International Journal of Applied Mathematics

Volume 38 No. 10s 2025,

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

INVENTORY MODEL FOR A NON-INSTANTANEOUS DETERIORATING ITEM WITH PRESERVATION TECHNOLOGY INVESTMENT, TRADE CREDIT AND PARTIAL BACKORDERING

Anamika Sharma[1] and Geetanjali Sharma[2*]
[1]Banasthali Vidyapith, Rajasthan- -304022, India, roma.kaushik@gmail.com
[2]Banasthali Vidyapith, Rajasthan- -304022, India, geetanjali.bu@gmail.com
ORCID ID 0000-0001-7814-1637
***Corresponding Author:** Geetanjali Sharma[2*]

Abstract

This research formulates an inventory optimization deterioration model for gradual degrading stock with demand varying dynamically in response to changing in time and price. Through the combined use of shelf-life extension technology, Credit facility financing, and Partial order backlog, the model provides holistic approach to optimizing inventory management. Preservation technology investment is analysed for its impact on reducing deterioration rates, thereby extending the shelf life of products. Trade credit terms are evaluated as a financial strategy to enhance cash flow and align payment cycles with revenue generation. Additionally, the model includes partial backordering to manage shortages effectively, balancing customer service levels with inventory costs. Analytical methods are employed to derive optimal policies for ordering, pricing, and backordering, aiming to maximize the overall profit. This study aims to explore how preservation technology and trade credit can reduce product deterioration and provide financial flexibility for retailers in perishable goods industries like food, pharmaceuticals, and electronics. This approach determines the Optimal pricing and preservation technology expenditure to achieve maximum profitability. Computational results and parameter variations are analysed for validation of the model

Keywords: non-instantaneous deterioration, trade credit, preservation technology, partial backlogging

AMS subject classification code: 90 Operations Research, mathematical programming

Introduction and literature review

In today's highly competitive and dynamic market environment, managing inventory effectively is crucial for businesses, particularly those dealing with perishable or deteriorating items. Traditional inventory models often assume instantaneous deterioration, which does not adequately reflect the reality faced by many industries where items deteriorate over time. This study bridges the existing research gap in managing non-instantaneous deteriorating goods by formulating a detailed inventory model that accounts as a consequence in which demands depends on time-selling pricing strategy. The perishability of products such as food, pharmaceuticals, and high-tech components necessitates innovative approaches to inventory management. Investing in preservation technology has become a crucial strategy for mitigating the effects of deterioration, thereby extending product

Received: August 06, 2025

shelf life and improving inventory management. thereby extending the usable life of products and reducing waste. However, the decision to invest in such technology must be balanced against its costs and the anticipated benefits in terms of reduced deterioration rates.

Financial strategies also play a critical role in inventory management. Trade credit, a common practice where suppliers extend payment terms to buyers, can significantly affect a firm's cash flow and inventory decisions. By aligning payment cycles with revenue generation, trade credit helps firms manage their working capital more effectively. This research analyzes the outcome of trade credit on stock management strategies, and how it can be leveraged to enhance profitability.

Furthermore, partial back ordering addresses situations where demand exceeds supply. Instead of losing sales entirely, businesses can fulfil backorders once new stock arrives, maintaining customer satisfaction while balancing inventory costs. Integrating partial back ordering into the model provides a more realistic and flexible approach to managing shortages.

Focusing on non-instantaneous deteriorating inventory, our findings intends to formulate a robust inventory model that leverages trade credit spending on product preservation, and partial backordering. The focus is to empower the planners with a robust methodology for optimizing stock policies, given the complex demand dynamics influenced by time and price. This paper reviews the relevant literature on inventory management, preservation technology, trade credit, and back-ordering strategies. We present the mathematical formulation of our model, followed by the derivation of optimal policies. Numerical examples illustrate the model's practical application, and Parametric analyses evaluate the influence of key parameters on inventory performance. Our results have several implications of our findings and Potential avenues for further research. The concept of non-instantaneous deteriorating inventory recognizes that items remain stable for a certain period before deterioration sets in. Pioneering work by the inventory model framework introduced by Ghare and Schrader (1963) has been influential in the context of exponential deterioration items, laying the groundwork for subsequent research on deteriorating inventory. Goyal (1985) investigated first-time trade credit with an inventory model. More recent studies, such as those by Balkhi et al. (2001), have expanded on this by considering non-instantaneous deterioration and its impact on inventory policies. These models often assume that deterioration follows a known function over time, allowing for more accurate inventory levels and order quantity predictions. Huang (2006) investigated an inventory model for limited storage space devices with a trade credit policy and also considered a sustainable environment. Gupta and Wang's (2009) analysis of retail operations under uncertainty reveals. The optimal policy structure is unaffected by variations in credit terms the optimal policy framework, the policy parameter's value, however, is influenced. In contrast, a continuous-time Framework was proposed for optimizing inventory levels. A study by Min et al. (2010) formulated a quantity optimization approach concerning perishable goods in which demand rates have dependency on current inventory levels and flexible payment terms resulting in benefits for the retailer from a Specified payment delay from the supplier and extends it to customers to enhance market competitiveness.

Received: August 06, 2025

Research conducted by Chung et al. (2014) developed an EPQ framework tailored to perishable goods, incorporating supplier-retailer credit arrangements., Under a delayed payment arrangement with the supplier the retailer grants payment delay facilities to purchasers Research by Molamohamadi et al. (2014) focused on formulating an EPQ inventory model with backordering, assuming delayed payments and instantaneous deterioration. Research executed by Jaggi et al. (2015) initiated an inventory model to minimize costs by optimizing cycle duration and stock-in time. Building on Mahata's (2012) research, Sarkar et al. (2015) presented through advancements by incorporating dynamic product deterioration including trade-credit policies for suppliers and retailers, with hybrid trade credit arrangement that is featuring full credit from suppliers to retailers and partial credit from retailers to customers. This promotes need of store to place bulk orders, which enables the company to lease more storage space. This gives the direction to Innovates a two-warehouse inventory model for non-instantaneous degrading commodities with permitted payment delays in light of inflation's impact a scenario. A pricing and inventory control model was introduced by Mahimahi et al. (2017) for gradual decaying products, incorporating a dual-tier credit system. In this model, a credit chain is formed, with the supplier granting credit to the retailer and the retailer granting this facility to customers. Partially backlogged shortages and a probabilistic demand function are utilised. Giri and Sharma (2016) investigated an inventory system featuring demand which is linear and vary over time, two-tier credit financing, and flexibility for shortages. They analyzed various scenarios investigating the link between supplier and retailer credit periods, they determined the conditions for an optimal outcome.

Anchal et al. (2016) investigated an inventory decision-making framework for degrading items, focusing on the optimal inventory replenishment policy for retailers accompanied by payment flexibility. Tiwari et al. (2016) Suggested an inventory management strategy, highlighting in which suppliers offer acceptable payment delays to retailers to boost demand in a competitive business environment. Geetha and Udayakumar (2018) proposed proposes two distinct inventory models for managing non-instantaneous decaying inventory one for single-warehouse scenarios and another for two-warehouse scenarios. The goal was to identify the most suitable replenishment duration and Optimal inventory replenishment quantity. Lashgari et al. (2018) designed an EOQ model for products with non-instantaneous decay incorporating a dual payment structure. According to it this scheme combines advance payments with delayed payments, where the advance payment for a portion of the order, with the balance due after delivery or within a specified period. after receiving the order. Chung et al. (2019) researched an inventory control model that incorporates delayed payment policy with cash discounts in inflationary environments. Babangida and Baraya (2019) presented inventory framework for perishable goods included Bi-component demand pattern and holding costs dependent on time, with delayed payment options, assuming time-dependent quadratic demand until degradation occurs. Research by Lin et al. (2019) focused on developing an inventory model that considers both two-stage deterioration and supplier financing allowing for shortages without partial backlogging. An EOQ model for decaying goods

Received: August 06, 2025

under a dual-level payment delay arrangement was examined by Mashud et al. (2019)., considering two demand functions with fully backlogged shortages.

Shaikh and Cardenas-Barron (2020) explored an inventory system with non-instantaneous deterioration and price-dependent demand. An inventory model with advertisement-relient demand and trade credit was investigated by Md. Mashud et al. (2020). Sharma et al. (2020) aimed to identify optimal retail strategies to maximizing total profit Showing the presence of a unique the Most effective outcome. Kumar et al(2020a) formulated a inventory cost optimization model with two warehouses and supplier financing for inventory items In the presence of uncertain demand and product spoilage. Kumar et al. (2020b) proposed a model for a Dual-warehouse setup with dynamic storage costs and Time-dependent demand decline under trade credit arrangements, also considered partially backlogged. Nayak et al. (2021) proposed a Fuzzy logic-based inventory replenishment approach Stocked product management with Weibull degradation and Demand with a constant slope under fully backlogged shortages. They used graded mean preference integration strategy and Fuzzy optimization problems targeting to total cost reduction per unit time. Das et al. (2021) developed a non-instantaneous deteriorating inventory model incorporating preservation technology Multiple payment periods, and variable time dependent demand on inventory level and selling price.

