

**COMPARATIVE STUDY OF MHD NANOFUID FLOW  
OVER A THIN NEEDLE WITH EXPONENTIALLY  
DEPENDENT HEAT SOURCE AND ZERO MASS FLUX**

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**Abstract:** In this paper, the nanofluid stream over a meager needle which extends in the outspread heading within the sight of exponential space-based heat source (ESHS) and thermal based heat source (THS) is examined. The Brownian movement and thermophoresis impacts are accounted to study the nanofluids. Streams with dynamic control and with latent control have been thought about through diagrams and outlines. The resulting nonlinear differential framework is dealt with by means of shooting strategy. The effects of controlling boundaries on stream profiles are examined and portrayed with the guide of diagrams. Results show that as the ESHS and THS boundaries increment, the warm field increments. Notwithstanding, ESHS peculiarity is profoundly persuasive than THS peculiarity on energy transport and its inclination.

**Keywords:** Thin needle; Exponential space-based heat source; Thermal based heat source; Active control; Passive control; Zero mass flux.

## **INTRODUCTION**

This area of study has recently drawn a lot of attention from researchers because of its possible applications in contemporary and industrial engineering. The potential to enhance heat transfer rates in heat exchangers and other industrial applications is a major motivating factor. The typical working fluids, which are inefficient and made of chemicals based on ethylene, kerosene, water, or glycols, have low heat conductivity. Improving the fluids' thermal conductivity has been the focus of much research on this problem.

## **Literature Review**

Choi [1] was the one who came up with the novel idea of "nanofluids" for the very first time, which eventually proved to be the answer to this complicated issue. The sort of fluids that include nanoparticles with a size ranging from one to one hundred nanometres is known as nanofluids. Due to the circumstance that these solid nanoparticles comprised metallic or metal oxide components, they were able to considerably improve certain qualities such as current

conductivity and specific heat capacity when they were spread into the base fluid. Because of their enhanced thermal characteristics, nanofluids have been the subject of a significant amount of study and applications in the field of thermodynamics.

In this particular piece of writing, we make use of the most current model of nanofluid [2]. The addition of Brownian wave and thermophoresis effects into the convective boundary layer analysis of nanofluids is an extension of the work that has been done in this subject in the past.

The problem of Lorentz forces operating nanofluids and forcing convection over an extended sheet was investigated by Sheik Holeslami and others. Hayat et al. conducted a study in which they included Brownian motion and thermophoresis in order to explore the impact of MHD on the flow of nanofluids of the trice grade. Hsiao [5] investigated the MHD flow of micropolar fluids in the vicinity of a nonlinear stretchable surface (1). On the other hand, Ramana Reddy et al. [6] investigated the Brownian signal and thermophoresis possessions on unsteady flow over a reduction surface with slip effects.

Makinde and Animasaun [7] have also investigated thermophoresis, Brownian motion bioconvection flow, and quartic chemical reactions in a UHPRS. Each of these processes was investigated.

Chen and Smith [8] had the opportunity to conduct a comprehensive investigation on the forced, steady, laminar, incompressible flow that occurred across tiny needles. Because of the slight variations in wall temperature and surface heat flux, they concentrated on the temperature profiles and heat assignment qualities that were caused by these changes. In order to regulate the Prandtl number and the amount to which needle size has an effect, they conducted a test that on condition that you with your flow temperature profile. Ishak et al. conducted an investigation into the flow in the boundary layer over a thin needle that was moving in conjunction with an aligned stream via the process of stretching and shrinking. According to the finite difference approach that was used in the research, there is a dual solution that exists in opposition to the travelling needle and the free stream.

An investigation of the flow of Upper Convicted Maxwell (UCM) nanofluids was carried out by Mahanthesh and colleagues [10] using Cattaneo-Christov heat flux models across a melting surface. The researchers took into consideration important space-dependent internal heat sources. The purpose of this research is to investigate the influence that significant space-dependent heat sources have on the dynamics and transport properties of nanofluids throughout various processes.

Several studies have been conducted around the world investigating various aspects of nanofluid dynamics. Many research groups (e.g., Kumar et al. [11], Sheikholeslami et al. [12], Hayat et al. [13, 14], Koriko et al. [15], and Mahanthesh et al. [17], significantly contributed to this issue. In another study, Nield and Kuznetsov [18] developed a model in which the concentration of nanoparticles at one end was controlled similar to varying temperature. In their second work [19], they switched the boundary condition used to a more realistic model, accounting for both Brownian Motion and Thermophoresis Effects. The assumption of this

model is that nanofluid was treated as a thin layer, no regular mass transfer takes place at the plate and concentration gradient could change in any moment even while both sides have been described adequately by Fick's second law.

