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A LAGRANGIAN-BASED OPTIMIZATION APPROACH TO SVM FOR SENTIMENT ANALYSIS

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Abstract

This study presents a Lagrangian-based optimization approach for sentiment classification using the Soft-Margin Support Vector Machine (SVM). Textual inputs, such as product reviews and tweets, undergo standard preprocessing including tokenization, stop-word removal, lemmatization, and vectorization to transform them into numerical feature vectors. The classification model is formulated using the primal soft-margin SVM, which incorporates slack variables to handle misclassifications in non-linearly separable data. To solve the constrained optimization, the Lagrangian dual formulation is employed, introducing dual multipliers for both margin and slack constraints. A small-scale experiment on a sentiment-labeled dataset demonstrates the derivation and solution of the dual problem, highlighting the identification of support vectors, computation of optimal model parameters, and formulation of the final decision boundary. Results show that the dual approach effectively captures sentiment polarity with clear margin separation, validating the robustness and interpretability of Lagrangian-optimized SVMs in text classification tasks.

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1. INTRODUCTION

Sentiment analysis, the task of automatically determining the polarity of textual content, has become a critical component in applications ranging from product review mining to social media monitoring. The challenge lies in accurately classifying text with nuanced expressions, varying linguistic structures, and potential noise from informal writing styles. Among the numerous machine learning methods developed for text classification, the Support Vector Machine (SVM) has proven to be one of the most robust and theoretically grounded approaches (Cortes & Vapnik, 1995; Cristianini & Shawe-Taylor, 2000).

The standard SVM aims to find an optimal separating hyperplane that maximizes the margin between two classes. In realistic scenarios, particularly in sentiment classification, datasets are rarely linearly separable, necessitating the use of the *soft-margin* formulation (Chen et al., 2004). This approach introduces slack variables to tolerate certain classification errors, thereby enhancing generalization capabilities. The optimization problem in soft-margin SVMs is typically expressed in its *primal form*, but is more efficiently solved via the *Lagrangian dual formulation*, which leverages the kernel trick for high-dimensional mappings and yields sparse solutions involving only support vectors (Mangasarian & Musicant, 2001; Hwang et al., 2011).

Lagrangian-based optimization offers additional benefits beyond computational convenience. By introducing dual multipliers for margin and slack constraints, the dual problem structure provides deeper interpretability of the decision boundary and its dependence on the support vectors (Yan & Li, 2019; Liu et al., 2023). Moreover, this framework allows integration with advanced optimization strategies such as augmented Lagrangian methods (Yan & Li, 2019) and primal–dual dynamics (Kosaraju et al., 2018), which improve convergence properties and robustness to adversarial data perturbations (Biggio et al., 2012).

In the context of sentiment analysis, where high-dimensional and sparse feature representations are common due to bag-of-words or word embedding schemes, the Lagrangian dual SVM formulation is especially appealing. Its ability to operate effectively in such feature spaces, coupled with the theoretical guarantees on margin maximization and error control, makes it an ideal choice for handling noisy and semantically complex text data (Rosasco et al., 2004; Jaggi, 2013).

Building on these foundations, this study proposes a Lagrangian-based optimization approach for sentiment classification using the soft-margin SVM. We present a detailed derivation of the dual problem, outline its solution procedure, and demonstrate its effectiveness on a sentiment-labeled dataset. Through experimental results, we highlight the method's capability to identify support vectors, compute optimal parameters, and establish a decision boundary that achieves clear separation between sentiment classes.

2. METHODOLOGY

2.1. Dataset Selection

A publicly available sentiment-labeled dataset was used for experimentation, consisting of short text samples such as product reviews and tweets. Each text instance was labeled with its corresponding sentiment polarity (positive or negative).

2.2. Pre-processing of Text Data

To transform raw text into numerical features suitable for SVM classification, the following preprocessing steps were applied:

Tokenization – Splitting sentences into individual words or tokens.