Babangida and Baraya (2021) investigated an inventory model for gradual decaying items demand Bi-phase demand rates, Time-sensitive holding expenses, and delayed payment option with partial backlogging. Mashud et al. (2021) examined Price-advertisement responsive demand model, gradual decaying model with preservation investment, partial lost sales, and delayed payment. Kumar et al. (2022a) developed Dual inventory models for Weibull-distributed deteriorating items with price-sensitive demand and lead-time under advance payment and post-payment scenarios. Kumar and Paikray (2022) proposed an inventory model for Perishable goods with trapezoidal demand function in both crisp and Stochastic environments. Tripathy et al. (2022) discussed a Stock management model based on dynamic trade credit policy for non-instantaneous decay items, considering various financial scenarios. Kumar et al. (2022b) formed an inventory model incorporation with trade credits, inventory-dependent demand, partial backlogging, and non-instantaneous degradation under inflation. Nayak and Sahoo (2023) developed an EOQ model assuming demand and other relevant factors Hatibaruah and Saha (2023) determined the optimal cycle time, Expenditure in preservation technology, frequency of advertisements, and timing of peak stock status and shortages to Optimize the system's average profit per unit period. A Strategy was proposed to extract most effective approach from the constructed model. Nayak et al. (2024) studied an EOQ model with continuous deterioration, power-pattern demand, Weibull amelioration, allowable total backlog shortages, and imprecise expenses in crisp and fuzzy scenarios, developing a mathematical inventory model and solution strategy. Tripathi (2024) presented Inventory control system with selling price-dependent demand, considering decaying and payment delay policy. Patra et al. (2024a) recommended a model with power-pattern demand, cost in preservation technology, and trade credit, including a learning effect on holding costs. Patra

Received: August 06, 2025

et al. (2024b) examined a retailer's stock system to determine the optimal plan for cost optimization under given constraints, including power-pattern demand, Persistent degradation, and uncertain costs. Shah et al. (2024) aimed to find Optimal stock control plan for 5. Phased deteriorating products with multivariate stochastic demand, incorporating dual-level trade credit financing and preservation technology. The demand of the decaying item is affected by selling price, inventory, along with Ad repetition, and the supplier includes price discounts and trade credit programs to increase revenue.

Table 1: literature review table

Authors	Demand pattern	Deterioration	Preservation technology	Shortages	Trade credit
Dye and Hsieh (2012)	Constant	Time dependent	consider	Partial backordering	Not consider
He and Huang (2013)	Price dependent	Constant	consider	Not consider	Not consider
Zhang et al. (2014)	Price dependent	Constant	consider	Not consider	Not consider
Lu et al. (2016)	Price and stock dependent	Constant	Not consider	Not consider	Not consider
Li et al. (2019)	Price dependent	Non instantaneous	consider	Not consider	Not consider
Mishra et al. (2018)	Price dependent	Constant	consider	Not consider	consider
Khanra et al. (2013)	Time dependent	Constant	Not consider	Partial backordering	consider
Annadurai et al. (2013)	Credit period dependent	Constant	Not consider	Partial backordering	consider
Shaikh et al. (2021)	Stock dependent	Constant	Not consider	Partial backordering	consider
Tripathi et al. (2022)	Time dependent	Constant	Not consider	Partial backordering	consider
Present paper	Time and price	Constant	consider	Partial backordering	consider

Our review of the literature indicates that delay payment policies for non-instantaneous decaying items with partial shortages have not been explored. This study fills that gap by presenting a In-depth inventory framework that considers these elements.

Assumptions

- Demand is modelled as D(p,t) = a + ct - bp, capturing price- time sensitive dynamics.
- The decaying rate is supposed to be constant.

Received: August 06, 2025

- For the duration of the study, deteriorated products are neither replaced nor repaired.
- The decaying rate is influenced via expenditure on preservation techniques, represented through the function $m(\xi) = e^{-d\xi}$ and satisfy the specific conditions $\frac{\partial m(\xi)}{\partial \xi} < 0, \frac{\partial^2 m(\xi)}{\partial \xi^2} > 0$. Our literature review reveals a gap in research on trade credit policies for deteriorating items with non-instantaneous decay including partial shortages. Our research bridges this gap by introducing a comprehensive inventory model that incorporates these factors.
- During stock-out periods, the backordering rate dependent on the time until stock is again refilled., given by $\frac{1}{1+\delta(T-t)}$, Where δ is the backordering coefficient, and (T − t) signifies the time until replenishment. Where δ is the backordering coefficient, and (T − t) signifies the time until replenishment.

Notations

Table 2 - Represents the Symbols that are used in this inventory model

Parameters	Units	Descriptions
$I_1(t)$	-	Inventory level in-between the time 0 to t_1.
$I_2(t)$	-	Inventory level during the time t_1 to t_2.
$I_3(t)$	-	Inventory level during the time t_2 to T.
t_1	Week/cycle	Time where items are started to deteriorate.
t_2	Week/cycle	Time where shortage started.
T	Week/cycle	Total cycle time.
a	-	Scaling factor of demand.
b	-	Scaling factor of demand.
c	-	Scaling factor of demand.
p	Rupees/item	Selling price.
θ	-	Deterioration rate.
δ	-	Backlogging parameter.
ξ	Rupees/unit	Preservation technology cost.
d	-	sensitive parameter of investment to the deterioration rate.
S	Kg/item	Initial inventory level.
B	Kg/item	Maximum shortage quantity per cycle.
C_h	Rupees/item	Holding cost per unit item.
O	Rupees/item	Ordering cost per unit item.
S_c	Rupees/item	Shortage cost per unit item.
S_l	Rupees/item	Lost sale cost per unit item.
e	Rupees/item	Purchasing cost per unit item.
M	Weeks/cycle	Trade credit period time.
I_e	%	Interest earns
I_c	%	Interest charge
Decision variable		
ξ	Rupees/item	Preservation technology cost

Received: August 06, 2025

p	Rupees/item	Selling price

Now in this section we discussed mathematical model of this problem and explain the solution after forming differential equations-

Mathematical modelling

Fig:1- represents the stock inventory level, the x-axis corresponds to time, and the y-axis symbolizes the inventory level. Firstly, till time $t1$, inventory decreases because of demand then inventory decreases as a result of demand and deterioration. After that, at the time $t2$ inventory becomes zero, and at T, the maximum shortage becomes high. The differential equations of this inventory model are written below-

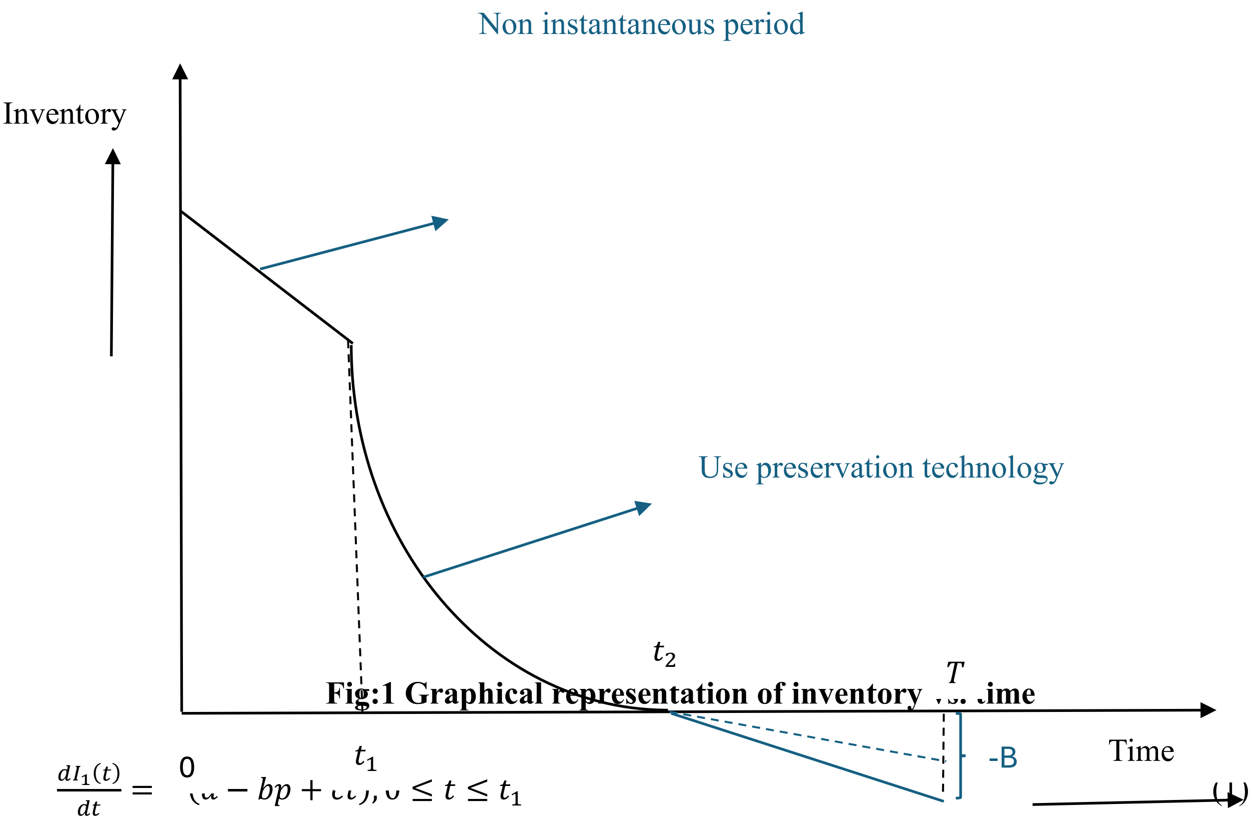

Fig:1 Graphical representation of inventory vs. time

$$\frac{dI_1(t)}{dt} = (a - bp + ct), 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI_2(t)}{dt} = -\theta m(\xi) I_2(t) - (a - bp + ct), t_1 \leq t \leq t_2 \quad (2)$$

$$\frac{dI_3(t)}{dt} = -\frac{D(p,t)}{1+\delta(T-t_1)}, t_2 \leq t \leq T \quad (3)$$

Equation (1) represent differential equation for interval $[0, t_1]$,

Equation (2) represents differential equation for interval $[t_1, t_2]$ and

Equation (3) represents differential equation for interval $[t_2, T]$.