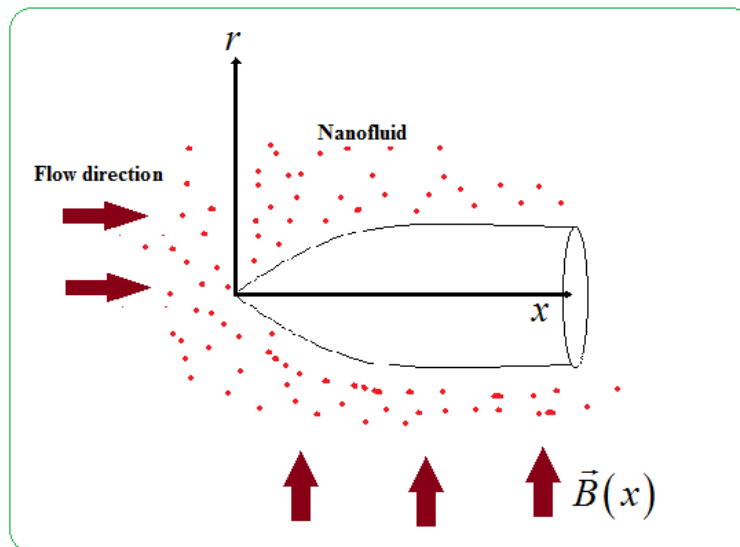


Figure 1.1: Physical model of the flow

Following the completion of following study, these prototypes have been endangered to a significant amount of development. Nield and Kuznetsov investigated their models to see how they performed in a variety of different situations. Among these situations were the initial convection that occurred in a thin layer of nanofluid [20], the conventional convective boundary layer flows of nanofluids [21], and the thermal instability that occurred in nanofluid-saturated porous layers [22].

The authors Rahman et al. [23] used Buongiorno's model to conduct a quantitative investigation flow of the borderline layer and heat transmission characteristics of a nanofluid finished a porous surface that was severely contracting and expanding. This was done in order to determine the flow features of the nanofluid. The circumstances surrounding the slip were of the highest kind. A convectively heated non-linearly stretched sheet was the source of the flow boundary layer that Mustafa et al. [24] investigated. This layer was created by the sheet being stretched in a non-linear fashion. Researchers Haq et al. [25] looked into the combined effects of thermal radiation and thermal slip on magnetohydrodynamic (MHD) boundary layer stagnation-point flow across a porous surface that was immersed in nanoparticles. Specifically, they were interested in how these two factors interrelate with one another. The researchers found that thermal slip had a negative impact on the temperature profile, while radiation had a positive impact. While Zaimi et al. [26] investigated the flow of nanofluids over a porous stretching/contracting sheet, Dhanai et al. [27] investigated the combined effects of magnetic fields and viscous indulgence on mass allocation in a boundary layer nanofluid flow that was induced by a power-law stretching/contracting sheet. Both of these studies were conducted in

order to better understand the relationship between the two phenomena. In the journal Nanofluids, both of these findings were published for scientific publication.

The major emphasis of this work is on the influence that enhanced space-dependent heat sources (ESHS) and thermal heat sources (THS) have on the flow of nanofluids over a thin needle. We compare the results of active control (AC) flow with those of passive control (PC) flow data in order to acquire an understanding of the effect that these enhanced heat sources have on the dynamics of the nanofluid. This is done in order to gain a better knowledge of the influence that these heat sources have.

### MATHEMATICAL FORMULATION

How about we consider consistent incompressible nanofluid stream over persistently uprooting slight needle as shown in figure 1, where  $(x, r)$  are the barrel shaped coordinate framework, which are determined toward hub and range separately, and the slim needle's size is  $\beta$ .

We accept that the needle is so thin to surpass the limit layer of the stream and the strain potential along the needle is dismissed. Albeit, the ramifications of cross over arch of the needle in huge over the stream. Temperature of the needle  $T_w$  and the encompassing temperature of the stream  $T_\infty$  are constants where  $T_w > T_\infty$ . consider  $U_w$  is the moving speed of the needle and  $U_\infty$  is the consistent free stream speed of the stream. Pressure angle along the needle is unimportant for example  $\frac{\partial p}{\partial x} = 0$ . Also,  $r = h(x)$  depicts the sweep of the needle, where  $r$  and  $x$  address the radial and axial coordinates. Under these suppositions the limit layer conditions alongside reasonable limit conditions in tube shaped arranges are

