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MOUSSA CONJECTURE: INSIGHTS ON THE DYNAMICS OF 3N+1 PROBLEM WITH A PROPOSAL OF A NOVEL REPRESENTATION OF POSITIVE INTEGERS

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Abstract

Collatz conjecture, a longstanding problem in number theory, has intrigued mathematicians due to its simple formulation yet complex behavior. In this study, Moussa Model of Positive Integers is introduced, a novel mathematical framework that extends and generalizes the Collatz conjecture. This model classifies integers into distinct sets, including decaying sequences, keys, subkeys, and their multiples, each following specific transformation rules. Moussa conjecture is introduced, which postulates that an infinite number of possible functions—each with distinct formulations and variations—converge to 1, thereby providing a broader perspective on integer dynamics. To validate this model, a computational approach using Python was employed, testing the framework across a defined numerical range. The results confirm that all tested integers adhere to Moussa model, reinforcing its structural validity. Additionally, multiple variations of transformation rules are explored, demonstrating that different formulations consistently align with the proposed model. A significant outcome of this study is that if Moussa conjecture is proven, the Collatz conjecture would be directly established as a special case within this generalized framework. This study suggests potential applications of Moussa model in cryptography, steganography, and encryption systems, particularly through the representation of keys as infinite structured sets. By approaching number theory through this innovative perspective, new mathematical problems may be addressed, and additional research directions may emerge.

Math. Subject Classification: 11Y16, 11B83, 11Y55

Keywords: Collatz conjecture, Moussa conjecture, number theory, integer sequences, mathematical modeling, cryptography.

1. Introduction

The Collatz conjecture is one of the best-known unsolved problems in number theory. It was stated by German Mathematician Lothar Collatz in 1937. It is also known as 3n+1 problem, the Syracuse problem, Haase's problem, Kakutani's problem, and Ulam's problem ((Orús-Lacort & Jouis, 2022); (Lagarias, 1985)). It is defined as follows:

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$$F(n) = \begin{cases} n/2 & n\ is\ even \\ 3n+1 & n\ is\ odd \end{cases}$$

Applying this equation to any given positive integer n generates a sequence of fluctuating positive integers that eventually reaches the 4-2-1 cycle. Although this conjecture remains unproven, Paul Erdős famously remarked about the 3n+1 problem, "Mathematics is not yet ready for such problems" (Kimms, 2021). Despite this, many mathematicians have extensively studied the problem and achieved significant insights. Most researchers contend that the conjecture is true, based on experimental evidence and heuristic reasoning. A particularly notable recent contribution was made by Terence Tao, who demonstrated that most orbits of the Collatz map exhibit almost bounded behavior (Schwob et al., 2021).

Andrei et al. (2000) introduced the concept of "chain subtrees" within Collatz's tree, followed by a characterization theorem and an exploration of specific subclasses of numbers that serve as labels for certain chain subtrees. Briscese & Calogero (2024) proposed and investigated conjectures analogous to the Collatz conjecture, including the 5n+1, 7n+1, and 11n+1 conjectures. Their analysis suggests the potential to introduce an infinite family of analogous conjectures. Furthermore, they proposed a generalized case of the Collatz conjecture, broadening its scope and applicability.

This study aims to provide insights into the dynamics of the underlying structure of the 3n+1 problem and its relationship to the properties of positive integers. It introduces a novel model for categorizing positive integers and presents the Moussa conjecture as a generalized framework that describes an infinite number of functions with a finite number of rules, following the same principles as the Collatz conjecture. The structure of this paper is as follows: Section 2 discusses the dynamics of the 3n+1 problem. Section 3 introduces Moussa Model of Positive integers and its validation. Section 4 presents Moussa conjecture and different variations. Finally, Section 5 summarizes the study's findings and offers future recommendations in the conclusion.

2. Dynamics of 3n+1 problem

An elementary observation regarding the Collatz conjecture is that its rules divide the set of positive integers into two categories: even and odd numbers. For even numbers, the transformation n→$\frac{n}{2}$ is applied, while for odd numbers, the rule n→3n+1 is used, which converts the odd number into an even number. For any n, where n is a positive integer, to eventually reach the 4→2→1 cycle, it must at some point enter the sequence 2^x, where x is a natural number. For instance, the sequence 2^0, 2^1, 2^2, 2^3, ……., 2^x represents what is defined as the Decaying Sequence, the only sequence that allows a number to reach 1 solely through repeated divisions by 2.

