

Predictive Mathematical Modelling of Heat Transfer and Phase Change for Solidification Processes in Sustainable Building

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Abstract

This study develops a comprehensive predictive mathematical modelling framework for analyzing heat transfer and phase change phenomena in sustainable buildings. The work integrates multiple aspects of thermal behaviour, including heating and cooling loads, ground temperature distribution, periodic heat transfer through walls, roofs, and windows, as well as the effects of ventilation, furnishings, and auxiliary heating. Special emphasis is given to solidification and phase change processes, which are critical in applications such as concrete hydration, phase change materials (PCMs), and waste solidification for sustainable construction. Governing equations are formulated using thermodynamic principles, heat conduction theory and response functions. Computational investigations demonstrate how temperature gradients, solar radiation, and soil depth influence building energy performance, while solidification modelling highlights the relationship between cooling rates, interface dynamics, and material strength development. The proposed framework provides valuable predictive insights for optimizing material selection, improving energy efficiency and enhancing durability in green building design. Overall, this research bridges building physics, thermal energy storage and materials science, offering engineers and architects a robust tool for advancing sustainable construction technologies.

Keywords: Sustainable modelling, heat transfer, phase change, solidification, BIM

1 Introduction

Building construction and operation are significant contributors to greenhouse gas emissions, energy consumption, and resource wastage. The global demand for sustainable buildings has surged due to increasing environmental concerns, resource depletion, and urbanization. Sustainable building design “green building” has emerged as a crucial solution to mitigate these impacts[1]. Sustainable buildings may be designed to minimize

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their environmental impact throughout their lifecycle from construction to demolition. This necessitates a shift toward designing and constructing buildings that are resource-efficient, energy-efficient, and low in emissions as demonstrated in following flowchart and table 1.

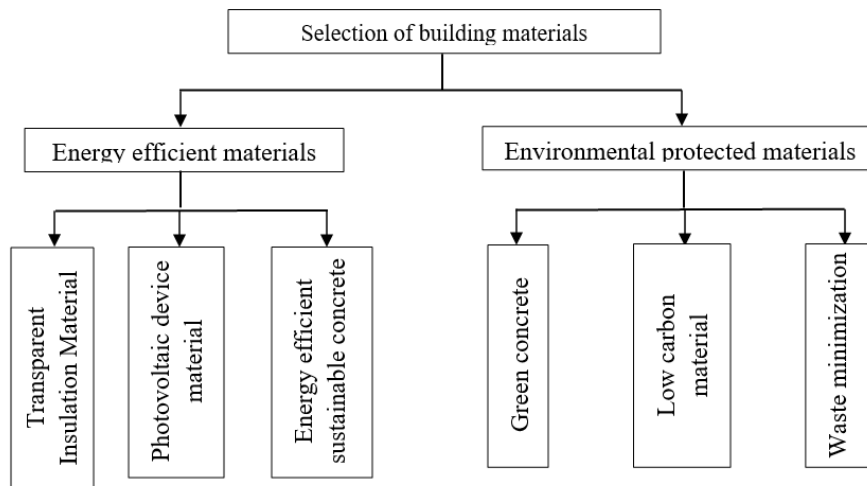
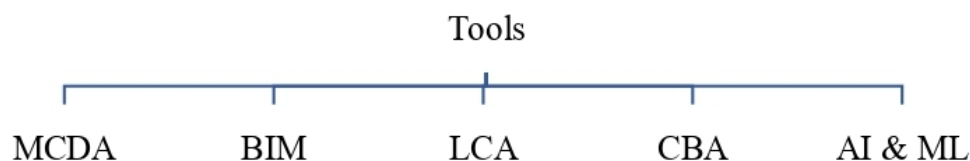


Table 1: Benefits of energy-efficient materials.

Transparent insulation materials	Photovoltaic materials	Energy-efficient sustainable concrete
Reduce energy consumption up to 40%	Reduce energy consumption up to 30%	Reduce energy-intensive heating and cooling

Decision making modelling for the optimization of material selection consists of following five tools:



Where, MCDA = Multi-Criteria Decision Analysis, BIM = Building Information Modelling, LCA = Life Cycle Assessment, CBA = Cost-Benefit Analysis, AI & ML = Artificial Intelligence and Machine Learning

To address these challenges, advanced modelling and technology have emerged as key tools in the design, construction, and operation in sustainable buildings as shown in Fig. 1. These technologies are focused on optimizing the use of resources, reducing environmental footprints, and improving energy performance. Building information modelling (BIM) is a digital representation of the physical and functional characteristics of a building. BIM software has revolutionized the design, construction, and operation of buildings by improving collaboration, accuracy, and efficiency in decision-making[2].

