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CONSTRUCTION AND CHARACTERIZATION OF A NEW DISCRETE WRAPPED LENGTH BIASED EXPONENTIAL DISTRIBUTION

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Abstract

In the present study, the discretization procedure for the currently existing continuous circular model is applied in order to develop a new discrete circular distribution. Here, we discretized the Wrapped Length Biased Exponential Distribution to create a New Discrete Wrapped Length Biased Distribution. Also, for the newly developed discrete circular model, the probability mass function, distribution function, characteristic function, and population characteristics are generated.

Keywords: Characteristic function; Distribution function; Trigonometric moments.

1. INTRODUCTION

The analysis of angular data relies significantly on circular models. Many continuous circular models are documented in the existing literature. A diverse and valuable set of circular data models is offered by wrapped circular models [1], stereographic circular and semicircular models [2], and offset circular and semicircular models [3]. However, there aren't many discrete wrapped circular models that are invariant of zero direction and sense of rotation in the literature. The Discrete Wrapped Exponential model [4], the Discrete Wrapped Cauchy model [5] and the Characteristics of Wrapped Poisson model [6] provide a wealth of inspiration to develop the new discrete circular model. This work uses discretization on the Wrapped Length Biased Exponential Distribution model to create a new discrete circular model named Discrete Wrapped Length Biased Exponential Distribution [7].

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2. CIRCULAR DISTRIBUTIONS

A probability distribution with a total probability defined on the unit circle is called a circular distribution. It is a method of allocating probability to various directions because each point on the unit circle represents a direction. A circular random variable's (θ) range, expressed in radians, is taken on $[0, 2\pi)$.

Apparently, there are two sorts of circular distributions: discrete, which assign probabilities to a finite number of directions, and continuous, which assign probabilities to an infinite number of directions. Wrapping a linear distribution is one way to generate circular models. In the following we discuss wrapped discrete circular random variables.

2.1. WRAPPED DISCRETE CIRCULAR RANDOM VARIABLE

For a discrete integer-valued random variable *Y*, applying reduction modulo $2\pi m\left(m \in \mathbf{Z}^{+}\right)$ maps the integers onto the group of m^{th} roots of unity, a subgroup of the unit circle.

i.e $\theta = 2\pi y \left(\mathrm{mod}\, 2\pi m\right)$

More specifically, θ denotes a mapping from the integer set G—a group under '+' to the set of m^{th} roots of unity G', which is a group under '$\cdot$'. The mapping is given by:

$$\theta(x) = e^{\frac{2\pi i y}{m}} \quad \text{where } y \in G, \quad e^{\frac{2\pi i y}{m}} \in G'$$

θ is then referred to as a wrapped discrete circular random variable.

Evidently θ is a homomorphism.

1) $\theta(y_1 + y_2) = e^{\frac{2\pi i (y_1 + y_2)}{m}} = e^{\frac{2\pi i y_1}{m}} . e^{\frac{2\pi i y_2}{m}} = \theta(y_1).\theta(y_2)$ where $y_1, y_2 \in G$

2) $\theta(0) = e^{\frac{2\pi i (0)}{m}} = e^{0} = 1$ where $0 \in G$, $1 \in G'$

Given that θ has a limited number of elements, they are represented by

As θ has a limited number of elements, they are represented by

$$\theta = \left\{ \frac{2\pi r}{m} \Big/ r = 0, 1, 2, \ldots m-1 \right\}$$

which on the unit circle is a lattice.

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2.2. PROBABILITY MASS FUNCTION

Assuming that θ is a wrapped discrete circular random variable, the probability mass function of θ, represented by $Pr\left(\theta = \frac{2\pi r}{m}\right)$, has the following characteristics.

1. $Pr\left(\theta = \frac{2\pi r}{m}\right) \geq 0$
2. $\sum_{r=0}^{m-1} Pr\left(\theta = \frac{2\pi r}{m}\right) = 1$
3. $Pr(\theta) = Pr(\theta + 2\pi h)$ For any integer h (i.e Pr is periodic function)

2.3. DISTRIBUTION FUNCTION

Assuming that θ is a wrapped discrete circular random variable, the probability distribution function of θ, represented by $F_w(\theta)$, has the following characteristics.

