International Journal of Applied Mathematics

Volume 37 No. 1 2024, 21–28
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)
doi: http://dx.doi.org/10.12732/ijam.v37i1.2

INTEGRAL CONJUGATION CONDITIONS FOR A DISCONTINUOUS FILTRATION FLOW VIA A GEOBARRER IN THE CASE OF ITS BIOCOLMATATION

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Abstract

In the article, modified conjugation conditions of non-ideal contact have been derived for the filtration rate when passing through a thin porous geo-barrier. Their feature is taking into account the influence on the change in the porosity of the geo-barrier by the dynamic bioclamation processes. Accounting for this effect leads to the appearance under conjugation conditions of time derivatives on microbial biomass concentration. As a result, the filtration flow of the pore fluid may be discontinuous when passing through the geo-barrier. The case of conjugation conditions has been considered when the concentration of microorganisms over the thickness of the inclusion can be approximated by a linear dependence.

MSC 2020: 35K61, 35Q35, 35Q92

Key Words and Phrases: porous medium, geochemical barrier, conjugation condition, biocolmatation

Received: July 26, 2023

1. Introduction

An important problem today is household waste and, apparently, the tasks of designing, building, and functioning of their storage facilities [1]. One of the design elements of waste storage facilities is geo-barriers [2]. The importance of geo-barriers is justified by their functions - preventing the spread of harmful substances outside the waste storage facility that can cause soil and water pollution [3]. Such geo-barriers may consist of recycled construction waste (crushed granite and concrete) without the use of artificial geomembranes [4]. However, the main structural materials of geobarriers are clay soils (most often bentonites) and artificial geomembranes [5].

Waste storage is associated with the danger of soil and groundwater contamination with organic chemicals [5]. The presence of such substances in a porous environment contributes to the intensification of the development of microorganisms. If we continue to talk about the construction of mathematical models of interconnected processes in environments with geo-barriers, then the development of microorganisms changes the characteristics of hydraulic conductivity of porous media [6]. In the work [7], the survey of natural experiments is given, which show how that the development of microorganisms in the pores of the porous core influence the characteristics of the hydroconductivity of the core itself. The paper [8] investigates the effect of bioclogging on the change in the porosity of the medium, and on its hydraulic conductivity in the two-dimensional case. These dependencies require both the modification of the filtration equations and the spread of pollution in environments with geo barriers, as well as the conjugation conditions on the geo barriers themselves. Thus, in the work [5] when passing through a geomembrane, the assumption is made about the continuity of pollution flows, but at the same time, a break in the function of the concentration of pollutants is possible. Meanwhile, the phenomenon of changes in hydraulic conductivity of the geobarrier depending on bioclogging is not taken into account, although field experiments will indicate the importance of such a phenomenon. Examples of modifications of conjugation conditions in case of non-linear dependencies of geobarrier parameters on physical and chemical influences are given in works [9, 10, 11, 12].

In the works [13, 14] mathematical models of filtration and filtration consolidation in a porous environment with fine inclusions were built, considering the effect of bioclogging. The first paper takes into account the dynamics of biomass based on the Monod equation, and the second one takes into account the diffuse equation for the development and spread of microorganisms. When deriving the conjugation conditions for heads, the dependence of the filtration coefficient on porosity and, accordingly, on biomass is considered. However, it does not take into account the dynamics of changes in porosity over time in accordance with the method of modeling interconnected processes in porous media, which is given in works [15]. In the works [17, 16], this dynamic change in porosity is considered in the equations of soil filtration and compaction in case of the development of microorganisms. However, a homogeneous soil massif without geo barriers was considered there. Therefore, the aim of this article is to derive the conjugation conditions for the filtration flow through the geo barrier, considering the dynamic change in the porosity of the geo barrier material from the biomass of microorganisms.

2. Conjugation condition taking into account dynamic bioclogging

Consider a non-deformable clay geobarrier of thickness d with variable porosity σ. The change in porosity is determined by the change in the concentration of biomass $B = B(\xi, t)$. That is,

$$\sigma = \sigma\left(\xi, B\right).$$

We derive the conjugation condition on the basis of the equation of the discontinuity of the liquid component of the soil [15] (let us consider the case of a completely saturated porous medium)

$$\frac{d\left(\rho_p \sigma\right)}{dt} + \frac{\partial}{\partial \xi}\left(\rho_p u\right),\ \xi \in (0; d)\,,$$

where ρ_p is pore fluid density; u is filtration rate.