The structure of the Collatz process ensures that every odd number is converted into an even number via the operation 3n+1. This even number (p) may then be tested for membership in the decaying sequence by checking if x= $\frac{ln(n)}{\ln(2)}$ is an integer. If x is an integer, the even number n directly transitions through the sequence down to 1. Otherwise, the iterative process continues, dividing even numbers by 2 and reapplying the 3n+1 rule to odd numbers, until the number

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eventually enters the decaying sequence 2^x shown in **Figure 1**. It can also be noticed that number 1 which is equivalent to 2^0 is the only odd number in the decaying sequence and this justifies why there is a cycle in 4→2→1.

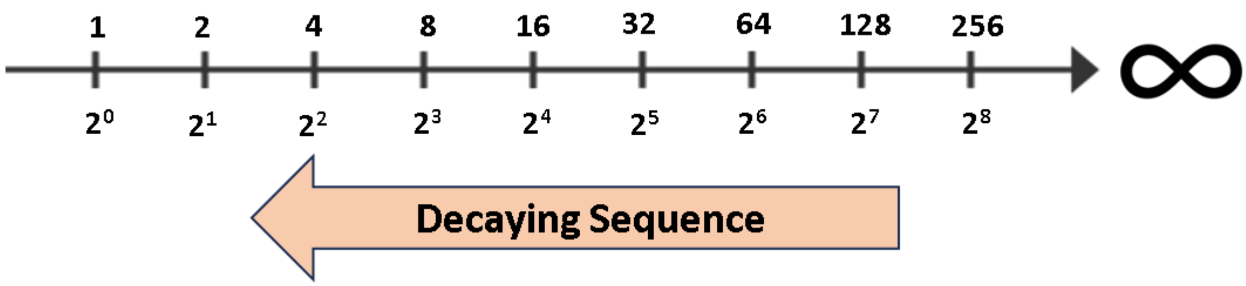

Figure 1: Decaying Sequence

For odd numbers, applying the reverse of the 3n+1 rule to numbers in the decaying sequence 2^x will allow identifying the odd numbers that produce these even numbers. The odd numbers that yield even numbers in the decaying sequence can be determined using the following equation:

$$\text{Odds} = \frac{2^x - 1}{3}$$

Table 1 illustrates several examples of applying this equation. Analyzing the results reveals a distinct pattern: when x is an even integer (excluding x= 0), the equation produces an odd integer. However, when x is an odd integer, the result is a rational number.

Lemma 1: Let $X \in N$, then: $\frac{2^x-1}{3} \in Z_{odd}$ if X is even and X > 0 and: $\frac{2^x-1}{3} \in Q \setminus Z$ if X is odd.

Proof:

Case 1: X even, X > 0.

- Divisibility by 3, observe the pattern of powers of 2 modulo 3:

  $2^1 \equiv 2 \pmod 3$, $2^2 \equiv 1 \pmod 3$, $2^3 \equiv 2 \pmod 3$, $2^4 \equiv 1 \pmod 3$, …….

  In general, $2^{2n} \equiv 1 \pmod 3$. Thus, if x is even, say x = 2n with n > 0,

  Then $2^x = 2^{2n} \equiv 1 \pmod 3 \Longrightarrow 3 \mid (2^x - 1)$.

  Hence

$$\frac{2^x-1}{3} \text{ is an integer.}$$

- Oddness.

For $x \geq 1$, 2^x is even, so $2^x - 1$ is odd. Since 3 is also odd, the quotient of an odd integer by an odd integer (when it divides exactly) must itself be odd. Formally, if

$2^x - 1 = (2k+1)$, $3 = (2m+1)$, and $(2m+1) \mid (2k+1)$, then the quotient must be odd. Therefore,

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$$\frac{2^x-1}{3} \in Z_{odd}$$

Case 2: X odd

- Again using $2^1 \equiv 2 \pmod 3$, $2^3 \equiv 2 \pmod 3$, it has been found that whenever x is odd, $2^x \equiv 2 \pmod 3$. Thus $2^x - 1 \equiv 2 - 1 \equiv 1 \pmod 3$, which means $3 \nmid (2x-1)$. Consequently, $\frac{2^x-1}{3}$ is not an integer (it has a nonzero remainder upon division by 3). It is obviously a rational number, so $\frac{2^x-1}{3} \in Q \setminus Z$

Hence the statement of the lemma is proved:

- **If x is even and x > 0:** $\frac{2^x-1}{3}$ is an **odd integer**.
- **If x is odd:** $\frac{2^x-1}{3}$ is a **non-integer rational**.