Stop-word Removal – Eliminating frequently occurring words (e.g., "is," "the," "and") that do not contribute to sentiment.

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Lemmatization – Reducing words to their canonical form (e.g., "running" → "run").
Vectorization – Converting the processed tokens into numerical feature vectors using term frequency–inverse document frequency (TF-IDF).

2.3. Formulation of Soft-Margin SVM

The sentiment classification task was modeled using the primal soft-margin SVM formulation, expressed as:

$$\min_{w,b,\xi} \ \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\xi_i$$

Subject to : $y_i(w^T x_i + b) \geq 1 - \xi_i,$ [Marginal Constrain]

$\xi_i \geq 0$ [Non negativity of Slack]

Where:

- w is the weight vector
- b is the bias term
- ξ_i are slack variables for misclassification
- C is a regularization parameter
- (x_i, y_i) are the feature vectors and sentiment labels.

2.4. Lagrangian Dual Formulation

To handle constraints efficiently, the Lagrangian was introduced as:

$$L(\mathrm{w}, \mathrm{b}, \xi, \alpha, \mu) = \frac{1}{2}\ \|w\|^2 + C\sum_{i=1}^{n}\xi_i \ - \sum_{i=1}^{n}\alpha_i\{y_i(w^T x_i + b) \geq 1 - \xi_i\} - \sum_{i=1}^{n}\mu_i\xi_i$$

with Lagrange multipliers $\alpha_i \geq 0$ for the slack variable constraints: $\mu_i \geq 0$
By taking partial derivatives and applying Karush-Kuhn-Tucker (KKT) conditions, the dual problem was derived as:

$$\max_{\alpha}\sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i,j=1}^{n}\alpha_i\alpha_j y_i y_j \ \langle x_i, x_j\rangle$$

Subject to: $0 \leq \alpha_I \leq C, \quad \sum_{I=1}^{n}\alpha_i y_i = 0$

2.5. Optimization and Model Training

- The dual optimization problem was solved using quadratic programming.
- Support vectors were identified as data points with non-zero αi.
- The optimal decision boundary was computed as:

$$f(x) = sign(\sum_{i=1}^{n}\alpha_i y_i(x_i x + b))$$

3. PROPOSED EXPERIMENT

3.1. An proposed Lagrangian-Optimized SVM for Sentiment Classification Algorithm

Input:

- Raw text data: $D = \{(t_1, y_1), (t_2, y_2), \ldots, (t_n, y_n)\}$, where $y_i \in \{-1, +1\}$ Regularization parameter: $C > 0$
- Kernel function $K(x_i, x_j) = \langle\phi(x_i), \phi(x_j)\rangle$ (optional)

$$Extract\ features\ xi\ from\ text\ ti;\ define\ kernel\ K\ (if\ used)$$

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$$\downarrow$$
$$Set\ up\ primal\ SVM\ objective\ with\ regularization\ and\ slack$$
$$\downarrow$$
$$Form\ Lagrangian\ with\ multipliers\ \alpha\ _i\ , \mu_i$$
$$\downarrow$$
$$Derive\ dual\ objective\ by\ eliminating\ w, b, , \xi i\ (get\ constraints)$$
$$\downarrow$$
$$Use\ QP\ or\ SMO\ to\ solve\ for\ optimal\ \alpha^*$$
$$\downarrow$$
$$Compute\ w\ and\ b\ from\ \alpha^*$$
$$\downarrow$$
$$Classify\ new\ instances\ using\ the\ decision\ function\ \hat{y}(x)$$

Output:

- Weight vector w
- Bias term b
- Prediction function $\hat{y} = sign(w^T x + b)$

4. ALGORITHM IMPLIMENTAION

This section presents a comprehensive framework for implementing a Lagrangian-optimized Support Vector Machine (SVM) tailored for sentiment classification. The methodology includes data pre-processing, feature extraction, SVM formulation, and Lagrangian based optimization using dual variables to solve the classification problem efficiently.

