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ON SOLVING VOLTERRA INTEGRO-DIFFERENTIAL EQUATION USING NEW MODIFIED ADOMIAN DECOMPOSITION METHOD

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Abstract

A novel and reliable semi-analytical method to solving Volterra integro-differential equations is presented in this study. A new modified Adomian Decomposition Method is described in this paper. By presenting some examples and plotting the error function and comparison between the exact and approximate solutions, we show the ability, simplicity, and effectiveness of this method.

Math. Subject Classification: 65R20, 65R99, 45G99

Key Words and Phrases: Volterra integro-differential equation, modified Adomian decomposition method (ADM), new modified Adomian Decomposition method

1. Introduction

The Adomian Decomposition Technique (ADM), developed by Manafianheris [1], separates an equation into linear and nonlinear components. Equations involving nonlinear functions are solved using this technique. To provide solutions that take the form of recursive series, Adomian polynomials are used. Saray [2], discovered that problems involving these equations may be reduced to a set of algebraic equations by using a method for solving Volterra integro-differential equations. The solution proved to be more successful than similar strategies that required less computing work. Olayiwola, et al. [3] described how the modified variational iteration approach converges to the precise solution after an iteration for the solution of the class of initial and boundary value problems. As a result, the approach is effective and trustworthy for solving bantu-type differential equations. Alaje, et al. [4] discovered that an analytical strategy of modified initial guess Homotopy perturbation is used to solve the Korteweg-de vries equation. The Banach fixed point theorem was used to demonstrate the method's convergence as well as a sequence of arbitrary orders.

The multi-wavelets Galerkin method may be used to tackle second-order problems that are both linear and nonlinear. Volterra integro-differential equations are resolved using the operational integration matrices and the wavelet transform matrix. Ibrahim, et al. [5] looked at the original solution to the second-order nonlinear Fredholm integro-differential equation, which involved applying the Simpson method to turn the Fredholm IDE into a collection of nonlinear algebraic equations. Siweilam [6] created the Variational Iteration Methodology (VIM), which resolves the bulk of difficulties encountered in computing Adomian polynomials using the Adomian decomposition method, to solve integro-differential equations, which are difficult to solve analytically.

According to Asiya, et al. ([7], [8], [13]), a comparison of numerical and analytical solutions to Fredholm integro-differential equations is successful when utilizing a modified Adomian decomposition method. According to Asiya, et al. [9], the system of general Fredholm Integro-differential (FID) equations was solved using the Direct Computation Method (DCM). It is crucial to note that additional techniques should be used for systems with different or separate kernels. While any form of kernel can be utilised with the DCM to solve the system of general Fredholm Integro-differential (FID) equations and also according to Asiya, et al. [10], using the Adomian decomposition method and Modified Adomian decomposition method, we looked at the approximation solution of the nonlinear Volterra-Fredholm integro-differential equation. The proposed techniques have been successfully used to solve the integro-differential equation and its system, and they require a great deal less computation time than conventional approaches. Another benefit is that it has been shown that theoretical conclusions, such as convergence and the uniqueness of the technique suggester's answers to the problem at hand, are accurate. Furthermore, only a limited number of times are required to provide a successful outcome. Also, according to Asiya, et al. [11], the Series Solution Method is a powerful technique that is capable of handling higher-order linear or non-linear Volterra Integro-differential equations. In [12], authors implemented the technique suggested by HPM, ADM, MADM, and VIM for obtaining the approximate solution of linear fuzzy integro-differential equations along with a fuzzy parametric form with suitable initial conditions and source functions. In this study, the fuzzy MADM's left bound of errors at $z = 0.5$ is competitive with the fuzzy ADM, HPM, and VIM. However, when it came to the remaining errors, fuzzy MADM outperformed fuzzy ADM.