Table 1: Instances for applying reverse 3n+1 on decaying sequence

X	2^x	$2^x - 1/3$	Notes
0	**1**	**0**	**Excluded**
1	**2**	**1/3**	**Rational**
2	**4**	**1**	**Odd**
3	**8**	**7/3**	**Rational**
4	**16**	**5**	**Odd**
5	**32**	**31/3**	**Rational**
6	**64**	**21**	**Odd**
7	**128**	**127/3**	**Rational**
8	**256**	**85**	**Odd**
9	**512**	**511/3**	**Rational**
10	**1024**	**341**	**Odd**

By proving Lemma 1, it becomes evident that the decaying sequence comprises two distinct types of numbers based on the parity of x:

1. **Effective Numbers**: These are numbers in the sequence that yield odd numbers when x is even. Examples include 4, 16, 64, 256, 1024, ….

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2. **Auxiliary Numbers**: These serve as transitional numbers in the sequence and are obtained by dividing effective numbers by 2 when x is odd. Examples include 2, 8, 32, 128, 512, ….

Additionally, the number 1 is a fundamental or elementary number in the sequence. At this stage, another category can be defined, referred to as **Keys**.

Key numbers are a set of odd integers that, upon reaching them through the direct application of the Collatz conjecture, immediately transition into the decaying sequence, thereby guaranteeing eventual arrival at the 4→2→1 cycle. Keys are mathematically defined by the following equation:

$$\text{Key} = \frac{4^x - 1}{3} \text{ where } x \in Z^+$$

Each key corresponds to an effective number in the decaying sequence. It is important to note that 4^x can be expressed equivalently as 2^{2x} where x is a positive integer $4^x = 2^{2^x} = 2^{2x}$. This relationship is illustrated in Figure 2 with instances of key numbers and their corresponding effective numbers within the decaying sequence.

At this stage, the number line of positive integers is examined and categorized into three distinct sets:

1. **Decaying Sequence (2^x)**: This set comprises effective numbers, auxiliary numbers, and the elementary number 1. All numbers in this sequence are even, except for 1, which serves as the starting point of the sequence.
2. **Keys**: These are odd numbers that act as direct entry points into the decaying sequence via effective numbers, ultimately converging into the inevitable 4→2→1 cycle.
3. **Remaining Numbers**: This set includes all positive integers (both even and odd) that do not belong to either the decaying sequence or the set of keys.

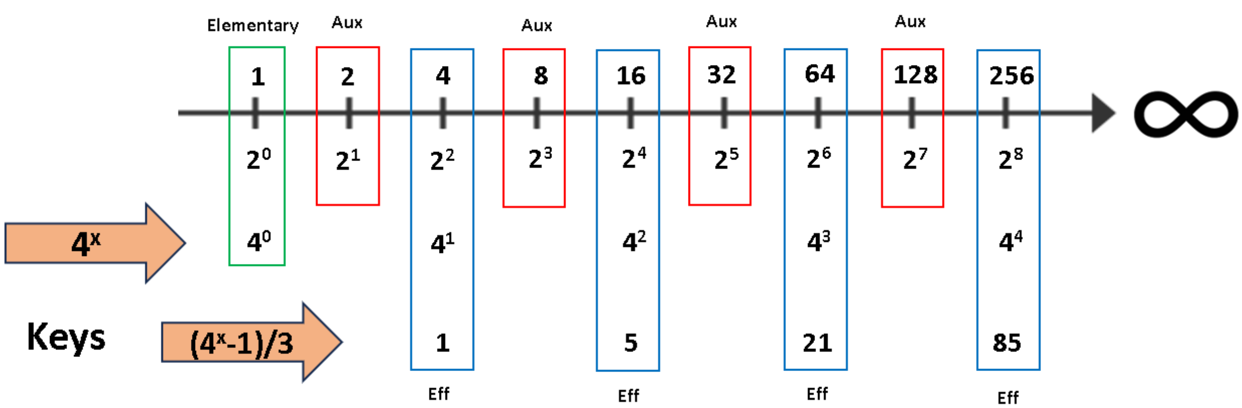

Figure 2: Keys and corresponding Effective numbers (Eff), Auxiliary numbers (Aux)

For the **remaining numbers**, each even number undergoes division by 2 until it becomes odd. Subsequently, this odd number is converted back to an even number using the transformation 3n+1, as defined by the Collatz conjecture. This process continues iteratively until an odd

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number is reached that serves as a key leading to the decaying sequence. After thoroughly analyzing the remaining numbers, additional subsets can be identified as follows:

4. **Key Multiples**: This subset consists of numbers that are multiples of all keys, expressed as Key $\times 2^x$, where $x \in Z^+$. All key multiples are even numbers. **Table 2** provides examples of various key multiples.