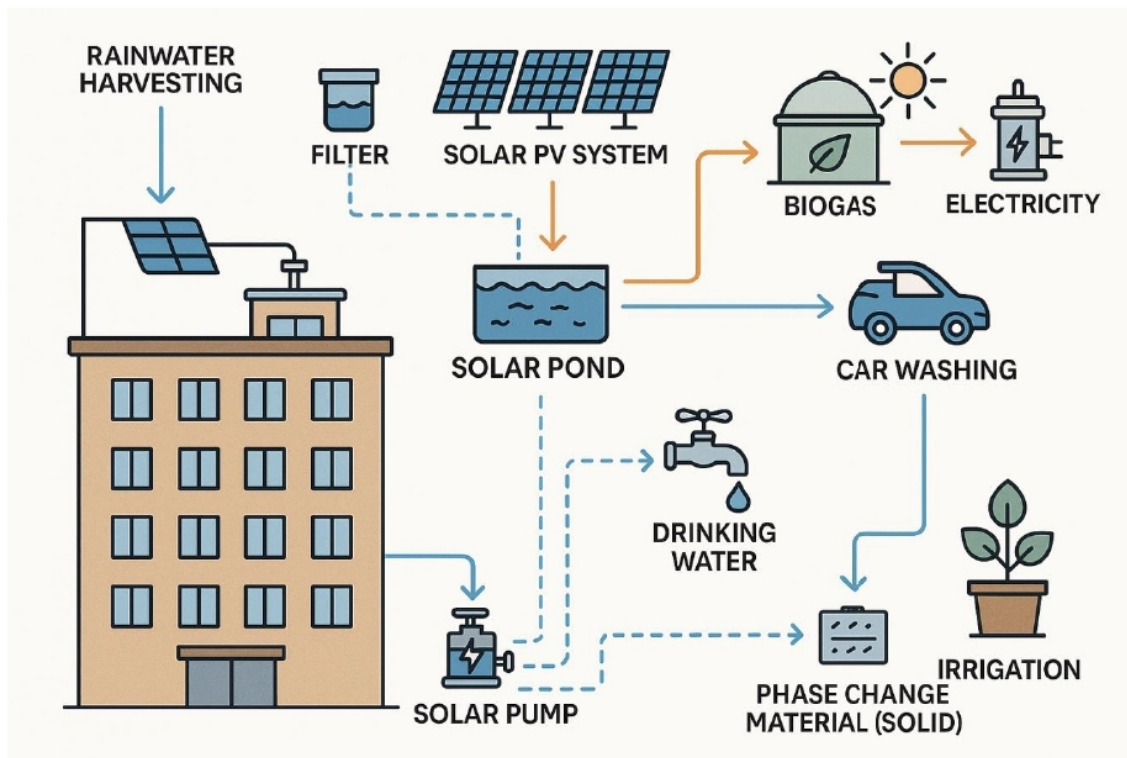


Figure 1: Operations in sustainable buildings

It allows for the creation of 3D models that integrate architectural, structural, and MEP (Mechanical, Electrical, Plumbing) systems, enabling architects, engineers, and contractors to visualize and analyze various aspects of the building before construction begins. BIM can be used to reduce building's overall energy consumption, perform energy simulations, allowing for optimization of heating, cooling, and lighting systems. It supports the analysis of material quantities and environmental impacts, ensuring the selection of sustainable materials that contribute to energy efficiency and low environmental impact. Energy efficiency in buildings is a global priority due to increasing energy demands and the need to reduce greenhouse gas emissions. Among various strategies, the utilization of phase change materials (PCMs) and controlled solidification processes has emerged as a promising approach to enhance passive heating and cooling, minimize HVAC (heating ventilation and air conditioned) loads, and store renewable thermal energy. Then BIM software helps us in conducting lifecycle assessments (LCA) to evaluate the environmental impact of a building throughout its lifecycle, from construction to demolition.

2 Mathematical modelling for heating/cooling load of a building

The heating/cooling load of a building may be expressed by

$$MC \frac{\partial \theta}{\partial t} = Q_{wl} + Q_r + Q_{wd} + Q_h - Q_f - Q_v - Q_g \quad (1)$$

where $\frac{\partial \theta}{\partial t}$ is the rate of change of temperature and $M, C, Q_{wl}, Q_r, Q_{wd}, Q_h, Q_f, Q_v, Q_g$ denote the mass of the building component, specific heat capacity, periodic heat transfer

through walls, periodic heat transfer through roof, periodic heat transfer through window, auxiliary heating, heat transfer by furnishings, ventilation/infiltration, and heat conducted by the underlying ground, respectively[3].

3 Mathematical modelling for ground temperature

The temperature distribution in ground is periodic in nature on the basis of sol-air temperature due to the combined effect of solar intensity and ambient temperature. Following Kaushika and Srivastava [4], the temperature of the ground may be defined as:

$$\theta(x, t) = \int_0^\infty \theta_a(t - \tau) h(x, \tau) d\tau, \quad (2)$$

where $h(x, \tau)$ is the ground response to a unit impulse input of $\theta_a(t)$ (response function). Let the ground response $U(x, t)$ be related to $h(x, t)$ as:

$$U(x, t) = \int_0^\infty h(x, t) dt. \quad (3)$$

Further, ground response $U(x, t)$ is characterized by the heat conduction equation given by

$$\frac{\partial U(x, t)}{\partial t} = \frac{K}{\rho c} \frac{\partial^2 U(x, t)}{\partial t^2} = \alpha \frac{\partial^2 U(x, t)}{\partial t^2} \quad (4)$$

and the energy balance at the ground surface may be defined as:

$$-K \left. \frac{\partial \theta(x, t)}{\partial x} \right|_{x=0} = h [\theta_a(t) - U(0, t)] \quad (5)$$

With boundary conditions

$$\theta_a(t) = \begin{cases} 0, & t < 0, \\ 1, & t \geq 0, \end{cases} \quad (6)$$

and initial condition

$$U(x, 0) = 0 \quad (7)$$

Let us assume that $U(x, t)$ is finite as $x \rightarrow \infty$.

Now solving eq.(4) by using Laplace transform we get

$$\bar{U}(x, p) = \frac{\mu}{p(\mu + \sqrt{p})} e^{-\gamma\sqrt{p}}, \quad (8)$$

where

$$\alpha = \frac{\mu^2 K}{h}, \quad \alpha \gamma^2 = x^2. \quad (9)$$

Now taking inverse Laplace transformation of eq.(8), we get

$$U(x, t) = \left[\operatorname{erfc}\left(\frac{\gamma}{2\sqrt{t}}\right) - e^{\gamma\mu + \mu^2 t} \operatorname{erfc}\left(\frac{\gamma}{2\sqrt{t}} + \mu\sqrt{t}\right) \right] \quad (10)$$

Now differentiate eq.(10) with respect to t , we get

$$h(x, t) = \frac{2}{\sqrt{\pi}} \left[e^{-\gamma^2/(4t)} \frac{1}{2} \mu t^{-1/2} - \mu^2 \frac{\sqrt{\pi}}{2} e^{\gamma\mu + \mu^2 t} \operatorname{erfc} \left(\frac{\gamma}{2\sqrt{t}} + \mu\sqrt{t} \right) \right] \quad (11)$$

Here $h(x, t)$ is independent from nature of $\theta_a(t)$.

Now putting $\theta_a(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos(m\omega t - r_m)$ in eq.(2), we get

$$\theta(x, t) = \int_0^{\infty} \left(a_0 + \sum_{m=1}^{\infty} a_m \cos(m\omega(t - \tau) - r_m) \right) h(x, \tau) d\tau \quad (12)$$

Using eq. (11) and putting, $d_0 = m\omega = \tan(\mu^2 \delta)$, $d_1 = (m\omega t - r_m)$ in eq.(12), we get

$$\theta(x, t) = a_0 + \sum_{m=1}^{\infty} J_m \quad (13)$$

Where $J_m = A \left(\frac{\mu}{\sqrt{d_0}} \right) \cos \left[d_1 - \frac{\pi}{4} - \left(a\sqrt{\frac{d_0}{2}} \right) \right] - A \left(\frac{\mu}{\sqrt{d_0}} \right) \cos \left[d_1 + \delta_1 - \frac{\pi}{4} - \left(a\sqrt{\frac{d_0}{2}} \right) \right] \cos \delta$
 $+ A \cos \delta \cos \left[d_1 + \delta - \left(a\sqrt{\frac{d_0}{2}} \right) \right]$ and $A = a_m e^{-\gamma\sqrt{\frac{d_0}{2}}}$; a is a phase parameter.
 Now on simplification we get

$$\theta(x, t) = a_0 + \sum_{m=1}^{\infty} B_m \exp(-m^{1/2} \alpha x) \cos(m\omega t - r_m - m^{1/2} \alpha x - \mu_m) \quad (14)$$

Where

$$B_m = \left[(1 + m^{1/2} \beta)^2 + m\beta^2 \right]^{-1/2}, \quad a_m = \frac{(k\omega\rho c)^{1/2}}{h\sqrt{2}} = \frac{b_0^{1/2}}{\mu\sqrt{2m}},$$

$$\alpha = \left(\frac{\omega\rho c}{2k} \right)^{1/2}, \quad \mu_m = \tan^{-1} \left(\frac{m^{1/2} \beta}{1 + m^{1/2} \beta} \right).$$

This expression is the same as that obtained by Kaushika N.D. and Srivastava[4] (1980), Khatry et al. [5] based on periodic analysis. It establishes the equivalence of the two methods for a periodic input. Further using source <https://cds.climate.copernicus.eu/datasets>, the ground temperature at surface and inside earth at different soil depth has been evaluated by using formulas

$$\theta(0, t) = \text{daily mean temp.} + \text{surface amplitude temp.} \cos\left(\frac{2\pi}{24} t\right), t = 0, 1, \dots, 23 \quad (15)$$

$$\theta(x, t) = \theta_m + S_0 e^{-x/\delta} \cos\left(\omega t - \frac{x}{\delta}\right) \quad (16)$$

where $\theta(0, t)$ is known as ground temperature, $\theta(x, t)$ is the soil wave temperature at x meter depth on daily basis, θ_m is daily mean temperature ($^{\circ}\text{C}$) at surface which is also known as shallow depth for a strictly periodic day, S_0 is the surface amplitude temperature, $\omega = \frac{2\pi}{24} h^{-1}$, $\delta = \sqrt{\frac{2\pi}{\omega}}$ is known as damping depth measured in meters, κ is the soil thermal diffusivity lies between $(0.6 \times 10^{-6}) - (1.2 \times 10^{-6}) \text{ m}^2/\text{s}$ for moist loam. Here $\frac{x}{\delta}$ is known as phase lags.

