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Some Results On Majority Roman Domination

S. Anandha Prabhavathy [1] , A. Anitha [2] , I. Sahul Hamid [3]

[1] Easwari Engineeering College
Assistant Professor, Department of Mathematics
Ramapuram
Chennai - 600089, INDIA
e-mail: prabhamaths6@gmail.com

[2] Thiagarajar College of Engineeering
Madurai
625015, INDIA
e-mail: anithavalli@tce.edu

[3] The Madura College
Madurai
625011, INDIA
e-mail: sahulmat@yahoo.co.in

Abstract

A Majority Roman Dominating Function (MRDF) on a graph G=(V,E) is a function $f : V \to \{-1, +1, 2\}$ satisfying the conditions that (i) the sum of its function values over at least half the closed neighborhood is at least

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one and (ii) every vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight of a MRDF is the sum of its function values over all vertices. The *Majority Roman Domination Number* of a graph G, denoted by $\gamma_{MR}(G)$, is defined as $\gamma_{MR}(G) = min\{w(f) \mid f\ is\ a\ Majority\ Roman\ Dominating\ Function\ of\ G\}$. In this paper, we found some results on Majority Roman Domination In Graphs.

Math. Subject Classification: 05C15, 05C69.
Key Words and Phrases: Majority domination, Majority Roman Domination.

1 Introduction

By a graph $G = (V, E)$, we mean a finite, non-trivial, connected, and undirected graph with neither loops nor multiple edges. The order and size of G are denoted by n and m respectively. For graph theoretic terminology we refer to Chartand and Lesniak [1].

The study of domination is one of the fastest growing areas within graph theory. A subset D of vertices is said to be a *dominating set* of G if every vertex in V either belongs to D or is adjacent to a vertex in D. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. Survey of several advanced topics on domination is given in the book edited by Haynes et al. [2].

For a real valued function $f : V \to R$ on V,*weight of f* is defined to be $w(f) = \sum_{v\in V} f(v)$ and also for a subset $S \subseteq V$, we define $f(S) = \sum_{v\in S} f(v)$. Therefore $w(f) = f(V)$. Further, for a vertex $v \in V$, let $f[v] = f(N[v])$ for notation convenience. A function $f : V \to \{-1, +1\}$ is called a *majority dominating function* if $f[v] \geq 1$ for at least half of the vertices in G. The *majority domination number* of G is denoted by $\gamma_{maj}(G)$ and is defined as $\gamma_{maj}(G) = min\{w(f) \mid f\ is\ a\ majority\ dominating\ function\ of\ G\}$. Majority domination was first introduced by Broere et al. in [3] and fur-

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ther studied in [4].

H. Abdollahzadeh Ahangar et al. [5] defined a Signed Roman dominating function (SRDF) on a graph $G = (V, E)$ to be a function $f : V \to \{-1, +1, 2\}$ satisfying the conditions that (i) the sum of its function values over any closed neighborhood is at least one and (ii) every vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$. The Signed Roman Domination Number of G, denoted $\gamma_{SR}(G)$, is the minimum weight of a SRDF in G.

S. Anandha Prabhavathy [6] defined a Majority Roman Dominating Function (MRDF) on a graph G=(V,E) is a function $f : V \to \{-1, +1, 2\}$ satisfying the conditions that (i) the sum of its function values over at least half the closed neighborhood is at least one and (ii) every vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight of a MRDF is the sum of its function values over all vertices. The *Majority Roman Domination Number* of a graph G, denoted by $\gamma_{MR}(G)$, is defined as $\gamma_{MR}(G) = min\,\{w(f) \mid f\ is\ a\ Majority\ Roman\ Dominating\ function\ of\ G\}$.

2 Majority Roman Domination In Graphs

Theorem 1. *Let G be a connected graph of order n such that exactly one vertex in G has degree $n-1$. Then $\gamma_{MR}(G) \leq 4$.*

Proof. Let G be a connected graph of order n such that exactly one vertex say x in G has degree $n-1$. If $n = 2k$ is even then partite the vertex into three sets V_1,V_{-1}, V_2 with $|V_2| = 1$, $|V_{-1}| = k-1$, and $|V_1| = k$ such that (i) number of edges induced by V_1 and V_{-1} is as large as possible and (ii) $x \in V_2$. Then each vertex v in V_1 is adjacent to at least as many vertices in V_1 as in V_{-1}. Now by assigning 2 to x, the value 1 to each vertex in V_1 and the value -1 to each vertex in V_{-1} we produce a MRDF. Hence $\gamma_{MR}(G) \leq 2 + (1)(k) - 1(k-1) = 3$. Now if $n = 2k+1$ is odd, then $G - v$ has even number of vertices where $v \neq x$. Then as discussed above we

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have $\gamma_{MR}(G-v) \leq 3$. Further by assigning 1 to the vertex v, we obtain the MRDF of G. Hence $\gamma_{MR}(G) \leq \gamma_{MR}(G-v)+1 = 4$ $\square$

Theorem 2. *Let G be a regular graph of order n. Then* $\gamma_{MR}(G) \geq \frac{\lceil \frac{n}{2} \rceil (r-1)+n(2-r)}{r+1}$.