4.1. Data Pre-processing

The input textual data (e.g., product reviews, tweets) undergoes standard pre-processing steps including:

- Tokenization
- Stop-word removal
- Lemmatization or stemming
- Vectorization (e.g. word embeddings)

Let the processed dataset be represented as a matrix:

$$X = [x_1, x_2, \dots\dots x_n]^T \in\ \mathbb{R}^{n\times d}$$

where $x_i \in R^d$ is the feature vector for the i^{th} sample and n is the number of samples.
Each data point has a label $y_i \in \{-1, +1\}$ indicating negative or positive sentiment.

4.2. SVM Formulation [Primal Problem Soft Margin SVM]

The primal optimization problem for a linear SVM is as given below:

$$\min_{w,b,\xi}\ \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\xi_i$$

Subject to : $y_i(w^T x_i + b) \geq 1 - \xi_i,$ [Marginal Constrain]

$\xi_i \geq 0$ [Non negativity of Slack]

Where:

- $w \in \mathbb{R}^d$ is the weight vector
- $b \in \mathbb{R}$ is the bias term

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- $\xi_i$are slack variables for misclassification
- $C > 0$ is a regularization parameter

4.3. Lagrangian Dual Problem with Multiple Multipliers

To solve this constrained optimization problem, we apply the method of Lagrange multipliers here we introduce dual variables:

- $\alpha_i \geq 0$ for the margin constraints: $y_i(w^T x_i + b) \geq 1 - \xi_i, where\ \ \xi_i \geq 0$

- $\mu_i \geq 0$ for the slack variable constraints: $\xi_i \geq 0$

$$L(\mathrm{w}, \mathrm{b}, \xi, \alpha, \mu) = \frac{1}{2}\ \|w\|^2 + C \sum_{i=1}^{n} \xi_i\ - \sum_{i=1}^{n} \alpha_i \{y_i(w^T x_i + b) \geq 1 - \xi_i\} - \sum_{i=1}^{n} \mu_i \xi_i$$

4.4. Dual Formulation [Compute Partial Derivative and to set 0]

We minimize the Lagrangian with respect to the primal variables $w, b,$ and ξ.Taking partial derivatives of L and setting them to zero:

$$\frac{\delta c}{\delta w} = w - \sum_{i=1}^{n} \alpha_i y_i x_i = 0 \Rightarrow w = \sum_{i=1}^{n} \alpha_i y_i x_i$$

$$\frac{\delta c}{\delta b} = -\sum_{i=1}^{n} \alpha_i y_i = 0$$

$$\frac{\delta c}{\delta \xi_i} = C - \alpha_i - \mu_i = 0\ \Rightarrow\ \alpha_i \leq C$$

Dual Problem: Using Lagrange Multipliers

$$\max_{\alpha} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j y_i y_j\ \langle x_i, x_j \rangle$$

Subject to: $0 \leq \alpha_I \leq C, \quad \sum_{I=1}^{n} \alpha_i y_i = 0$

4.5. Back into Lagrangian to get Dual

1st Term: $\frac{1}{2}\ \|w\|^2 = \frac{1}{2}\ \|\sum_{i=1}^{n} \alpha_i y_i x_i\|^2$

2nd Term: $C \sum \xi_i - \sum \alpha_i \xi_i - \sum \mu_i \xi_i = \sum \xi_i (C - \alpha_i - \xi_i) = 0$

(from earlier, $C - \alpha_i - \xi_i = 0$) So all ξ_i terms cancel. $\alpha_i y_i (\sum_j \alpha_j y_j x_j)^T$

3rd term: $-\sum \alpha_i y_i w^T x_i = -\sum_i \alpha_i y_i (\sum_j \alpha_j y_j x_j)^T x_i = \sum_i \sum_j \alpha_i \alpha_j\ y_i\, y_j {x_i}^T x_j$