For 12 August 2025 which was the rainy day in Bareilly, computational values of ground temperature at surface and at 5 cm, 10 cm, 15 cm, 20 cm soil depth at daily mean temperature (30°C) and surface amplitude= (4.50°C) are obtained as:

$$\begin{aligned}\theta(0, t) &= [34.5, 34.34, 33.89, 33.18, 32.25, 31.16, 30, 28.83, 27.75, 26.81, 26.10, \\ &\quad 25.65, 25.50, 25.65, 26.10, 26.82, 27.75, 28.84, 30, 31.17, 32.25, 33.18, 33.89, 33.34] \\ \theta(0.05, t) &= [24.28, 24.89, 25.78, 26.25, 27.29, 28.82, 30.19, 31.63, 32.09, 34.28, 33.85, \\ &\quad 34.64, 36.43, 33.70, 34.39, 33.23, 32.63, 30.66, 29.64, 28.94, 27.38, 25.90, 25.95, 25.39] \\ \theta(0.10, t) &= [24.34, 25.22, 25.69, 26.20, 27.64, 27.98, 29.07, 31.27, 31.75, 32.73, 33.38, \\ &\quad 34.24, 34.22, 34.32, 33.91, 32.77, 32.09, 31.20, 29.24, 28.19, 26.94, 26.10, 25.66, 28.81] \\ \theta(0.15, t) &= [24.47, 24.82, 25.14, 25.86, 26.43, 28.14, 28.86, 30.10, 31.45, 31.84, 32.88, \\ &\quad 33.11, 33.53, 32.90, 32.82, 31.97, 31.22, 30.01, 28.91, 27.78, 26.81, 25.96, 24.99, 24.71] \\ \theta(0.20, t) &= [23.98, 24.09, 24.65, 24.73, 26.03, 27.01, 27.91, 29.02, 30.12, 31.22, 31.95, \\ &\quad 32.22, 32.52, 32.19, 32.01, 31.02, 34.38, 29.20, 27.88, 27.22, 25.68, 25.15, 24.33, 23.65]\end{aligned}$$

Variation in ambient temperature along with ground temperature in Bareilly on 12th Aug 2025 at different soil depths have been depicted in Fig. 2

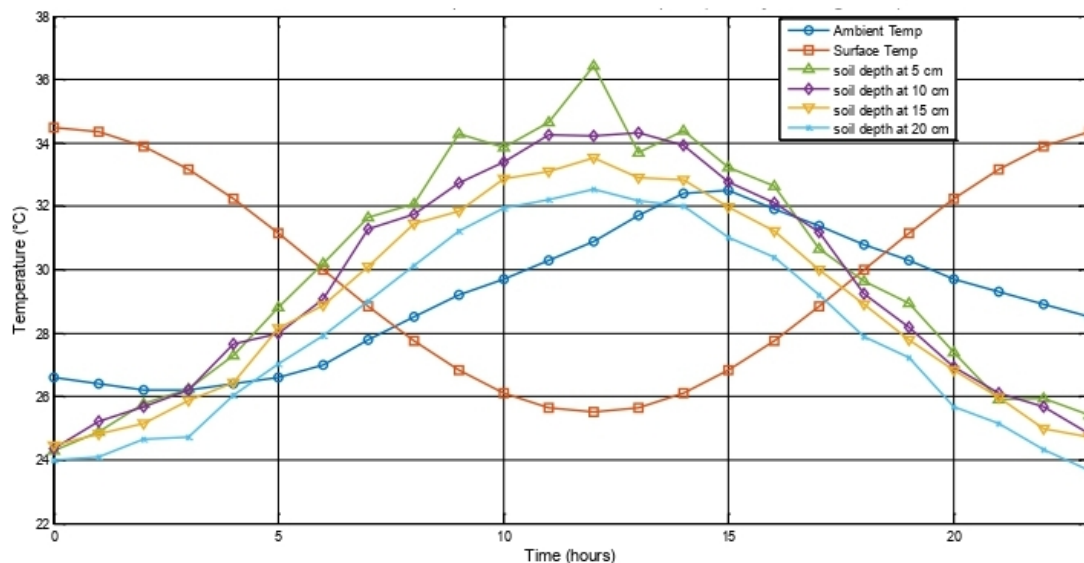


Figure 2: Ambient temperature ground temperature at different depths in Bareilly on 12-08-2025

In this modelling of temperature and intensity analysis, the result shows that soil depth significantly moderates temperature fluctuations. While the surface undergoes sharp day-night variations, deeper layers remain cooler and more stable, providing a buffered environment for biological processes and root systems. This thermal lag and damping effect highlight the soil's role in regulating ground-atmosphere energy exchange.

4 Mathematical modelling for temperature distribution in walls and roof

The temperature distribution in walls and roof may be expressed by one-dimensional Fourier heat conduction equation given by

$$\kappa^{(n)} \frac{\partial^2 \theta^{(n)}(x, t)}{\partial x^2} = \frac{\partial \theta^{(n)}(x, t)}{\partial t} \quad (17)$$

Where $\kappa^{(n)}$ is the thermal diffusivity of n^{th} wall material given by $\kappa^{(n)} = \frac{K^{(n)}}{\rho^{(n)} c^{(n)}}$

At the outer surface of n^{th} wall, the boundary conditions can be written as :

$$-\kappa^{(n)} \left. \frac{\partial \theta^{(n)}(x, t)}{\partial x} \right|_{x=0} = \alpha^{(n)} I^{(n)}(t) + h_1^{(n)} (\theta_a(t) - \theta^{(n)}(0, t)) - \varepsilon \Delta R = h_1^{(n)} (\theta_{sa}^{(n)}(t) - \theta^{(n)}(0, t)) \quad (18)$$

Sol-air temperature $\theta_{sa}^{(n)}$ for n^{th} wall/roof may be evaluated by using formula

$$\theta_{sa}^{(n)}(t) = \theta_a^{(n)}(t) + \frac{\alpha^{(n)} I^{(n)}(t) - \varepsilon \Delta R}{h_0^{(n)}} \quad \text{where } \Delta R = \sigma (\theta_a^{(n)}(t) - \theta_{sky}^{(n)}(t)) \quad (19)$$