Proof. Let f be a MRDF of G. Since for at least half of the vertices $v \in V$, $f(N[v]) \geq 1$, we have

$$\sum_{v\in V} f(N[v]) \geq 1(\lceil \tfrac{n}{2} \rceil) + (-r+2)(n - \lceil \tfrac{n}{2} \rceil)$$

Also the $\sum_{v\in V} f(N[v])$ counts the value $f(v)$ exactly $deg\ v+1$ times for each vertex $v \in V$. That is, $\sum_{v\in V} f(N[v]) = \sum_{v\in V} f(v)(deg\ v+1)$. Hence,

$$\sum_{v\in V} f(v)(r+1) \geq (1)(\lceil \tfrac{n}{2} \rceil) + (-r+2)(n - \lceil \tfrac{n}{2} \rceil)$$

$$= \lceil \tfrac{n}{2} \rceil (r-1) + n(2-r)$$

$$f(V) \geq \frac{\lceil \frac{n}{2} \rceil (r-1)+n(2-r)}{r+1}$$

$\square$

Theorem 3. *Let G be a graph and $\Delta(G) = 3$. Then $\gamma_{MR}(G) \geq -\lceil \frac{n}{2} \rceil$.*

Proof. Let f be a γ_{MR} - function of G. Then each vertex $u \in V_{-1}$ is adjacent to at least one vertex $v \in V_2$. Also $|V_1| \geq 0$. Since maximum degree is 3, we have $|V_{-1}| \leq 3|V_2|$. Again since $\Delta(G) = 3$, if we assign $f(v_i) = 2$ for each odd i,then we have $f(N[v_i]) \geq 1$, for each even i. Therefore $|V_2| \leq \lceil \frac{n}{2} \rceil$. Hence $\gamma_{MR}(G) \geq -\lceil \frac{n}{2} \rceil$. $\square$

Theorem 4. *Let$G$=$K_{2n} - M$; where M is a perfect matching in the complete graph K_{2n}. Then $\gamma_{MR}(G) = 0$.*

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Proof. Let $V(K_{2n}) = (v_1, v_2, ..., v_{2n})$ and $M = (v_1v_2, v_3v_4, v_5v_6, ..., v_{2n-1}v_{2n})$. Now define a function $f : V \to \{-1, 1, 2\}$ by

(i) $2n \equiv 0(mod\ 6)$

$$f(v_i) = \begin{cases} 2 & \text{if } 1 \le i \le 2n/3, \\ -1 & \text{otherwise.} \end{cases}$$

(ii) $2n \equiv 2(mod\ 6)$

$$f(v_i) = \begin{cases} 2 & \text{if } 1 \le i \le \lfloor 2n/3 \rfloor, \\ 1 & \text{if } i = \lfloor 2n/3 \rfloor + 1, \\ -1 & \text{otherwise.} \end{cases}$$

(iii) $2n \equiv 4(mod\ 6)$

$$f(v_i) = \begin{cases} 2 & \text{if } 1 \le i \le \lfloor 2n/3 \rfloor - 1, \\ 1 & \text{if } i = \lfloor 2n/3 \rfloor \ and\ i = \lfloor 2n/3 \rfloor + 1, \\ -1 & \text{otherwise.} \end{cases}$$

Then it is easy to verify that f is a MRDF. Hence $\gamma_{MR}(G) \le 0$. Now let g be any MRDF of G with $\gamma_{MR}(G) = g(V)$. Let $v_i \in N_g$ and let v_j be the vertex which is not adjacent to v_i. Hence $g(V) = g(N[v_i]) + g(v_j) \ge 1 - 1 = 0$. □

Theorem 5. *Let G denote the friendship graph with t-triangles.Then*

$$\gamma_{MR}(G) = \begin{cases} 4 - t & \text{if } t\ is\ even, \\ 3 - t & \text{if } t\ is\ odd. \end{cases}$$

Proof. Let u be the central vertex of G and let $(v_1, v_2, ..., v_{2t})$ be the vertices in the triangles. Now define $f : V \to \{-1, 1, 2\}$ by $f(u) = 2$ and

$$f(v_i) = \begin{cases} 1 & \text{if } 3 \le i \le t + 3\ and\ i \equiv 1(mod2), \\ -1 & \text{otherwise.} \end{cases}$$