$$\left. \begin{aligned} \frac{\partial}{\partial x}(rU) + \frac{\partial}{\partial r}(rV) &= 0 \\ U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial r} &= \frac{\nu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial U}{\partial r} \right) - \frac{\sigma B^2 U}{\rho} \\ U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial r} &= \alpha r^{-1} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \tau \left( D_{BM} \frac{\partial T}{\partial r} \frac{\partial B}{\partial r} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial r} \right)^2 \right) + \frac{Q_t^*}{\rho c_p} (T - T_\infty) + \frac{Q_e^*}{\rho c_p} (T_w - T_\infty) \exp \left( -\frac{n U_\infty r^2}{\nu x} \right) \\ U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial r} &= D_{BM} r^{-1} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) + \frac{D_T}{T_\infty} r^{-1} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \end{aligned} \right\} \quad (1.1)$$

$$\left. \begin{aligned} U = U_w, V = 0, T = T_w, & \left\{ \begin{aligned} C = C_w, & \text{for active control} \\ D_{BM} \frac{\partial C}{\partial r} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial r} = 0, & \text{for passive control} \end{aligned} \right. \text{ at } r = h(x) \\ U \rightarrow U_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty & \text{ as } r \rightarrow \infty \end{aligned} \right\} \quad (1.2)$$

Here, 'B' means remotely applied attractive power field,  $D_{BM}$  addresses Brownian movement dissemination,  $D_T$  is thermophoretic dispersion,  $Q_t^*$  = thermal based heat source steady,  $Q_e^*$  = dramatically space based heat source consistent,  $n$  = exponential index.

To tackle the set-up of PDEs in (1) alongside limit conditions (2) we expect to present a bunch of similitude factors to change (1-2) into a bunch of Tributes. They are as per the following

$$\left. \begin{aligned} \xi &= \frac{U_\infty r^2}{vx}, \psi = xf(\xi), U = r^{-1} \frac{\partial \psi}{\partial r}, V = -r^{-1} \frac{\partial \psi}{\partial x}, \\ \theta(\xi) &= \frac{T - T_\infty}{T_w - T_\infty}, h(x) = \sqrt{\frac{v cx}{U_\infty}}, \varphi(\xi) = \begin{cases} \frac{C - C_\infty}{C_w - C_\infty}, & \text{for Active Control (A.C.)} \\ \frac{C - C_\infty}{C_\infty}, & \text{for Passive Control (P.C.)} \end{cases} \end{aligned} \right] \quad (1.3)$$

After that, by substituting (3) into equations (1-2), it is possible to construct the remaining nonlinear ordinary differential equations in this way:

$$\left. \begin{aligned} 2(\xi f'')' + ff'' - Mf' &= 0 \\ Pr^{-1}(\xi \theta')' + f\theta' + 2\xi(N_B \theta' \varphi' + N_T (\theta')^2) + Q_t \theta + Q_e e^{-n\xi} &= 0 \\ 2(\zeta \varphi')' + Le f \varphi' + 2 \frac{N_T}{N_B} (\xi \theta')' &= 0 \end{aligned} \right] \quad (1.4)$$

$$\left. \begin{aligned} f(c) = \frac{\gamma c}{2}, f'(c) = \frac{\gamma}{2}, \theta(c) = 1, \begin{cases} \varphi(c) = 1 & \text{for Active Control (A.C.)} \\ N_B \varphi'(c) + N_T \theta'(c) = 0 & \text{for Passive Control (P.C.)} \end{cases} \\ f' \rightarrow \frac{(1-\gamma)}{2}, \theta \rightarrow 0, \varphi \rightarrow 0 \end{aligned} \right] \quad (1.5)$$

The magnetic factor is denoted by M, the Prandtl number is denoted by Pr, the Brownian motion parameter is denoted by NB, the thermophoresis parameter is denoted by NT, the thermally based heat source constant is denoted by =, and the exponentially space based heat source constant is shown by =. The Lewis number, denoted by the letter Le, and the velocity ratio parameter, where the two are described by.

$$M = \frac{\sigma B_0^2}{\rho U_0}, B = B(x) = B_0 x^{-0.5} N_B = \frac{\tau D_{BM} (N_w - N_\infty)}{v}, \gamma = \frac{U_w}{U_\infty}, U_0 = U_w + U_\infty \neq 0,$$

$$Q_e = \frac{Q_e^*}{(\rho c_p)_{nf}} \sqrt{\frac{v}{(T_w - T_\infty)}}, Le = \frac{\alpha}{D_{BM}}, N_T = \frac{\tau D_T (T_{wall} - T_\infty)}{T_\infty v}, Pr = \frac{v}{\alpha}, Q_t = \frac{Q_t^*}{(\rho c_p)_{nf}} \sqrt{\frac{v}{(T_w - T_\infty)}}$$

To see the patterns of intensity and mass transaction of the liquid arrangement of the ongoing paper, creators have presents terms like Nusselt number Nu which assigns heat transmission rate and Sherwood number Sh which presents mass transmission rate (wall species dissemination rate).