With boundary conditions

Received: August 06, 2025

$$I_1(0) = S, I_1(t_1) = I_2(t_1), I_2(t_2) = 0, I_3(t_2) = 0, I_3(T) = -B \qquad (4)$$

Solution of equations (1), (2) and (3) using these conditions are:

$$I_1(t) = S - (a - bp)t - \frac{ct^2}{2} \qquad (5)$$

$$I_2(t) = \left(\frac{(a-bp)}{\theta m(\xi)} - \frac{c}{\left(\theta m(\xi)\right)^2}\right)\left(e^{\theta m(\xi)(t_2-t)} - 1\right) + \frac{c}{\theta m(\xi)}\left(t_2 e^{\theta m(\xi)(t_2-t)} - t\right) \qquad (6)$$

$$I_3(t) = \left(\frac{a-bp}{\delta} + \frac{cT}{\delta} + \frac{c}{\delta^2}\right) log\left(\frac{1+\delta(T-t)}{1+\delta(T-t_2)}\right) + \frac{c}{\delta}(t - t_2) \qquad (7)$$

and also $B = \left(\frac{a-bp}{\delta} + \frac{cT}{\delta} + \frac{c}{\delta^2}\right) log\left(1 + \delta(T - t_2)\right) + \frac{c}{\delta}(t_2 - T)$ (8)

$$S = (a - bp)t_1 + \frac{ct_1^2}{2} + \left(\frac{a-bp}{\theta m(\xi)} + \frac{c}{\left(\theta m(\xi)\right)^2}\right)\left(e^{\theta m(\xi)(t_2-t)} - 1\right) + \frac{c}{\theta m(\xi)}\left(t_2 e^{\theta m(\xi)(t_2-t)} - t_1\right)$$

(9)

Total quantity per unit cycle is

$$Q = S + B = (a - bp)t_1 + \frac{ct_1^2}{2} + \left(\frac{a-bp}{\theta m(\xi)} + \frac{c}{\left(\theta m(\xi)\right)^2}\right)\left(e^{\theta m(\xi)(t_2-t)} - 1\right) + \frac{c}{\theta m(\xi)}\left(t_2 e^{\theta m(\xi)(t_2-t)} - t_1\right) + \left(\frac{a-bp}{\delta} + \frac{cT}{\delta} + \frac{c}{\delta^2}\right) log\left(1 + \delta(T - t_2)\right) + \frac{c}{\delta}(t_2 - T)$$

(10)

Now calculate inventory costs:

1. Ordering cost = O
   (11)
2. Holding cost for the entire cycle is represented by equation (12)

   Holding cost= $C_h \int_0^{t_1} I_1(t)dt + C_h \int_{t_1}^{t_2} I_2(t)dt$

   $$= C_h\left(-\frac{(a-bp)t_1^2}{2} - \frac{ct_1^3}{6}\right) + C_h\left(\frac{(a-bp)}{\theta m(\xi)}\left(-\frac{1}{\theta m(\xi)} - t_2 + \frac{e^{\theta m(\xi)(t_2-t_1)}}{\theta m(\xi)} + t_1\right) + \frac{c}{\theta m(\xi)}\left(-\frac{t_2}{\theta m(\xi)} - \frac{t_2^2}{2} + \frac{t_2 e^{\theta\, m(\xi)(t_2-t)}}{\theta m(\xi)} + \frac{t_1^2}{2}\right) - \frac{c}{\left(\theta m(\xi)\right)^2}\left(-\frac{1}{\theta m(\xi)} - t_2 + \frac{e^{\theta m(\xi)(t_2-t_1)}}{\theta m(\xi)} + t_1\right)\right) \qquad (12)$$

3. Purchasing cost = eQ (13)
4. Shortage cost = $-S_c \int_{t_2}^{T} I_3(t)dt$

$$= -S_c\left(\left(\frac{a-bp}{\delta}+\frac{cT}{\delta}+\frac{c}{\delta^2}\right)\left(t_2 - T - (1+\delta T)\, log(1+\delta\,(T-t_2)\right) + \frac{c}{\delta}\left(\frac{T^2-t_2^2}{2} - t_2(T - t_2)\right)\right) \qquad (14)$$

5. Lost sale cost = $S_l \int_{t_2}^{T} D\left(1-\frac{1}{1+\delta(T-t)}\right)dt$

$$= S_l((a-bp)\left(T-t_2-\frac{log\left(1+\delta(T-t_2)\right)}{\delta}\right) + c\left(\frac{T^2}{2}-\frac{T}{\delta}-\frac{t_2^2}{2}+\frac{t_2}{\delta}+\frac{1+\delta T}{\delta^2}\,log\left(1+\delta(T-t_2)\right)\right)$$
(15)

6. Preservation technology cost = ξT (16)
7. Sales revenue = $Dpt_2 + pB$

$$= Dpt_2 + p\left(\left(\frac{a-bp}{\delta}+\frac{cT}{\delta}+\frac{c}{\delta^2}\right) log\left(1+\delta(T-t_2)\right) + \frac{c}{\delta}(t_2 - T)\right) \qquad (17)$$

Here three cases

Table 3- Represents different cases for credit period.

Case- I	Case- II	Case- III
$0 \le M \le t_1 \le t_2 \le T$	$0 \le t_1 \le M \le t_2 \le T$	$0 \le t_1 \le t_2 \le M \le T$

For case I - when $0 \le M \le t_1 \le t_2 \le T$

Interest charge- The cost incurred by a buyer for delaying payment beyond the trade credit period. It represents the financial penalty or additional expense due to late payment.

$$IC = eI_c \int_M^{t_2} I(t)\,dt = I_c\left(\int_M^{t_1} I_1(t)\,dt + \int_{t_1}^{t_2} I_2(t)\,dt\right)$$

$$= I_c e\left(\left(S(t_1-M)+\frac{a-bp}{2}(M^2-t_1^2)+\frac{c}{6}(M^3-t_1^3)\right)+\left(\frac{1}{\theta m(\xi)}\left(e^{\theta m(\xi)(t_2-t_1)}-1\right)+(t_1-t_2)\right)\left(\frac{a-bp}{\theta m(\xi)}-\frac{c}{\left(\theta m(\xi)\right)^2}\right)+\frac{c}{\theta m(\xi)}\left(\frac{1}{\theta m(\xi)}\left(t_2e^{\theta m(\xi)(t_2-t_1)}-t_2\right)+\frac{t_1^2}{2}-\frac{t_2^2}{2}\right)\right) \qquad (18)$$

Interest earns- The financial benefit a buyer gains when they utilize the trade credit period to delay payment, thus allowing them to use the funds elsewhere to generate income or interest.

Received: August 06, 2025

$$IE = pI_e \int_0^M Dt\,dt = I_e p \int_0^M (a - bp + ct)\,tdt = I_e p \left(\frac{(a-bp)M^2}{2} + \frac{cM^3}{3}\right) \tag{19}$$

For case II - $0 \le t_1 \le M \le t_2 \le T$

$$IC = eI_c \int_M^{t_2} I_2(t)\,dt = I_c e\Bigg(\bigg(\frac{1}{\theta m(\xi)}\left(e^{\theta m(\xi)(t_2-M)} - 1\right) + (M - t_2)\bigg)\bigg(\frac{a-bp}{\theta m(\xi)} - \frac{c}{\left(\theta m(\xi)\right)^2}\bigg) + \frac{c}{\theta m(\xi)}\left(\frac{1}{\theta m(\xi)}\left(t_2 e^{\theta m(\xi)(t_2-M)} - t_2\right) + \frac{M^2}{2} - \frac{t_2^2}{2}\right)\Bigg) \tag{20}$$

$$IE = pI_e \int_0^M Dt\,dt = I_e p \int_0^M (a - bp + ct)\,tdt = I_e p \left(\frac{(a-bp)M^2}{2} + \frac{cM^3}{3}\right) \tag{21}$$

For case III - $0 \le t_1 \le t_2 \le M \le T$

$$IC = 0$$

$$IE = pI_e \left[\int_0^M Dt\,dt + \int_0^M D dt\right] = I_e p \left[\int_0^M (a - bp + ct)\,tdt + \int_0^M (a - bp + ct)dt\right] = I_e p\left[\left(\frac{(a-bp)M^2}{2} + \frac{cM^3}{3}\right) + (a - bp)M + \frac{cM^2}{2}\right] \tag{22}$$