Swinbank (1963) has given relation between sky temperature and ambient temperature given by:

$$\theta_{sky}^{(n)}(t) = 0.0552 \theta_a^{1.5}(t) \quad (20)$$

While Whillier (1967) has given relation

$$\theta_{sky}^{(n)}(t) = \theta_a^{(n)}(t) - 6 \quad (21)$$

$$\theta_{sky}^{(n)}(t) = \theta_a^{(n)}(t) - 12 \quad (22)$$

Where θ_{sky} and θ_a both are in Kelvin. But the commonly used relation between sky temperature and ambient temperature for Bareilly (Latitude 28.3670°N and Longitude 79.4300°E) is given by:

$$\theta_{sky}^{(n)}(t) = \theta_a^{(n)}(t) - 10 \quad (23)$$

This relation for New Delhi (Latitude 28.7041°N and Longitude 77.1025°E) and Hyderabad (Latitude 17.4065°N and Longitude 78.4772°E) is given by:

$$\theta_{sky}^{(n)}(t) = \theta_a^{(n)}(t) - 6 \quad (24)$$

Following source www.worldweatheronline.com, the ambient temperature, sol air temperature and Solar intensity on 12 August 2025 for Bareilly, New Delhi & Hyderabad have been represented by Table 2 and Table 3 represents Monthly solar insolation & intensity of Bareilly, New Delhi and Hyderabad in 2024. Let us assume that

$\theta_{ab} (^{\circ}C)$ = Ambient temperature of Bareilly, $\theta_{sab} (^{\circ}C)$ = Sol-air temperature of Bareilly

$\theta_{ad} (^{\circ}C)$ = Ambient temperature of New Delhi, $\theta_{sad} (^{\circ}C)$ = Sol-air temperature of New Delhi

$\theta_{ah} (^{\circ}C)$ = Ambient temperature of Hyderabad, $\theta_{sah} (^{\circ}C)$ = Sol-air temperature of Bareilly

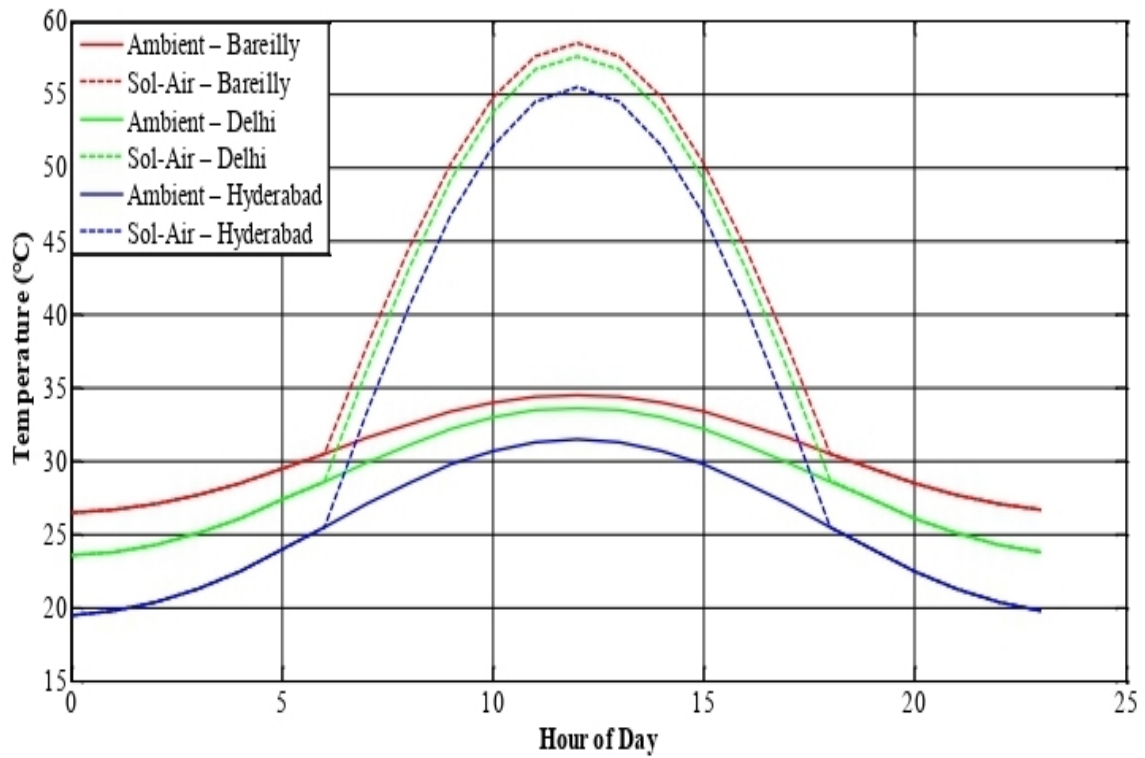
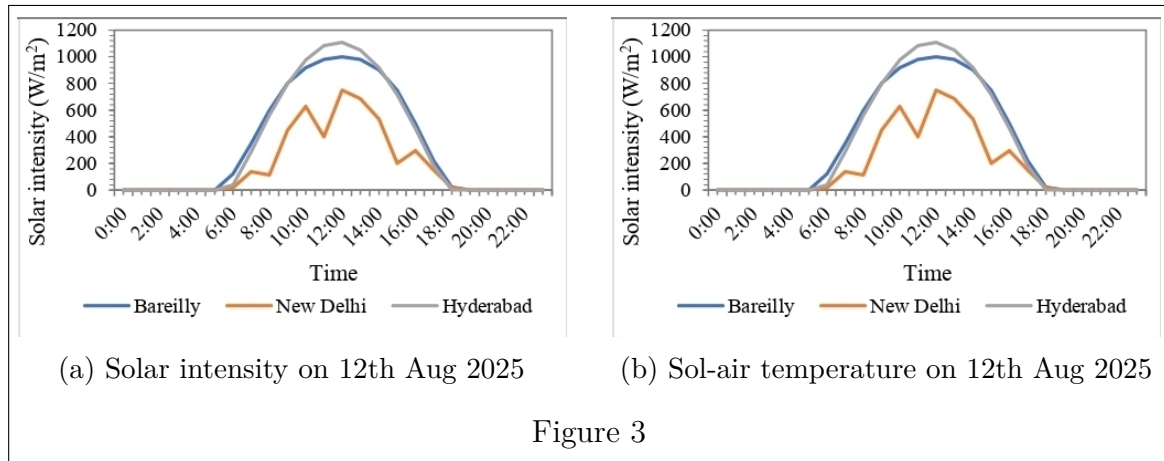
Table 2: Ambient temp., sol-air temp. and solar Intensity on 12-08-2025 at horizontal surface for Bareilly, New Delhi and Hyderabad