Additionally, we have determined that  $Nu \propto -\theta'(c)$  and  $Sh \propto -\varphi'(c)$ . So we expect that local Nusselt numeral  $Nur = -\theta'(c)$  and local Sherwood numeral  $Shr = -\varphi'(c)$ .

### SOLUTION METHODOLOGY:

The creators have carried out the Runge - Kutta strategy for 6th request alongside the cycle procedure of Nachtsheim and Swigert shooting interaction to tackle the normal

differential conditions (4) with the BCs (5) mathematically. To get the arrangement creators have expect the upsides of  $f''(c), \theta'(c)$  and  $\varphi(c)$  and considered the step length of  $\xi = 0.005$ . The methodology is gone on until the outcome up to the ideal level of exactness of  $10^{-6}$  has accomplished. The calculation of the cycle is as per the following:

1. Reduce the suit of Tribute in (1) into an arrangement of first order ODEs.
2. Orient the BCs and ICs into right structure.
3. To accomplish the underlying thought of  $f''(c), \theta'(c)$  and  $\varphi(c)$  creators utilized Nachtsheim and Swigert shooting process.
4. Continue with past method until the BCs are fulfilled.
5. Apply R-K strategy for sixth request to address first order ODEs.
6. Obtain the necessary request of intermingling of the arrangement.

**CODE VERIFICATION:**

Using the above mentioned scheme we solved the 1<sup>st</sup> equations of (6) with the value of  $\gamma=0$  and  $M=0$  and the boundary conditions are

$f(c)=0, f'(c)=0, f'(\infty)=\frac{1}{2}$  and obtained the values of  $f''(c)$  with various values of ‘c’. Then compare our findings with the Chen and Smith [2] and Ishak et al. [3]. Here we observe our results have excellent degree of agreement with the already established work.

Table 1.1: Various values of  $f''(c)$  for various values of ‘c’

‘c’	Present research	Chen and Smith [2]	Ishak et al. [3]
0.1	1.2887052	1.28881	1.288801
0.01	8.49239011	8.49244	8.492412
0.001	62.1620478	62.16372	62.16371

**RESULT AND DISCUSSION:**

After completing the solution methodology, authors have staged the outcomes of the study through graphs and tables. The influence of the exponential power index ‘n’, exponential space based heat source  $Q_0$  and applied thermal dependent heat source  $Q_1$  on the characteristics of the flow system have been discussed here. The deliberations are obtained by considering the magnitudes of the parameters as and the rest of the factors are mentioned accordingly. The outcomes produced under two distinct kinds of boundary conditions and compared how well they performed concurrently. One is zero mass flux at the needle surface and the other is constant nanoparticle mass at the boundary.