Now the total profit for this model is

$$TP = \frac{1}{T}(SR - OC - HC - PC - SC - LSC - PTC - IC + IE)$$

$$Total\,profit = \begin{cases} TP_1, 0 \le M \le t_1 \le t_2 \le T \\ TP_2, 0 \le t_1 \le M \le t_2 \le T \\ TP_3, 0 \le t_1 \le t_2 \le M \le T \end{cases} \tag{23}$$

$$TP_1(p,\xi) = \frac{1}{T}\Bigg(\bigg(Dpt_2 + p\bigg(\Big(\frac{a-bp}{\delta} + \frac{cT}{\delta} + \frac{c}{\delta^2}\Big) log\big(1 + \delta(T - t_2)\big) + \frac{c}{\delta}(t_2 - T)\bigg)\bigg) - O - \Bigg(C_h\Big(-\frac{(a-bp)t_1^2}{2} - \frac{ct_1^3}{6}\Big) + C_h\bigg(\frac{(a-bp)}{\theta m(\xi)}\Big(-\frac{1}{\theta m(\xi)} - t_2 + \frac{e^{\theta m(\xi)(t_2-t_1)}}{\theta m(\xi)} + t_1\Big) + \frac{c}{\theta m(\xi)}\Big(-\frac{t_2}{\theta m(\xi)} - \frac{t_2^2}{2} + \frac{t_2 e^{\theta m(\xi)(t_2 - t)}}{\theta m(\xi)} + \frac{t_1^2}{2}\Big) - \frac{c}{\left(\theta m(\xi)\right)^2}\Big(-\frac{1}{\theta m(\xi)} - t_2 + \frac{e^{\theta m(\xi)(t_2-t_1)}}{\theta m(\xi)} + t_1\Big)\bigg)\Bigg) - \Bigg(e\bigg((a - bp)t_1 + \frac{ct_1^2}{2} + \Big(\frac{a-bp}{\theta m(\xi)} + \frac{c}{\left(\theta m(\xi)\right)^2}\Big)\big(e^{\theta m(\xi)(t_2-t_1)} - 1\big) + \frac{c}{\theta m(\xi)}\big(t_2 e^{\theta m(\xi)(t_2-t_1)} - t_1\big) + \Big(\frac{a-bp}{\delta} + \frac{cT}{\delta} + \frac{c}{\delta^2}\Big) log\big(1 + \delta(T - t_2)\big) + \frac{c}{\delta}(t_2 - T)\bigg)\Bigg) - \Bigg(-S_c\bigg(\Big(\frac{a-bp}{\delta} + \frac{cT}{\delta} + \frac{c}{\delta^2}\Big)\big(t_2 - T - (1 +$$

$$\delta T)\, log(1+\delta\,(T-t_2)) + \frac{c}{\delta}\left(\frac{T^2-t_2^2}{2} - t_2(T-t_2)\right)\Bigg)\Bigg)\Bigg) - \Bigg(S_l\Bigg((a-bp)\Big(T - t_2 -$$

$$\frac{log\big(1+\delta(T-t_2)\big)}{\delta}\Big) + c\left(\frac{T^2}{2} - \frac{T}{\delta} - \frac{t_2^2}{2} + \frac{t_2}{\delta} + \frac{1+\delta T}{\delta^2} log\big(1+\delta(T-t_2)\big)\right)\Bigg)\Bigg) - (\xi T) -$$

$$\Bigg(I_c e\Bigg(\Bigg(S(t_1-M) + \frac{a-bp}{2}(M^2 - t_1^2) + \frac{c}{6}(M^3 - t_1^3)\Bigg) + \Bigg(\frac{1}{\theta m(\xi)}\left(e^{\theta m(\xi)(t_2-t_1)} - 1\right) + (t_1 -$$

$$t_2)\Bigg)\left(\frac{a-b\,p}{\theta m(\xi)} - \frac{c}{\big(\theta m(\xi)\big)^2}\right) + \frac{c}{\theta m(\xi)}\left(\frac{1}{\theta m(\xi)}\left(t_2 e^{\theta m(\xi)(t_2-t_1)} - t_2\right) + \frac{t_1^2}{2} - \frac{t_2^2}{2}\right)\Bigg)\Bigg) +$$

$$\left(I_e p\left(\frac{(a-bp)M^2}{2} + \frac{cM^3}{3}\right)\right)\Bigg) \qquad (24)$$

$$TP_2(p,\xi) = \frac{1}{T}\Bigg(\Bigg(Dpt_2 + p\left(\left(\frac{a-bp}{\delta} + \frac{cT}{\delta} + \frac{c}{\delta^2}\right) log\big(1+\delta(T-t_2)\big) + \frac{c}{\delta}(t_2 - T)\right)\Bigg) -$$

$$O - \Bigg(C_h\left(-\frac{(a-bp)t_1^2}{2} - \frac{ct_1^3}{6}\right) + C_h\Bigg(\frac{(a-bp)}{\theta m(\xi)}\left(-\frac{1}{\theta m(\xi)} - t_2 + \frac{e^{\theta m(\xi)(t_2-t_1)}}{\theta m(\xi)} + t_1\right) +$$

$$\frac{c}{\theta m(\xi)}\left(-\frac{t_2}{\theta m(\xi)} - \frac{t_2^2}{2} + \frac{t_2 e^{\theta\, m(\xi)(t_2-t)}}{\theta m(\xi)} + \frac{t_1^2}{2}\right) - \frac{c}{\big(\theta m(\xi)\big)^2}\left(-\frac{1}{\theta m(\xi)} - t_2 + \frac{e^{\theta m(\xi)(t_2-t_1)}}{\theta m(\xi)} +\right.$$

$$\left.t_1\right)\Bigg)\Bigg) - (eQ) - \Bigg(-S_c\Bigg(\left(\frac{a-bp}{\delta} + \frac{cT}{\delta} + \frac{c}{\delta^{\,2}}\right)\big(t_2 - T - (1+\delta T)\, log(1+\delta\,(T-t_2)) +$$

$$\frac{c}{\delta}\left(\frac{T^2-t_2^2}{2} - t_2(T-t_2)\right)\Big)\Bigg)\Bigg) - \Bigg(S_l((a-bp)\left(T - t_2 - \frac{log\big(1+\delta(T-t_2)\big)}{\delta}\right) + c\left(\frac{T^2}{2} - \frac{T}{\delta} - \frac{t_2^2}{2} +\right.$$

$$\left.\frac{t_2}{\delta} + \frac{1+\delta T}{\delta^2} log\big(1+\delta(T-t_2)\big)\right)\Bigg) - (\xi T) - \Bigg(I_c e\Bigg(\Bigg(\frac{1}{\theta m(\xi)}\left(e^{\theta m(\xi)(t_2-M)} - 1\right) + (M -$$

$$t_2)\Bigg)\left(\frac{a-bp}{\theta m(\xi)} - \frac{c}{\big(\theta m(\xi)\big)^2}\right) + \frac{c}{\theta m(\xi)}\left(\frac{1}{\theta m(\xi)}\left(t_2 e^{\theta\, m(\xi)(t_2-M)} - t_2\right) + \frac{m^2}{2} - \frac{t_2^2}{2}\right)\Bigg)\Bigg) +$$

$$\left(I_e p\left(\frac{(a-bp)M^2}{2} + \frac{cM^3}{3}\right)\right)\Bigg) \qquad (25)$$

$$\begin{aligned}
&TP_3(p,\xi) = 1/T\Bigg(\bigg(Dpt_2 + p\left(\left(\frac{a-bp}{\delta}+\frac{cT}{\delta}+\frac{c}{\delta^2}\right)log\big(1+\delta(T-t_2)\big)+\frac{c}{\delta}(t_2-T)\right)\bigg) - O - \\
&\Bigg(C_h\left(-\frac{(a-bp)t_1^2}{2}-\frac{ct_1^3}{6}\right)+C_h\Bigg(\frac{(a-bp)}{\theta m(\xi)}\left(-\frac{1}{\theta m(\xi)}-t_2+\frac{e^{\theta m(\xi)(t_2-t_1)}}{\theta m(\xi)}+t_1\right)+\frac{c}{\theta m(\xi)}\bigg(-\frac{t_2}{\theta m(\xi)}- \\
&\frac{t_2^2}{2}+\frac{t_2e^{\theta\, m(\xi)(t_2-t)}}{\theta m(\xi)}+\frac{t_1^2}{2}\bigg)-\frac{c}{\left(\theta m(\xi)\right)^2}\left(-\frac{1}{\theta m(\xi)}-t_2+\frac{e^{\theta m(\xi)(t_2-t_1)}}{\theta m(\xi)}+t_1\right)\Bigg)\Bigg)-(eQ)- \\
&\Bigg(-S_c\Bigg(\left(\frac{a-bp}{\delta}+\frac{cT}{\delta}+\frac{c}{\delta^2}\right)\big(t_2-T-(1+\delta T)\,log(1+\delta\,(T-t_2)\big)+\frac{c}{\delta}\bigg(\frac{T^2-t_2^2}{2}-t_2(T- \\
&t_2)\bigg)\Bigg)\Bigg)-\bigg(S_l((a-bp)\left(T-t_2-\frac{log\big(1+\delta(T-t_2)\big)}{\delta}\right)+c\left(\frac{T^2}{2}-\frac{T}{\delta}-\frac{t_2^2}{2}+\frac{t_2}{\delta}+\frac{1+\delta T}{\delta^2}log\big(1+\right. \\
&\left.\delta(T-t_2)\big)\right)\bigg)-(\xi T)-\left(I_ep\left(\frac{(a-bp)M^2}{2}+\frac{cM^3}{3}\right)\right)\Bigg) \qquad (26)
\end{aligned}$$

Equation (24), (25) and (26) represents total profit for Case-I, Case-II and Case-III respectively.