Since geobarriers located at a considerable distance from pollution sources are characterized by low concentrations of pore chemical solution, than we will consider the density of the pore liquid to be constant $\rho_p = const$. Thus, in the cross-section of a thin clay geobarrier, taking into account the implementation of Darcy's law, we have the following filtration problem

$$\frac{\partial \sigma}{\partial B}\frac{\partial B}{\partial t} = \frac{\partial}{\partial \xi}\left(k_h\left(\sigma\right)\frac{\partial h}{\partial \xi}\right) = 0,\ 0 < \xi < d,\ t > 0, \tag{1}$$

$$h\left(x, t\right)\big|_{x=0} = h^{-}\left(t\right),\ h\left(x, t\right)\big|_{x=d} = h^{+}\left(t\right),\ t > 0. \tag{2}$$

Here, $k_h\left(\sigma\right)$ is the filtration coefficient, which depends on the porosity; h^{-}, h^{+} is pressure values, which in this case may depend on time t.

From equation (1) we have

$$k_h\left(\sigma\right)\frac{\partial h}{\partial \xi} = \int_0^{\xi} \frac{\partial \sigma}{\partial B}\frac{\partial B}{\partial t} dz + h_1,$$

where h_1 is an unknown function that may depend on time. Then

$$h\left(\xi, t\right) = \int_0^{\xi} \frac{1}{k_h\left(\sigma\right)} \int_0^{\varsigma} \frac{\partial \sigma}{\partial B}\frac{\partial B\left(z, t\right)}{\partial t} dz d\varsigma + h_1 \int_0^{\xi} \frac{d\varsigma}{k_h\left(\sigma\right)} + h_2, \tag{3}$$

where h_2 is also an unknown function that depends only on time.

Then, from (3) and boundary conditions (2) we have

$$\begin{cases} h(0,t) = h_2 = h^-, \\ h(d,t) = \int_0^d \frac{1}{k_h(\sigma)} \int_0^\varsigma \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz d\varsigma + h_1 \int_0^d \frac{d\varsigma}{k_h(\sigma)} + h_2 = h^+. \end{cases}$$

From this system of equations, we have

$$h_1 = \frac{1}{\int_0^d \frac{d\varsigma}{k_h(\sigma)}} \left(h^+ - h^- - \int_0^d \frac{1}{k_h(\sigma)} \int_0^\varsigma \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz d\varsigma \right).$$

Then from the problem (1), (2) we have

$$h(\xi,t) = h^- + \int_0^\xi \frac{1}{k_h(\sigma)} \int_0^\varsigma \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz d\varsigma$$

$$+ \frac{\int_0^\xi \frac{d\varsigma}{k_h(\sigma)}}{\int_0^d \frac{d\varsigma}{k_h(\sigma)}} \left(h^+ - h^- - \int_0^d \frac{1}{k_h(\sigma)} \int_0^\varsigma \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz d\varsigma \right).$$

For the final derivation of the conjugation condition, we need to know the filtration rates. According to Darcy's law, we have

$$u = -k_h(\sigma) \frac{\partial h}{\partial \xi} = -\int_0^\xi \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz$$

$$- \frac{1}{\int_0^d \frac{d\varsigma}{k_h(\sigma)}} \left([h] - \int_0^d \frac{1}{k_h(\sigma)} \int_0^\varsigma \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz d\varsigma \right).$$

Here $[h] = h^+ - h^-$ is the pressure jump when passing through the inclusion.

So, we get the following conjugation conditions for non-ideal contact

$$u^- = u|_{\xi=0} = -\frac{1}{\int_0^d \frac{d\varsigma}{k_h(\sigma)}} \left([h] - \int_0^d \frac{1}{k_h(\sigma)} \int_0^\varsigma \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz d\varsigma \right), \tag{4}$$

$$u^+ = u|_{\xi=d} = -\int_0^d \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz$$

$$- \frac{1}{\int_0^d \frac{d\varsigma}{k_h(\sigma)}} \left([h] - \int_0^d \frac{1}{k_h(\sigma)} \int_0^\varsigma \frac{\partial \sigma}{\partial B} \frac{\partial B(z,t)}{\partial t} dz d\varsigma \right). \tag{5}$$

In this case, unlike the previous work [10], the filtration flow when passing through the geobarrier can be discontinuous and in the general case $u^- \neq u^+$. Similar conditions were obtained taking into account the chemical suffusion of the geobarrier material [11]. And, as denoted in the cited work, different values of the filtration speed at $\xi = 0$ and $\xi = d$ does not mean that the laws of conservation are not fulfilled. This means that in the direction normal to the thin inclusion, the amount of liquid at the inlet and outlet differs by the amount of change over time in the pore volume of the geobarrier material. And in our case of a non-deformable geobarrier, the change in the volume of the pore space depends on the change in the volume of the pore space depends on the change in the volume of the biomass of microorganisms.