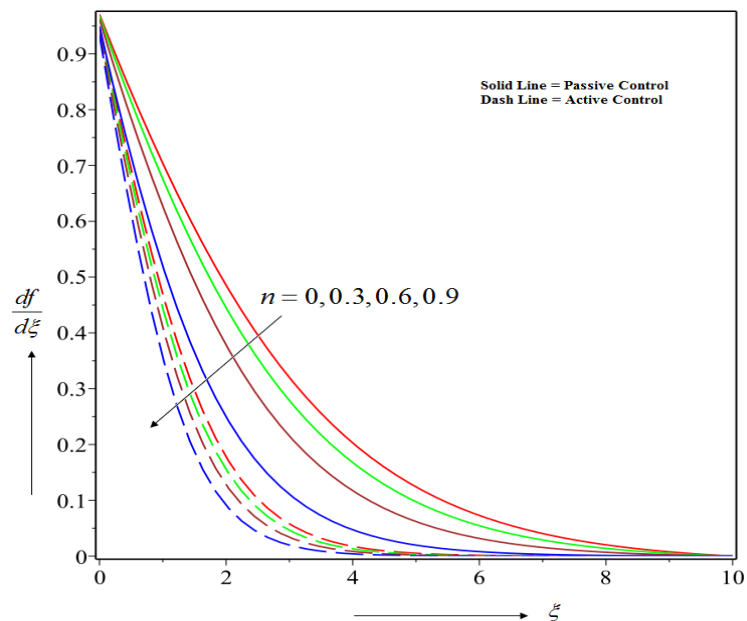


Figure 1.2: influence of ‘n’ on velocity

In the above flow system heat transfer and mass transfers are calculated by Nusselt numeral ( $Nur$ ) and Sherwood number ( $Shr$ ) respectively. The influence of  $n, Q_e$  and  $Q_i$  are staged in table 02. After carefully consideration the result in table 02, it is observed that exponential power index has significant impact on physical quantities. We observe that increasing has diminishing influence on  $Nur$  and  $Shr$ . Also, we find that  $Nur$  in case of passive control is much higher than in case of active control. Also we notice the same effects of  $Q_e$  and  $Q_i$  on  $Nur$ . Conversely,  $Shr$  is less affected by the exponential power index. But on the contrary,  $Q_e$  and  $Q_i$  have raising effects on mass transfer rate. Also, in case of active control mass transfer rate is higher than passive control.

Table 1.2: Effects on Physical Quantities

$n$	$Q_e$	$Q_i$	$Nur$		$Shr$	
			AC	PC	AC	PC
0	0.2	0.2	0.674012	0.741549	0.415526	0.312645
0.1			0.625541	0.725489	0.401156	0.295648
0.2			0.602487	0.714589	0.382218	0.282328
0.3			0.584417	0.691125	0.354411	0.254485
0.4			0.561248	0.689584	0.332267	0.218897
0.1	0	0.2	0.641891	0.764415	0.375541	0.285561

	0.1		0.634578	0.749986	0.382216	0.289477
	0.2		0.625541	0.725489	0.401156	0.295648
	0.3		0.601254	0.711598	0.412322	0.301125
	0.4		0.584519	0.702591	0.421558	0.325998
0.1		0	0.658474	0.741215	0.394785	0.274189
	0.2	0.1	0.635889	0.735487	0.400225	0.288447
		0.2	0.625541	0.725489	0.401156	0.295648
		0.3	0.614478	0.712125	0.412894	0.301002
		0.4	0.611213	0.708941	0.421105	0.318547

