(G) = 0$. Let $2 \leq a = b$. If $a = b = 2$, then $G = K_3$ satisfies the required property. So let $b \geq 3$. Let $G = K_{2a}$. Then by Theorem 2.12, $f_{\gamma a}(G) = \gamma_a(G) = a$. So let $2 < a < b$.

Let a copy of order two be $P_i: u_i, v_i$ $(1 \leq i \leq a)$. Let G be the graph created from P_i $(1 \leq i \leq b)$ and K_{b-a} by joining each u_i $(1 \leq i \leq a)$ with each vertex of $V(K_{b-a})$ and joining each v_i $(1 \leq i \leq a)$ with each vertex of $V(K_{b-a})$. The graph G displays in Figure 2.4. First we prove that $\gamma_a(G) = b$. It is easily observed that $V(K_{b-a})$ is a subset of every γ_a-set of G. Let $H_i = \{u_i, v_i\}$ $(1 \leq i \leq a)$. Then it is easily seen that every γ_a-set of G contains at least one vertex from each H_i $(1 \leq i \leq a)$ and so $\gamma_a(G) \geq b - a + b = b$. Let $D = V(K_{b-a}) \cup \{u_1, u_2, \ldots, u_a\}$. Then D is a γ_a-set of G. As a result, $\gamma_a(G) = b$.

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Next we then demonstrate that $f_{\gamma a}(G) = a$. By Observation 2. 7, $f_{\gamma a}(G) \leq b - (b-a) = a$. Since every γ_a -set contains at least one vertex from each H_i $(1 \leq i \leq a)$ and $V(K_{b-a})$ is a subset of every γ_a -set $S = V(K_{b-a}) \cup \{c_1, c_2, \dots, c_a\}$, where $c_i \in H_i$ $(1 \leq i \leq a)$. Let T be a forcing subset S with $|T| < a$. Then there exists $c_j \in S$ such that $c_j \notin T$. Let d_j be a vertex of H_j such that $d_j \neq c_j$. Then $S_1 = (S - \{c_j\}) \cup \{d_j\}$, where $d_j \in H_j (1 \leq j \leq a)$. Then S_1 is a γ_a-set of G properly containing S_1, this is a contradiction. As a result $f_{\gamma a}(S) = a$. Since true for all γ_a-sets S of G, we have $f_{\gamma a}(G) = a$. ■

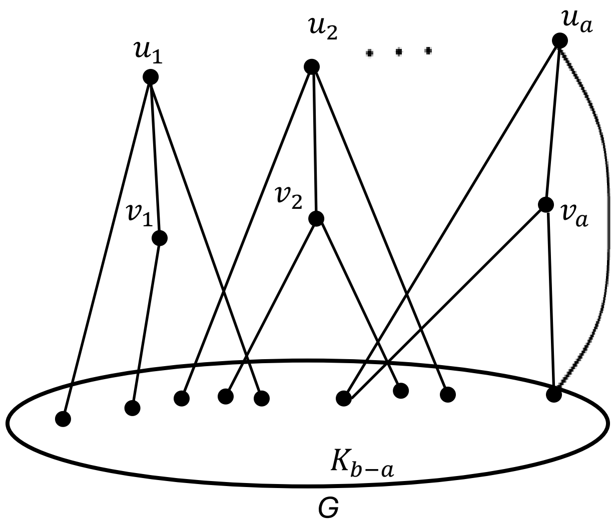

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Figure 2.4

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