So the Dual Lagrangian becomes $L_D(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j\ y_i y_j {x_i}^T x_j$

Such that $0 \leq \alpha_I \leq C, \quad \sum_{I=1}^{n} \alpha_i y_i = 0$

This is a Quadratic Programming Problem and the dual formulation of the SVM
This dual problem is solved using Quadratic Programming (QP) solvers. Once the optimal $\alpha\ _i$values are obtained, we compute:

$$w = \sum_{I=1}^{n} \alpha_i y_i\, x_i$$

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$b = y_k w^T x_k$ for any support vector x_k such that $0 < \alpha_k < C$

Then, for any new input x, the sentiment is predicted by:

$$\hat{y} = sign(w^T x + b)$$

4.6. Kernel Extension (Optional)

For non-linear separability, we apply the kernel trick using a mapping ϕ(x), leading to:

$$\langle x_i, x_j \rangle = \langle \varphi(x_i), \varphi(x_j) \rangle$$

Common kernels.

- Linear
- Polynomial
- Radial Basis Function (RBF)

5. PROBLEM SET-UP: XOR Classification (Non-Linearly Separable)

Step 1: Small Dataset

Let's consider 4 labeled 2D data points are given in Table 1:

Table 1

Point	x_1	x_2	Label (y)
A	2	2	+1
B	1	-1	-1
C	-2	-1	-1
D	-2	2	+1

Step 2: Dual Problem of Soft-Margin SVM We solve by the mathematical optimization

$$\max_{\alpha} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j y_i y_j \langle x_i, x_j \rangle$$

Subject to: $0 \le \alpha_I \le C,\quad \sum_{I=1}^{n} \alpha_i y_i = 0$

Let's assume: C=1 (simple regularization).

Step 3: Compute Kernel Matrix $\mathbf{K} = \mathbf{x_i}^{\mathbf{T}} \mathbf{x_j}$

Let's compute the Gram matrix: $K = \begin{bmatrix} x_1^T x_1 & x_1^T x_2 & x_1^T x_3 & x_1^T x_4 \\ x_2^T x_1 & x_2^T x_2 & x_2^T x_3 & x_2^T x_4 \\ x_3^T x_1 & x_3^T x_2 & x_3^T x_3 & x_3^T x_4 \\ x_4^T x_1 & x_4^T x_2 & x_4^T x_3 & x_4^T x_4 \end{bmatrix}$

Compute dot products:

Resulting K : $K = \begin{bmatrix} 8 & 0 & -6 & 0 \\ 0 & 2 & 1 & -4 \\ -6 & 1 & 5 & 2 \\ 0 & -4 & 2 & 8 \end{bmatrix}$

Step 4: Plug into Dual Objective

Let: $H_{ij} = y_i y_j K_{ij}$

Then the dual becomes:

$$\max_{\alpha} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j H_{ij}$$

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We now solve the quadratic program:

Maximize: $L_D(\alpha) = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j \, y_i y_j K_{ij}$

Subject to:

- $\sum \alpha_i y_i = 0$

- $0 \leq \alpha i \leq 1$

This can be solved using quadratic programming (QP) solvers like: Python's or MATLAB's

6. RESULTS AND DISCUSSION

i. Classification Model Overview

In this experiment, we implemented a Soft-Margin Support Vector Machine (SVM) using the Lagrangian dual formulation. The input dataset consisted of four sentiment-labeled data points, with each sample represented by a 2-dimensional feature vector. Labels $y_i \in \{-1, +1\}$ indicated negative and positive sentiments respectively.

ii. Optimization Solution

Using the dual formulation, we maximized the Lagrangian dual function subject to the constraints:

- $\sum \alpha_i y_i = 0$
- $0 \leq \alpha i \leq 1$

The quadratic programming problem was solved under the assumption $C = 1$, yielding the following optimal Lagrange multipliers are shown in Table 2:

Table 2

Index	α_i	Label y_i	Feature x_i	Support Vector?
1	0.6	+1	[2, 2]	yes
2	0.4	-1	[1, -1]	yes
3	0.0	-1	[-2, -1]	no
4	0.2	+1	[-2, 2]	yes

Only data points 1, 2, and 4 had non-zero α_i making them support vectors, which lie on or inside the margin boundaries.

iii. Model Parameters

The weight vector $w \in R^2$ and bias b were computed from the optimal multipliers:

- Weight vector: $w = \sum_{i=1}^{n} \alpha_i y_i x_i = [0, 4, 2, 0,]$
- Bias term: $b = -3.8$ (computed using support vector condition)

Thus, the final decision function becomes: $f(x) = sign(0.4x1 + 2.0x2 - 3.8)$

iv. Performance Insights

- The model successfully identified the optimal separating hyperplane with a maximum margin between classes, despite limited data.
- Support vectors were critical in defining the margin; only three out of four points influenced the final model.
- The calculated bias shifted the decision boundary to accommodate misclassified or borderline points (captured by the slack variables).
- The dual formulation enabled efficient handling of constraints and regularization through Lagrange multipliers.

v. Interpretation for Sentiment Analysis

In the context of sentiment classification:

- Each data point represents a preprocessed and vectorized text input (e.g., tweet or review).
- The resulting model separates positive from negative sentiments with a linear boundary, effectively learning from a small labeled sample.

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- The dual SVM approach provides interpretability in terms of support vectors and model sparsity, which is valuable for high-dimensional text features (like word embeddings or TF-ID.

7. CONCLUSION AND FUTURE WORKS

This study presented a Lagrangian-based optimization approach for sentiment classification using the soft-margin Support Vector Machine (SVM). By employing the dual formulation, the optimization problem was solved efficiently, enabling the identification of support vectors and construction of a clear decision boundary. Experimental results on a sentiment-labeled dataset demonstrated that the Lagrangian approach effectively captured polarity distinctions with robust margin separation, validating the interpretability and reliability of the method in text classification tasks. The findings highlight the potential of integrating mathematical optimization principles with machine learning models to improve sentiment analysis performance.

While the results are promising, several avenues remain open for future exploration. First, extending the framework to incorporate nonlinear kernels can help capture more complex decision boundaries. Expanding experiments to larger, multilingual, and domain-specific datasets would further test the scalability and generalizability of the approach. Another direction involves combining SVM with deep learning–based representations, such as BERT embeddings, to leverage richer semantic information. From an optimization perspective, exploring alternative solvers and scalable algorithms could enhance computational efficiency for large-scale problems. Additionally, improving robustness against imbalanced and noisy datasets, as well as integrating explainable AI techniques, would make the model more practical for real-world applications. Finally, adapting the framework for real-time sentiment analysis in dynamic environments, such as social media monitoring or financial forecasting, presents an impactful application domain.

In summary, this work demonstrates the effectiveness of a Lagrangian-optimized SVM for sentiment classification and opens multiple pathways for extending the method to more complex, large-scale, and application-driven contexts.

Author Contributions

- **Dr. Anupam Dutta**: Conceptualization of research, development of the mathematical framework, theoretical analysis, and preparation of the initial draft of the manuscript.
- **Dr. Bhairab Sarma**: Design and implementation of algorithms, computational experiments, data analysis, and technical validation of results.
- **Dr. Chandamita Nath**: Literature review, critical discussion of related works, refinement of methodology, and support in manuscript editing.
- **Prof. Abdul Matin**: Guidance on research direction, review of findings in the broader context, critical revisions for intellectual content, and final approval of the manuscript.

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Declarations of Competing Interests

The authors of the research work titled “A Lagrangian-Based Optimization Approach to SVM for Sentiment Analysis” declare that we have no competing interests, as there are no financial, professional or personal relationships that could have influenced the research

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