The physical phenomena in engineering may all be solved using integral-differential equations. The third-order derivatives of unknown functions are contained in integro-differential equations (IDEs) known as third-order IDEs. Haar functions are employed in integro-differential equations, both linear and nonlinear, to approximate the third-order derivative. Lower-order derivatives and the solution to the mystery are produced through integration. Several partial differential equations are both linear and nonlinear that Chen [14] and Rohaninasab, et al. [15] have been utilized to solve. It has been proven to be a successful technique for getting numerical solutions. The Legendre collocation spectral method may be used to solve high-order linear Volterra-Fredholm integro-differential equations under mixed situations. Nonlinear Volterra integral and integro-differential equations may be used to study a variety of scientific topics, including heat transport, the spread of infectious illnesses, semiconductor neutron diffusion, and others (see [16]). Non-orthogonal polynomials can also be decomposed using the Laplace Adomian approach. The Adomian decomposition technique is a summation of an infinite convergent series without any restrictive constraints. When solving functional equations that are no longer valid, the Laplace-Adomian decomposition method combines two efficient techniques. The modified Laplace Adomian Decomposition Technique (LADM), which uniformly distributes the source function before performing Laplace Adomian Decomposition, is used to solve the Volterra integral and integro-differential equations. To estimate the solutions of nonlinear partial differential equations, the Laplace transform employs the decomposition method.

Many researchers have explored third-order integro-differential equations, notably the nonlinear variety in closed form. The answer is then integrated to acquire the lower-order derivatives, while the trapezoidal approach is used to derive the unknown function itself. The power of series and canonical polynomials is used to approximate the largest derivatives in the topics studied. The polynomial issues presented by Olayiwola and Kareem [17] may be solved analytically using a variety of approaches; however, some of these approaches are challenging and call for several iterations that may be challenging to solve and take a long time to arrive at an approximation. This method is applied in numerous fields, including engineering, economics, chemical kinetics, fluid mechanics, etc. Olayiwola, et al. [18] explained how to develop Maple code for the method and simulation of the generalized Burger-equation Fisher's solution. With less computation, the results were produced. The Homotopy perturbation method was used to solve Integro-differential equations with two-point boundary conditions, and the numerical results obtained proved to be a very accurate algorithm for solving problems of linear Fourth-order Integro-differential equations.

The Laplace transform employs the decomposition method to approximate the solutions of nonlinear partial differential equations. According to Amin, et al. [19], many scholars have explored third-order integro-differential equations, especially the nonlinear variety in closed form. The answer is then integrated to create the lower-order derivatives, while the trapezoidal approach is used to derive the unknown function itself. The power of series and canonical polynomials approximate the largest derivatives in the topics studied. This method is utilized in numerous areas, including engineering, economics, chemical kinetics, fluid mechanics, etc.

In this paper, we introduce a new modification of decomposition method and make further progress beyond the achievements made so far in this regard. Several examples concerning Volterra integro-differential equations are tested, and the results suggest that this new idea proposes a promising tool for the computation of integro-differential equations both linear and nonlinear.

2. **Description of the new modified Adomian Decomposition Method**

Since the beginning of the 1980s, the Adomian decomposition method has been applied to a wide class of integro-differential equations. To illustrate the procedure, consider the following linear Volterra integro-differential equations of the second kind given by

$$\phi^n(z) = g(z) + \lambda \int_0^z K(z,t)\phi(t)dt$$

and the non-linear Volterra integro-differential equation of the second kind as follows:

$$\phi^n(z) = g(z) + \lambda \int_0^z K(z,t)G(\phi(t))dt,$$

$$\phi^n(z) = g(z) + \lambda \int_0^z K(z,t)(L(\phi(t)) + N(u(t)))dt,\ \lambda \neq 0, \tag{1}$$