3. Dependences of changes in porosity on the dynamics of microorganisms

The porosity of the medium σ will depend on the concentration B of the biomass of microorganisms per unit volume of the porous medium.That is

$$\sigma\left(\xi, B\right) = \sigma_0\left(\xi\right) - \sigma_B\left(B\right).$$

Here σ_0 is porosity in the absence of biomass; $\sigma_B\left(B\right)$ is a function of conversion of biomass concentration into volume.

To convert biomass concentration into volume, similarly to the works [13, 14], we will use the assumption that 80% of biomass consists of water, and the remaining 20% of dry mass is 50 percent carbon (bio-carbon) [18]. That is,

$$\sigma_B\left(B\right) = \frac{B}{0.8\rho_w + 0.2\frac{\rho_c}{2}}, \tag{6}$$

where ρ_w is the density of water; ρ_c is density of bio-carbon.

According to formula (6)

$$\frac{\partial \sigma}{\partial B} = const.$$

We denote it as $\beta = \frac{\partial \sigma}{\partial B}$.

Next, at

$$B\left(\xi, t\right)\big|_{\xi=0} = B^-\left(t\right), \; B\left(\xi, t\right)\big|_{\xi=d} = B^+\left(t\right)$$

we approximate the function B by a linear dependence

$$B\left(\xi, t\right) = B^- + \frac{\xi}{d}\left(B^+ - B^-\right)$$

or

$$B\left(\xi, t\right) = B^- + \frac{\xi}{d}\left[B\right]$$

on the thickness of the geobarrier. Then

$$\int_0^\xi \frac{\partial B\left(z,t\right)}{\partial t} dz = \int_0^\xi \left(\frac{\partial B^-}{\partial t} + \frac{z}{d}\left[\frac{\partial B}{\partial t}\right]\right) dz = \frac{\partial B^-}{\partial t}\xi + \frac{\xi^2}{2d}\left[\frac{\partial B}{\partial t}\right];$$

$$\int_0^d \frac{\partial B\left(z,t\right)}{\partial t} dz = \frac{\partial B^-}{\partial t} d + \frac{d}{2}\left[\frac{\partial B}{\partial t}\right] = \frac{d}{2}\left(\frac{\partial B^+}{\partial t} + \frac{\partial B^-}{\partial t}\right).$$

As a result, from conditions (4), (5), we have

$$u^- = u\big|_{\xi=0} = -\frac{1}{\int_0^d \frac{d\varsigma}{k_h(\sigma)}}\left(\left[h\right] - \beta\int_0^d \frac{1}{k_h\left(\sigma\right)}\left(\frac{\partial B^-}{\partial t}\varsigma + \frac{\varsigma^2}{2d}\left[\frac{\partial B}{\partial t}\right]\right) d\varsigma\right);$$

$$u^+ = u|_{\xi=d} = -\beta\frac{d}{2}\left(\frac{\partial B^+}{\partial t} + \frac{\partial B^-}{\partial t}\right)$$
$$-\frac{1}{\int_0^d \frac{d\varsigma}{k_h(\sigma)}}\left([h] - \beta\int_0^d \frac{1}{k_h(\sigma)}\left(\frac{\partial B^-}{\partial t}\varsigma + \frac{\varsigma^2}{2d}\left[\frac{\partial B}{\partial t}\right]\right)d\varsigma\right).$$

So, in this case, the rate of filtration when passing through the geobarrier will depend on the jump in pressure, the jump in the dynamic change in the biomass of microorganisms, as well as the dynamic change in this biomass at the boundary of the geobarrier. In addition, an important component of the derived conjugation conditions is the presence of an integral over the thickness of the geobarrier from the variable filtering coefficient. In classical conditions of conjugation, such effects are not considered. However, if the filtration coefficient is considered constant and the influence of biomass on the change in porosity is not taken into account, then from the above conditions we will obtain the classical conditions of conjugation of non-ideal contact.

4. Conclusions

Conjugation conditions are an integral part of mathematical models of processes in porous media with fine inclusions. In the article, the conjugation conditions for pressures on a thin geobarrier have been modified, taking into account the dynamics of the biomass of microorganisms. In the future, relevant boundary-value problems with the derived conjugation conditions, establishing the degree of influence of the change in time of bioclogging processes on the dynamics of pressure jumps need to be researched.

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