Location / time	Bareilly			New Delhi			Hyderabad		
	θ_{ab} (°C)	θ_{sab} (°C)	I (W/m ²)	θ_{ad} (°C)	θ_{sad} (°C)	I (W/m ²)	θ_{ah} (°C)	θ_{sah} (°C)	I (W/m ²)
0	26.6	26.60	0	32	29.89	0	22	20.47	0
1	26.4	26.40	0	32	29.89	0	21	19.49	0
2	26.2	26.20	0	31	28.91	0	22	20.47	0
3	26.2	26.20	0	31	28.91	0	22	20.47	0
4	26.4	26.40	0	30	27.93	0	22	20.47	0
5	26.6	26.60	0	30	27.93	0	22	20.47	0
6	27.0	27.20	120	30	28.72	18	22	21.77	37.02
7	27.8	57.55	350	30	34.01	139	22	30.59	289.13
8	28.5	79.50	600	27	29.89	112	23	41.11	561.43
9	29.2	97.20	800	27	32.00	450	24	50.38	798.19
10	29.7	107.90	920	27	32.00	627	23	55.64	976.71
11	30.3	113.60	980	27	31.00	400	23	59.35	1082.76
12	30.9	115.90	1000	27	31.00	750	23	60.25	1108.39
13	31.7	115.00	980	28	30.00	686	24	59.25	1051.69
14	32.4	108.90	900	28	30.00	533	26	56.50	916.88
15	32.5	96.25	750	28	30.00	200	26	49.41	714.23
16	31.9	74.40	500	28	30.00	297	25	39.55	460.63
17	31.4	50.10	220	28	27.00	150	25	29.89	184.83
18	30.8	32.50	20	28	27.00	16	23	21.46	0.10
19	30.3	30.30	0	27	27.00	0	23	21.46	0
20	29.7	29.70	0	27	27.00	0	22	20.47	0
21	29.3	29.30	0	27	27.00	0	22	20.47	0
22	28.9	28.90	0	27	28.00	0	22	20.47	0
23	28.5	28.50	0	27	28.00	0	22	20.47	0

Table 3: Monthly solar insolation and intensity of Bareilly, New Delhi and Hyderabad in 2024

Month	Bareilly		New Delhi		Hyderabad	
	Solar insolation (kWh/m ²)	Solar intensity (W/m ²)	Solar insolation (kWh/m ²)	Solar intensity (W/m ²)	Solar insolation (kWh/m ²)	Solar intensity (W/m ²)
January	60	2500.00	64	2666.67	131	5458.33
February	94	3916.67	88	3666.67	153	6375.00
March	137	5708.33	135	5625.00	172	7166.66
April	158	6583.33	150	6250.00	174	7250.00
May	169	7041.67	164	6833.33	168	7000.00
June	144	6000.00	120	5000.00	115	4791.66
July	95	3958.33	80	3333.33	108	4500.00
August	129	5375.00	117	4875.00	122	5083.33
September	109	4541.66	122	5083.33	137	5708.33
October	113	4708.33	104	4333.33	144	6000.00
November	77	3208.33	71	2958.33	135	5625.00
December	50	2083.33	50	2083.33	84	3500.00

The variation in solar intensity and sol-air temperature on 12th August 2025 for Bareilly, New Delhi and Hyderabad has been represented by Fig. 3. Comparative variation in air & sol-air temperature in Bareilly, Delhi and Hyderabad on 12th August 2025 has been represented in Fig. 4. Monthly global, beam and diffuse radiation for Bareilly, Delhi and Hyderabad have been taken from the site <https://power.larc.nasa.gov/api/temporal/hourly/point>.



Using these data for 12th Aug 2024, the solar irradiance on walls and roof may be evaluated by using the formulations for the tilted surface outlined in Duffie and Beckman [6] and Pawan Kumar [7] as follows:

Radiation intensity on tilted surface is

$$I_t^{(n)}(t) = I_{hb}^{(n)}(t)R_b^{(n)}(t) + I_{hd}^{(n)}(t)R_d^{(n)}(t) + I_{hg}^{(n)}(t)R_r^{(n)}(t) \quad (25)$$

where $R_b^{(n)}(t) = \frac{\cos \theta_i^{(n)}}{\cos \theta_z^{(n)}}$, $R_d^{(n)}(t) = \frac{1 + \cos \beta^{(n)}}{2}$, and $R_r^{(n)}(t) = \frac{\rho_g (1 - \cos \beta^{(n)})}{2}$ are known as the tilt factors for beam radiation, diffuse radiation, and reflected radiation respectively for the n^{th} wall/roof such that

$$\omega_s \text{ (in degree)} = 15 (t - 12) \quad (26)$$

$$\delta \text{ (in degree)} = 23.45 \sin\left(\frac{360(284 + N)}{365}\right) \quad (27)$$

where N is the number of calendar day and

$$\cos \theta_z^n = \cos \phi \cos \delta \cos \omega_s + \sin \delta \sin \phi \quad (28)$$

ϕ is the latitude of the building, δ is the solar declination angle, ω_s is the solar hour angle. The value of $\cos \theta_z^n$ for each wall/roof is constant but the angle of incidence θ_i^n ($0 \leq \theta_i^n \leq 90^\circ$) varies with the solar hour angle ω_s as follows:

For $\cos \omega_s > \frac{\tan \delta}{\tan \phi}$:

$$\begin{aligned} \cos \theta_i^n &= \cos \delta \cos \omega_s \sin \phi \cos \gamma \sin \beta - \sin \delta \cos \phi \cos \gamma \sin \beta + \cos \delta \sin \omega_s \sin \gamma \sin \beta \\ &\quad + \cos \beta \cos \phi \cos \delta \cos \omega_s + \cos \beta \sin \phi \sin \delta \end{aligned} \quad (29)$$

For $\cos \omega_s < \frac{\tan \delta}{\tan \phi}$:

$$\begin{aligned} \cos \theta_i^n &= -\cos \delta \cos \omega_s \sin \phi \cos \gamma \sin \beta + \sin \delta \cos \phi \cos \gamma \sin \beta + \cos \delta \sin \omega_s \sin \gamma \sin \beta \\ &\quad + \cos \beta \cos \phi \cos \delta \cos \omega_s + \cos \beta \sin \phi \sin \delta \end{aligned} \quad (30)$$

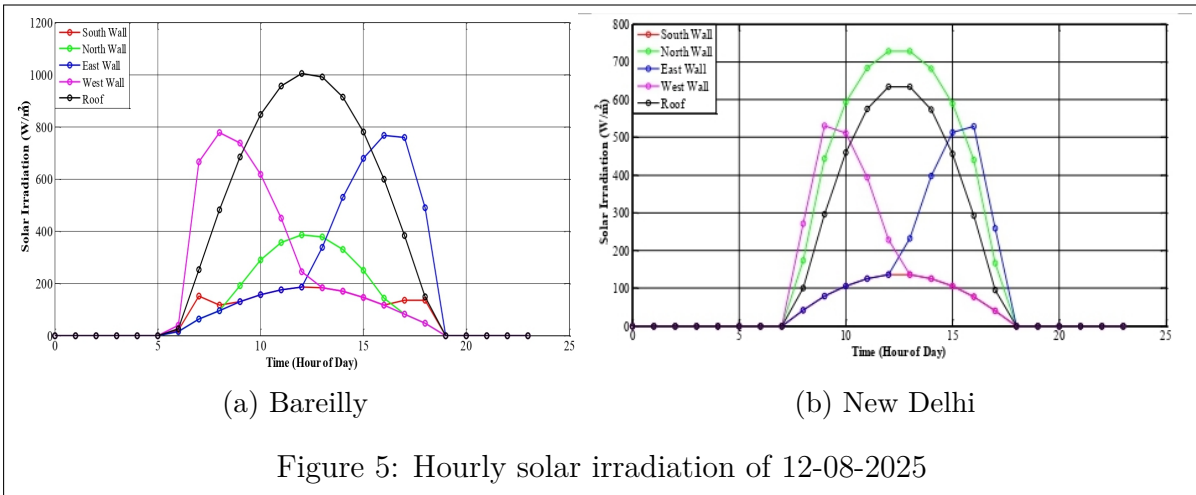
The angle of inclination β for each wall has been taken as 90° . The surface azimuth angle (γ) for the wall due south ($n = 1$), north ($n = 2$), east ($n = 3$) and west ($n = 4$) is 0° , 180° , -90° , and 90° , respectively. $N = 5$ represents the roof of the building. The computational results of solar intensity on the walls and roof for the Bareilly, New Delhi and Hyderabad regions for 12th August 2024 have been represented by Table 4.