1. Solution methodology

The concavity of the function $TP(p,\xi)$ can be verified using the Hessian matrix (H):$H = \begin{bmatrix} \frac{\partial^2 TP}{\partial^2 p} & \frac{\partial^2 TP}{\partial p \partial \xi} \\ \frac{\partial^2 TP}{\partial \xi \partial p} & \frac{\partial^2 TP}{\partial \xi^2} \end{bmatrix}$

profit function $TP(p,\xi)$ will be the maximum for the values of p,ξ the Hessian matrix's principal determinants must fulfill at p,ξ as

$H_{11} < 0$ and $H_{22} < 0$.

The Hessian matrix's strong nonlinearity makes closed-form proofs hard to obtain. Consequently, numerical methods were used to verify the concavity of TP(p,ξ).

Numerical analysis

Numerical examples have a significant impact in understanding the inventory models. They provide clarity, validate theories, aid in decision-making, and enhance problem-solving skills. Whether in an educational setting or a business environment, numerical examples are essential for understanding and optimizing inventory management practices. We have used mathematica-13 software to validate and solve numerical section for this model.

Table 4- Represents numerical values for case-I

Parameters	Values	Parameters	Values
a	50	δ	0.01
b	0.01	C_h	50 Rupees
c	1	I_c	0.5 %
O	50 rupees	I_e	0.35 %
d	0.015	S_c	5 Rupees
e	1000 rupees	S_l	20 Rupees
θ	0.1 %	M	6 Weeks
t_1	10 Weeks	t_2	20 Weeks
T	50 Weeks		

Optimal Solution for case -I

Parameters	Values
ξ (Preservation technology cost)	25.8789 Rupees
p (selling Price)	3561.34 Rupees
Total profit	346391 Rupees

$$H_{11} = \frac{1}{T}\left(-bI_cM^2 - \frac{2bLog(1+(T-t_2)\delta)}{\delta}\right) = -0.0961646 < 0$$

$$H_{22} = -0.548342 < 0$$

Table 5- Represents numerical values for case-II

Parameters	Values	Parameters	Values
a	50	δ	0.01
b	0.01	C_h	50 Rupees
c	1	I_c	0.5 %
O	50 rupees	I_e	0.35 %
d	0.015	S_c	5 Rupees
e	1000 rupees	S_l	20 Rupees
θ	0.1 %	M	14 Weeks
t_1	10 Weeks	t_2	20 Weeks
T	50 Weeks		

Optimal solution for case -II

Parameters	Values
ξ (Preservation technology cost)	25.0362 Rupees
p (selling Price)	3400.4 Rupees
Total profit	446429 Rupees

$$H_{11}=\frac{1}{T}p\Bigg(-\frac{1}{2}bI_cM^2p+I_c\left(\frac{cM^3}{3}+\frac{1}{2}M^2(a-bp)\right)-\frac{1}{2}bC_ht_1^2-\frac{1}{2}bpt_2^2+\frac{1}{2}(a-bp)t_2^2+$$

$$\frac{ct_2^3}{3}+\frac{c(t_2-T)}{\delta}+\frac{1}{\theta}bC_he^{d\xi}\left(t_1-t_2-\frac{e^{d\xi}}{\theta}+\frac{1}{\theta}\left(\mathrm{e}^{\mathrm{e}^{-d\xi}(-t_1+t_2)\theta+d\xi}\right)\right)+\frac{1}{\theta}\Bigg(be\mathrm{e}^{d\xi}I_e\Bigg(M-t_2+$$

$$\frac{\mathrm{e}^{d\xi}\left(-1+\mathrm{e}^{\mathrm{e}^{-d\xi}(-M+t_1)\theta}\right)}{\theta}\Bigg)\Bigg)+\left(\frac{c}{\delta^2}+\frac{a-bp}{\delta}+\frac{cT}{\delta}\right)Log[1+(T-t_1)\delta]-\frac{bpLog[1+(T-t_1)\delta]}{\delta}+$$

$$bS_l\left(T-t_2-\frac{Log[1+(T-t_1)\delta]}{\delta}\right)-e\Bigg(-bt_1-\frac{b\mathrm{e}^{d\xi}\left(-1+\mathrm{e}^{\mathrm{e}^{-d\xi}(-t_1+t_2)\theta}\right)}{\theta}-\frac{bLog[1+(T-t_2)\delta]}{\delta}\Bigg)-$$

$$\frac{bS_h(-T+t_2-(1+T\delta)Log[1+(T-t_2)\delta])}{\delta}\Bigg)=-0.731998<0$$

$$H_{22}=-2.37193<0$$

Table 6- Represents numerical values for case-III

Parameters	Values	Parameters	Values
a	50	δ	0.01
b	0.01	C_h	50 Rupees
c	1	I_c	0.5 %
O	50 rupees	I_e	0.35 %
d	0.015	S_c	5 Rupees
e	1000 rupees	S_l	20 Rupees
θ	0.1 %	M	30 Weeks
t_1	10 Weeks	t_2	20 Weeks
T	50 Weeks		

Optimal solution for case-III

Parameters	Values
ξ (Preservation technology cost)	19.2408 Rupees
p (selling Price)	3467.29 Rupees
Total profit	780361 Rupees

$$H_{11}=\frac{1}{T}\left(-bI_cM^2-bt_2^2-\frac{2bLog[1+(T-t_2)\delta]}{\delta}\right)=-0.153495<0$$

$$H_{22}=-0.465187<0$$

Received: August 06, 2025

Graphical representation

The graphical representations are showing below for optimal solutions in different cases.

For case -I

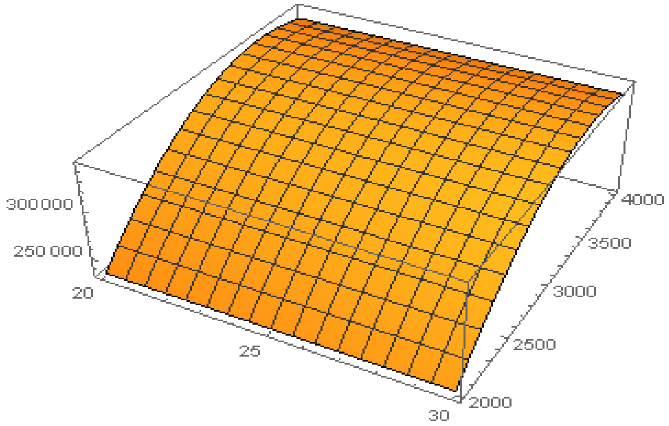

Fig:2- shows concavity of total profit with respect to ξ and p for case-I ($0 \leq M \leq t_1 \leq t_2 \leq T$) x-axis is represented by ξ (Preservation technology cost), y-axis represented by p (selling price and z-axis represented by total profit.

For case-II

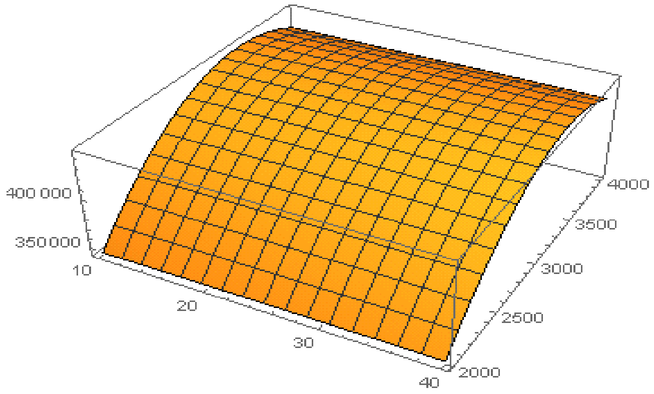

Fig:3- shows concavity of total profit with respect to ξ and p for case-II ($0 \leq t_1 \leq M \leq t_2 \leq T$) x-axis is represented by ξ (Preservation technology cost), y-axis represented by p (selling price and z-axis represented by total profit.

For case-III

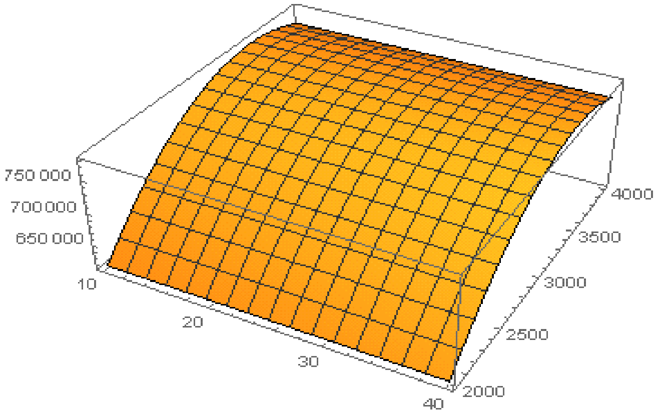

Fig:4- shows concavity of total profit with respect to ξ and p for case III($0\le t_1\le t_2\le M\le T$) x-axis is represented by ξ (Preservation technology cost), y-axis represented by p (selling price and z-axis represented by total profit.