where $\phi^n(z)$ indicate the nth derivative of $\phi(z)$ such as $\phi^n = \frac{d^n\phi}{dz^n}$, initial conditions $\phi^{(p)}(0) = d_p; 0 \leq p \leq (n-1)$ such as $\phi(0)$, $\phi^{'}(0)$, $\phi^{''}(0)$, $\ldots$, $\phi^{n-1}(0)$, the function $g(z)$ are given real valued functions, $K(z,t)$ is the kernel of integral equation, λ is suitable constant and d_p are constants that define the initial conditions. The function $G(\phi(z))$ is a non-linear function of $\phi(z)$ such as $\phi^2(z)$, $\sin(\phi(z))$, and $e^{\phi(z)}$. Because the equation in (1) combines the differential operator and the integral operator, it is necessary to define initial conditions for the determination of the particular solution $\phi(z)$ of the nonlinear Volterra integro-differential equation.

In the decomposition method, we usually express the solution of (1) in a series form defined by

$$\phi(z) = \sum_{i=0}^{\infty} \phi_i(z). \tag{2}$$

Moreover, the decomposition method identifies the nonlinear term $N(\phi(z))$ by the decomposition series

$$N(\phi(z)) = \sum_{i=0}^{\infty} A_i(z), \tag{3}$$

where $A_j(z)$ is the so-called Adomian polynomials, which can be evaluated by the following formula:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i\phi_i\right)\right]_{\lambda=0}, \tag{4}$$

where $n = 0, 1, 2, 3, \ldots$ and $i = 2, 3, 4, \ldots$.

Substituting (2) and (3) into both sides of (1) gives

$$\sum_{i=0}^{\infty}\phi_i^n(z) = g(z) + \lambda\int_0^z K(z,t)\left[L\left(\sum_{i=0}^{\infty}\phi_i(t)\right) + \sum_{i=0}^{\infty}A_i(t)\right]dt. \tag{5}$$

In the following, we outline the basic feature of the standard decomposition method and the modified decomposition method.

The Standard Adomian Decomposition Method (SADM)

By the standard decomposition method, the components $\phi_0(z)$, $\phi_1(z)$, $\phi_2(z)$, $\ldots$ of the solution $\phi(z)$ of (1), are completely determined in the following recurrence manner:

$$\phi_0(z) = g(z) \tag{6}$$

$$\phi_{i+1}^n(z) = \lambda\int_0^z K(z,t)(L(\phi_i) + A_i)dt, \quad i \geq 0 \tag{7}$$

Having determined the components $\phi_0(z)$, $\phi_1(z)$, $\phi_2(z)$, $\ldots$ the solution $\phi(z)$ in a series form defined by (2) follows immediately.

The Modified Adomian Decomposition Method (MADM)

The standard decomposition method by Adomian ([7], [8]) was modified by Wazwaz [20]. The modification is based on the assumption that the function $g(z)$ can be divided into two parts, namely, $g_0(z)$ and $g_1(z)$. Under this assumption we set

$$g(z) = g_0(z) + g_1(z).$$

Accordingly, a slight variation was proposed only for the components $\phi_0(z)$ and $\phi_1(z)$. The suggestion was that only the part $g_0(z)$ will be assigned to the zeroth component $\phi_0(z)$, whereas the remaining part $g_1(z)$ will be combined with the other terms given into (7) to define $\phi_1(z)$. Consequently, the modified recursive relation

$$\begin{aligned}
\phi_0(z) &= g_0(z),\\
\phi_1(z) &= g_1(z) + \lambda \int_0^z K(z,t)(L(\phi_0) + A_0)dt,\\
\phi_2(z) &= \lambda \int_0^z K(z,t)(L(\phi_1) + A_1)dt,\\
&\vdots\\
\phi_{i+1}(z) &= \lambda \int_0^z K(z,t)(L(\phi_i) + A_i)dt, \quad i \geq 0,
\end{aligned}$$

was developed.

This Modified Decomposition Method is a lower order comparison of the Adomian Decomposition Method. The Modified Adomian Decomposition Method gives exact solutions that can be derived from using two iterations only for $\phi_0(z)$ and $\phi_1(z)$ ([8]-[10]).