Table 2: Instances of Keys and their multiples

Keys	Keys multiples
5	{10,20,40,80,160,320,640,1280}
21	{42,84,168,336,672,1344}
85	{170,340,680,1360,2720}
341	{682,1364,2728,5456,10912}
1365	{2730,5460,10920,21840}

5. **Subkeys**: These are odd numbers that are not keys, identified by applying the reverse Collatz transformation on even numbers of key multiples to yield an odd result. Subkeys are defined mathematically as follows:

$$\text{Subkey} = \frac{(Key * 2^x) - 1}{3}$$

where key and x will yield an odd positive integer. Not all even numbers in those multiples produce odd results when reverse Collatz is applied. For instance, the even number 10 is a multiple of 5 (a key) and can be derived from 20 by division by 2 or from (10−1) / 3, which equals 3. In this case, 3 is identified as a subkey. **Figure 3** presents examples of various even numbers and their corresponding subkeys. Moreover, all keys and their multiples that are divisible by 3 for instance {21, 1365, 87381, …} do not have any subkeys that cannot be obtained from them. Those sequences can be defined as **Golden sequences.**

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Key	x	key* 2^x	sub keys
5	0	5	
	1	10	3
	2	20	
	3	40	13
	4	80	
	5	160	53
	6	320	
	7	640	213
	8	1280	
	9	2560	853
	10	5120	
	11	10240	3413

Key	x	key* 2^x	subkeys
85	0	85	
	1	170	
	2	340	113
	3	680	
	4	1360	453
	5	2720	
	6	5440	1813
	7	10880	
	8	21760	7253
	9	43520	
	10	87040	29013
	11	174080	
	12	348160	116053

Figure 3: Subkeys from multiples of keys, on the left subkeys obtained from key 5 and on the right subkeys obtained from key 85

6. **Subkey Multiples**: This set includes numbers that are multiples of all subkeys, expressed as Subkey$\times 2^x$, where $x \in Z^+$. Similar to key multiples, all subkey multiples are even numbers.

3. Moussa Model of positive integers

All positive integers are either even or odd numbers, odd numbers whether Keys or Subkeys, and even numbers whether Decaying sequence with the exception of 1 or Multiples of Keys or Subkeys. **Table 3** shows the model of positive integers.

Table 3: Positive integer mathematical model

Category (Set)	Equation
Decaying Sequence (D.S)	2^x, $x \in N$
Keys (K)	$\frac{4^x-1}{3}$, $x \in Z^+$
Keys multiples (K.M)	$K * 2^x$, $x \in Z^+$
Subkeys (S.K)	$\frac{(Key*2^x)-1}{3}$, key and x will output odd integer
Subkeys multiples (S.K.M)	$S.K * 2^x$, $x \in Z^+$

$$Z^+ = \{D.S \cup K \cup K.M \cup S.K \cup S.K.M\}$$

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It has been observed that the number 1 serves as the first key, and its multiples form the decaying sequence. Furthermore, the decaying sequence, along with keys, subkeys, and their respective multiples, extends infinitely. Subkeys, however, can be categorized into distinct levels based on their multiples, which adhere to the same conditions that define subkeys. **Figure 4** illustrates an example of a key, its multiples, subkeys, the multiples of subkeys, and the hierarchical levels of subkeys. It is important to note that these numbers extend infinitely to encompass all positive integers.

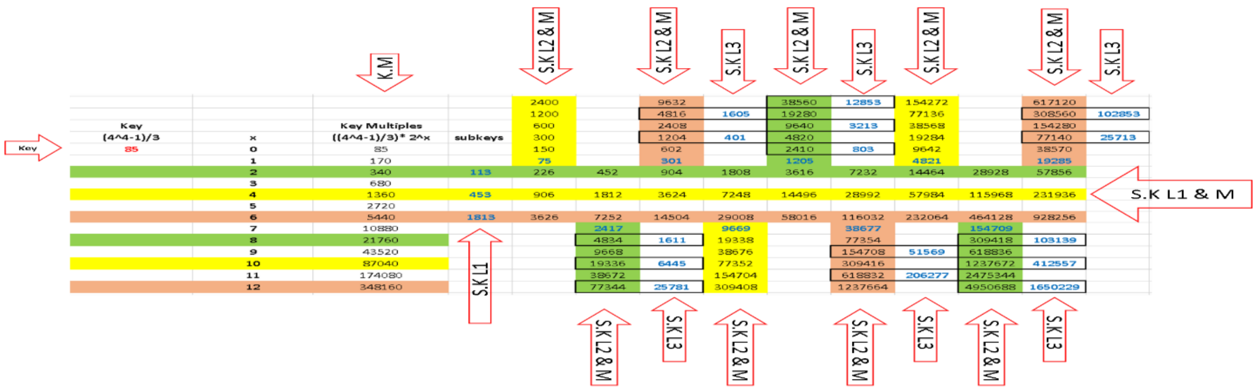

Figure 4: Key 85 network of subkeys levels and multiples

From this model, the following statement can be derived: ***"Any positive integer can be represented as a series of subkeys and their multiples until it reaches the primary key, which directly enters the decaying sequence."*** This implies that for any given number, its series will consist of one key, along with a finite number of subkeys and their multiples. Assuming the validity of this statement and that all positive integers conform to this model, the Collatz Conjecture would be effectively proven.