Now, compare the values of total profit, selling price and preservation technology cost for different cases by graphs which are shown below

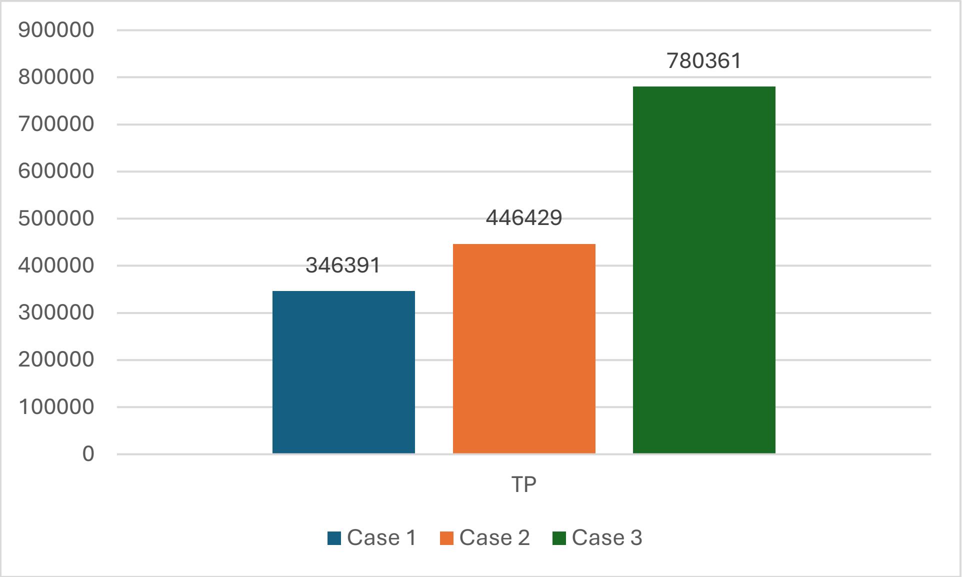

Fig: 5- Graph of total profit for all three cases

From above it is clear that for this model we get maximum profit in case- III and minimum profit in case- I it means we get maximum profit when trade credit period is between shortage time.

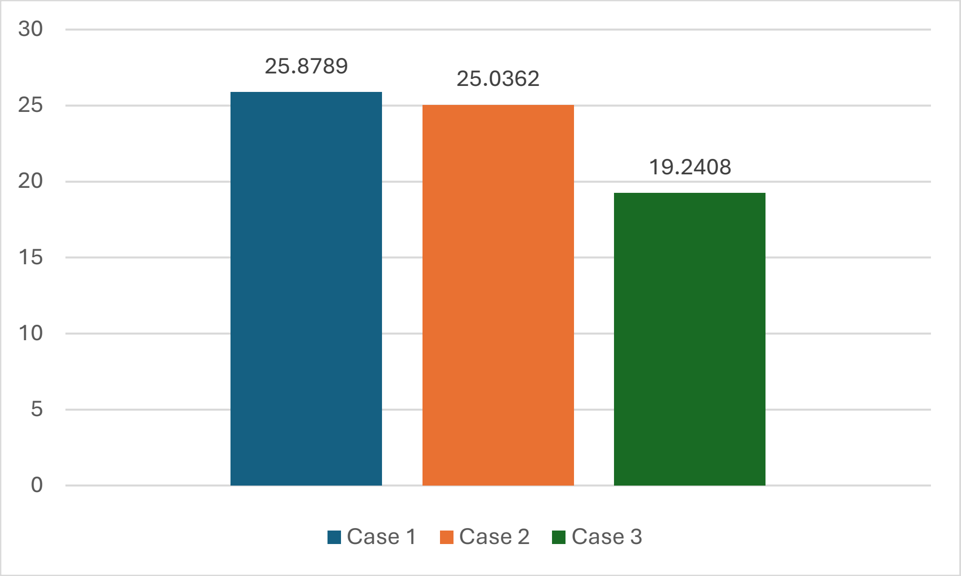

Fig:6- graph of preservation technology cost for all three cases

From above it is clearly seen that we get maximum cost in case I, and minimum cost in case- III.

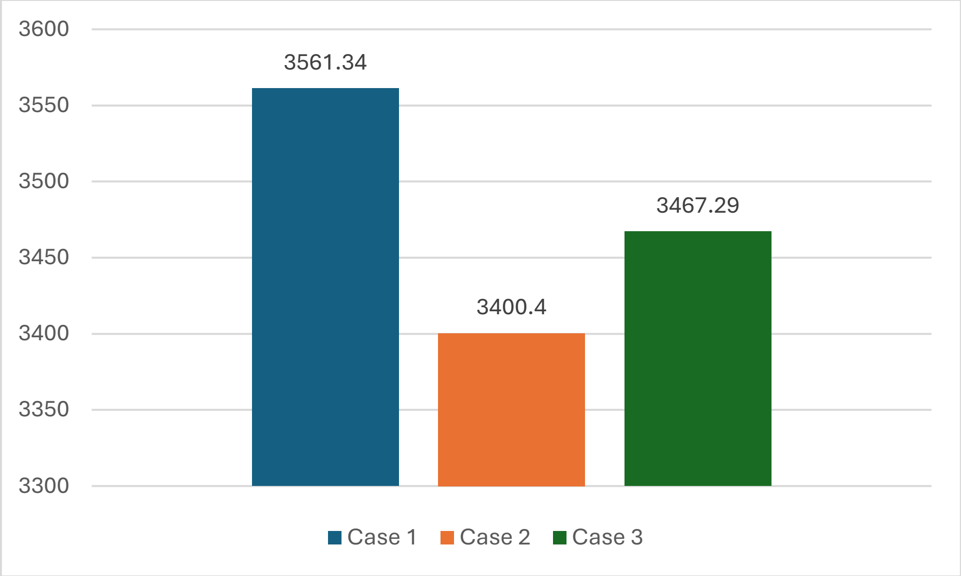

Fig:7 – Graphical Representation of Selling Prices in all cases

From above clearly seen that selling price is maximum for case- I and minimum for case- II.

Sensitivity analysis

Received: August 06, 2025

Sensitivity analysis enables us to evaluate to analyse the impact of input variability on model output. it reveals how model outputs respond to input adjustments. This is particularly important in decision-making processes, where it's crucial to know which variables have the most influence on the results and how changes in these variables can affect the overall performance, this model's sensitivity is evaluated in this section.

Table 7- Represents sensitivity analysis for case-I $0 \leq M \leq t_1 \leq t_2 \leq T$

Parameters	% Change	ξ	p	Total profit
a	+20 %	29.5299	4058.5	497541
	+10 %	27.7608	3809.85	419042
	0	25.8789	3561.34	346391
	-10%	24.53	3398.38	287644
	-20%	21.709	3064.86	218641
b	+20 %	25.4376	3015.04	263740
	+10 %	25.6592	3263.34	301275
	0	25.8789	3561.34	346391
	-10%	26.0967	3925.6	401615
	-20%	26.3126	4380.98	470737
O	+20 %	25.8789	3561.34	346390
	+10 %	25.8789	3561.34	346391
	0	25.8789	3561.34	346391
	-10%	25.8789	3561.34	346391
	-20%	25.8789	3561.34	346391
θ	+20 %	38.0337	3561.34	346379
	+10 %	32.2329	3561.34	346384
	0	25.8789	3561.34	346391
	-10%	18.8549	3561.34	346398
	-20%	11.0027	3561.34	346406
C_h	+20 %	26.0361	3561.5	346171
	+10 %	25.9576	3561.42	346281
	0	25.8789	3561.34	346391
	-10%	25.8	3561.25	346501
	-20%	25.7209	3561.17	346611
I_c	+20 %	26.0486	3588.86	335285
	+10 %	25.9767	3575.09	340829
	0	25.8789	3561.34	346391
	-10%	25.7496	3547.59	351970
	-20%	25.5811	3533.86	357568
S_l	+20 %	25.8787	3561.37	346227
	+10 %	25.8788	3561.35	346309
	0	25.8789	3561.34	346391
	-10%	25.879	3561.32	346472
	-20%	25.8792	3561.3	346554

Table 8- Represents sensitivity analysis for case-II $0 \leq t_1 \leq M \leq t_2 \leq T$

Parameters	% Change	ξ	p	Total profit
a	+20 %	28.519	3899.64	621272
	+10 %	26.8281	3650	530589
	0	25.0362	3400.4	446429
	-10%	23.1298	3150.84	368793
	-20%	21.093	2901.33	297683
b	+20 %	24.7069	2649.84	319545
	+10 %	24.8171	2858.33	354772
	0	25.0362	3400.4	446429
	-10%	25.145	3761.79	507572
	-20%	25.2535	4213.53	584029
O	+20 %	25.0362	3400	446429
	+10 %	25.0362	3400	446429
	0	25.0362	3400.4	446429
	-10%	25.0362	3400	446429
	-20%	25.0362	3400	446429
θ	+20 %	42.5271	3445.3	446410
	+10 %	37.19909	3400.4	446417
	0	25.0362	3400.4	446429
	-10%	18.0212	3572.3	446436
	-20%	10.1599	3876.4	446444
C_h	+20 %	25.548	3872.2	446204
	+10 %	25.2931	3762.3	446316
	0	25.0362	3400.4	446429
	-10%	24.7771	3276.45	446542
	-20%	24.5158	3289.5	446655
I_c	+20 %	25.9868	3404.16	445017
	+10 %	25.5148	3402.28	445723
	0	25.0362	3400.4	446429
	-10%	24.5507	3398.51	447136
	-20%	24.0582	3396.63	447845
S_l	+20 %	25.0359	3897.3	446265
	+10 %	25.0361	3598.2	446347
	0	25.0362	3400.4	446429
	-10%	25.0363	38794.09	446511
	-20%	25.0364	3517.34	446593