Suppose if θ is a wrapped discrete circular random variable then distribution function of θ is denoted by $F_w(\theta)$ and its definition is

$$F_w(\theta) = \sum_{r=0}^{k} Pr\left(\theta = \frac{2\pi r}{m}\right) \qquad \text{where } k = 0,1,...m-1 \tag{2.1}$$

The process of converting continuous functions, models, variables, and equations into their discrete equivalents is known as discretization in mathematics. Depending on the real-world scenario, this procedure is typically used to mimic a specific discrete event.

3. DISCRETIZATION OF A CONTINUOUS DISTRIBUTION

In the present work, we modify current continuous circular models using discretization from [8] to create discrete circular models. The procedure is as follows:

To create a probability mass function $Pr\left(\theta = \frac{2\pi r}{m}\right)$ on a set of evenly spaced points for $r = 0,1,2...m-1$ with a fixed integer m equal to or larger than 2, consider a continuous circular distribution with probability density function (pdf) $f(\theta)$ on the circle.

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$$Pr\left(\theta=\frac{2\pi r}{m}\right)=\frac{f\left(\frac{2\pi r}{m}\right)}{\sum_{r=0}^{m-1} f\left(\frac{2\pi r}{m}\right)} \tag{3.1}$$

shows the corresponding discrete circular model's probability mass function (pmf), $m=1$ is eliminated since it is degenerate.

Here, we use the discretization of a continuous circular model to generate a new discrete circular model.

4. DISCRETE WRAPPED LENGTH BIASED EXPONENTIAL DISTRIBUTION

A wrapped length biased exponential distribution's probability density function is

$$f(\theta)=\frac{e^{-\frac{1}{\lambda}(\theta-2\pi)}}{\lambda^{2}\left(e^{\frac{2\pi}{\lambda}}-1\right)}.\left[\theta+\frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)}\right] \tag{4.1}$$

Where λ >0 is a parameter.

The probability mass function of the Discrete Wrapped Length Biased Exponential Distribution is now defined using equation (3.1) as

$$Pr\left(\theta=\frac{2\pi r}{m}\right)=\frac{e^{-\frac{1}{\lambda}(\theta-2\pi)}\left[\theta+\frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)}\right]}{\sum_{r=0}^{m-1} e^{-\frac{1}{\lambda}(\theta-2\pi)}\left[\theta+\frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)}\right]} \tag{4.2}$$

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$$= \frac{e^{-\frac{\theta}{\lambda}}\left[\theta + \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}} - 1\right)}\right]}{\sum_{r=0}^{m-1} e^{-\frac{\theta}{\lambda}}\left[\theta + \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}} - 1\right)}\right]}$$

Consider $\sum_{r=0}^{m-1} \theta e^{-\frac{\theta}{\lambda}}$

$$= \sum_{r=0}^{m-1} \frac{2\pi r}{m} e^{-\frac{2\pi r}{m\lambda}}$$

$$= \frac{2\pi}{m} e^{-\frac{2\pi}{m\lambda}} + \frac{4\pi}{m} e^{-\frac{4\pi}{m\lambda}} + \frac{6\pi}{m} e^{-\frac{6\pi}{m\lambda}} + \ldots\ldots\ldots\ldots + \frac{2\pi(m-1)}{m} e^{-\frac{2\pi(m-1)}{m\lambda}}$$

$$= \frac{2\pi}{m} e^{-\frac{2\pi}{m\lambda}}\left[1 + 2e^{-\frac{2\pi}{m\lambda}} + 3\left(e^{-\frac{2\pi}{m\lambda}}\right)^2 + \ldots\ldots\ldots\ldots + (m-1)\left(e^{-\frac{2\pi}{m\lambda}}\right)^{m-2}\right]$$

By Arithmetic- Geometric progression

$$= \frac{2\pi}{m} e^{-\frac{2\pi}{m\lambda}}\left[\frac{1 - m\left(e^{-\frac{2\pi}{m\lambda}}\right)^{m-1} + me^{-\frac{2\pi}{\lambda}} - e^{-\frac{2\pi}{\lambda}}}{\left(1 - e^{-\frac{2\pi}{m\lambda}}\right)^2}\right] \qquad (4.3)$$

Consider $\sum_{r=0}^{m-1} e^{-\frac{\theta}{\lambda}} = \sum_{r=0}^{m-1} e^{-\frac{2\pi r}{m\lambda}}$