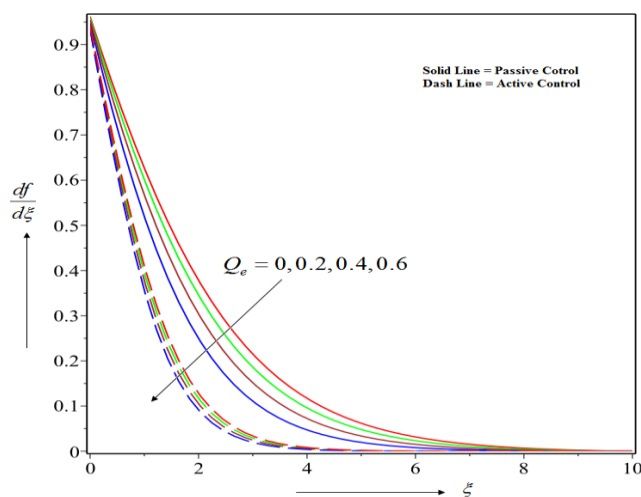


Figure 1.3: influence of  $Q_e$  on velocity

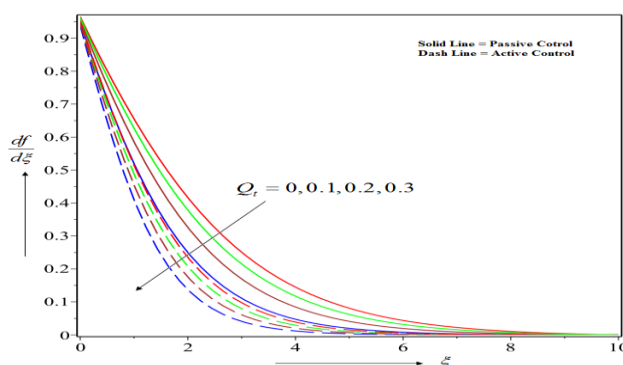


Figure 1.4: influence of  $Q_t$  on velocity

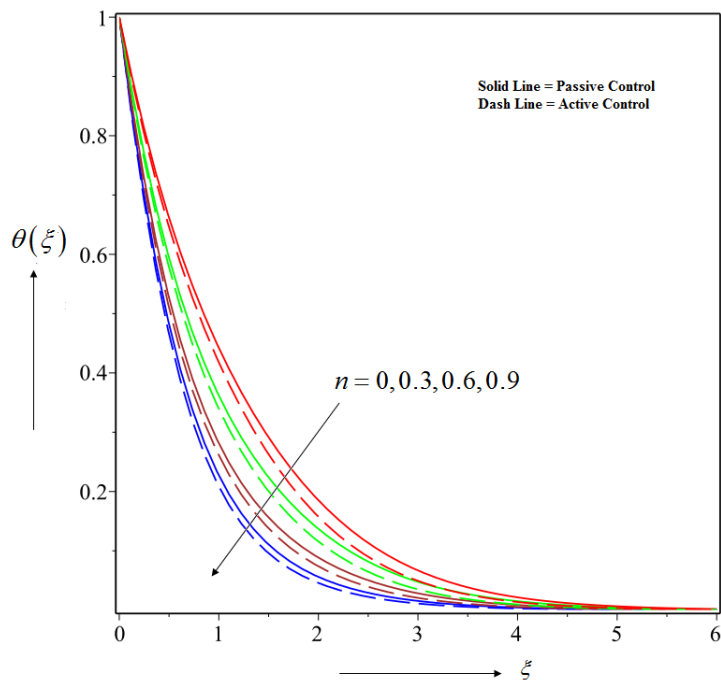


Figure 1.5: influence of 'n' on Temperature

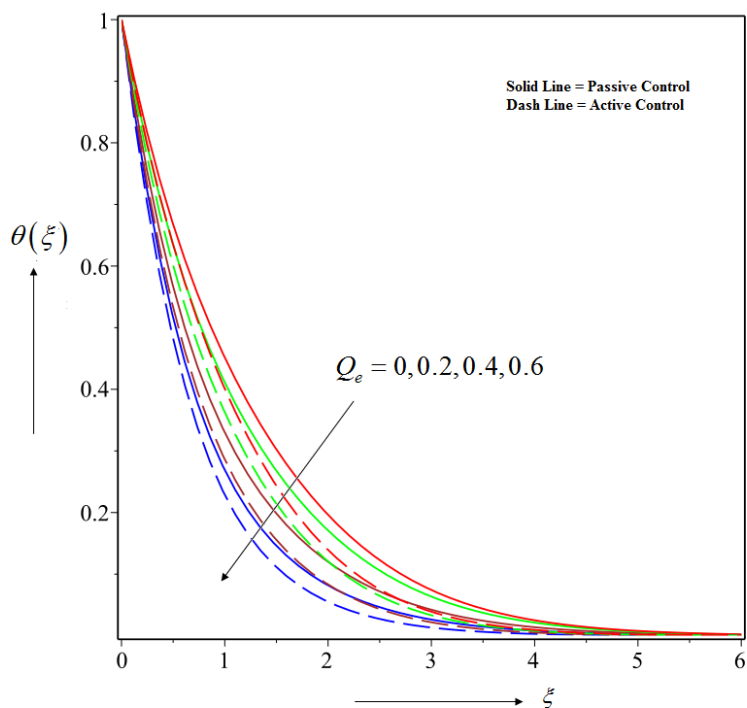


Figure 1.6: influence of  $Q_e$  on Temperature

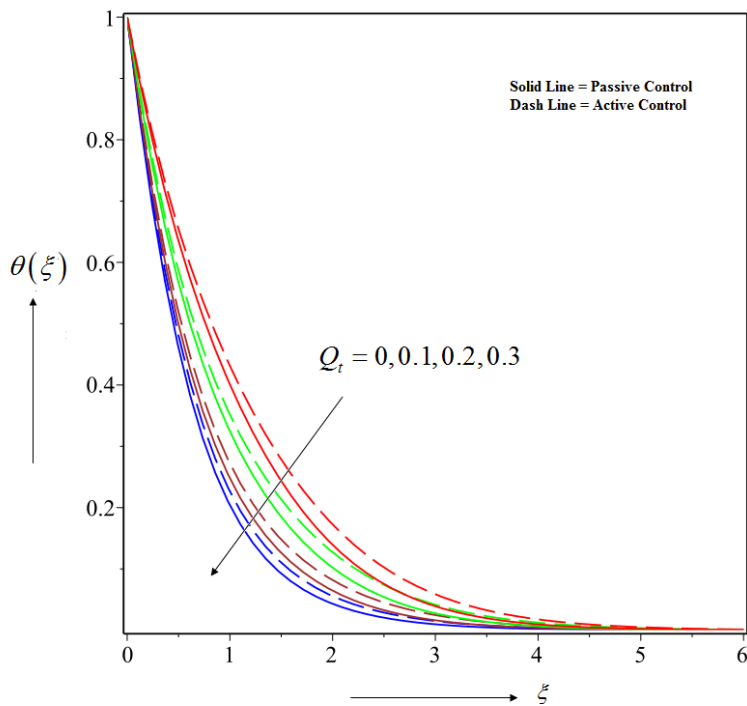


Figure 1.7: influence of  $Q_t$  on Temperature

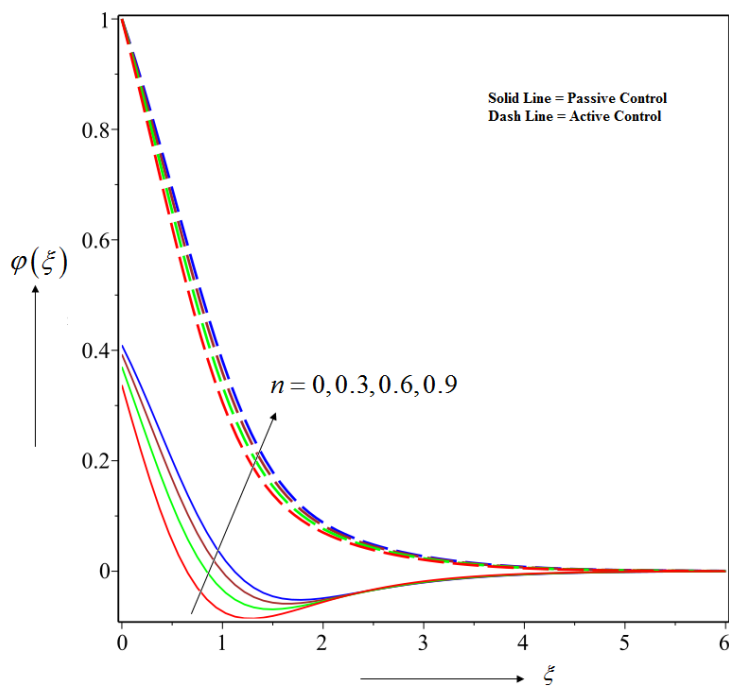


Figure 1.8: influence of 'n' on nanoparticle density

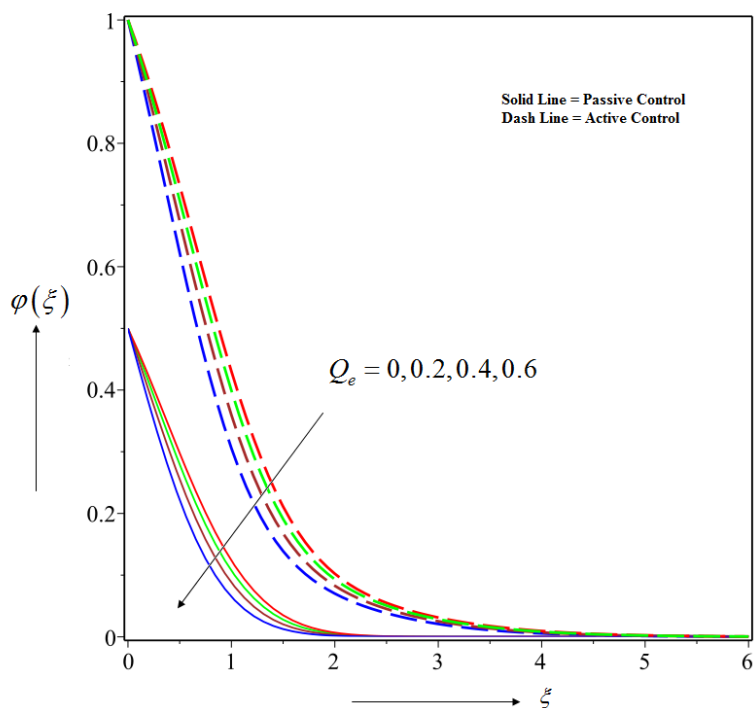


Figure 1.9: influence of  $Q_e$  on nanoparticle density

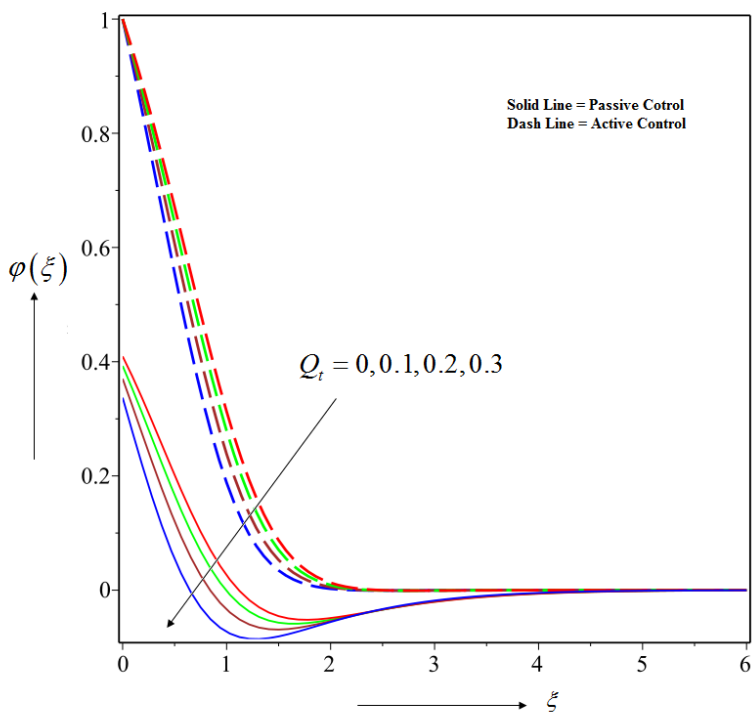


Figure 1.10: influence of  $Q_t$  on nanoparticle density

In figure 02-04, the speed of the stream reduces with the higher  $n, Q_e$  and  $Q_i$ . Likewise in all figure, it is seen that stream with uninvolved control has higher speed than stream with dynamic control. The impact of  $Q_e$  on the air conditioner and PC stream is more unmistakable as displayed in figure 03. In figure 02, it is seen that the impact on 'n' is critical in the event of PC stream. The distinction of speed between two sorts of stream is diminishes with higher  $Q_i$  as organized in figure 04. The impacts of  $n, Q_e$  and  $Q_i$  on temperature of the stream are introduced in figures 05-01. The high temperature of the stream is persistently all