3.1. Model validation

A Python script has been developed to validate the proposed novel model of positive integers, which is provided in the Appendix A section. The primary purpose of this script is to generate all integers up to a user-defined limit, categorize them according to the proposed model, and verify that all numbers within the specified range conform to the model. The script also identifies any missing numbers that do not fit into the model. The main input required for running the script is the numerical limit or range to be analyzed, and the primary outputs include:

1. **Target Set**: This consists of all integers from 1 to the defined limit, compiled by merging the arrays of all categories derived from the calculations.
2. **Missing Numbers Set**: This contains any numbers within the defined range that are not included in the target set.

An illustrative scenario is presented where the script is applied to validate the first 20000 positive integers. After conducting multiple trials, the following insights were noted:

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- **Expansion Beyond the Limit**: When applying the Collatz function, certain numbers expand to reach maximum values far exceeding the specified limit. For example:
  - The number 27 reaches a maximum value of 9232.
  - The number 31 reaches a maximum value of 9232.
  - The number 37 reaches a maximum value of 112.

  To accommodate this growth, an additional **allowance parameter** is introduced. This parameter provides sufficient space for numbers to expand to their maximum values. The allowance parameter is multiplied by the limit, and the script iterates until this extended range is reached. Without this parameter, many numbers would be omitted from the target set.

- **Keys Larger Than Their Numbers**: Certain numbers have keys that are larger than themselves. For instance:
  - The number 75 has a main key of 85.
  - The number 19417 has a main key of 21845.

  To address this limitation, a **key allowance parameter** is incorporated, enabling the inclusion of such numbers in the target set. To sum up, the script requires the following user-defined inputs:

1. **Limit**: The range of positive integers to be analyzed.
2. **Allowance**: A parameter to extend the range, allowing numbers to grow to their maximum values.
3. **Key Allowance**: A parameter to account for numbers with keys larger than themselves.

The primary outputs of the script include:

1. **Target Set**: A sequence of integers from 1 to the limit, ensuring all numbers conform to the model.
2. **Missing Numbers Array**: A list of numbers that are not included in the target set.

Additionally, the script generates auxiliary arrays, including:

- The **decaying sequence**.
- **Keys** and their multiples.
- **Subkeys** at various levels and their multiples.

The script terminates when the subkey level array becomes empty, which occurs when a limit is imposed. However, if no limit is specified, all categories would continue to expand indefinitely, eventually encompassing all positive integers.

The script also identifies numbers that attain significantly large maximum values relative to their initial values without requiring brute-force computations for all integers. These numbers remain missing from the target set until the allowance parameter is appropriately adjusted. The script

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was tested to validate the model for ranges up to $\mathbf{2^{30}}$, and all positive integers within this range conformed to the correctness of the proposed model.

4. Moussa Conjecture

"Let M(q) be a function where q is a positive integer. The function M(q) is governed by L distinct rules, where L is a finite number. These rules partition the set of positive integers along the number line according to specific conditions. One of these rules must include the transformation $\frac{q}{2}$ whenever q is even. For odd numbers, classification occurs based on predefined conditions, assigning them to the remaining (L−1) rules. Based on that, there exist infinitely many possible formulations of M(q) that, when iteratively applied, converge to 1. Each of these functions adheres to the same structural framework as **Moussa's Model of Positive Integers**, which consists of **keys and subkeys** with variation in numerical composition based on the imposed rules"

This conjecture serves as a generalization of the **Collatz conjecture** and extends the work of (Briscese & Calogero, 2024) who proposed conjectures analogous to the Collatz problem. It is crucial to note that not all transformation rules will necessarily lead to convergence to 1. To provide a comprehensive understanding of **Moussa's conjecture**, three distinct variations will be introduced. Additionally, the code utilized in these trials is provided in Appendix B for further reference and verification.