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$$=1+e^{-\frac{2\pi}{m\lambda}}+\left(e^{-\frac{2\pi}{m\lambda}}\right)^{2}+.............+\left(e^{-\frac{2\pi}{m\lambda}}\right)^{m-1}$$

Since $\left(\because \left| e^{-\frac{2\pi}{m\lambda}} \right| < 1\right)$

Then by Geometric progression

$$=\left[\frac{1-e^{-\frac{2\pi}{\lambda}}}{1-e^{-\frac{2\pi}{m\lambda}}}\right] \tag{4.4}$$

By using (4.3) and (4.4)

$$Pr\left(\theta=\frac{2\pi r}{m}\right)=c\,e^{-\frac{\theta}{\lambda}}\left[\theta+\frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)}\right] \text{ Where } m\in Z^{+}, r=0,1,2,......m-1 \text{ and } \lambda>0 \tag{4.5}$$

Where c is a constant which is defined as

$$c=\frac{\left[1-e^{\frac{-2\pi}{m\lambda}}\right]^{2}\left[e^{\frac{2\pi}{\lambda}}-1\right]}{\frac{2\pi}{m}\left[e^{\frac{2\pi}{\lambda}}-1\right]\left[e^{\frac{-2\pi}{m\lambda}}-me^{\frac{-2\pi}{\lambda}}+(m-1)e^{\frac{-2\pi}{\lambda}}e^{\frac{-2\pi}{m\lambda}}\right]+2\pi\left[1-e^{\frac{-2\pi}{m\lambda}}\right]\left[1-e^{\frac{-2\pi}{\lambda}}\right]} \tag{4.6}$$

5. DISTRIBUTION FUNCTION FOR DISCRETE WRAPPED LENGTH BIASED EXPONENTIAL DISTRIBUTION

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The Discrete Wrapped Exponential Distribution's distribution function is described as $F_w(\theta) = \sum_{r=0}^{k} Pr\left(\theta = \frac{2\pi r}{m}\right)$ Where $k = 0,1,2,...m-1$

By using (4.5) and (4.6)

$$= c\sum_{r=0}^{k} e^{-\frac{\theta}{\lambda}} \left[\theta + \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}} - 1\right)}\right]$$

$$= c\left[\sum_{r=0}^{k} \theta e^{-\frac{\theta}{\lambda}} + \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}-1}\right)}\sum_{r=0}^{k} e^{-\frac{\theta}{\lambda}}\right] \tag{5.1}$$

Consider $\sum_{r=0}^{k} \theta e^{-\frac{\theta}{\lambda}}$

$$= \sum_{r=0}^{k} \frac{2\pi r}{m} e^{-\frac{2\pi r}{m\lambda}}$$

$$= \frac{2\pi}{m} e^{-\frac{2\pi}{m\lambda}} + \frac{4\pi}{m} e^{-\frac{4\pi}{m\lambda}} + \frac{6\pi}{m} e^{-\frac{6\pi}{m\lambda}} + ...+ \frac{2\pi k}{m} e^{-\frac{2\pi k}{m\lambda}}$$

$$= \frac{2\pi}{m} e^{-\frac{2\pi}{m\lambda}} \left[1 + 2e^{-\frac{2\pi}{m\lambda}} + 3\left(e^{-\frac{2\pi}{m\lambda}}\right)^2 + ...+ k\left(e^{-\frac{2\pi}{m\lambda}}\right)^{k-1}\right]$$

By Arithmetic- Geometric progression

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$$=\frac{2\pi}{m}e^{-\frac{2\pi}{m\lambda}}\left[\frac{1-\left(e^{-\frac{2\pi}{m\lambda}}\right)^{k}-k\left(e^{-\frac{2\pi}{m\lambda}}\right)^{k}\left(1-e^{-\frac{2\pi}{m\lambda}}\right)}{\left(1-e^{-\frac{2\pi}{m\lambda}}\right)^{2}}\right]$$

$$=\frac{2\pi}{m}\left[\frac{e^{-\frac{2\pi}{m\lambda}}-\left(e^{-\frac{2\pi}{m\lambda}}\right)^{k+1}-k\left(e^{-\frac{2\pi}{m\lambda}}\right)^{k+1}\left(1-e^{-\frac{2\pi}{m\lambda}}\right)}{\left(1-e^{-\frac{2\pi}{m\lambda}}\right)^{2}}\right] \tag{5.2}$$