Similarly computational results of solar intensity intensity on walls and roof in Bareilly, New Delhi and Hyderabad region for 12th Aug 2025 has been presented in Fig. 5 & Fig. 6 respectively.

Proceeding in this way, the graphical representation of monthly irradiation on the walls and roof in Bareilly, New Delhi and Hyderabad has been depicted in Fig. 7 and Fig. 8(a), respectively, and the total monthly solar irradiation of the walls and roof in Bareilly, New Delhi and Hyderabad has been represented in Fig. 8(b).

Table 4: Solar intensity on walls / roof in Bareilly, New Delhi and Hyderabad region for 12th Aug 2024

Time	Bareilly					New Delhi					Hyderabad				
	South	North	East	West	Roof	South	North	East	West	Roof	South	North	East	West	Roof
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1.33	48.93	175.27	1.33	0	5.72	104.03	392.28	5.72	0	3.71	2.58	9.57	3.71	0
6	39.48	146.24	748.98	39.48	24.8	45.64	140.98	768.42	45.64	3.8	36.27	171.90	732.36	36.27	0
7	79.85	103.63	805.81	79.85	251.7	85.50	94.42	803.54	85.50	217.5	79.79	173.42	800.16	79.79	201.8
8	178.26	115.94	754.20	115.94	480.8	197.20	120.63	740.24	120.63	448.3	118.52	161.51	749.19	118.52	450.8
9	279.56	145.03	633.73	145.03	683.6	295.01	148.44	611.25	148.44	655.6	149.84	149.52	627.87	149.52	673.1
10	350.16	165.09	462.61	165.09	845.4	360.89	167.00	434.02	167.00	823.8	201.11	170.60	455.58	170.60	851.8
11	384.23	174.73	257.00	174.73	954.7	389.45	175.01	224.74	175.01	941.2	224.85	180.31	248.76	180.31	974.2
12	379.10	173.28	173.28	312.79	1004.1	378.47	171.93	171.93	343.45	999.6	219.14	177.97	177.97	331.50	1032
13	335.16	160.84	160.84	511.03	990	328.80	157.97	157.97	537.05	995	184.42	163.76	163.76	526.98	1021.1
14	255.89	138.27	138.27	670.54	913.6	244.38	134.09	134.09	689.58	927.8	138.64	153.69	138.64	681.59	942.3
15	148.02	107.14	107.14	775.13	780	132.52	101.95	101.95	784.73	802.4	104.37	166.48	104.37	778.55	801.1
16	69.65	116.46	69.65	804.46	598.5	63.84	122.13	63.84	800.43	627.7	63.39	176.01	63.39	793.75	607.1
17	28.76	150.34	28.76	699.36	382	22.90	149.06	22.90	659.36	416	19.21	153.87	19.21	625.11	374.3
18	0	0	0	0	147.8	0	0	0	0	183.4	0	0	0	0	121.9
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