The first variation is defined as follows:

$$\mathbf{M(q)} = \begin{cases} \frac{q}{2}, & q \,\%\, 2 = 0 \\ \frac{q-1}{4}, & q\text{-}1 \,\%\, 4 = 0 \\ \frac{3q+1}{2}, & q\text{-}1 \,\%\, 4 \neq 0 \end{cases}$$

In this variation, the rules divide the positive integers into even numbers and odd number. Unlike Collatz conjecture, the odd numbers in this variation are divided by 2 rules into numbers divisible by 4 and numbers not divisible by 4 after subtracting 1. **Table 4** contain trials of different positive integersand their sequence converging to 1. Notice that for the variation tables instances: Keys are in redbold, D.S are in blue bold

Table 4: First variation instances

Positive integer	Sequence
37364378129	[37364378129, 9341094532, 4670547266, 2335273633, 583818408, 291909204, 145954602, 72977301, 18244325, 4561081, 1140270, 570135, 855203, 1282805, 320701, 80175, 120263, 180395, 270593, 67648, 33824, 16912, 8456, 4228, 2114, 1057, 264, 132, 66, **33**, **8, 4, 2, 1**]
4892066732014	[4892066732014, 2446033366007, 3669050049011, 5503575073517, 1375893768379, 2063840652569, 515960163142, 257980081571, 386970122357, 96742530589, 24185632647, 36278448971, 54417673457,

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	13604418364, 6802209182, 3401104591, 5101656887, 7652485331, 11478727997, 2869681999, 4304522999, 6456784499, 9685176749, 2421294187, 3631941281, 907985320, 453992660, 226996330, 113498165, 28374541, 7093635, 10640453, 2660113, 665028, 332514, 166257, 41564, 20782, 10391, 15587, 23381, 5845, 1461, 365, 91, 137, 34, **17**, **4, 2, 1**]
7845612248	[7845612248, 3922806124, 1961403062, 980701531, 1471052297, 367763074, 183881537, 45970384, 22985192, 11492596, 5746298, 2873149, 718287, 1077431, 1616147, 2424221, 606055, 909083, 1363625, 340906, 170453, 42613, 10653, 2663, 3995, 5993, 1498, 749, 187, 281, 70, 35, 53, 13, 3, **5**, **1**]
12325367123	[12325367123, 18488050685, 4622012671, 6933019007, 10399528511, 15599292767, 23398939151, 35098408727, 52647613091, 78971419637, 19742854909, 4935713727, 7403570591, 11105355887, 16658033831, 24987050747, 37480576121, 9370144030, 4685072015, 7027608023, 10541412035, 15812118053, 3953029513, 988257378, 494128689, 123532172, 61766086, 30883043, 46324565, 11581141, 2895285, 723821, 180955, 271433, 67858, 33929, 8482, 4241, 1060, 530, 265, 66, **33**, **8, 4, 2, 1**]

The second variation is defined as follows:

$$\mathbf{M(q)} = \begin{cases} \frac{q}{2}, & q \,\%\, 2 = 0 \\ \frac{q-1}{6}, & q\text{-}1 \,\%\, 6 = 0 \\ \frac{5q+1}{2}, & q\text{-}1 \,\%\, 6 \neq 0 \end{cases}$$

In this variation, the rules divide the positive integers into even numbers and odd number. Odd numbers in this variation are divided by 2 rules into numbers divisible by 6 and numbers not divisible by 6 after subtracting 1. **Table 5** contain trials of different positive integers and their sequence convergingto 1.

Table 5: Second variation instances

Positive integer	**Sequence**
37364378129	[37364378129, 93410945323, 15568490887, 2594748481, 432458080, 216229040, 108114520, 54057260, 27028630, 13514315, 33785788, 16892894, 8446447, 1407741, 3519353, 8798383, 1466397, 3665993, 9164983, 1527497, 3818743, 636457, 106076, 53038, 26519, 66298, 33149, 82873, 13812, 6906, 3453, 8633, 21583, 3597, 8993, 22483, 3747, 9368, 4684, 2342, 1171, 195, 488, 244, 122, 61, 10, 5, **13**, **2, 1**]
4892066732014	[4892066732014, 2446033366007, 6115083415018, 3057541707509,

	7643854268773, 1273975711462, 636987855731, 1592469639328, 796234819664, 398117409832, 199058704916, 99529352458, 49764676229, 124411690573, 20735281762, 10367640881, 25919102203, 4319850367, 719975061, 1799937653, 4499844133, 749974022, 374987011, 62497835, 156244588, 78122294, 39061147, 6510191, 16275478, 8137739, 20344348, 10172174, 5086087, 847681, 141280, 70640, 35320, 17660, 8830, 4415, 11038, 5519, 13798, 6899, 17248, 8624, 4312, 2156, 1078, 539, 1348, 674, 337, 56, 28, 14, **7**, **1**]
7845612248	[7845612248, 3922806124, 1961403062, 980701531, 163450255, 27241709, 68104273, 11350712, 5675356, 2837678, 1418839, 236473, 39412, 19706, 9853, 1642, 821, 2053, 342, 171, 428, 214, 107, 268, 134, 67, 11, 28, 14, **7**, **1**]
12325367123	[12325367123, 30813417808, 15406708904, 7703354452, 3851677226, 1925838613, 320973102, 160486551, 401216378, 200608189, 33434698, 16717349, 41793373, 6965562, 3482781, 8706953, 21767383, 3627897, 9069743, 22674358, 11337179, 28342948, 14171474, 7085737, 1180956, 590478, 295239, 738098, 369049, 61508, 30754, 15377, 38443, 6407, 16018, 8009, 20023, 3337, 556, 278, 139, 23, 58, 29, 73, 12, 6, **3**, **8, 4, 2, 1**]