Table 9- Represents sensitivity analysis for case-III $0 \leq t_1 \leq t_2 \leq M \leq T$

Parameters	% Change	ξ	p	Total profit
a	+20 %	22.5974	3966.84	1052242
	+10 %	20.9674	3717.05	911499

	0	19.2408	3467.29	780361
	-10%	17.4047	3217.56	658826
	-20%	15.4438	2967.85	546894
b	+20 %	19.1159	2904.01	634365
	+10 %	19.1784	3160.05	700721
	0	19.2408	3467.29	780361
	-10%	19.303	3842.82	877711
	-20%	19.365	4312.22	999414
O	+20 %	19.2408	3467.29	780361
	+10 %	19.2408	3467.29	780361
	0	19.2408	3467.29	780361
	-10%	19.2408	3467.29	780361
	-20%	19.2408	3467.29	780361
θ	+20 %	36.7317	3467.29	780343
	+10 %	31.3955	3467.29	780349
	0	19.2408	3467.29	780361
	-10%	12.2167	3467.29	780368
	-20%	4.36453	3467.29	780376
C_h	+20 %	20.4198	3467.52	779896
	+10 %	195396	3467.35	780244
	0	19.2408	3467.29	780361
	-10%	18.9391	3467.23	780478
	-20%	18.6345	3467.18	780595
I_c	+20 %	15.3452	3224.42	750382
	+10 %	17.4355	3398.284	779282
	0	19.2408	3467.29	780361
	-10%	20.4583	3532.598	803923
	-20%	24.3837	3943.34	810293
S_l	+20 %	19.2406	3467.31	780197
	+10 %	19.2407	3467.3	780279
	0	19.2408	3467.29	780361
	-10%	19.2408	3467.28	780442
	-20%	19.2409	3467.27	780524

Observations

For all cases- on increasing in values of different parameters we get different situations which are shown below-

Table 10 Observational Findings Table

	Case- I			Case- II			Case- III		
	ξ	p	TP	ξ	p	TP	ξ	p	TP
a	↑	↑	↑	↑	↑	↑	↑	↑	↑
b	↓	↓	↓	↓	↓	↓	↓	↓	↓

Received: August 06, 2025

O	constant	constant	constant	constant	constant	constant	constant	constant	constant
θ	↑	constant	↓	↑	↑	↓	↑	constant	↓
C_h	↑	↑	↓	↑	↑	↓	↑	↑	↓
I_c	↑	↑	↓	↑	↑	↓	↓	↓	↓
S_l	↓	↑	↓	↓	fluctuate	↓	↓	↑	↑

Now explore the of all involved parameters on the decision variable-

- While increasing in scaling parameter and then for all the three cases investment in preservation technology, selling price and total profitability are also increasing.
- When increase in parameter (ordering cost) then in all three cases investment in preservation technology, selling price and total profitability are decreasing.
- When increase in parameter (deterioration rate) then for all three cases preservation technology investment is increasing, total profit is decreasing while selling price is constant for case-I and III but increase in case II.
- When increase in parameter (holding cost) then preservation technology investment and selling price both are increasing in all three cases while total profit is decreasing for all cases.
- When increase in parameter (interest charge) then preservation technology investment and selling price both are increasing in all three cases while total profit is decreasing for all cases.
- When increase in parameter (lost sale cost) then for all three cases preservation technology investment is decreasing while total profit is decreasing for cases I and II but increasing in case-3 and selling price is increasing for case -I and III but fluctuating for case-II

Managerial insights-

This model is useful for industries dealing with perishable or time-sensitive goods, such as food, pharmaceuticals, chemicals, and fashion. Businesses in these sectors can benefit from preservation investments to reduce spoilage, trade credit financing to improve cash flow, and partial backordering to manage shortages efficiently.

E-commerce, retail supply chains, and logistics companies can also use this model to optimize inventory control and reduce costs. By integrating these strategies, companies can enhance profitability, minimize waste, and ensure better customer satisfaction while maintaining financial stability.

The results of this model recommend that managers strategically 5. employ preservation methods to decrease product decay rates and maximize shelf life., ultimately minimizing waste and lowering costs. They should also leverage trade credit financing to align payment cycles with revenue, improving cash flow and financial stability.

Additionally, managers are encouraged to adopt partial backordering strategies to handle shortages efficiently while maintaining customer satisfaction. By integrating these approaches, businesses can optimize inventory levels, enhance profitability, and achieve a balancing holding costs with other expenses, demand fulfilment, along with financial flexibility.

Conclusion

The current work presents a comprehensive model designed for non-instantaneous decaying items, where demand is influenced by both time and selling price. By spending on preservation technology, financing through trade credit and Partial backorder management str, this model provides robust framework for optimizing inventory management in industries dealing with perishable goods. The key findings of this research highlight the significant benefits of investing in preservation technology to reduce deterioration rates, thereby extending the shelf life of products and minimizing waste. Additionally, trade credit terms have been shown to improve cash flow management and align payment cycles with revenue, enhancing overall financial performance. Partial backordering strategies offer a pragmatic approach to managing shortages, maintaining customer satisfaction, and balancing inventory costs. Numerical illustrations and sensitivity evaluation demonstrate the practical applicability of this model, offering valuable insights into the interplay between preservation investments, trade credit terms, and backordering policies. The results indicate that a strategic combination of these factors can lead to substantial improvements in inventory performance and profitability. While this study provides a solid foundation for managing non-instantaneous deteriorating inventories, several areas warrant further exploration to optimize the model's feasibility and robustness.

Advantages- The advantages of this model are written below-

- By incorporating preservation technology investment, the model effectively reduces deterioration rates, leading to extended shelf life and reduced waste.
- The inclusion of trade credit financing helps businesses align payment cycles with revenue streams, enhancing liquidity and financial stability.
- The integration of partial back ordering allows firms to balance stock levels and manage shortages efficiently, reducing overall holding and shortage costs.
- A strategic combination of preservation investment, trade credit, and back ordering contributes to better inventory performance and increased profitability.

Limitations- The limitations of this model are written below-

- The model assumes that demand is influenced solely by time and selling pric,. which could potentially not fully capture complex consumer behaviour and market fluctuations.
- The model does not explicitly address dynamic pricing strategies, competitive influences, or changing economic conditions.
- It primarily considers a single-product inventory system, limiting its applicability to businesses managing multiple perishable items.

Received: August 06, 2025

- The model assumes deterministic lead times, which may not reflect real-world uncertainties in supply chains.
- While preservation technology reduces deterioration, the model does not explicitly account for environmental impacts, such as carbon footprint or waste disposal strategies.
- Although theoretically sound, the integration of trade credit terms, preservation investment, and partial back ordering may require complex decision-making and higher initial investment, which can be a challenge for small businesses.
- Future research could consider the following directions: dynamic Market Conditions, Multiple Products and multi-stage supply chains under demand and lead time uncertainty, Sustainability and Environmental Impact, Technological Advancements, and two or more warehouses.

References

[1] Anchal, A., Isha, S., & Smita, R. (2016). A partial backlogging inventory model for non-instantaneous decaying items under trade credit financing facility. Indian Journal of Science and Technology, 9(34), 1-11.

[2] Annadurai, K., & Uthayakumar, R. (2013). Two-echelon inventory model for deteriorating items with credit period dependent demand including shortages under trade credit. Optimization Letters, 7, 1227-1249.

[3] Babangida, B., & Baraya, Y. M. (2019). An inventory model for non-instantaneous deteriorating item with time dependent quadratic demand and linear holding cost under trade credit policy. ABACUS: Journal of the Mathematical Association of Nigeria, 46(1), 191-217.

[4] Babangida, B., & Baraya, Y. M. (2021). An EOQ model for non-instantaneous deteriorating item with two-phase demand rates, time-dependent linear holding cost and time-dependent partial backlogging rate under trade credit policy. The Transactions of the Nigerian Association of Mathematical Physics, 17, 131-144.

[5] Balkhi, Z. T., Goyal, S. K., & Giri, B. C. (2001). Some notes on the optimal production stopping and restarting times for an EOQ model with deteriorating items. Journal of the Operational Research Society, 52(11), 1300-1301.

[6] Chung, K. J., Cárdenas-Barrón, L. E., & Ting, P. S. (2014). An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three-layer

Received: August 06, 2025

supply chain system under two levels of trade credit. International Journal of Production Economics, 155, 310-317.

[7] Chung, K. J., Liao, J. J., Lin, S. D., Chuang, S. T., & Srivastava, H. M. (2019). The inventory model for deteriorating items under conditions involving cash discount and trade credit. Mathematics, 7(7), 596.

[8] Das, S. C., Manna, A. K., Rahman, M. S., Shaikh, A. A., & Bhunia, A. K. (2021). An inventory model for non-instantaneous deteriorating items with preservation technology and multiple credit periods-based trade credit financing via particle swarm optimization. Soft Computing, 25(7), 5365-5384.