Consider $\sum_{r=0}^{k} e^{-\frac{\theta}{\lambda}} \quad =\sum_{r=0}^{k} e^{-\frac{2\pi r}{m\lambda}}$

$$=\left[1+e^{-\frac{2\pi}{m\lambda}}+\left(e^{-\frac{2\pi}{m\lambda}}\right)^{2}+\ldots\ldots\ldots\ldots+\left(e^{-\frac{2\pi}{m\lambda}}\right)^{k}\right]$$

Since $\left(\because\left|\, e^{-\frac{2\pi}{m\lambda}}\,\right|<1\right)$

By Geometric progression

$$=\frac{\left(1-e^{-\frac{2\pi(k+1)}{m\lambda}}\right)}{\left(1-e^{-\frac{2\pi}{m\lambda}}\right)}$$

(5.3)

By using (5.2) and (5.3)

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$$\therefore F_w(\theta)=\frac{\frac{2\pi}{m}\left[e^{\frac{2\pi}{\lambda}}-1\right]\left[e^{\frac{-2\pi}{m\lambda}}-e^{\frac{-2\pi(k+1)}{m\lambda}}-k\,e^{\frac{-2\pi(k+1)}{m\lambda}}+k\,e^{\frac{-2\pi(k+2)}{m\lambda}}\right]+2\pi\left[1-e^{\frac{-2\pi(k+1)}{m\lambda}}\right]\left[1-e^{\frac{-2\pi}{m\lambda}}\right]}{\frac{2\pi}{m}\left[e^{\frac{2\pi}{\lambda}}-1\right]\left[e^{\frac{-2\pi}{m\lambda}}-me^{\frac{-2\pi}{\lambda}}+(m-1)e^{\frac{-2\pi}{\lambda}}e^{\frac{-2\pi}{m\lambda}}\right]+2\pi\left[1-e^{\frac{-2\pi}{m\lambda}}\right]\left[1-e^{\frac{-2\pi}{\lambda}}\right]}$$

6. CHARACTERISTIC FUNCTION FOR THE DISCRETE WRAPPED LENGTH BIASED EXPONENTIAL DISTRIBUTION

The Discrete Wrapped Length Biased Exponential Distribution's characteristic function is described as

$$\varphi_\theta(p)=E(e^{ip\theta})=\rho_p e^{i\mu_p} \quad \text{Where } p\in\mathbf{Z}$$

$$=\alpha_p+i\beta_p$$

$$=\sum_{r=0}^{m-1}e^{ip\theta}\,pr\left(\theta=\frac{2\pi r}{m}\right)$$

$$\therefore \varphi_\theta(p)=\frac{c}{\left(1-e^{\frac{2\pi}{m}\left(ip-\frac{1}{\lambda}\right)}\right)^2}\left[\begin{array}{l}\frac{2\pi}{m}\left[e^{\frac{2\pi}{m}\left(ip-\frac{1}{\lambda}\right)}-m\,e^{2\pi\left(ip-\frac{1}{\lambda}\right)}+(m-1)e^{\frac{2\pi}{m}\left(ip-\frac{1}{\lambda}\right)}e^{2\pi\left(ip-\frac{1}{\lambda}\right)}\right]+\\ \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)}.\left[1-e^{2\pi\left(ip-\frac{1}{\lambda}\right)}\right]\left[1-e^{\frac{2\pi}{m}\left(ip-\frac{1}{\lambda}\right)}\right]\end{array}\right] \tag{6.1}$$

$$=\alpha_p+i\beta_p$$

By using (6.1)

$$\alpha_p=\frac{c}{\left(1-e^{\frac{-2\pi}{m\lambda}}\right)^2}A$$

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$$\beta_p = \frac{c}{\left(1-e^{\frac{-2\pi}{m\lambda}}\right)^2} B$$

Here

$$A = \left[\begin{array}{l} \frac{2\pi}{m} e^{-\frac{2\pi}{\lambda}} \cos\left(\frac{2\pi p}{m}\right) - 2\pi e^{-\frac{2\pi}{\lambda}} + 2\pi\, e^{-\frac{2\pi}{m\lambda}} e^{-\frac{2\pi}{\lambda}} \cos\left(\frac{2\pi p}{m}\right) - \frac{2\pi}{m}\, e^{-\frac{2\pi}{m\lambda}} e^{-\frac{2\pi}{\lambda}} \cos\left(\frac{2\pi p}{m}\right) + \\ \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)} - \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)} e^{-\frac{2\pi}{m\lambda}} \cos\left(\frac{2\pi p}{m}\right) - \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)} e^{-\frac{2\pi}{\lambda}} + \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)} e^{-\frac{2\pi}{m\lambda}} e^{-\frac{2\pi}{\lambda}} \cos\left(\frac{2\pi p}{m}\right) \end{array}\right]$$