The third variation is defined as follows:

$$\mathbf{M(q)} = \begin{cases} \frac{q}{2}, & q \,\%\, 2 = 0 \\ \frac{q}{3}, & q \,\%\, 3 = 0 \\ 3q + 1, & q \,\%\, 3 \neq 0 \end{cases}$$

In this variation, the rules divide the positive integers into even numbers and odd number. Odd numbers in this variation are divided by 2 rules into numbers divisible by 3 and numbers not divisible by 3. **Table 6** contain trials of different positive integers and their sequence converging to 1.

Table 6: Third variation instance

Positive integer	**Sequence**
37364378129	**[37364378129, 112093134388, 56046567194, 28023283597, 84069850792, 42034925396, 21017462698, 10508731349, 31526194048, 15763097024, 7881548512, 3940774256, 1970387128, 985193564, 492596782, 246298391, 738895174, 369447587, 1108342762, 554171381, 1662514144, 831257072, 415628536, 207814268, 103907134, 51953567, 155860702, 77930351, 233791054, 116895527, 350686582, 175343291, 526029874, 263014937, 789044812, 394522406, 197261203, 591783610, 295891805, 887675416, 443837708, 221918854, 110959427, 332878282, 166439141,**

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Through an in-depth analysis of the variations and sequences, it has been observed that all generated sequences conform to Moussa's Model of Positive Integers, which comprises decaying sequences, keys, subkeys, and their multiples. However, the specific numbers involved vary depending on the rules of each variation. Additionally, while all sequences ultimately converge to 1, the number of steps required for convergence differs due to the varying transformation rules in M(q).

As a result, Moussa's conjecture serves as a generalized extension of the Collatz conjecture, demonstrating that an infinite number of possible functions—each with distinct formulations and variations—also converge to 1. Furthermore, Moussa's model is applicable to multiple variations, not solely the Collatz variation. If Moussa's conjecture is formally proven, then the Collatz conjecture would be directly validated as a special case within this broader framework.

5. Conclusion

This study presents new insights into the dynamics of the Collatz problem, introducing a novel model of positive integers formulated through distinct sets and their governing equations. A validation of this model is conducted using Python, providing computational verification. Moussa's conjecture is proposed as a generalization of the Collatz conjecture, with multiple variations explored under different transformation rules. Additionally, a Python script is included to facilitate trials and further analysis.

Moussa's conjecture inherently validates the Moussa Model of Positive Integers, demonstrating that various variations align with this structure until convergence to 1. If Moussa's conjecture is rigorously proven, the Collatz conjecture would consequently be established as a special case. Conversely, if all positive integers adhere to Moussa's model, then the Collatz conjecture would be definitively verified.

Future research should involve a comprehensive exploration of Moussa's model and conjecture, aiming to uncover deeper insights into number theory. The present study validates the model up to a specific numerical limit, ensuring that all numbers within this range conform to Moussa's model. An essential next step is the development of an algorithm capable of representing any number as a set comprising {key, subkey levels}. For instance, under the Collatz variation, the number 37 can be expressed as {5, 13, 17, 11, 7, 37}. Establishing such an algorithm would enable further investigation into the intricate relationships between number sequences and their structural properties.

Moreover, keys can be represented as infinite sets or arrays, extending up to specific subkey levels. This concept has potential applications in cryptography, steganography, and encryption systems, offering new methodologies for secure data encoding. By analyzing number theory from this novel perspective, existing mathematical problems may be resolved, while new questions emerge for further exploration. This study introduces a transformative approach to understanding number theory, opening avenues for future theoretical and applied research.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author used ChatGPT in order to Proofread the manuscript to improve its readability. After using this tool, the author reviewed and edited the content as needed and took full responsibility for the content of the publication.