[9] Dye, C. Y., & Hsieh, T. P. (2012). An optimal replenishment policy for deteriorating items with effective investment in preservation technology. European Journal of Operational Research, 218(1), 106-112.

[10] Ghare, P.M., and Schrader, G.H., (1963), “A model for exponentially decaying inventory system”, International Journal of Production Research, 21 , 449-460.

[11] Giri, B. C., & Sharma, S. (2016). Optimal ordering policy for an inventory system with linearly increasing demand and allowable shortages under two levels trade credit financing. Operational Research, 16, 25-50.

[12] Goyal, S. K. (1985). Economic order quantity under conditions of permissible delay in payments. Journal of the operational research society, 335-338.

[13] Gupta, D., & Wang, L. (2009). A stochastic inventory model with trade credit. Manufacturing & Service Operations Management, 11(1), 4-18.

[14] Hatibaruah, A., & Saha, S. (2023). An inventory model for two-parameter Weibull distributed ameliorating and deteriorating items with stock and advertisement frequency dependent demand under trade credit and preservation technology. Opsearch, 60(2), 951-1002.

[15] He, Y., & Huang, H. (2013). Optimizing inventory and pricing policy for seasonal deteriorating products with preservation technology investment. Journal of Industrial engineering, 2013(1), 793-568.

Received: August 06, 2025

[16] Huang, Y. F. (2006). An inventory model under two levels of trade credit and limited storage space derived without derivatives. Applied Mathematical Modelling, 30(5), 418-436.

[17] Jaggi, C. K., Yadavalli, V. S., Verma, M., & Sharma, A. (2015). An EOQ model with allowable shortage under trade credit in different scenario. Applied Mathematics and Computation, 252, 541-551.

[18] Khanra, S., Mandal, B., & Sarkar, B. (2013). An inventory model with time dependent demand and shortages under trade credit policy. Economic Modelling, 35, 349-355.

[19] Kumar, B. A., & Paikray, S. K. (2022). Cost optimization inventory model for deteriorating items with trapezoidal demand rate under completely backlogged shortages in crisp and fuzzy environment. RAIRO-Operations Research, 56(3), 1969-1994.

[20] Kumar, B. A., Paikray, S. K., & Dutta, H. (2020). Cost optimization model for items having fuzzy demand and deterioration with two-warehouse facility under the trade credit financing. AIMS Math, 5(2), 1603-1620.

[21] Kumar, B. A., Paikray, S. K., & Mishra, U. (2020). Two-storage fuzzy inventory model with time dependent demand and holding cost under acceptable delay in payment. Mathematical Modelling and Analysis, 25(3), 441-460.

[22] Kumar, B. A., Paikray, S. K., & Padhy, B. (2022). Retailer's optimal ordering policy for deteriorating inventory having positive lead time under pre-payment interim and post-payment strategy. International Journal of Applied and Computational Mathematics, 8(4), 165.

[23] Kumar, S., Singh, S. R., & Sharma, R. K. (2022). Inventory Models for Non-Instantaneous Deterioration Items with Stock Dependent Demand under Trade Credit Policy with Inflation. The International Journal of Engineering and Sciences, 11(2), 62-71.

[24] Lashgari, M., Taleizadeh, A. A., & Sadjadi, S. J. (2018). Ordering policies for non-instantaneous deteriorating items under hybrid partial prepayment, partial trade credit and partial backordering. Journal of the Operational Research Society, 69(8), 1167-1196.

[25] Li, G., He, X., Zhou, J., & Wu, H. (2019). Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items. Omega, 84, 114-126.

[26] Lin, F., Jia, T., Wu, F., & Yang, Z. (2019). Impacts of two-stage deterioration on an integrated inventory model under trade credit and variable capacity utilization. European Journal of Operational Research, 272(1), 219-234.

[27] Lu, L., Zhang, J., & Tang, W. (2016). Optimal dynamic pricing and replenishment policy for perishable items with inventory-level-dependent demand. International Journal of Systems Science, 47(6), 1480-1494.

[28] Maihami, R., Karimi, B., & Fatemi Ghomi, S. M. T. (2017). Effect of two-echelon trade credit on pricing-inventory policy of non-instantaneous deteriorating products with probabilistic demand and deterioration functions. Annals of Operations Research, 257, 237-273.

[29] Mashud, A. H. M., Uddin, M. S., & Sana, S. S. (2019). A two-level trade-credit approach to an integrated price-sensitive inventory model with shortages. International Journal of Applied and Computational Mathematics, 5, 1-28.

[30] Mashud, A. H. M., Wee, H. M., & Huang, C. V. (2021). Preservation technology investment, trade credit and partial backordering model for a non-instantaneous deteriorating inventory. RAIRO-Operations Research, 55, S51-S77.

[31] Md Mashud, A. H., Hasan, M. R., Wee, H. M., & Daryanto, Y. (2020). Non-instantaneous deteriorating inventory model under the joined effect of trade-credit, preservation technology and advertisement policy. Kybernetes, 49(6), 1645-1674.

[32] Min, J., Zhou, Y. W., & Zhao, J. (2010). An inventory model for deteriorating items under stock-dependent demand and two-level trade credit. Applied Mathematical Modelling, 34(11), 3273-3285.

[33] Mishra, U., Tijerina-Aguilera, J., Tiwari, S., & Cárdenas-Barrón, L. E. (2018). Retailer's joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments. Mathematical Problems in Engineering, 2018(1), 6962417.

[34] Molamohamadi, Z., Arshizadeh, R., & Ismail, N. (2014). An EPQ inventory model with allowable shortages for deteriorating items under trade credit policy. Discrete Dynamics in Nature and Society, 2014(1), 476085.

[35] Nayak, D. K., & Sahoo, A. K. (2023). An Fuzzy EOQ Model for deteriorating items having time dependent demand under partially backlogged shortages. In Proceedings of the Jangjeon Mathematical Society (Vol. 26, No. 3, pp. 269-290).

[36] Nayak, D. K., Routray, S. S., Paikray, S. K., & Dutta, H. (2021). A fuzzy inventory model for Weibull deteriorating items under completely backlogged shortages. Discrete & Continuous Dynamical Systems-Series S, 14(7).

[37] Nayak, D. K., Sahoo, A. K., & Paikray, S. K. (2024). A deteriorating inventory model with amelioration under power pattern demand in crisp and fuzzy environments. Mathematical Foundations of Computing.

[38] Patra, S. K., Paikray, S. K., & Dutta, H. (2024a). An inventory model under power pattern demand having trade credit facility and preservation technology investment with completely backlogged shortages. Journal of Industrial and Management Optimization, 20(8), 2652-2679.

[39] Patra, S. K., Paikray, S. K., & Kumar, B. A. (2024b). A Retailer’s Deteriorating Inventory Model with Amelioration and Permissible Backlogging Under Power Pattern Demand. Journal of the Indian Society for Probability and Statistics, 25(2), 597-619.

[40] Sarkar, B., Saren, S., & Cárdenas-Barrón, L. E. (2015). An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. Annals of Operations Research, 229, 677-702.

[41] Shah, N. H., Keswani, M., Khedlekar, U. K., & Prajapati, N. M. (2024). Non-instantaneous controlled deteriorating inventory model for stock-price-advertisement dependent probabilistic demand under trade credit financing. Opsearch, 61(1), 421-459.

[42] Shaikh, A. A., & Cárdenas-Barrón, L. E. (2020). An EOQ inventory model for non-instantaneous deteriorating products with advertisement and price sensitive demand under order quantity dependent trade credit. Investigación Operacional, 41(2), 168-187.

[43] Shaikh, A. A., Cárdenas-Barrón, L. E., Manna, A. K., Céspedes-Mota, A., & Treviño-Garza, G. (2021). Two level trade credit policy approach in inventory model with expiration rate and stock dependent demand under nonzero inventory and partial backlogged shortages. Sustainability, 13(23), 13493.

[44] Sharma, A. K., Tiwari, S., Yadavalli, V. S., & Jaggi, C. K. (2020). Optimal trade credit and replenishment policies for non-instantaneous deteriorating items. RAIRO-Operations Research, 54(6), 1793-1826.

[45] Tiwari, S., Cárdenas-Barrón, L. E., Khanna, A., & Jaggi, C. K. (2016). Impact of trade credit and inflation on retailer's ordering policies for non-instantaneous deteriorating items in a two-warehouse environment. International Journal of Production Economics, 176, 154-169.

[46] Tripathi, R. P. (2024). EOQ model for time dependent demand with deterioration, inflation, shortages and trade credits. International Journal of Information and Decision Sciences, 16(1), 73-89.

[47] Tripathy, M., Sharma, G., & Sharma, A. K. (2022). An EOQ inventory model for non-instantaneous deteriorating item with constant demand under progressive financial trade credit facility. Opsearch, 59(4), 1215-1243.

[48] Udayakumar, R., & Geetha, K. V. (2018). An EOQ model for non-instantaneous deteriorating items with two levels of storage under trade credit policy. Journal of Industrial Engineering International, 14, 343-365.

[49] Zhang, J., Bai, Z., & Tang, W. (2014). Optimal pricing policy for deteriorating items with preservation technology investment. Journal of industrial & management optimization, 10(4).

Received: August 06, 2025