It is important to note here that there were some conclusions made in ([20], [21]). First, the slight variation in reducing the number of terms of ϕ_0 will result in a reduction of the computational work and will accelerate the convergence. Second, this slight variation in the definition of the components ϕ_0 and ϕ_1 may provide the solution by using two iterations only. Third, there is no need sometimes to evaluate the so-called Adomian polynomials required for the nonlinear equations.

The New Modified Adomian Decomposition Method (NMADM)

The modification was carried out by decomposing the source term function into series of the form (1), and the new recursive relation was obtained as the theoretical aspect of the method:

$$
\begin{aligned}
\phi_0(z) &= g_0(z), \\
\phi_1(z) &= g_1(z) + g_2(z) + \lambda \int_0^z K(z,t)(L(\phi_0(t)) + A_0)dt, \\
\phi_2(z) &= g_3(z) + g_4(z) + \lambda \int_0^z K(z,t)(L(\phi_0(t) + \phi_1(t)) + A_1)dt, \\
&\vdots \\
\phi_{i+1}(z) &= g_{2(i+1)}(z) + g_{2(j+1)-1}(z) + \lambda \int_0^z K(z,t)(L(\phi_i(t) \\
&\quad + \phi_{i-1}(t)) + A_1)dt.
\end{aligned} \tag{8}
$$

In case of non-linear, the newly modified Adomian decomposition method (NMADM) accelerates the convergence of the solution (MADM) faster than Standard Adomian Decomposition Method (SADM). Assuming that the nonlinear function is $N(\phi(z))$ can be evaluated by using the equation(4). Therefore, below are few of Adomian polynomials:

$$
\begin{aligned}
A_0 &= N(\phi_0), \\
A_1 &= \phi_1 N^{'}(\phi_0), \\
A_2 &= \phi_2 N^{'}(\phi_0) + \frac{1}{2!}\phi_1^2 N^{''}(\phi_0), \\
A_3 &= \phi_3 N^{'}(\phi_0) + \phi_1\phi_2 N^{''}(\phi_0) + \frac{1}{3!}\phi_1^3 N^{'''}(\phi_0), \\
A_4 &= \phi_4 N^{'}(\phi_0) + \left(\frac{1}{2!}\phi_2^2 + \phi_1\phi_3\right) N^{''}(\phi_0) + \frac{1}{2!}\phi_1^2\phi_2 N^{'''}(\phi_0) \\
&\quad + \frac{1}{4}\phi_1^4 N^{(iv)}(\phi_0).
\end{aligned}
$$

Two important observations can be made here. First, A_0 depends only on ϕ_0, A_1 depends only on ϕ_0 and ϕ_1, A_2 depends only on ϕ_0, ϕ_1, and ϕ_2, and so on.

Secondly, substituting these $A_i's$ in (8) gives:

$$
\begin{aligned}
N(\phi) &= A_0 + A_1 + A_2 + A_3 + \ldots \\
&= N(\phi_0) + (\phi_1 + \phi_2 + \phi_3 + \ldots)N'(\phi_0) \\
&+ \frac{1}{2!}(\phi_1^2 + 2\phi_1\phi_2 + 2\phi_1\phi_3 + \phi_2^2)N''(\phi_0) \\
&+ \frac{1}{3!}(\phi_1^3 + 3\phi_1^2\phi_3 + 6\phi_1\phi_2\phi_3 + \ldots)N'''(\phi_0) + \ldots \\
&= N(\phi_0) + (\phi - \phi_0)N'(\phi_0) + \frac{1}{2!}(\phi - \phi_0)^2 N''(\phi_0) + \ldots .
\end{aligned}
$$

3. Results and discussions

3.1. *Results.* Some research results are given in 3 different cases of examples.

EXAMPLE 3.1. Consider the first order linear Volterra integro-differential equation:

$$\phi'(z) = 1 - \int_0^z \phi(t)dt, \quad 0 \le z, \quad t \le 1$$

with initial condition $\phi(0) = 0$ and exact solution is $\phi(z) = \sin z$.