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7. Appendix A

8. The following Python code proves that for any given upper limit number defined by the user, all numbers will obey Moussa model of positive integers.
9. The code:
10.

```
# user inputs
limit = 20000
```

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```
allowance = 1500
key_allowance = 2
#**********************
D_S = []
keys = []
multiples_1 = []
remaining_source = []
S_K_1 = []
n_D_S = 0
n_keys = 2
n_multi = 0
#******************************
# for Decaying sequence
while True:
   value = 2**n_D_S
   if value > limit:  # Check against the limit
      break  # Exit the loop if the value is greater or equal to the limit
   D_S.append(value)  # Append the value to the list
   n_D_S += 1  # Increment n_D_S
# Output the results
print("Decaying Sequence:", D_S)
# for Keys
while True:
   value = int(((4**n_keys)-1)/3)
   if value > key_allowance*limit:  # Check against the limit
      break  # Exit the loop if the value is greater or equal to the limit
   keys.append(value)  # Append the value to the list
   n_keys += 1  # Increment x
print("Keys:", keys)
```

11. for value in keys:

```
   n_multi = 1
   while True:
      multiple = value* (2**n_multi)
      if multiple > allowance*limit:  # Check against the limit
         break  # Exit the loop if the value is greater or equal to the limit
      multiples_1.append(multiple)  # Append the value to the list
      n_multi += 1

# Output the result
print("Multiples", multiples_1)

# Loop through the input array
```

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```
for value in multiples_1:
    if (value - 1) % 3 == 0 and value % 2 == 0:  # Check if (value - 1) is divisible by 3
        remaining_source.append(value)  # Append to the result array

print("Remaining_source:", remaining_source)

for value in remaining_source:
    result = int((value - 1) / 3)
    S_K_1.append(result)

print("Sub Keys L1:", S_K_1)

current_array = S_K_1  # Start with S_K_1
current_array_name = "S_K_1"
iteration = 1  # Track iterations

while current_array:  # Continue until current array is empty
    # Step 1: Generate multiples as value * (2^x)
    multiples = []
    for value in current_array:
        x = 0
        while True:
            multiple = value * (2 ** x)
            if multiple > allowance*limit:  # Stop if the multiple exceeds the limit
                break
            multiples.append(multiple)
            x += 1
    multiples_name = f"{current_array_name}_multiples"
    globals()[multiples_name] = multiples  # Dynamically define the multiples array
    print(f"{multiples_name}: {multiples}")
```

12. # Step 2: Check multiples and create next array

```
    next_array = []
    for value in multiples:
        if (value - 1) % 3 == 0 and value % 2 == 0:  # Check the condition
            next_value = int((value - 1) / 3)
            next_array.append(next_value)

    # Define the next array dynamically
    next_array_name = f"S_K_{iteration + 1}"
    globals()[next_array_name] = next_array
    print(f"{next_array_name}: {next_array}")
```

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```
        # Step 3: Update loop variables
        current_array = next_array
        current_array_name = next_array_name
        iteration += 1

    # Final output
    print("Processing complete.")

    # Initialize a set to hold all unique values
    all_values = set()

    # Add values from each array to the set
    all_values.update(D_S)
    all_values.update(keys)
    all_values.update(multiples_1)
    all_values.update(S_K_1)
```

13. # Dynamically include all S_K_n and their multiples

```
    for i in range(1, iteration + 1):
        sk_name = f"S_K_{i}"
        sk_multiples_name = f"S_K_{i}_multiples"

        if sk_name in globals():
            all_values.update(globals()[sk_name])

        if sk_multiples_name in globals():
            all_values.update(globals()[sk_multiples_name])

    # Convert the set to a sorted list
    merged_sorted_array = sorted(all_values)

    # Output the result
    print("Merged and Sorted Array:", merged_sorted_array)

    new_set = set(sorted(merged_sorted_array)[:limit])

    all_numbers = set(range(1, limit+1))

    # Convert the merged_sorted_array into a set
    merged_set = set(merged_sorted_array)

    # Find the missing numbers using set difference
```

```
missing_numbers = sorted(all_numbers - merged_set)

print("Decying Sequence:", D_S)
print("Keys:", keys)
print("Target set:", new_set)

# Output the missing numbers
print("Missing Numbers:", missing_numbers)
```

14. Appendix B

15.
```
import time
user_input = int(input("Enter positive integer: "))
num_digits = len(str(user_input))
star_time = time.time()
solution = []
solution.append(user_input)

result = user_input
# rules
while result != 1:
    if result % 2 = = 0:
        result = result //2
    elif (result-1) % 4 = = 0:
        result = (result -1)//4
    elif (result-1) % 4 != 0:
        result = (result*3+1)//2
    solution.append(result)

end_time= time.time()
print(num_digits)
print(solution)
print(len(solution)-1)
print(end_time - star_time)
```