Then it is easy to verify that f is a MRDF with

$$\gamma_{MR}(G) \le \begin{cases} 4 - t & \text{if } t\ is\ even, \\ 3 - t & \text{if } t\ is\ odd. \end{cases}$$

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Now let g be any MRDF with $\gamma_{MR}(G) = g(V)$. If $u \in N_g$, then $\gamma_{MR}(G) \geq 1$. Now suppose $u \notin N_g$. If $g(u) = +1$, then $\gamma_{MR}(G) \geq 1 + t$. If $g(u) = -1$, then $\gamma_{MR}(G) \geq -1 + t$. Now if $g(u) = 2$. Since $deg\ v_i = 2$,for all i, and $|N_g| \geq \lceil \frac{n}{2} \rceil$, we have $|V_1| \geq \lfloor \frac{t}{2} \rfloor + 1$. Therefore $\gamma_{MR}(G) \geq 2(1) + 1(\lfloor \frac{t}{2} \rfloor + 1) - 1(2t + 1 - \lfloor \frac{t}{2} \rfloor - 1 - 1)$. □

Theorem 6. *Given any positive integer $k \geq 1$, there exists a graph G with $n = 6k$ vertices such that $\gamma_{MR}(G) = k$*

Proof. Let $G = C_n \circ K_1$, where $n \equiv 0(mod 3)$, $n \geq 3$. Let $(v_1, v_2, ..., v_n, v_1)$ be the vertices of C_n and let $(u_1, u_2, ..., u_n)$ be the pendant vertices of C_n. Now define a function $f : V \to \{-1, 1, 2\}$ by

$$f(v_i) = \begin{cases} 2 & \text{if } \ i = 1 \ and \ i \equiv 1(mod\ 3), \\ -1 & otherwise. \end{cases}$$

and

$$f(u_i) = \begin{cases} -1 & \text{if } \ i = 1 \ and \ i \equiv 1(mod\ 3), \\ +1 & otherwise. \end{cases}$$

Then it is easy to verify that f is a MRDF with $\gamma_{MR}(G) \leq k$. Now let g be a MRDF with $\gamma_{MR}(G) = g(V)$. Now suppose $|V_{-1}| = \frac{n}{2} + 1$, then there exists adjacent vertices $v_i u_j$, with $f(v_i) = f(u_j) = -1$ which is a contradiction, by the definition of MRDF. Hence $|V_{-1}| \leq \frac{n}{2}$. Also since $|V_{-1}| \leq 3\,|V_2|$, we have $3\,|V_2| = \frac{n}{2}$. Therefore $|V_2| = \frac{n}{6}$. Since $|V_{+1}| + |V_{-1}| + |V_2| = n$, we have $|V_1| \geq \frac{n}{3}$. Thus $\gamma_{MR}(G) \geq \frac{n}{3} - \frac{n}{2} + \frac{n}{3} = k$. □

Theorem 7. *Let G be the graph obtained by adding 5 pendant vertices to each vertices of C_n. Then $\gamma_{MR}(G) = -3n$.*

Proof. Let $(v_1, v_2, ..., v_n, v_1)$ be the vertices of C_n and let $(u_1, u_2, ..., u_{5n})$ be the pendant vertices. Now define a function $f : V \to \{-1, 1, 2\}$ by $f(v_i) = 2$ for all $1 \leq i \leq n$ and $f(u_i) = -1$ for all $1 \leq i \leq 5n$. Then it is easy to verify that f is a MRDF with $\gamma_{MR}(G) \leq 2(n) - 1(5n) = -3n$. Now let g be any MRDF with $\gamma_{MR}(G) = g(V)$. If $g(v_i) = 1\ or - 1$ for any i, then we must assign 1 to all pendant

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vertices for the corresponding v_i. Therefore $g(v_i) = 2$ for all i. Thus $\gamma_{MR}(G) \geq -3n$. □

Corollary 8. *Given any positive integer $k \geq 3$, there exists a graph G with $n = 6k$ vertices such that $\gamma_{MR}(G) = \gamma_{SR}(G) = -3n$.*

Proof. Let G be the graph obtained in the Theorem 2.7. Then $\gamma_{MR}(G) = -3n$. In the similar way one can easily prove that $\gamma_{SR}(G) = -3n$. Thus $\gamma_{MR}(G) = \gamma_{SR}(G) = -3n$.