through the stream. In figure 05, temperature of the stream with PC is higher than in the event of stream with AC and is diminishes with higher 'n'. Comparative impacts are seen in figure 06. Likewise, with higher  $Q_e$  the temperature contrast between two kinds of stream decreases. In figure 07, a contrary impact is seen is noticed. Here the temperature of the stream with AC is higher than the stream with PC. Ultimately, the impacts of the elements in center around the nanoparticle thickness are introduced in figures 08-10. It is seen that in figure 08, the focus in the stream with PC is not exactly that of the stream with AC. Additionally 'n' assists with hoisting the centralization of the liquid and has more tremendous impact in the event of PC stream. However, in figure 09, we see that the impact of  $Q_e$  is more critical in the event of AC stream. Additionally, centralization of the stream is reduced with raised  $Q_e$  and  $Q_i$  as displayed in figures 09-10.

### CONCLUSION:

This numerical study examines the continuous, two-dimensional viscous flow of incompressible nanofluid flow over a thin, tapering needle in two dimensions.

A flow problem which admits a hydrodynamic slip velocity is addressed herein, brought into an intermediate expression within the general surface movement speed. Second, by including actual boundary constraints at the surface that do not allow for normal flux, the dynamics of nanoparticles are taken into account. Thereby, Brownian motion and thermophoresis effects can be taken into reflection. 'bvp4c' from MATLAB, which solves borderline value difficulties, has been utilized in conjunction with the Newton method and a classical shooting approach to solve the model. The impacts of dynamic and passive controls on nanoparticles, besides other parameters involved, are thoroughly analyzed and discussed. The following are the highlights of the investigation:

1. Heat move of the stream is higher in the event of PC stream than that of AC stream. Likewise  $n, Q_e$  and  $Q_i$  have decreased the intensity move capacity of the liquid.
2. In the event of mass exchange capacity of the stream, AC stream has higher greatness of mass exchange than PC stream. Likewise, where dramatic power record diminishes the mass exchange pace of the stream,  $Q_e$  and  $Q_i$  raise the mass exchange pace of the stream.
3. The impacts of  $Q_e$  on the speed of the stream with PC are more precarious than other stream.

4. In the event of the impact of  $Q_i$  on the temperature of the stream is switched concerning two sorts of the stream. I. e. temperature of the air conditioner stream is higher than PC stream.
5. The remarkable power list 'n' raised the nanoparticle thickness of the stream while other two variables diminished it in the stream.

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