TABLE 1. Comparison of numerical results for Example 1.

x	**Exact**	**MADM**	**NMADM**
0	0	0	0
0.1	0.099833416646828	0.099833416646827	0.099833416646828
0.2	0.198669330795061	0.198669330795060	0.198669330795061
0.3	0.295520206661340	0.295520206661340	0.295520206661340
0.4	0.389418342308651	0.389418342308650	0.389418342308651
0.5	0.479425538604203	0.479425538604203	0.479425538604203
0.6	0.564642473395035	0.564642473395035	0.564642473395035
0.7	0.644217687237691	0.644217687237691	0.644217687237691
0.8	0.717356090899523	0.717356090899523	0.717356090899523
0.9	0.783326909627483	0.783326909627483	0.783326909627483
1.0	0.841470984807897	0.841470984807897	0.841470984807897

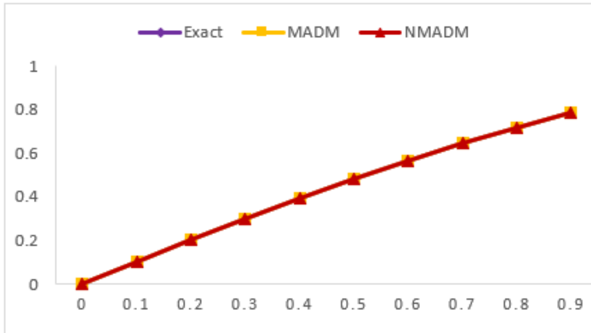

FIGURE 1. Comparison curve results for Example 1.

EXAMPLE 3.2. Consider the second order linear Volterra integro-differential equation:

$$\phi^{''}(z) = 1 + \int_0^z (z-t)\phi(t)dt,$$

with initial conditions $\phi(0) = 1$, $\phi^{'}(0) = 0$ and exact solution is $\phi(z) = \cosh z$.

TABLE 2. Comparison of numerical results for Example 2.

x	Exact	MADM	NMADM
0	1.000000000000000	1.000000000000000	1.000000000000000
0.1	1.005004168055804	1.005004168055804	1.005004168055804
0.2	1.020066755619076	1.020066755619076	1.020066755619076
0.3	1.045338514128861	1.045338514128861	1.045338514128861
0.4	1.081072371838455	1.081072371838455	1.081072371838455
0.5	1.127625965206381	1.127625965206381	1.127625965206381
0.6	1.185465218242268	1.185465218242268	1.185465218242268
0.7	1.255169005630943	1.255169005630943	1.255169005630943
0.8	1.337434946304845	1.337434946304845	1.337434946304845
0.9	1.433086385448775	1.433086385448775	1.433086385448775
1.0	1.543080634815244	1.543080634815244	1.543080634815244

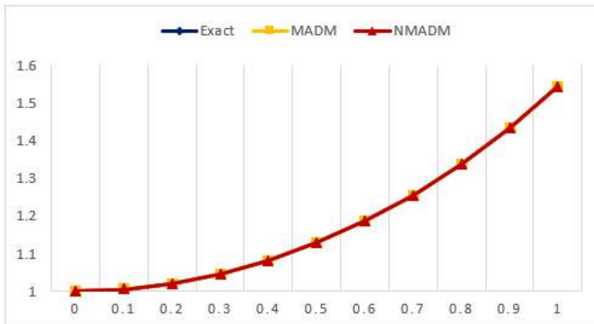

FIGURE 2. Comparison curve results for Example 2.

EXAMPLE 3.3. Consider the third order linear Volterra integro-differential equation:

$$\phi^{'''}(z) = 1 - \frac{1}{2}z^2 + \int_0^z \phi(t)dt,$$

with initial conditions $\phi(0) = 1$, $\phi(1) = e + 1$, $\phi^{'}(0) = 2$, $\phi^{'}(1) = e + 1$ and exact solution is $\phi(z) = e^z + x$.