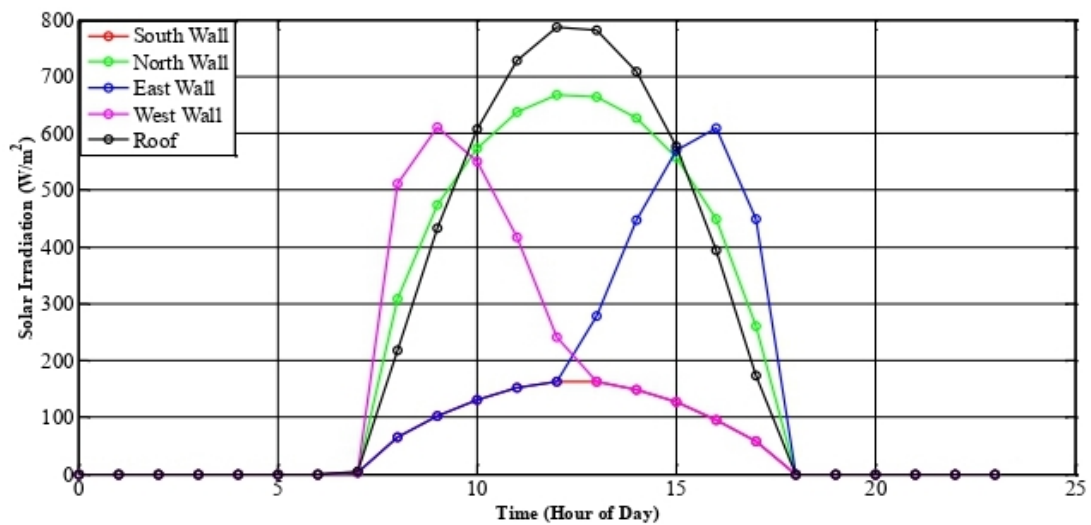


Figure 6: Hourly solar irradiation in Hyderabad of 12-08-2025

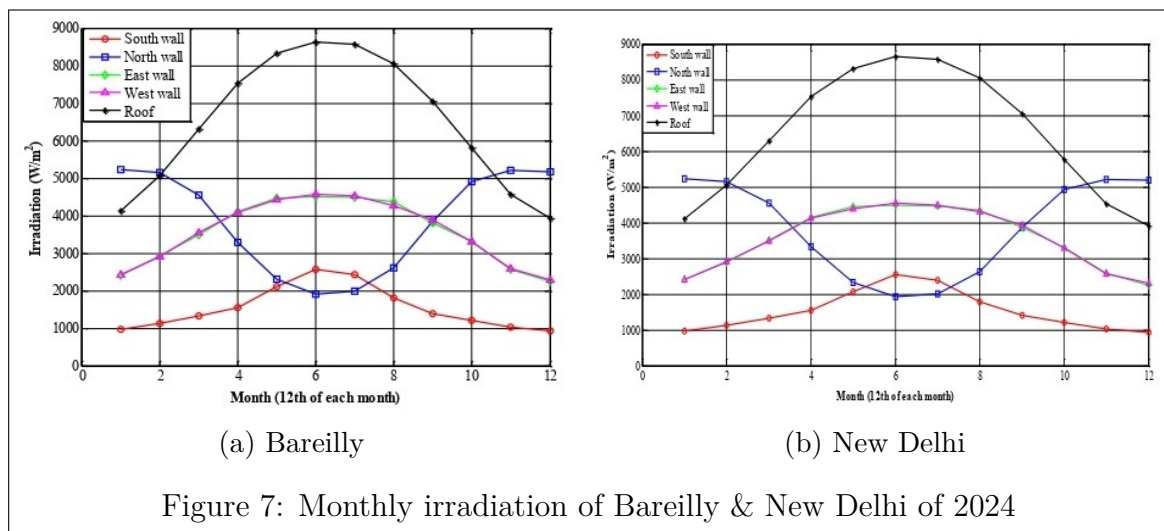


Figure 7: Monthly irradiation of Bareilly & New Delhi of 2024

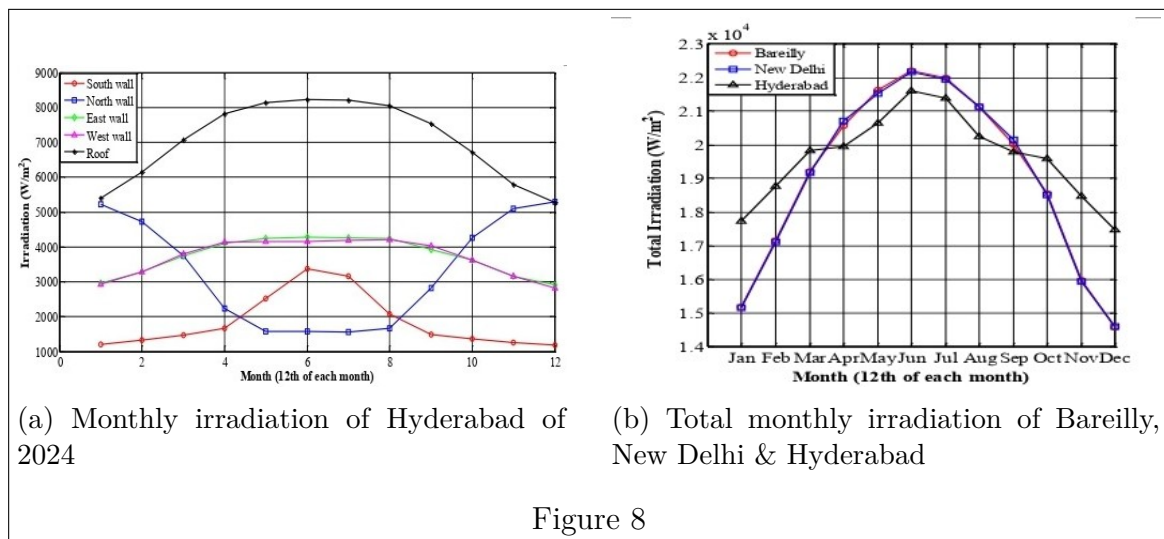


Figure 8

5 Mathematical modelling for solidification

Solidification process is the setting of concrete when water and cement undergo hydration reaction. It gives structural strength, stability and durability. It controls over shape and form. This research presents a comprehensive mathematical modelling framework to understand and predict the behavior of solidification phenomena [8, 9]. The solidification process in building application may be expressed as shown in Table 5.

Table 5: Solidification in building applications

Concrete and cement alternatives	Road and pavement construction
 • Geopolymer concrete • Solidification of calcium carbonate waste • Magnesium phosphate cement (MPC)	 • Solidified waste aggregates • Reclaimed asphalt pavement (RAP) with solidification • Soil-cement pavement
Energy-efficient and smart buildings	Hazardous and industrial waste management
 • Phase change materials (PCMs) • Solidification of sludge and wastewater residues	 • Encapsulation of toxic waste • Solidification of sludge and wastewater residues

For mathematical modelling of the solidification process, let us consider that the solidification in the mould be occupied by a semi-infinite half-space for $x > 0$. For $t > 0$, the phase change temperature distribution in liquid, solid and mould represented by θ_l , θ_s and θ_m respectively are governed by the following equations:

$$\rho_m c_m \frac{\partial \theta_m}{\partial t} = k_m \frac{\partial^2 \theta_m}{\partial x^2}, \quad -\infty < x < 0 \quad (31)$$

$$\rho_s c_s \frac{\partial \theta_s}{\partial t} = k_s \frac{\partial^2 \theta_s}{\partial x^2}, \quad 0 < x < a \quad (32)$$

$$\rho_l c_l \frac{\partial \theta_l}{\partial t} = k_l \frac{\partial^2 \theta_l}{\partial x^2}, \quad a < x < \infty \quad (33)$$

where (ρ_m, ρ_s, ρ_l) , (c_m, c_s, c_l) , (k_m, k_s, k_l) represent density, specific heat and thermal conductivity of mould, solid and liquid respectively.

Equations (32) and (33) may also be represented as:

$$\frac{\partial \theta_s}{\partial t} = \alpha_s^2 \frac{\partial^2 \theta_s}{\partial x^2}, \quad \frac{\partial \theta_l}{\partial t} = \alpha_l^2 \frac{\partial^2 \theta_l}{\partial x^2} \quad (34)$$

where,

$$\alpha_s^2 = \frac{k_s}{\rho_s c_s}, \quad \alpha_l^2 = \frac{k_l}{\rho_l c_l}$$

are diffusivity constants of solid and liquid melt respectively.

The physical properties of the mould and melt are such that

$$\rho_m c_m \gg \rho_s c_s \approx \rho_l c_l, \quad k_m \gg k_s \approx k_l$$

and there is no thermal resistance between the melt and the mould. The mould acts as an ideal heat sink such that

$$\theta_m(x, t) = \theta_0, \quad x < 0, t > 0$$

and the significant region of heat transfer is confined to the solidified melt such that

$$\left(\frac{\partial^2 \theta_s}{\partial x^2} \right)_{x=0} = h(\theta_s - \theta_0), \quad \theta_s(x, 0) = \theta_{ph} \quad (35)$$

where θ_{ph} represents the phase transformation temperature of solid melt at $t = 0$.

For $x = 0, t > 0$ we have:

$$k_s \frac{\partial \theta_s}{\partial x} = h(\theta_s - \theta_m), \quad \left(\frac{\partial \theta_s}{\partial x} \right)_{x=0} = h(\theta_s - \theta_0) \quad (36)$$

At the solid-liquid interface $x = a(t)$, we have

$$\theta_l(a, t) = \theta_s(a, t), \quad \theta_s(0, t) = \theta_m(0, t), \quad \rho_l L \frac{\partial a}{\partial t} = k_l \frac{\partial \theta_l}{\partial x} - k_s \frac{\partial \theta_s}{\partial x} \quad (37)$$

On solving set of equations (34) by taking Laplace transform of both sides, we get

$$p\bar{\theta}_s(x, p) - \theta_s(x, 0) = \alpha_s^2 \frac{d^2 \bar{\theta}_s(x, p)}{dx^2}, \quad p\bar{\theta}_l(x, p) - \theta_l(x, 0) = \alpha_l^2 \frac{d^2 \bar{\theta}_l(x, p)}{dx^2} \quad (38)$$

This set of equations may be written as:

$$\alpha_s^2 \frac{d^2 \bar{\theta}_s(x, p)}{dx^2} - p \bar{\theta}_s(x, p) = -\theta_s(x, 0), \quad \frac{d^2 \bar{\theta}_l}{dx^2} - \frac{p}{\alpha_l^2} \bar{\theta}_l = -\frac{\theta_{ph}}{\alpha_l^2} \quad (39)$$