Theorem 9. *Let G_1,G_2 and G_3 be complete graphs of order r, s and t respectively with $r \geq s \geq t$. Let $V(G_1) = \{v_1, v_2, ..., v_r\}$, $V(G_2) = \{w_1, w_2, ..., w_s\}$ and $V(G_3) = \{u_1, u_2, ..., u_t\}$. Let G be the graph obtained from G_1,G_2 and G_3 by adding the edges v_1w_1 and w_2u_2.*

(i) If $r > s + t$, then $\gamma_{MR}(G) = 7 - (s + t)$

(ii) If $r \leq s + t$, then $\gamma_{MR}(G) = 5 - r$

Proof. **Case 1.** $r > s + t$

Now define the function $f : V \to \{-1, +1, 2\}$ by

(i) r is even

$$f(v) = \begin{cases} 2 & \text{if v=v}_1, w_1, u_2, \\ +1 & \text{if } 2 \leq i \leq \frac{r}{2}, \\ -1 & \text{otherwise.} \end{cases}$$

(ii) r is odd

$$f(v) = \begin{cases} 2 & \text{if v=v}_1, v_{r-1}, w_1, u_2, \\ +1 & \text{if } 2 \leq i \leq \left\lfloor \frac{r}{2} \right\rfloor - 1, \\ -1 & \text{otherwise.} \end{cases}$$

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Then it is easy to verify that f is a MRDF of G. Hence $\gamma_{MR}(G) \leq 7-(s+t)$

Now, let g be any MRDF of G with $\gamma_{MR}(G) = g(V)$. Since $r > s+t$ and $n = r+s+t$, there exists a vertex v_i in $V(G_1)$, which belongs to N_g. Therefore $\gamma_{MR}(G) \geq 1+2+2+(-1)(s-1)+(-1)(t-1) = 7-(s+t)$.

Case 2. $r = s+t$

Now define the function $f : V \to \{-1, +1, 2\}$ by

(i) s and t are even

$$f(v) = \begin{cases} 2 & \text{if v=v}_1, w_1, u_1, \\ +1 & \text{ifv=w}_i,\ (2 \leq i \leq \frac{s}{2}) \ \text{ and } \ v = u_j,\ (2 \leq j \leq \frac{t}{2}), \\ -1 & \text{otherwise.} \end{cases}$$

(ii) s and t are odd

$$f(v) = \begin{cases} 2 & \text{if v=v}_1, w_{s-1}, w_1, u_1, u_{t-1}, \\ +1 & \text{if v=w}_i,\ \ (2 \leq i \leq \left\lfloor \frac{s}{2} \right\rfloor - 1) \ \text{ and } \ v = u_j,\ \ (2 \leq j \leq \left\lfloor \frac{t}{2} \right\rfloor - 1), \\ -1 & \text{otherwise.} \end{cases}$$

(iii) s is even and t is odd

$$f(v) = \begin{cases} 2 & \text{if v=v}_1, w_1, u_1, u_{t-1}, \\ +1 & \text{if v=w}_i,\ \ (2 \leq i \leq \frac{s}{2}) \ \text{ and } \ v = u_j,\ \ (2 \leq j \leq \left\lfloor \frac{t}{2} \right\rfloor - 1), \\ -1 & \text{otherwise.} \end{cases}$$

(iv) s is odd and t is even

$$f(v) = \begin{cases} 2 & \text{if v=v}_1, w_1, w_{s-1}, u_1, \\ +1 & \text{if v=w}_i,\ \ (2 \leq i \leq \left\lfloor \frac{s}{2} \right\rfloor - 1) \ \text{ and } \ v = u_j,\ \ (2 \leq j \leq \frac{t}{2}), \\ -1 & \text{otherwise.} \end{cases}$$

Then it is easy to verify that f is a MRDF of G. Hence $\gamma_{MR}(G) \leq 5-r$

Now, let g be any MRDF of G with $\gamma_{MR}(G) = g(V)$. If $N_g \cap V(G_1) = \phi$, then $V(G_2) \cup V(G_3) \subseteq N_g$. Hence $\gamma_{MR}(G) \geq 1+1+2-1(r-1) = 5-r$. Now if $N_g \cap V(G_1) \neq \phi$, then as dicussed in Case 1, $\gamma_{MR}(G) \geq 1+2+2+(-1)(s-1)+(-1)(t-1) = 7-(s+t)$. Therefore $\gamma_{MR}(G) \geq 5-r$.

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Case 3. $r < s + t$

Now define the function $f : V \to \{-1, +1, 2\}$ as in Case 2, we obtain the MRDF of G. Hence $\gamma_{MR}(G) \leq 5 - r$. Now, let g be any MRDF of G with $\gamma_{MR}(G) = g(V)$. Hence $|N_g| \geq \lceil \frac{n}{2} \rceil$. Also $n = r+s+t > 2r$ and hence $|N_g| > r$. Therefore N_g has non empty intersection with at least two of the sets $V(G_1), V(G_2)$ *and* $V(G_3)$. Since $r \geq s \geq t$, we have $\gamma_{MR}(G) \geq 5 - r$. $\square$

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