TABLE 3. Comparison of numerical results for Example 3.

x	Exact	MADM	NMADM
0	1.000000000000000	1.000000000000000	1.000000000000000
0.1	1.205170918075648	1.205170918075648	1.205170918075648
0.2	1.421402758160170	1.421402758160170	1.421402758160170
0.3	1.649858807576003	1.649858807576003	1.649858807576003
0.4	1.891824697641270	1.891824697641270	1.891824697641270
0.5	2.148721270700128	2.148721270700128	2.148721270700128
0.6	2.422118800390509	2.422118800390509	2.422118800390509
0.7	2.713752707470476	2.713752707470476	2.713752707470476
0.8	3.025540928492468	3.025540928492468	3.025540928492468
0.9	3.359603111156950	3.359603111156950	3.359603111156950
1.0	3.718281828459046	3.718281828459046	3.718281828459046

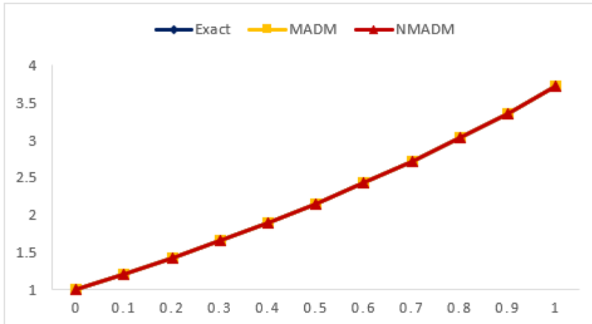

FIGURE 3. Comparison curve results for Example 3.

3.2. *Discussion.* This research work has introduced a new approach to the modification of Adomian Decomposition Method. This new method applies effectively to the solution of Volterra Integro-differential equations. The approximate solutions of three numerical examples obtained with the help of MADM and NMADM, in Tables 1-3, respectively. From the numerical results, it is clear that the NMADM is more efficient and accurate. The graphical comparison of exact and approximate solutions is shown in Figures 1-3, respectively.

In Examples 2 and 3, both MADM and NMADM can fulfil the exact solution effectively but in example 1, it is of certain difficulty to obtain the exact solution through the first finite terms of the series solution when using MADM. Whereas, ADM needs quite a little computational work to obtain the exact solution. If the exact solution exists in the zeroth component, MADM needs extensive workload for the suitable choice of $\phi_0(z)$ and $\phi_1(z)$, while the new modification can proceed in a fixed manner. The proposed method converges faster. Also, selection of the Taylor series expansion of the source term needs to be of high order to make the selection of the Taylor series expansion of the source term. The accuracy is also improved through an increase in the selection of terms of the Taylor series expansion. The result obtained compared well with the exact and in most cases, they converge directly to the exact in low number of iterations. It is therefore worthy to state that the method is elegant and sufficiently applicable to the solution of Volterra integro-differential equations.

4. **Conclusion**

This study developed a new method for modifying the Adomian Decomposition Technique. The Volterra Integro-Differential Equation is easily solved using this novel approach. The Taylor series extension of the source word must be chosen with care to broaden the selection as much as feasible. To boost the convergence tendency, we broaden the Taylor series of the source term with additional options. The proposed method converges more quickly to exact than existing methods. The idea has been shown to be computationally efficient in applying the proposed technique to several linear Volterra integro-differential equations. Also, through the examples, it is seen that the new modification suits for those integro-differential equations where the exact solution appears as part of the zeroth term. It is also interesting to point out that one can obtain the approximate solution of integro-differential equations as mentioned in [10-12] simply by slightly varying the modification of this work. Also, this method is useful for finding an accurate approximation of the exact solution.

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Conflicts of interest.

The authors declare no conflicts of interest regarding the publication of this paper.

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