Solution of the set of equations (35) is given by:

$$\bar{\theta}_s(x, p) = \frac{(\theta_0 - \theta_{ph})h}{p(\sqrt{p/\alpha_s^2} + h)} e^{-\sqrt{p/\alpha_s^2} x} + \frac{\theta_{ph}}{p}, \quad \bar{\theta}_l(x, p) = \frac{(\theta_0 - \theta_{ph})h}{p(\sqrt{p/\alpha_s^2} + h)} e^{-R_x \sqrt{p}} + \frac{\theta_{ph}}{p} \quad (40)$$

where

$$R_x = \frac{a}{\alpha_s} - \frac{a}{\alpha_l} + \frac{x}{\alpha_s}.$$

Now taking inverse Laplace transform of (40), we get

$$\begin{aligned} \theta_s(x, t) &= \theta_{ph} + (\theta_0 - \theta_{ph})h \left[\operatorname{erfc} \left(\frac{x}{2\alpha_s \sqrt{t}} \right) - e^{hx+h^2\alpha_s^2 t} \operatorname{erfc} \left(\frac{x}{2\alpha_s \sqrt{t}} + h\alpha_s \sqrt{t} \right) \right], \\ \theta_l(x, t) &= \theta_{ph} + (\theta_0 - \theta_{ph})h \left[\operatorname{erfc} \left(\frac{R_x}{2\sqrt{t}} \right) - e^{hx+h^2 t} \operatorname{erfc} \left(\frac{R_x + h\sqrt{t}}{2\sqrt{t}} \right) \right] \end{aligned} \quad (41)$$

Here we have taken $\operatorname{erfc}(f(t)) = 1 - \operatorname{erf}(f(t))$.

Since at the solid interface $x = a(t)$, temperature is continuous

$$\theta_l(a, t) = \theta_s(a, t).$$

In addition, release of the latent heat L by the solidified melt requires that

$$\rho_l L \frac{da}{dt} = k_l \frac{\partial \theta_l}{\partial x} - k_s \frac{\partial \theta_s}{\partial x} \quad (42)$$

Differentiating equation (42) with respect to x and finding $\frac{\partial \theta_s}{\partial x}$ at $x = a$, then substituting the values of $\frac{\partial \theta_s}{\partial x}$ and $\frac{\partial \theta_l}{\partial x}$ at $x = a$ in equation (42), we get the following relation between a and t :

$$a^2 = 4\lambda^2 \alpha_s t \quad (43)$$

where $\alpha_s = \frac{k_s}{c_s \rho_s}$ is the diffusivity of the solid melt and λ is a constant.

The above analysis shows that a^2 is proportional to t . This solution shows the relation between thickness of solid melt (a) and the time (t). When the cooling starts, the thickness of solid melt parabolically increases with respect to time. Accurate modelling must reconcile the microscopic scale of dendritic growth and grain refinement with the macroscopic scale of heat flow and casting geometry. Moreover, model calibration requires high-quality thermophysical property data, and predictive accuracy can be hindered by uncertainties in boundary conditions, thermal gradients, and kinetic parameters. Addressing these challenges demands a comprehensive modelling approach — one that integrates theoretical formulation, numerical simulation, experimental verification, and uncertainty quantification.

6 Mathematical modelling for periodic heat transfer through window

Periodic heat transfer through a window is the sum of heat conduction through window glazing and the solar heat gain through window, given by

$$Q_{wd}(t) = Q_c(t) + Q_s(t) \quad (44)$$

where $Q_{wd}(t)$ represents the window heat flux, $Q_c(t)$ is the conduction through window glazing, and $Q_s(t)$ is the solar heat gain through window, given by

$$Q_c(t) = U_w A_w (\theta_{out}(t) - \theta_{in}(t)) \quad (45)$$

$$Q_s(t) = A_w S_{hgc} I_w(t) \quad (46)$$

where U_w (W/m²K) is the overall heat transfer coefficient of the window, A_w (m²) is the area of the window, $\theta_{out}(t)$, $\theta_{in}(t)$ are the outside and inside air temperatures, S_{hgc} is the solar heat gain coefficient (dimensionless), and $I_w(t)$ (W/m²) is the incident solar radiation on the window.

7 Mathematical modelling for heat flux by auxiliary heating source

Almost all the electrical energy used by a room heater is converted into heat. Some heat is lost with exhaust gases and incomplete combustion of a gas furnace/boiler, while a heat pump moves heat from outdoors to indoors.

The mathematical formulation for a heater delivering power $Q_h(t)$ is given by:

$$Q_{heater}(t) = \eta_h Q_h(t) \quad (47)$$

where $Q_h(t)$ (W) is the rated heat input. It may be constant or controlled by heater efficiency η_h , which tells us how much of the input energy of a heater actually becomes useful inside the room. It always lies between 0 and 1. Mathematically,

$$\eta_h = \frac{\text{useful heat output to the room}}{\text{energy input to the heater}} \quad (48)$$

In steady-state control, we have

$$Q_{heater}(t) = \text{Heat losses through envelope} + \text{Ventilation} \quad (49)$$

8 Heat Transfer by Furnishings

Furnishings act as lumped thermal masses. It may be defined by:

$$Q_f(t) = m_f c_p \frac{d\theta_f(t)}{dt} \quad (50)$$

where m_f (kg) is the mass of furnishings, c_p (J/kg·K) is the specific heat capacity, and $\theta_f(t)$ (°C) is the furniture surface temperature.

Coupled with convective heat transfer to room air, we have

$$q_c(t) = h_f A_f (\theta_f(t) - \theta_{\text{air}}(t)) \quad (51)$$

where h_f (W/m²·K) is the convective heat transfer coefficient and A_f (m²) is the exposed surface area of furniture.

9 Heat loss/gain due to ventilation and infiltration

The sensible load due to airflow is given by

$$Q_v(t) = \dot{m}_a(t) C_{pa} (\theta_{\text{out}}(t) - \theta_{\text{in}}(t)) \quad (52)$$

where $\dot{m}_a(t)$ (kg/s) is the mass flow rate of outdoor air and $C_{pa} \approx 1005$ J/kg · K is the specific heat of air.

If expressed in terms of air changes per hour (ACH):

$$\dot{m}_a(t) = \rho_a V_r \frac{\text{ACH}}{3600} \quad (53)$$

where $\rho_a \approx 1.2$ kg/m³ is the density of air and V_r (m³) is the room volume.

The latent load (if humidity difference is considered) is given by

$$Q_{\text{lat}}(t) = \dot{m}_a(t) h_{fg} (W_{\text{out}}(t) - W_{\text{in}}(t)) \quad (54)$$

where h_{fg} (J/kg) is the latent heat of vaporization, and $W_{\text{out}}(t), W_{\text{in}}(t)$ (kg water/kg dry air) are humidity ratios.

10 Summary and Conclusion

This research presents a comprehensive mathematical modelling framework to analyze heat transfer and phase change during solidification processes in sustainable building applications. By integrating thermodynamic principles, fluid dynamics, and phase-change dynamics, the study develops predictive models for heating and cooling loads, ground

temperature distribution, heat conduction through walls and roofs, window heat transfer, auxiliary heating, furnishings, and ventilation effects. Computational investigations, including case studies for Bareilly, New Delhi, and Hyderabad, highlight variations in ambient and ground temperatures, solar irradiation, and their impacts on energy performance. The solidification modelling further explains hydration, crystallization, and strength development in materials such as concrete, geo-polymer cement, and phase change materials, emphasizing their role in enhancing structural durability and energy efficiency. The findings confirm that advanced mathematical modelling, combined with sustainable materials and technologies like BIM, can significantly optimize resource use, reduce energy consumption, and minimize environmental impact, ultimately paving the way for greener, more efficient, and durable building systems.

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