And

$$B = \left[\begin{array}{l} \frac{2\pi}{m} e^{-\frac{2\pi}{\lambda}} \sin\left(\frac{2\pi p}{m}\right) + 2\pi e^{-\frac{2\pi}{\lambda}} + 2\pi\, e^{-\frac{2\pi}{m\lambda}} e^{-\frac{2\pi}{\lambda}} \sin\left(\frac{2\pi p}{m}\right) - \frac{2\pi}{m}\, e^{-\frac{2\pi}{m\lambda}} e^{-\frac{2\pi}{\lambda}} \sin\left(\frac{2\pi p}{m}\right) - \\ \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)} e^{-\frac{2\pi}{m\lambda}} \sin\left(\frac{2\pi p}{m}\right) + \frac{2\pi}{\left(e^{\frac{2\pi}{\lambda}}-1\right)} e^{-\frac{2\pi}{m\lambda}} e^{-\frac{2\pi}{\lambda}} \sin\left(\frac{2\pi p}{m}\right) \end{array}\right]$$

Clearly $\rho_p = \sqrt{\alpha_P{}^2 + \beta_p{}^2}$ and $\mu_p = \tan^{-1}\left(\frac{\beta_p}{\alpha_p}\right)$

Here μ_1 denotes circular mean direction which is given by

$$\mu_1 = \tan^{-1}\left[\frac{\beta_1}{\alpha_1}\right]$$

Now ρ_1 stands for the concentration in the direction of the mean, which is described as

$$\rho_1 = \sqrt{\alpha_1{}^2 + \beta_1{}^2}$$

In general $\mu_1 = \mu$ and $\rho_1 = \rho$.

The circular variance is expressed by the symbol V_o and can be defined as $V_o = 1-\rho$.

Additionally, the definition of the circular standard deviation is

$$\sigma_o = \sqrt{-2\log(1-V_o)}$$

7. CENTRAL TRIGONOMETRIC MOMENTS

The definition of pth Central trigonometric moment of θ is

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$$\varphi_\theta^*(p) = E\left(e^{ip(\theta-\mu)}\right)$$

$$= E\left(e^{ip\theta}e^{-ip\mu}\right) = e^{-ip\mu}E\left(e^{ip\theta}\right)$$

$$= \left(\cos p\mu - i\sin p\mu\right)\left(\alpha_p + i\beta_p\right)$$

$$= \left(\alpha_p \cos p\mu + i\beta_p \cos p\mu - i\alpha_p \sin p\mu + \beta_p \sin p\mu\right)$$

$$= \alpha_p \cos p\mu + \beta_p \sin p\mu + i(\beta_p \cos p\mu - \alpha_p \sin p\mu)$$

$$= \alpha_p^* + i\beta_p^*$$

where $\alpha_p^* = \alpha_p \cos p\mu + \beta_p \sin p\mu$

$\beta_p^* = \beta_p \cos p\mu - \alpha_p \sin p\mu$

The Discrete Wrapped Length Biased Exponential Distribution's skewness is now defined as

$$\gamma_1 = \frac{\beta_2^*}{(V_o)^{\frac{3}{2}}}$$

For the Discrete Wrapped Length Biased Exponential Distribution, the definition of kurtosis

is $\gamma_2 = \dfrac{\alpha_2^* - \left(1 - V_o\right)^4}{\left(V_o\right)^2}$

8. CONCLUSION

Through a suitable discretization process, we were able to successfully extend an existing continuous circular model into the discrete domain in this study. We created a new Discrete Wrapped Length Biased Distribution with clearly defined structural properties by discretizing the Wrapped Length Biased Exponential Distribution. The probability mass function, distribution function, characteristic function, and key population characteristics of the proposed model were all carefully calculated. These results demonstrate that the newly developed discrete circular distribution improves the family of discrete circular statistical models and opens up new theoretical and practical research opportunities by providing a practical and mathematically sound framework for representing circular data in discrete form.

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